Managerial Optimism and Debt Covenants*

Jakob Infuehr†  Volker Laux‡

University of Southern Denmark  University of Texas at Austin

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Abstract: This paper studies the effects of managerial optimism on the optimal design of debt covenants. We find that managers that are more optimistic about the future success of their investment ideas provide lenders with greater control rights via tighter debt covenants. This is optimal for managers although they understand that tighter covenants increase the probability of covenant violations and lead to excessive lender intervention. The broad reason for this result is that optimistic managers wish to write contracts that repay lenders more frequently in bad states rather than in good states and the only way to achieve this is by granting lenders more control rights via tighter covenants. Our model offers a novel explanation for the empirical evidence that covenants in debt contracts are set very tightly, are often violated, and sometimes renegotiated or even waived.

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†jakob.infuehr@mccombs.utexas.edu.
‡Corresponding author. volker.laux@mccombs.utexas.edu.
1 Introduction

Empirical and survey evidence suggests that entrepreneurs and executives are overly optimistic about the chances of success of their own projects.\(^1\) One explanation for this phenomenon is developed in Van den Steen (2004) who argues that individuals who forego other opportunities to start a new venture are typically those who overestimate the chances that their venture will be successful.\(^2\) Besides this "choice-driven" theory of optimism, there are several other explanations for managerial hubris that are grounded in the psychology literature (DeBondt and Thaler, 1995; Malmendier and Tate, 2005; Gervais, 2010).

In this manuscript, we take managerial optimism as given and study how it affects the optimal allocation of control rights in debt contracts. Accounting-based covenants transfer control rights to lenders if accounting signals fall below certain thresholds. A tighter covenant (i.e., a higher threshold) increases the likelihood that the lender gains control over the firm, permitting the lender to take actions against the manager’s will, such as liquidating the firm. Our model offers a novel explanation for the empirical evidence that covenants in debt contracts are set very tightly, are frequently violated, sometimes renegotiated or even waived (Chava and Roberts, 2008; Dichev and Skinner, 2002; Nini et al. 2012).

An owner manager raises capital from a lender to finance a project. If the project is continued to completion, it succeeds if the state of the world is good and fails if

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\(^1\)See, e.g., Larwood and Whittaker (1977), Cooper et al. (1988), Arabsheibani et al. (2000), Malmendier and Tate (2005), Landier and Thesmar (2009), and Ben-David et al. (2013).

\(^2\)See also De Meza and Southey (1996).
the state is bad. The manager is optimistic in the sense that she believes the prior probability of the good state is higher than what the lender believes. At an interim date, a public accounting signal is released that is imperfectly informative about the state and hence about the project’s success probability. Based on the signal, both players update their prior beliefs using Bayes’ rule. The party in control (either the manager or the lender) then chooses to continue or liquidate the project based on the available information. Similar to Aghion and Bolton (1992) and Dessein (2005), we assume the manager enjoys private benefits of control when the project is continued.

The debt contract includes a covenant that is contingent on the accounting signal. If the accounting signal lies below a certain threshold, the covenant is violated and the lender gains control over the liquidation decision. If the accounting signal lies above the threshold, the covenant is not violated and the manager remains in charge. After the signal is revealed and before the party in control takes the action, the two parties can renegotiate the contract.

We first study the case in which the parties cannot observe the true state of the world, and then extend the model to assume that the state is sometimes observable (but not verifiable) prior to the liquidation decision.

When the parties cannot observe the state, the contract is never renegotiated in equilibrium and the lender liquidates the project whenever the covenant is violated. Choosing a tighter covenant threshold has costs and benefits for the manager and the strength of these effects depends on her degree of optimism. First, an increase in the covenant threshold increases the probability that the lender gains control and discontinues the project, preventing the manager from enjoying private benefits.
Although a more optimistic manager believes that the good state is more likely, this does not imply that an optimist also believes that a tighter covenant will have a weak effect on the probability of losing control. In fact, the opposite can be true. The desire to receive private benefits can therefore result in a looser or tighter covenant when the manager is more optimistic. Second, since a higher covenant increases the probability of project liquidation, the manager is less likely to receive the residual cash flows when the state is good. This is a greater concern for the manager when she is more optimistic since an optimist believes that the project is more likely to succeed. This consideration induces a more optimistic manager to choose a looser covenant to prevent the lender from terminating projects that she deems valuable. Third, a tighter covenant renders the lender willing to reduce the face value of the debt, which increases the manager’s residual in case of success. This effect is more important for a manager who is more optimistic since a lower face value benefits the manager only when the project succeeds. As a result, more optimistic managers are more eager to lower the face value of the debt via tighter covenants.

We find that the third effect dominates so that managers that are more optimistic about their future success choose tighter covenants. The optimal covenant is so tight that it leads to socially excessive liquidations, not only from the optimistic manager’s perspective, but also from the lender’s perspective. The key behind this result is that, due to the heterogeneous priors, granting the lender more control reduces the manager’s perceived cost of financing. Specifically, since the manager is more optimistic about the project’s success probability than the lender, repaying the lender in case of success is a costly way to satisfy his participation constraint. The
optimist would like to promise the lender a large payment when the project fails, but this is infeasible since cash flows are then limited. The only way to reduce the face value of the debt and hence the repayment to the lender in case of success is by offering him a contract with a tighter covenant, allowing him to liquidate the project more often. A tighter covenant shifts the repayments from the good state to the bad state and thereby reduces the manager’s perceived financing costs. This effect, which we term the protection effect, is more important when the manager is more optimistic and is the reason for our main prediction that optimistic managers choose tighter covenants. One immediate implication of this result is that firms with more optimistic managers are more likely to be liquidated at an early stage relative to firms with less optimistic managers. This prediction does not follow because optimists are more likely to be unsuccessful relative to unbiased managers, but because optimists allow lenders to intervene more often to reduce their perceived cost of financing.

We then consider the case in which the parties can sometimes observe nonverifiable information about the state prior to the liquidation decision. The potential observability of the state has no effect on the optimal covenant threshold when the two players have homogeneous priors. When beliefs are heterogeneous, however, the prospect of observing the state induces the manager to set an even tighter covenant. The reason for this result is similar to the aforementioned protection effect. To reduce the perceived financing cost, the manager wishes to protect the lender in case the parties observe the bad state and the lender is more likely protected when the covenant threshold is high.

The optimal covenant can be so tight that the lender may find it optimal to
waive the covenant after a violation, even when the parties do not observe the state of the world. This finding seems surprising since one might have expected that the parties can achieve the same result simply by setting a loser covenant. However, the tight covenant is not meant to protect the lender when the accounting signal is the only relevant information but to protect him when nonverifiable information becomes available that renders liquidation optimal despite a high accounting signal.

A growing analytical literature examines the consequences of managerial overconfidence (see Gervais 2010 for an overview). The standard argument in the literature is that managers that are optimistic about their projects’ prospects overinvest relative to unbiased managers (e.g., Gervais et al. 2011; Malmendier and Tate, 2005). In our model, we obtain the opposite result: more optimistic managers design contracts that lead to underinvestment (i.e., excessive liquidations) relative to unbiased managers.

Landier and Thesmar (2009) show in a setting with binary states and binary accounting signals that optimistic managers prefer short-term debt over long-term debt and provide empirical evidence in support of this prediction. Short-term debt gives the lender greater control because it allows him to liquidate the firm when the accounting signal is low. The key assumptions in their model are that (i) a good state leads to a high signal with certainty, and (ii) the optimist believes the state is good with certainty. As a result, the optimist believes that the accounting signal will always be high so that short-term debt never leads to the loss of control and, hence, does not involve any costs. In contrast, in our setting, the accounting signal is continuous and only imperfectly informative of the state, and the optimistic man-
anger does not necessarily believe the state is good with certainty. In this setting, an increase in the covenant threshold always increases the manager’s risk of losing control, and this effect can even be stronger when the manager is more optimistic. Thus, more optimistic managers choose tighter covenants not because they underestimate the effects of tighter covenants on the likelihood of a covenant violation, but because of the aforementioned protection effect.

Dessein (2005) and Gârleanu and Zwiebel (2009) provide a rationale for tight debt covenants based on asymmetric information at the contracting stage. In these papers, "good-type" managers grant lenders greater control rights than "bad-type" managers to signal their type. In contrast, in our model, the allocation of control rights does not serve as a signalling device because information is symmetric at the contracting stage. Lu, Sapra, and Venugopalan (2018) provide an alternative explanation for the tightness of debt covenants that is based on the notion that managers are tempted to take excessive risks after raising debt. Offering the lender a tighter covenant reduces the face value of the debt, which allows the manager to commit herself to abstain from risk-shifting activities.

2 Model

A risk-neutral penniless manager-owner has a new investment idea that requires capital \( I > 0 \). To implement the idea, the manager raises debt from a risk-neutral lender (e.g., a bank or venture capitalist). The outcome of the project (if not liquidated at an intermediate date) depends on the state of the world, \( \omega \in \{0,1\} \), which is
initially not observable to any player. The project succeeds if the state is good, and fails if the state is bad. The manager may be more optimistic than the lender about the chances of success. Specifically, the manager’s prior belief that the state is good is $\beta_M$ and the investor’s prior is $\beta_L$, with $\beta_M \geq \beta_L$. The players’ prior beliefs are common knowledge, that is, they agree to disagree.

**Timing:** The game has five dates. At date 1, the manager and the lender agree on the terms of the debt contract and the lender provides capital $I$. At date 2, the firm’s accounting system generates a verifiable signal, denoted $S$, that is informative about the state $\omega$. At the same time, $\omega$ becomes publicly observable (but not verifiable) with probability $p \geq 0$. At date 3, the parties can engage in mutually beneficial contract renegotiations. At date 4, the project is either liquidated or continued and the financial contract in place determines who has control over this decision. At date 5, payoffs are realized and shared between the two parties according to the contract in place.

**Payoffs:** If the project is liquidated at date 4, cash flows are $L \in (0, I)$. If the project is continued, payoffs depend on the state $\omega$. Specifically, when the state is good, $\omega = 1$, the project succeeds and generates cash flows of $X$, with $X > I$, and when the state is bad, $\omega = 0$, the project fails and generates zero cash flows. All cash flows are verifiable. Further, if the project is continued, the manager enjoys private benefits of control $B \in (0, L)$, regardless of the state. The manager does not receive any private benefits if the project is liquidated since liquidation implies that the manager is no longer running the project.

**Accounting signal:** At date 2, the accounting system generates a public signal
that is informative of the state \( \omega \). The state-dependent distribution function of the signal is \( F_\omega (S) \) and the probability density function is \( f_\omega (S) > 0 \). The ex-ante density is \( f (S, \beta) = \beta f_1 (S) + (1 - \beta) f_0 (S) \). Conditional on \( S \), the posterior probability of the good state is

\[
\theta (S, \beta) = \frac{\beta f_1 (S)}{\beta f_1 (S) + (1 - \beta) f_0 (S)}.
\]

(1)

The signal \( S \) satisfies the monotone likelihood ratio property (MLRP), that is, \( \frac{f_1 (S)}{f_0 (S)} \) is strictly increasing in the signal \( S \) for all values of \( S \). Thus, higher signals indicate that the state is more likely good. Further, we assume \( \lim_{S \to -0} \frac{f_1 (S)}{f_0 (S)} = 0 \) and \( \lim_{S \to 1} \frac{f_1 (S)}{f_0 (S)} = \infty \), which implies that the signal becomes perfectly informative when it approaches 0 or 1.

At the same time, the state \( \omega \) is observable with probability \( p > 0 \). While the contract can be made contingent on \( S \), it cannot be made contingent on \( \omega \).

When the state is not observable so that the signal \( S \) is the only available information, the first-best decision is to continue the project if and only if the expected social value of continuation \( \theta (S, \beta) X + B \) equals or exceeds the liquidation value \( L \). Continuation is therefore efficient if the signal \( S \) exceeds a certain threshold, denoted \( S^{FB} \), which solves

\[
\theta (S^{FB}, \beta) X + B = L.
\]

(2)

When the manager is optimistic, \( \beta_M > \beta_L \), the two parties disagree on the first-best decision. Letting \( S_M^{FB} \) and \( S_L^{FB} \) denote the first-best thresholds from the
manager’s and the lender’s perspective, respectively, we obtain $S^F_M < S^F_L$ because for any signal $S$ the manager’s posterior probability of success is higher than the lender’s.

**Financial contract:** The debt contract specifies the amount $D$ the manager has to pay the lender at date 5 ($D$ is the debt’s face value). The contract also determines who has control over the liquidation decision at date 4 via a debt covenant. Specifically, if the accounting signal $S$ falls below a certain threshold, denoted $S_C$, the covenant is violated and the lender receives control. If $S \geq S_C$, the covenant is not violated and the manager remains in charge. The lending market is competitive and the manager has all the bargaining power at date 1. The manager therefore chooses the $(D, S_C)$ combination that maximizes her expected utility subject to the lender’s break even constraint. We assume that $\beta_L X \geq I$ to ensure that the manager is able to obtain financing even when the contract does not provide the lender with any control and the manager always continues the project. Thus, different to Aghion and Bolton (1992), in our setting there is no need to grant the lender control rights in order to obtain financing.

**Renegotiation:** After the signal $S$ is released and the state $\omega$ is observed (if it is observable), the parties can renegotiate the contract in place. At this time, the manager does not necessarily have all of the bargaining power because she can no longer shop around for the best offer. We assume the manager can make a take-it-or-leave-it offer with probability $\gamma \in [0, 1]$, and the lender can make a take-it-or-leave-it offer with probability $(1 - \gamma)$. Thus, $\gamma$ represents the manager’s bargaining power at date 3.
3 State is not observable ($p = 0$)

We start the analysis with the case in which the state $\omega$ is not observable, $p = 0$, and study the case with $p > 0$ in Section 4.

Consider the lender’s optimal action when he has control over the liquidation decision. When the lender liquidates the project, he receives the whole liquidation value $L$ because the debt’s face value $D$ exceeds $L$ in equilibrium (which follows from $D > I > L$). When the lender continues the project, he receives the face value $D$ when the project succeeds and nothing when it fails. The lender will therefore terminate the project if and only if

$$L > \theta (S, \beta_L) D.$$  \hfill (3)

Since his posterior $\theta(S, \beta_L)$ is increasing in the signal $S$, continuation is more attractive for the lender when the signal $S$ is higher.

When the manager retains control, she will prefer to continue the project because continuation yields her private benefits $B$ as well as the residual $X - D$ in case of success, whereas liquidation leaves her empty-handed.

Assume for now that the optimal covenant $S_C$ is such that the two parties will not want to renegotiate the contract after observing signal $S$ and that the lender prefers to liquidate the project whenever he gains control, that is, inequality (3) is satisfied for $S < S_C$. We show in Subsection 3.3 (and more fully in the appendix) that this is indeed the case in the optimal solution when $p = 0$. 

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The lender’s and the manager’s ex ante utilities are:

$$
\Upsilon = \beta_L \int_{S_C}^{1} D f_1 (S) dS + \int_{0}^{S_C} L f (S, \beta_L) dS - I, \quad (4)
$$

and

$$
U = (1 - \beta_M) \int_{S_C}^{1} B f_0 (S) dS + \beta_M \int_{S_C}^{1} (X - D + B) f_1 (S) dS, \quad (5)
$$

respectively.

The lender provides the required capital $I$ if he breaks even in equilibrium. The equilibrium face value of the debt $D$ therefore solves $\Upsilon = 0$. Inspection of (4) shows that the equilibrium $D$ declines as the covenant threshold $S_C$ increases. This is intuitive since a higher covenant threshold $S_C$ provides the lender with greater protection.

### 3.1 Optimal covenant with homogeneous priors

As a benchmark, consider the case in which the two parties have homogeneous priors, $\beta = \beta_M = \beta_L$. Substituting the lender’s participation constraint (4) into the manager’s ex ante utility (5) yields:

$$
U(\beta) = V(\beta, S_C) \equiv \beta X \int_{S_C}^{1} f_1 (S) dS + B \int_{S_C}^{1} f (S, \beta) dS + L \int_{0}^{S_C} f (S, \beta) dS - I. \quad (6)
$$

Since the lender receives his reservation utility of zero in expectation, the manager’s ex ante utility equals the total expected net payoff (including private benefits),
which we denote by $V$. Taking the first-order condition for the optimal covenant $S_C$ yields:

$$\frac{dV(\beta, S_C)}{dS_C} = (1 - \beta) f_0(S_C)(L - B) - \beta(X + B - L)f_1(S_C) = 0. \quad (7)$$

An increase in the covenant $S_C$ provides the lender with greater control and leads to more liquidations. A higher $S_C$ therefore reduces the risk that the project is continued in the bad state (type I error), but increases the risk that the project is liquidated in the good state (type II error). The optimal covenant, denoted $S_C^0$, trades off these costs and benefits and implements the first-best continuation decision given the available information; that is, $S_C^0 = S^{FB}$.\(^3\)

To study the effect of an increase in the prior $\beta$ on the optimal covenant $S_C$ we take the first derivative of (7) with respect to $\beta$:

$$\frac{d^2V(\beta, S_C)}{dS_C d\beta} = -f_0(S_C)(L - B) - (X + B - L)f_1(S_C) < 0. \quad (8)$$

Together with the second-order condition, $\frac{d^2V(\beta, S_C)}{dS_C d\beta} < 0$ implies that the optimal covenant $S_C^0$ declines as $\beta$ increases. Intuitively, when the prior $\beta$ is higher, the manager is less worried about the risk of continuing projects in the bad state, and is more worried about the risk of liquidating projects in the good state. This shift in the trade-off leads to a decline in the optimal covenant, as reported in the next proposition.

\(^3\)Note that for homogeneous priors, the first-order condition in (7) simplifies to (2).
Proposition 1 Suppose $p = 0$ and $\beta = \beta_M = \beta_L$. The optimal covenant is $S^0_C = S^{FB}$, and implements the first-best continuation decision. As the prior $\beta$ increases, the optimal covenant threshold declines, $dS^0_C / d\beta < 0$.

3.2 Optimal covenant with heterogeneous priors

When the manager is optimistic relative to the lender, $\beta_M > \beta_L$, the two players no longer agree on the first-best threshold. The first-best threshold from the manager’s perspective lies below the first-best threshold from the lender’s perspective, $S^{FB}_M < S^{FB}_L$. Thus, there exists a range of signals $S \in (S^{FB}_M, S^{FB}_L)$, for which the manager believes continuation is socially optimal, whereas the lender believes liquidation is socially optimal.

One might expect that the manager’s optimal contract involves a covenant $S_C$ that lies somewhere between $S^{FB}_M$ and $S^{FB}_L$ to take into account the beliefs of both players. However, this intuition is not correct. The optimal covenant exceeds both the lender’s and the manager’s first-best thresholds and increases as the manager’s prior $\beta_M$ increases.

Proposition 2 Suppose $p = 0$ and $\beta_M > \beta_L$. The optimal covenant $S^0_C$ solves

\[ \theta(S^0_C, \beta)L + B = L + (1 - \theta(S^0_C, \beta)) \frac{(\beta_M - \beta_L)}{\beta_M(1 - \beta_L)} B, \quad (9) \]

and (i) is higher than both the lender’s and the manager’s first-best thresholds, $S^0_C > S^{FB}_L > S^{FB}_M$, and (ii) increases when the manager is more optimistic, $dS^0_C / d\beta_M > 0$. 

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The manager chooses the covenant threshold that maximizes her ex ante utility (5) taking into account the lender’s participation constraint (4). The first order condition for an optimal choice of $S_C$ is given by:

\[
\frac{dU(\beta_M, S_C)}{dS_C} = - ((1 - \beta_M) f_0(S_C) + \beta_M f_1(S_C)) B \]  
\[-\beta_M (X - D(S_C, \beta_L)) f_1(S_C) \]
\[-\beta_M \int_{S_C}^{1} f_1(S) dS \frac{dD(S_C, \beta_L)}{dS_C} = 0.
\]

An increase in $S_C$ has costs and benefits and the strength of these effects depends on the manager’s prior $\beta_M$. In what follows we discuss each of these effects in turn.

**Probability of receiving B:** An increase in the covenant threshold $S_C$ increases the probability of a covenant violation by

\[(1 - \beta_M) f_0(S_C) + \beta_M f_1(S_C) > 0.\]

Thus, even from the perspective of the most optimistic manager (with $\beta_M = 1$), a tighter covenant increases the risk that the lender gains control and liquidates the project, preventing the manager from receiving private benefits $B$. This effect is captured in the first term in (10) and implies that the manager prefers a looser covenant $S_C$ over a stricter one. The question now is whether this effect is stronger or weaker when the manager is more optimistic. The answer depends on the sign of $f_0(S_C) - f_1(S_C)$. If $f_0(S_C) - f_1(S_C) > 0$, a more optimistic manager believes that
an increase in $S_C$ increases the probability of a covenant violation less quickly, but if $f_0(S_C) - f_1(S_C) < 0$, the opposite is true and a more optimistic manager believes that an increase in $S_C$ increases the probability of a covenant violation more quickly. Thus, when choosing $S_C$ the manager takes into account that a tighter covenant reduces the chance of receiving benefits $B$ and this is either a greater or weaker concern when she is more optimistic, depending on the sign of $f_0(S_C) - f_1(S_C)$.

**Probability of receiving $X - D$:** An increase in $S_C$ reduces the probability that the manager remains in control and hence the probability that she earns the residual $X - D$. Again, this effect implies that the manager prefers a looser covenant $S_C$ over a stricter one, as indicated in the second term in (10). The potential loss of the residual is a greater concern for the manager when she is more optimistic because the optimist believes that the project is more likely to succeed if continued. This consideration implies that more optimistic managers prefer a looser covenant to retain control more often and to prevent the lender from liquidating valuable projects.

**Face Value $D$:** The only benefit of a tighter covenant $S_C$ is that it renders the lender willing to reduce the face value $D$. This benefit is captured in the third term in (10) and is stronger when the manager is more optimistic because the size of $D$ is relevant only when the project succeeds. This last effect therefore implies that more optimistic managers wish to give the lender more control via a higher $S_C$.

It is instructive to compare this situation with the setting in Section 3.1, where the two players have homogeneous priors. Substituting the lender’s participation
constraint (4) into (5) yields

\[
U(\beta_M, S_C) = V(\beta_M, S_C) - \frac{\beta_M - \beta_L}{\beta_L} \left( I - L \int_0^{S_C} f_0(S) dS \right).
\]  

(11)

The manager’s utility in (11) differs from the total expected net payoff \( V \), defined in (6), only due to the second expression. The second expression arises because the manager believes that she is repaying the lender more than \( I \) in expectation. Of course, from the lender’s perspective, he is getting back exactly \( I \). The reason for this discrepancy is that the manager believes the project is more likely to succeed than what the lender believes. Taking the first-order condition of (11) with respect to the covenant threshold \( S_C \) yields

\[
\frac{dU(\beta_M, S_C)}{dS_C} = \frac{dV(\beta_M, S_C)}{dS_C} + \frac{\beta_M - \beta_L}{\beta_L} L f_0(S_C) = 0.
\]  

(12)

The first term in (12) capture how an increase in the covenant \( S_C \) affects the total expected payoff \( V(\beta_M, S_C) \) and has already been discussed in Section 3.1. The second term shows that an increase in \( S_C \) reduces the manager’s perceived cost of financing. As the covenant threshold increases, the lender is more likely to have control and to liquidate the project. The lender is then more often paid via the liquidation value \( L \), which allows the manager to lower the face value \( D \). The important point is that \( D \) is only paid when the state is good, whereas \( L \) is paid in good and bad states. The increase in \( L \) and decline in \( D \) therefore shift the repayments from the good to the bad state. This shift lowers the manager’s perceived financing cost because the manager
believes that the good state is more likely to occur than what the lender believes. In other words, due to the heterogeneous priors, satisfying the lender’s participation constraint via payments in the bad state is less costly than satisfying his participation constraint via payments in the good state. In contrast, for homogeneous beliefs, the way the lender is paid back does not directly affect the manager’s utility and the last term in (12) disappears. This effect, which we term the protection effect, creates a new role for the covenant $S_C$: The optimistic manager wishes to repay the lender via the liquidation value $L$ more frequently to shift payments from the good state to the bad state, and the only way to do so is by granting him more control rights. The optimal covenant therefore trades off the reduced cost of financing (due to the protection effect) with the costs of giving the lender "excessive" control that leads to inefficient liquidations. Due to this trade off, the manager’s optimal covenant threshold exceeds her first-best threshold, $S_C^0 > S^F_M$ (recall that $S^F_M$ maximizes $V(M;S_C)$).

The question now is how does this trade-off change when the manager becomes more optimistic. Taking the derivative of (12) with respect to $\beta_M$ yields:

$$\frac{d^2U(M;S_C)}{dS_C d\beta_M} = \frac{d^2V(M;S_C)}{dS_C d\beta_M} + \frac{L}{\beta_L} f_0(S_C).$$

(13)

From (8), we know that $\frac{d^2V(M;S_C)}{dS_C d\beta_M} < 0$, which implies that the covenant that maximizes the total expected payoff $V(M;S_C)$ declines as $\beta_M$ increases. Intuitively, when $\beta_M$ is larger, the manager believes the project is more likely successful and hence should be continued more often. The second term in (13) depicts how an
increase in $\beta_M$ affects the protection effect. When the manager is more optimistic, her perceived cost of financing increases and the protection effect becomes more important. That is, the optimistic manager is more eager to lower the perceived cost of financing and she can do so by choosing a tighter covenant. Substituting the first order condition (12) into (13) shows that the second effect dominates, so that $\frac{\partial U(\beta_M, S_C)}{dS_C d\beta_M} > 0$. As a result, more optimistic managers choose tighter covenants, confirming part (ii) of Proposition 2.

Proposition 2 also states that the optimal covenant exceeds the manager’s first-best level $S^{FB}_M$ as well as the lender’s first-best level $S^{FB}_L$. These results immediately follow from the previous result that $S^0_C$ increases with $\beta_M$. Starting with homogeneous priors, $\beta_M = \beta_L$, which implies that $S^0_C = S^{FB}_M = S^{FB}_L$, an increase in $\beta_M$ reduces $S^{FB}_M$ but leads to an increase in the optimal covenant $S^0_C$, whereas $S^{FB}_L$ remains constant, implying that $S^0_C > S^{FB}_L > S^{FB}_M$, for $\beta_M > \beta_L$.

3.3 Discussion

Since the optimal threshold $S^0_C$ exceeds the first-best threshold even from the lender’s perspective $S^{FB}_L$, the covenant is rather tight and the lender receives control for a wide range of signals. This result raises the question whether the lender will indeed liquidate the project when in control as initially assumed and, if so, whether the parties would not want to renegotiate the contract to implement the efficient interim decision. To answer these questions, let $S_X$ denote the threshold that solves

$$\theta(S_X, \beta_L) X = L.$$  (14)
Since $\theta(S, \beta_L)$ is increasing in the signal $S$, the lender prefers liquidation over continuation for any $S < S_X$ even when he received the total output $X$ in case of success. Recall that the optimal covenant $S^0_C$ is highest when the manager exhibits extreme hubris ($\beta_M = 1$). The highest possible covenant therefore maximizes $U(\beta_M = 1)$ and solves $\theta(S_C, \beta_L)(X + B) = L$, showing that $S^0_C$ is always less than $S_X$. Thus, in the optimal solution, the lender will indeed liquidate the project when in control.

Consider now the question of contract renegotiations. When the signal $S$ lies in the range $[S^{FB}_L, S^0_C]$, the covenant is violated and the lender receives control and, as just discussed, prefers to liquidate the project. Nevertheless, for these values of $S$, both parties agree that the first-best decision is to continue the project. One might therefore expect that the two parties renegotiate the contract to implement the efficient action and to share the surplus. This is, however, not the case because the manager’s benefit of control $B$ is not transferable. The most the manager can offer in an attempt to convince the lender to continue the project is a face value of $D = X$. But even for $D = X$ the lender prefers liquidation since, as just discussed, $S^0_C < S_X$. For similar reasons, the contract is also not renegotiated for any $S < S^{FB}_L$. Further, when the covenant is not violated, $S \geq S^0_C$, the manager receives control and continues the project. In this case, both parties agree that continuing the project is the socially optimal decision and there is again no room for renegotiation.

In the appendix, we also consider cases in which the covenant threshold $S_C$ takes extremely low or high values so that the parties renegotiate the contract ex post, and show that these extreme covenants are never optimal for $p = 0$. This discussion leads to the next proposition.
Proposition 3 When \( p = 0 \), the parties do not renegotiate the contract and the optimal covenant \( S_0^C \) leads to socially excessive liquidations even from the lender’s perspective for all \( S \in [S_L^{FB}, S_0^C) \).

4 State is observable with probability \( p > 0 \)

Empirical research has shown that debt covenants are often set very tightly and are frequently renegotiated or waived after a violation (Chava and Roberts, 2008; Dichev and Skinner, 2002; Roberts and Sufi, 2009; Nini et al. 2012). Our discussion in Section 3 can explain why covenants are set tightly but not why covenants are waived or renegotiated. The observation that contracts involve tight covenants only to have them waived by the lender later is puzzling because it raises the question why the parties did not set a looser covenant to begin with. We show that when the state \( \omega \) is observable at date 2 with positive probability \( p \), there are two reasons for why the lender may renegotiate and/or waive a covenant rather than exercise his right to liquidate the firm. First, the parties may learn that the state is good, so that continuation is efficient even when the covenant is violated. Second, and more surprisingly, the covenant was initially set so tightly that continuation is efficient even when the covenant is violated and the state is not observable.

We start the analysis with a discussion of the optimal actions at date 2 after the parties observe the state \( \omega \). We then discuss the optimal covenant threshold \( S_C \).
4.1 Action when $\omega$ is observed

When the parties observe the state $\omega$ at date 2, they implement the efficient decision regardless of who is in control. To see this, suppose the good state is observed. Neither the lender nor the manager wants to liquidate the project in this case. If the manager is in control, she will simply continue the project. If the lender is in control, he will waive the covenant without renegotiation, allowing the manager to continue the project. There is no scope for contract renegotiation in this case since any threats to enforce termination by the lender (or the manager) are not credible.

Suppose now the bad state is observed. If the manager is in control, the parties renegotiate the contract to implement liquidation. If the lender is in control, he simply liquidates the project, and there is again no scope for renegotiation. This discussion leads to the next proposition.

**Proposition 4** When the parties observe the state $\omega$ at date 2, they implement the efficient decision and continue the project if and only if the state is good. The covenant is waived without renegotiation when the lender has control and the good state is observed.

4.2 Optimal debt covenant

4.2.1 Homogeneous priors

We now determine the optimal debt covenant for the benchmark case with homogeneous beliefs, $\beta = \beta_L = \beta_M$. Similar to Proposition 1, we obtain the following results.
Proposition 5 For $\beta_M = \beta_L$, the optimal covenant is $S_C^* = S^{FB}$ for any $p \in [0, 1]$.

When the state $\omega$ is not observable at date 2, the contract is not renegotiated and the project is continued if and only if $S \geq S^{FB}$.

As discussed in Subsection 4.1, the parties always implement the efficient decision when they observe the state $\omega$ at date 2. Thus, the covenant $S_C$ only plays a role for the liquidation decision when the state is not observable. For reasons similar to those discussed in Section 3, the optimal covenant is $S_C = S^{FB}$ to ensure efficient decisions when the state is not observed.

4.2.2 Heterogeneous priors and $p = 1$

While the prospect of learning the state $\omega$ plays no role for the optimal covenant threshold when priors are homogeneous, $\beta_M = \beta_L$, it does play a key role when the manager is optimistic, $\beta_M > \beta_L$. To show this, we start the analysis with the special case in which the state $\omega$ is observable at date 2 with certainty, that is, $p = 1$.\footnote{The assumption that the parties can observe the state prior to the interim decision is common in the incomplete contracting literature; see, e.g., Aghion and Bolton (1992) and Garleanu and Zwiebel (2009).}

From Proposition 4, we know that the parties always implement the efficient action when they observe the state at date 2 regardless of who has control. One might therefore expect that for $p = 1$ the ex ante control right allocation is irrelevant. This is, however, not the case as the next proposition shows.

Proposition 6 Suppose $p = 1$ and $\beta_M > \beta_L$. Although the allocation of control rights has no effect on the project continuation decision, the manager finds it strictly
optimal to grant the lender unconditional control, that is, the optimal covenant is
\( S_C^* = 1 \).

To explain this result consider the manager’s ex ante utility for \( p = 1 \):
\[
U(p = 1) = \beta_M(X - D + B) + (1 - \beta_M)(\gamma L + (1 - \gamma)B) \int_{S_C}^{1} f_0(S)dS. \tag{15}
\]

From the manager’s perspective, the state is good with probability \( \beta_M \), in which
case she receives \( X - D + B \). The covenant \( S_C \) does not directly affect the manager’s
payoff when the state is good because the project is then always continued without
renegotiation regardless of who is in control. With probability \( (1 - \beta_M) \) the state
is bad, in which case the covenant \( S_C \) has a direct affect on the parties’ payoffs.
Specifically, when the manager remains in control (because \( S \geq S_C \)), she can exploit
her control rights to renegotiate with the lender to implement liquidation, which
yields her \( \gamma L + (1 - \gamma)B \) in expectation, where \( \gamma \) represents her bargaining power. In
contrast, when the lender receives control (because \( S < S_C \)), he can simply terminate
the project in the bad state without having to "bribe" the manager. Thus, giving
the lender more control via a higher \( S_C \) yields the lender a higher payoff in the bad
state because he can liquidate the project without having to renegotiate with the
manager. Since the lender breaks even in expectation he will return this expected
payoff in the form of a lower face value \( D \). This trade is beneficial from the manager’s
perspective because she believes the project is more likely to succeed than what the
lender believes. This is, of course, a variant of the protection effect discussed in
Section 3. Since a change in $S_C$ affects the parties’ payoffs in the bad state but does not affect the decision they implement, the protection effect causes the manager to grant the lender unconditional control when $p = 1$.

A contract that allocates full control to the lender can be interpreted as short-term debt that must be refinanced at date 2 (e.g., Park, 2000). Short term debt gives the lender control over the continuation decision since he can always refuse to provide additional capital to continue the project.

4.2.3 Heterogeneous priors and $p \in [0, 1]$

We are now ready to consider the general case in which $p$ lies in the range $[0, 1]$. The next proposition states the optimal covenant threshold $S_C^*$ as a function of $p$.

**Proposition 7** Suppose $\beta_M > \beta_L$ and $p \in [0, 1]$. There is a threshold $\hat{p}$, with $\hat{p} \in (0,1)$, such that:

(i) For $p < \hat{p}$, the optimal covenant $S_C^*$ solves

$$\theta (S_C^*, \beta_L) X + B = L + \left(1 - \theta (S_C^*, \beta_L)\right) \frac{(\beta_M - \beta_L)(B + (L - B)p\gamma)}{\beta_M (1 - \beta_L) (1 - p)}$$

and lies in the range $(S_L^{FB}, S_X)$;

(ii) For $p \geq \hat{p}$, the optimal covenant is $S_C^* = 1$.

Proposition 7 shows that we have to distinguish between the two cases in which $p < \hat{p}$ and $p \geq \hat{p}$. We will discuss each case in turn.

**Case $p < \hat{p}$**. When $p < \hat{p}$, the optimal covenant threshold lies below $S_X$, where $S_X$ is determined in (14). Similar to the intuition provided in Section 3, when the
parties do not observe the state at date 2, they do not renegotiate the contract and the project is liquidated if and only if the lender gains control. We obtain the following results for $p < \hat{p}$.

**Proposition 8** Suppose $p < \hat{p}$ and $\beta_M > \beta_L$. When the state $\omega$ is not observable at date 2, the contract is not renegotiated and the project is continued if and only if $S \geq S_C^*$, leading to socially inefficient liquidations from the lender’s perspective for all $S \in [S_L^{FB}, S_C^*)$. The optimal covenant threshold $S_C^*$, and hence the probability of inefficient liquidations, increases when

(i) the manager is more optimistic ($\beta_M$ is larger);

(ii) the state $\omega$ is more likely observed at date 2 ($p$ is larger).

The intuition for why the optimal covenant $S_C^*$ increases when the manager is more optimistic is similar to the intuition provided in Section 3.

To explain why the optimal covenant increases in $p$ (for $p < \hat{p}$), we rewrite the first derivative of the manager’s utility with respect to the covenant $S_C$ as

$$
\frac{dU}{dS_C} = (1 - p) \frac{dU(p = 0)}{dS_C} + p \frac{dU(p = 1)}{dS_C},
$$

where $U(p = 0)$ is the utility when all players know the state is never observable ($p = 0$) and $U(p = 1)$ is the utility when all players know the state is observable at date 2 with certainty ($p = 1$). From Propositions 2 and 6 we know that $U(p = 0)$ is maximized when $S_C = S_C^0 < 1$ and $U(p = 1)$ is maximized when $S_C = 1$. Since an increase in $p$ increases the weight placed on $U(p = 1)$ and reduces the weight placed on $U(p = 0)$, the manager chooses a tighter covenant $S_C$ as $p$ increases.
Case $p \geq \hat{p}$. For $p \geq \hat{p}$, the optimal contract gives the lender unconditional control, $S_C^* = 1$. Thus, as long as $p < \hat{p}$, $S_C^*$ increases with $p$ (Proposition 8) and jumps to $S_C^* = 1$ when $p \geq \hat{p}$. To see why the optimal covenant threshold is a corner solution for $p \geq \hat{p}$ we need to understand the players’ actions at date 2 when the signal $S$ is the only available information, that is, the state is not observable.

**Proposition 9** Suppose $p \geq \hat{p}$ and the covenant is chosen optimally, $S_C = 1$. When the state $\omega$ is not observable at date 2:

(i) the lender waives the covenant without renegotiation for all $S \geq S_D$, where $S_D$ satisfies $\theta(S_D, \beta_L) D = L$.

(ii) the parties agree to increase the face value $D$ in lieu of a covenant waiver for all $S \in [S_X, S_D)$,

(iii) the lender liquidates the project without renegotiation for all $S < S_X$.

Thus, the project is continued if and only if $S \geq S_X$, leading to socially inefficient liquidations from the lender’s perspective for all $S \in [S_X^F, S_X)$.

Proposition 9 shows that the parties continue the project for any signal $S \geq S_X$. Specifically, when the signal is relatively high, $S \geq S_D$, the lender prefers to continue the project even in the absence of renegotiation and simply waives the covenant. One may wonder if the lender could elicit a higher face value $D$ from the manager by threatening to liquidate the project. This is not feasible, however, since such a threat is not credible. If the signal lies in an intermediate range, $S \in [S_X, S_D)$, the lender prefers liquidation over continuation but the manager can and will convince the lender to waive the covenant by offering a higher face value $D$. If the signal is
low, $S < S_X$, the lender prefers liquidation and the manager cannot convince the lender to waive the covenant by offering a higher $D$. The contract is therefore not renegotiated and the lender liquidates the project.

There are two reasons for why setting $S_C = 1$ is optimal when $p \geq \hat{p}$. First, choosing any covenant threshold $S_C$ that lies in the range $[S_X, 1]$ leads to the same continuation/liquidation decision than choosing $S_C = 1$. This follows because for all $S \geq S_X$ the project will be continued regardless of who is in control. Second, an increase in $S_C$ increases the manager’s perceived ex ante utility via the protection effect. As a result, $S_C = 1$ dominates any other covenant that lies in the range $[S_X, 1]$.

Note that in the optimal solution, the lender sometimes waives the covenant without renegotiation even when there is no additional information besides the accounting signal. One might then argue that the covenant is set too tightly and should be lowered to avoid the need for a covenant waiver. This argument is not correct here, however, since the purpose of the high covenant is to protect the lender when the parties observe the bad state. The manager would have preferred to write a contract that transfers control rights to the lender when the state is bad but such a contract is not feasible because the state is not verifiable. Thus, giving the lender full control is a substitute for the nonverifiability of the state.
5 Empirical implications

Empirical studies find that debt covenants are set very tightly, are frequently violated, and are often renegotiated and/or waived. Our model offers a novel explanation for these observations based on managerial optimism.

The key prediction in our model is that debt covenants are set tighter when the manager is more optimistic. To the best of our knowledge, the relation between managerial optimism and covenant tightness has not been tested yet. However, Lerner and Merges (1998) examine 200 biotech alliances between financing firms (large pharmaceutical companies) and research firms (small biotechnology companies), and find that financing firms receive significantly more control rights when the research firm’s project is in its early stage at the time the alliance is formed. This finding is consistent with our results since optimistic biases about projects’ prospects are more likely to occur for projects that are in their early stages than for projects in their later stages when there is less uncertainty.

Further, our model predicts that firms of more optimistic managers are more likely liquidated at an early stage than firms of less optimistic managers. This prediction follows not because optimists take suboptimal actions but because they choose contracts with tight covenants to reduce their perceived cost of financing.

Finally, our model predicts that firms choose tighter debt covenants when additional (noncontractible) information about the project’s prospects is more likely to become available prior to the liquidation/continuation decision. This relation, however, occurs only when the manager is optimistic. There is a large literature...
showing that analyst following and media coverage increase information dissemination.\footnote{See, e.g., Roulstone (2003), Miller (2006), and Fang and Peress (2009).} Assuming that analyst following and media attention are good proxies for the probability that additional information about the project becomes available, our model predicts that firms with larger analyst following and greater media attention choose debt contracts with tighter covenants. This effect is stronger when the manager is more optimistic and is mute when the manager’s and the lender’s beliefs are homogeneous.

### 6 Conclusion

We develop a model to study the effects of managerial optimism on the optimal design of debt covenants. We find that managers that are more optimistic about their projects’ prospects give lenders greater control rights via tighter covenants. The tighter covenants lead to excessive project liquidations not only from the optimistic manager’s perspective but also from the unbiased lender’s perspective. Further, with heterogeneous priors, debt covenants are set tighter when the uncertainty about the project is more likely resolved prior to the liquidation decision.

The reason behind these results is that the covenant plays two roles in our setting. First, by allocating control to the lender via covenants, the manager can induce more efficient liquidation decisions, which increases her utility ex ante. Second, giving the lender more control allows the optimistic manager to lower her perceived cost of financing. This effect arises because from the optimist’s perspective a contract that
pays the lender in case of success is expensive. In order to reduce the face value of the
debt and hence the payment to the lender when the project succeeds, the manager
sets tighter covenants. Tighter covenants give the lender greater control and permit
him to liquidate the project and receive the liquidation value more frequently. As
a result, repayments to the lender are shifted from the good state to the bad state,
which reduces the manager’s financing costs. This effect is the driver behind our
results that managers set tighter covenants when they are more optimistic about
future performance and when the uncertainty about the project is more likely resolved
prior to the liquidation decision.

7 Appendix

7.1 Preliminaries: Ex ante utilities

In this subsection we derive the manager’s ex ante utility as a function of the covenant
$S_C$. To do so we proceed in three steps. In the first step, we determine the players’
ex post utilities when the state $\omega$ is observed. In the second step, we determine
the players’ ex post utilities when the state $\omega$ is not observed. We determine the
manager’s ex ante utility in the third step.

The next lemma states the player’s payoffs when the state is observed at date 2,
which occurs with probability $p$.

Lemma 1 Suppose the parties observe the state $\omega$ at date 2. When the manager has
control rights and:

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\(i\) \(\omega = 0\), the parties renegotiate the contract to implement liquidation. The expected payoff is then \(\gamma L + (1 - \gamma) B\) for the manager and \((1 - \gamma) (L - B)\) for the lender;

\(ii\) \(\omega = 1\), the manager continues the project and receives \(B + X - D\) and the lender receives \(D\).

When the lender has control rights and:

\(i\) \(\omega = 0\), the lender liquidates the project and receives \(L\) and the manager receives nothing;

\(ii\) \(\omega = 1\), the lender allows the manager to continue the project and receives \(D\) and the manager receives \(B + X - D\).

From Lemma 1, it follows that conditional on the covenant \(S_C\) and the face value \(D\), the lender’s and the manager’s expected ex post payoffs when the state is observed at date 2 are (where \(os\) stands for observed state):

\[
\Upsilon_{os}(S_C) = \int_{S_C}^{1} \left( \beta_L D f_1(S) + (1 - \beta_L) (1 - \gamma) (L - B) f_0(S) \right) dS + \int_{0}^{S_C} \left( \beta_L D f_1(S) + (1 - \beta_L) L f_0(S) \right) dS,
\]

and

32
\[ U_{os}(S_C) = \beta_M (X - D + B) + (1 - \beta_M) (\gamma L + (1 - \gamma) B) \int_{S_C}^{1} f_0(S) dS. \quad (19) \]

We next determine the players’ payoffs when the state is not observed. To do so we need to specify three thresholds, denoted by \( S_B, S_X, \) and \( S_D, \) which are defined by

\[
\begin{align*}
\theta(S_B, \beta_L) D + \theta(S_B, \beta_M) (X - D) + B &= L, \\
\theta(S_X, \beta_L) X &= L, \\
\theta(S_D, \beta_L) D &= L.
\end{align*}
\]

Note that \( S_B < S_X < S_D. \) We already discussed thresholds \( S_X \) and \( S_D \) in the main text. To explain threshold \( S_B \) suppose the manager has control rights. She will then continue the project unless the contract is renegotiated. Renegotiation to implement liquidation takes place only when the liquidation value \( L \) exceeds the sum of the expected continuation payoff for the lender (from the lender’s perspective) and the expected continuation payoff for the manager (from the manager’s perspective), which is the case if \( S < S_B. \) Otherwise, if \( S > S_B \) there is no room for renegotiation and the manager continues the project. Lemma 2 states the players’ payoffs as a function of signal \( S \) when the state \( \omega \) is not observed at date 2.

**Lemma 2** Suppose the state \( \omega \) is not observed at date 2. When the lender has control rights and:
(i) \( S < S_X \), the lender liquidates the project without renegotiation and receives \( L \) and the manager receives nothing.

(ii) \( S \in [S_X, S_D) \), the parties renegotiate the contract to implement project continuation and the lender expects to receive \( \gamma L + (1 - \gamma) \theta (S, \beta_L) X \) and the manager

\[
\gamma \left( \theta (S, \beta_M) \left( X - \frac{L}{\theta(S, \beta_L)} \right) + B \right) + (1 - \gamma) B.
\]

(iii) \( S \geq S_D \), the project is continued without renegotiation and the lender expects to receive \( \theta (S, \beta_L) D \) and the manager \( \theta (S, \beta_M) (X - D) + B \).

When the manager has control rights and:

(i) \( S < S_B \), the parties renegotiate the contract to implement liquidation and the lender expects to receive \( \gamma \theta (S, \beta_L) D + (1 - \gamma) (L - \theta (S, \beta_M) (X - D) - B) \) and the manager

\[
\gamma (L - \theta (S, \beta_L) D) + (1 - \gamma) (\theta (S, \beta_M) (X - D) + B).
\]

(ii) \( S \geq S_B \), the project is continued without renegotiation and the lender expects to receive \( \theta (S, \beta_L) D \) and the manager \( \theta (S, \beta_M) (X - D) + B \).

We are now ready to determine the manager’s ex ante utility as a function of the covenant \( S_C \). We have to distinguish between four cases that we discuss subsequently.

Case (i): \( S_C \in [S_B, S_X] \).

From Lemma 2, the parties do not renegotiate the contract when \( S_C \in [S_B, S_X] \) and the manager (lender) continues (liquidates) the project when in control, unless the state \( \omega \) is observed. When the state is observed, which occurs with probability \( p \), the parties’ payoffs are as in (19) and (18). The lender’s and the manager’s ex ante utilities are now given by
\[ Y_1 = (1 - p) \left( \beta_L \int_{S_C}^1 Df_1(S) dS + \int_0^{S_C} Lf(S, \beta_L) dS \right) + pY_{os}(S_C) - I, \quad (20) \]

and

\[ U_1 = (1 - p) \left( \beta_M \int_{S_C}^1 (X - D + B) f_1(S) dS + (1 - \beta_M) B \int_{S_C}^1 f_0(S) dS \right) + pU_{os}(S_C). \quad (21) \]

Substituting (20) into the manager’s ex ante utility (21) yields

\[ U_1 = (1 - p) \left( V(\beta_M, S_C) - \frac{\beta_M - \beta_L}{\beta_L} \left( I - L \int_0^{S_C} f_0(S) dS \right) \right) + pT(S_C), \quad (22) \]

where \( T(S_C) \) is given by

\[
T(S_C) = \beta_M (X + B) + (1 - \beta_M) L - I \\
- \frac{\beta_M - \beta_L}{\beta_L} \left( I - L \int_0^{S_C} f_0(S) dS - (1 - \gamma)(L - B) \int_{S_C}^1 f_0(S) dS \right).
\]

The multiplier of \((1 - p)\) in (22) is identical to (11). The term \( T(S_C) \) is the ex ante value when the state is observable.

Case (ii): \( S_C \in [S_X, S_D] \).

From Lemma 2, the parties renegotiate the contract to avoid inefficient liquidation
for all $S \in (S_X, S_C)$. The lender’s and manager’s ex ante utilities are

$$
\Upsilon_2 = (1-p) \int_0^{S_X} L f(S, \beta_L) \, dS + (1-p) \int_{S_X}^{S_C} (\gamma L + (1-\gamma) \theta(S, \beta_L) X) f(S, \beta_L) \, dS
$$

$$
+ (1-p) \int_{S_C}^1 \beta_L D f_1(S) \, dS + p \Upsilon_{os}(S_C) - I
$$

and

$$
U_2 = (1-p) \int_{S_X}^{S_C} \left( B + \gamma \theta(S, \beta_M) \left( X - \frac{L}{\theta(S, \beta_L)} \right) \right) f(S, \beta_M) \, dS
$$

$$
+ (1-p) \int_{S_C}^1 \left( \theta(S, \beta_M) (X - D) + B \right) f(S, \beta_M) \, dS + p U_{os}(S_C).
$$

After substituting $\Upsilon_2 = 0$ into the manager’s utility function, we obtain:

$$
U_2 = (1-p) \left( V(\beta_M, S_X) - \frac{\beta_M - \beta_L}{\beta_L} \left( I - L \int_0^{S_X} f_0(S) \, dS \right) \right) + p T(S_C).
$$

Case (iii): $S_C \geq S_D$.

From Lemma 2, the lender waives the covenant for all $S \in [S_D, S_C)$ when the state is not observed. The lender’s and manager’s ex ante utilities are:
\[ \Upsilon_3 = (1 - p) \int_0^{S_X} Lf(S, \beta_L) \, dS + (1 - p) \int_{S_X}^{S_D} (\gamma L + (1 - \gamma) \theta(S, \beta_L) X) f(S, \beta_L) \, dS \]

\[ + (1 - p) \int_{S_D}^{1} \theta(S, \beta_L) Df(S, \beta_L) \, dS + p \Upsilon_{os}(S_C) - I, \]

(26)

and

\[ U_3 = (1 - p) \int_{S_X}^{S_D} \left( B + \gamma \theta(S, \beta_M) \left( X - \frac{L}{\theta(S, \beta_L)} \right) \right) f(S, \beta_M) \, dS \]

\[ + (1 - p) \int_{S_D}^{1} (\theta(S, \beta_M) (X - D) + B) f(S, \beta_M) \, dS + pU_{os}(S_C). \]

Substituting \( \Upsilon_3 = 0 \) into the manager’s utility function yields:

\[ U_3 = (1 - p) \left( V(\beta_M, S_X) - \frac{\beta_M - \beta_L}{\beta_L} \left( I - L \int_0^{S_X} f_0(S) \, dS \right) \right) + p\Gamma(S_C). \]

(28)

Case (iv): \( S_C \in [0, S_B] \).

From Lemma 2, the parties will renegotiate the contract to avoid inefficient continuation (from the lender’s perspective) for all \( S \in [S_C, S_B] \). The lender’s and manager’s ex ante utilities are then:
\[ \Upsilon_4 = (1 - p) \int_{S_C}^{S_B} L f(S, \beta_L) \, dS \\
+ (1 - p) \int_{S_C}^{S_B} (\gamma \theta(S, \beta_L) D + (1 - \gamma) (L - \theta(S, \beta_M)(X - D) - B)) f(S, \beta_L) \, dS \\
+ (1 - p) \int_{S_B}^{1} \beta_L D f_1(S) \, dS + p\Upsilon_\text{os} - I. \] 

and

\[ U_4 = (1 - p) \int_{S_C}^{S_B} (\gamma (L - \theta(S, \beta_L) D) + (1 - \gamma) (\theta(S, \beta_M)(X - D) + B)) f(S, \beta_M) \, dS \\
+ (1 - p) \int_{S_B}^{1} (\theta(S, \beta_M)(X - D) + B) f(S, \beta_M) \, dS + pU_\text{os}(S_C). \] 

Substituting \( \Upsilon_4 = 0 \) into the manager's utility function to partially eliminate \( D \) yields:

\[ U_4 = (1 - p) \left( V(\beta_M, S_B) - \left( \frac{\beta_M - \beta_L}{\beta_L} \right) \left( I - L \int_0^{S_B} f_0(S) \, dS \right) \right) \]

\[ -(1 - p) \frac{\beta_M - \beta_L}{\beta_L} \left( \int_{S_C}^{S_B} (\gamma (L - \theta(S, \beta_L) D) + (1 - \gamma) (\theta(S, \beta_M)(X - D) + B)) f_0(S) \, dS \right) \]

\[ + pT(S_C). \]
7.2 Proofs

Proof of Propositions 7 and 2

We start the proof by determining the optimal covenant threshold for the four regions identified in Subsection 7.1.

Case 1: $S_C \in [S_B, S_X]$. Taking the first-order condition of (22) yields

$$
\frac{dU_1}{dS_C} = 0 = (1 - p) \left( (1 - \beta_M) f_0(S_C) (L - B) - \beta_M (X + B - L) f_1(S_C) + \frac{\beta_M - \beta_L}{\beta_L} L f_0(S_C) \right) \\
+ p \left( \frac{\beta_M - \beta_L}{\beta_L} (L - (1 - \gamma)(L - B)) f_0(S_C) \right),
$$

which can be rewritten as

$$
\frac{dU_1}{dS_C} \frac{1}{f_0(S_C) \beta_M} = 0 = - (1 - p) (X + B - L) \frac{f_1(S_C)}{f_0(S_C)} \\
+ \frac{1}{\beta_L} L - p(L - B) \left( (1 - \gamma) \frac{1}{\beta_L} + \gamma \frac{1}{\beta_M} \right) - \frac{1}{\beta_M} B.
$$

(33) further simplifies to (16).

We denote the threshold that solves (33) by $S_C^\#$. From (33), we obtain $\frac{dU_1}{dS_C} > 0$ when $\frac{f_1(S_C)}{f_0(S_C)} < \frac{f_1(S_C^\#)}{f_0(S_C^\#)}$ and $\frac{dU_1}{dS_C} < 0$ when $\frac{f_1(S_C)}{f_0(S_C)} > \frac{f_1(S_C^\#)}{f_0(S_C^\#)}$. Due to the MLRP, $\frac{dU_1}{dS_C} > 0$ when $S < S_C^\#$ and $\frac{dU_1}{dS_C} < 0$ when $S_C > S_C^\#$, which establishes that $S_C^\#$ is the optimal solution for the range $[S_B, S_X]$. Since $S_C^\# > S_L^F$ and $S_L^F > S_B$, we obtain $S_C^\# > S_B$. As shown further below, for values of $p$ for which $S_C^\#$ is the optimal solution ($p < \hat{p}$),
we obtain \( S^\#_C < S_X \).

*Case (ii):* \( S_C \in [S_X, S_D] \). Taking the first derivative of (25) with respect to \( S_C \) and simplifying yields

\[
\frac{dU_2}{dS_C} = p \frac{(\beta_M - \beta_L)}{\beta_L} (\gamma L + (1 - \gamma)B) f_0(S_C) \geq 0. \tag{34}
\]

For any \( S_C \) in the range \([S_X, S_D]\), the manager prefers \( S_C = S_D \) if \( p > 0 \) and is indifferent if \( p = 0 \).

*Case (iii):* \( S_C \geq S_D \). The derivative of (28) is identical to (34). Thus, for any \( S_C \) in the range \([S_D, 1]\), the manager prefers \( S_C = 1 \) if \( p > 0 \) and is indifferent if \( p = 0 \).

*Case (iv):* \( S_C \in [0, S_B] \). Since

\[
\frac{dT(S_C)}{dS_C} = \frac{\beta_M - \beta_L}{\beta_L} (L - (1 - \gamma)(L - B)) f_0(S_C) > 0,
\]

(31) shows that the manager strictly prefers \( S_C = S_B \) over any other \( S_C < S_B \) for any \( p \in [0, 1] \).

We now determine the optimal overall covenant \( S_C^* \). Suppose first that \( p = 0 \). In this case, the only relevant range is \([S_B, S_X]\) and the optimal covenant is \( S_C^* = S^\#_C \), which is identical to the \( S^0_C \) threshold defined in (9). Note that (9) can be written as

\[
\theta \left( S^0_C, \beta_L \right) = L - \frac{(\beta_M - \beta_M \beta_L) - (1 - \theta(S^0_C, \beta_L))(\beta_M - \beta_L)}{\beta_M (1 - \beta_L)} B, \tag{35}
\]

which shows that \( S^0_C < S_X \). We already have established that \( S^\#_C > S_B \) for any \( p \), implying that \( S^0_C > S_B \).
Suppose now that \( p > 0 \). The optimal covenant is then either \( S = S^\# \) or \( S = 1 \). We show in the proof of Proposition 8 that \( S^\# \) is increasing in \( p \). Next, we show that there exists a threshold \( \hat{p} \in (0, 1) \) such that for \( p > \hat{p} \) the corner solution \( S = 1 \) is optimal whereas \( S^\# \) is optimal for \( p < \hat{p} \). Since, as just established, for \( p = 0 \) the optimal covenant is \( S^\# \), the threshold \( \hat{p} \) must be greater than zero. To prove that \( \hat{p} < 1 \), we show first that there exists a threshold \( \check{p} \in (0, 1) \) such that \( S^\#_C(\check{p}) = S_X \). Substituting \( \theta(S_X, \beta_L) X = L \) and \( S^\#_C = S_X \) into (16) and solving for \( p \) yields

\[
\check{p} = \frac{B\beta_M (1 - \beta_L) - B (\beta_M - \beta_L) (1 - \theta(S_X, \beta_L))}{B\beta_M (1 - \beta_L) + \gamma (L - B) (\beta_M - \beta_L) (1 - \theta(S_X, \beta_L))} < 1, \tag{36}
\]

which shows that \( \check{p} \) exists with \( \check{p} \in (0, 1) \). Thus, for \( p \geq \check{p} \), the manager prefers \( S = S_X \) over any other \( S \) in the range \([S_B, S_X]\). We know from cases (ii) and (iii) that for any \( S \in [S_X, 1] \) the manager strictly prefers \( S = 1 \) when \( p > 0 \) and \( \beta_M > \beta_L \). Hence, \( S = 1 \) is the overall optimal solution for \( p = \check{p} \). This establishes that \( \check{p} < \hat{p} \). Thus, for all \( p < \hat{p} \), the optimal covenant is \( S^*_C = S^\#_C \) and for all \( p \geq \hat{p} \), the optimal covenant is \( S^*_C = 1 \).

**Proof of Proposition 8**

To show that \( S^*_C \) is increasing in \( \beta_M \) and \( p \) for \( p < \hat{p} \), we rearrange (16) to obtain

\[
\Gamma \equiv \theta(S^*_C, \beta_L) X + B - L - (1 - \theta(S^*_C, \beta_L)) \frac{(\beta_M - \beta_L) (B + (L - B) p \gamma)}{\beta_M (1 - \beta_L) (1 - p)} = 0. \tag{37}
\]
Applying the implicit function theorem yields:

\[ \frac{dS_C^*}{d\beta_M} = -\frac{d\Gamma/d\beta_M}{d\Gamma/dS_C} > 0 \text{ and } \frac{dS_C^*}{dp} = -\frac{d\Gamma/dp}{d\Gamma/dS_C} > 0, \quad (38) \]

where

\[ d\Gamma/d\beta_M = -(1 - \theta (S_C^*, \beta_L)) \left( \frac{\beta_L (B + (L - B) p_\gamma)}{\beta_M^2 (1 - \beta_L)(1 - p)} \right) < 0, \]
\[ d\Gamma/dp = -\frac{(1 - \theta (S_C^*, \beta_L)) (\beta_M - \beta_L) (B + (L - B) \gamma)}{\beta_M (1 - \beta_L)(1 - p)^2} < 0, \]
\[ d\Gamma/dS_C = \frac{d\theta (S_C, \beta_L)}{dS_C} \left( X + \frac{(\beta_M - \beta_L) (B + (L - B) p_\gamma)}{\beta_M (1 - \beta_L)(1 - p)} \right) > 0, \]

and \( \frac{d\theta(S_C, \beta_L)}{dS_C} > 0 \) follows from the MLRP.

**Proof of Proposition 6**

The proof follows immediately from the proof of Proposition 7 and hence is omitted.

**References**


