The Performance Measurement Trap*

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Abstract

This paper investigates the effect of performance measurement on the optimal effort allocation by salespeople when firms are concerned about retention of salespeople with higher abilities. It shows that introducing salespeople performance measurement may result in productivity, profit, and welfare losses when all market participants optimally respond to the expected information provided by the measurement and the (ex-post) optimal retention efforts of the firm cannot be (ex-ante) contractually prohibited. In other words, the dynamic inconsistency of the management problems of inducing the desired effort allocation by the salespeople and the subsequent firm's objective to retain high ability salespeople may result in performance measurement yielding an inferior outcome.

Keywords: Game Theory, Contract Design, Principal-Agent problem, Salesforce Compensation

"The worker is not the problem. The problem is at the top! Management!"
— W. Edwards Deming

1. Introduction

Measurement of the salespeople performance is one of the key tasks of many sales organizations. It is a complicated task because what a firm can measure is often not very well aligned with what it is trying to achieve in the long run. As Likierman (2009) argues, what is measured may potentially provide little insight into a firm's performance, and may potentially hurt the firm. In particular, firms may use what can be more easily measured, or what is more popular, and not activities that are difficult to measure, immeasurable, or less popular. For example, Likierman (2009) points out that the Net Promoter Score (Reichheld 2003, which measures the likelihood that customers will recommend a product) may only be a useful indicator when recommendations play a crucial role in the consumers' decisions, but the importance of customer recommendations may vary from industry to industry. As another example, the number of telephone calls of a salesperson may be easy to document, while the content and preparation for those telephone calls may not be as easy to measure but potentially a more important component of the selling effort.

It is well known that putting too much weight on visible measures not perfectly aligned with the organization's objectives may lead to a detrimental distortion of the salesperson's effort allocation. As a recent example from business practice, Wells Fargo's incentives for the employees to cross-sell accounts to customers lead to the adverse behavior of the employees, and in the Fall of 2016, Wells Fargo ended up considerably hurt both by the customer backlash and the regulatory actions.[1] Likewise, if a product manager is evaluated on the proportion of successful product introductions (which is easier to measure than the contribution of the manager to the overall profitability of a manufacturer), the manager may distort work toward pushing products that are likely to fail (Simester and Zhang, 2010). For an example related to the marketing of academic

[1]See, e.g., "Wells Fargo Fined $185 Million for Fraudulently Opening Accounts," by M. Corkery, *New York Times*, September 8, 2016.

ideas,[2] consider an assistant professor in a tenure track position at a university. The university wants to retain scholars who make and disseminate advancements in their field (e.g., teaching and research). But the university imperfectly observes the assistant professor's ability. Good assistant professors may receive offers from other universities, which may induce strong scholars to be productive in their rookie position. In addition, the university might try to measure some of the assistant professor's output directly, but imperfectly. For instance, the university may simply count publications and teaching ratings. This could be detrimental to the university's mission because the scholars could distort their efforts in favor of the measured variables, in contrast to making and disseminating advancements in their field more generally.

One may be inclined to ascribe the negative outcomes of measurements or incentives to boundedly rational behavior or the coordination issues within an organization's management. But what we show in this paper is that under some conditions, even fully rational agents designing and responding to the optimal incentive contracts cannot avoid being hurt by the very fact of the measurement's existence. Specifically, we show that the net effect of an additional dimension being measured can decrease productivity, profits, and welfare, even when the management fully accounts for the current and future effects when designing the optimal compensation structure. We formalize the following intuition: Suppose that the total effort that a particular salesperson would exert is essentially fixed (i.e., not easily changed by incentives), and that in the absence of additional performance measurement(s), salespeople are incentivized to allocate effort across two components in an efficient way. For example, in the absence of the number of telephone calls being measured, the salesperson allocates effort efficiently between the number of telephone calls made and the preparation for those calls. However, suppose the existing incentive to allocate effort efficiently (using the existing measurements) is not particularly strong, so that a salesperson would distort his effort allocation if a new, non-negligible incentive to do so is introduced.

For example, management may be able to observe and enforce the time spent at work, and any allocation of time across activities may not impose a disutility on the salespeople so far as they have to spend that time at work. In this case, a slight weight on overall output of the

[2] We thank the Associate Editor for the suggestion of this example.

organization, even if this output is only weakly correlated with the effort allocation of a given salesperson, is sufficient to induce the salesperson's optimal effort allocation. As a result, the system is working nearly perfectly. Different salespeople contribute differently to the total output due to their different abilities, but each does what he can (providing the effort he is capable of and efficiently allocating it across the effort components), and nothing can be done about the abilities themselves.

In order to simplify the presentation, we will identify the total effort the salesperson is capable of with the salesperson's ability. Now suppose that due to a random value of the outside option (e.g., due to the changing preferences for living close to the current employment area), each salesperson may leave the company at the beginning of each period, but this decision may, at least for some salespeople, be affected by the compensation that he is offered. Then, to encourage the better salespeople not to leave the organization through an appropriate retention policy, i.e., by offering a better next period contract to the better salespeople, management is interested in evaluating its salespeople's abilities.

Suppose further that the feasible measurement under consideration is the measurement of one, but not all, of the effort components. Note that without this additional performance measurement, given the optimal effort allocation, the salesperson's ability (total effort) is perfectly correlated with each of the effort components. Then a salesperson's ability could be perfectly judged from either component. This seems to provide a compelling argument for introduction of the performance measurement, even though it only measures one of the effort/performance components. However, knowing that management will offer a better contract in the next period to those salespeople who performed better on the measured component, if management starts measuring one of the dimensions of effort, and even if no weight is placed on it in the current compensation schedule, salespeople will distort effort allocation toward that dimension in hopes of securing a better contract offer in the next period. If this distortion is not costly for the salespeople, all effort may be allocated to only one component, thus, possibly having a negative effect on the total output. In the salesperson example above, the salesperson would prepare the telephone calls minimally, and would just maximize the number of telephone calls made.

But would not management then be able to adjust the current compensation package to eliminate this distortion, perhaps through a negative weight in the current contract on the performance component measured? The answer is: generally, no. The reason, again, is salespeople heterogeneity. Generally, one might expect that salespeople would have some independent and private estimate of their likelihood of staying with the organization next period. Therefore, the best management can hope to achieve with the optimally designed "reverse incentives" contract is that some salespeople, knowing that they are more likely than average to leave, will be incentivized by the reverse incentive to distort the effort towards the other component, while most of the remaining salespeople, having a higher likelihood to stay than management estimates, would still distort their effort toward the measured component. The outcome is that, while any aggregate mix of effort allocations can be achieved, almost all salespeople may end up allocating effort inefficiently.

Note that perfect information, if costlessly available to management, should be welfare enhancing. However, imperfect measurement, insofar as it measures some, but not all, of the components of productive output, is likely to distort effort allocation. While this measurement can be used for better retention and a better total effort incentive policy, these improvements are at the expense of the efficiency of the effort allocation. Furthermore, inability to commit to future contracts means that this asymmetric measurement problem may not be fully solvable.[3] Therefore, when deciding on whether to introduce a partial (or imperfect) measurement (i.e., a measurement that weighs some of the effort/output components heavier than others), the firm needs to decide whether the problems of the effort enforcement and salesperson retention are more important than the problem of effort allocation. Thus, the common practice to "collect the data first; decide what to do with it later" could be a treacherous path even if the management

[3]Management can offer long-term contracts, but the problem is that once management sees evidence of high ability and given the probability that the salesperson quits, it would then offer retention bonuses to some salespeople. That is to say, a commitment to never use the measurement is not renegotiation-proof. One could also envision "slavery contracts" that commit the salesperson to work forever. Although such contracts would solve the issue of commitment (since retention is no longer an issue), they would result in inefficiency since due to the random outside options (if they are not perfectly predictable by the salespeople), sometimes it is efficient for a salesperson to leave. From a profitability standpoint, such contracts would come at the expense of offering a higher base salary up front. Thus, the negative value of measurement would persist even if slavery contracts are allowed.

is fully rational and benevolent.

We present the paper in the context of incentives for salespeople, but the ideas presented here could equally apply to incentives to product managers for new product development (e.g., Simester and Zhang, 2010). More generally, the results here would apply to employees in an organization whose compensation depends on incentives.

The remainder of the paper is organized as follows. The next section discusses the related literature. Section 3 presents the model, and Section 4 considers the cases when measurement is not possible, and when measurement is available on the first period effort level. Section 5 presents the effect of different measurement technologies, and Section 6 concludes. The proofs are collected in the Appendix.

2. Related Literature

This paper builds on the extensive literature on the principal-agent and salesforce compensation problems.[4] This literature, in particular, explores the optimal weights the principal needs to place on measurements to account for the effort distortion across multiple tasks (e.g., Holmstrom and Milgrom, 1991, Hauser et al., 1994, Bond and Gomes, 2009).[5] For example, Holmstrom and Milgrom (1991) show that when the effort distortion between tasks is sufficiently severe, flat pay (i.e., contracts ignoring performance measurement) may be optimal, and discuss how it applies to the ongoing teacher compensation debate. Hauser et al. (1994) explore making the incentive scheme based on customer satisfaction measures as a way for employees to put effort on dimensions that have longer term implications. Bond and Gomes (2009) explore the ability of the principal to affect a multi-tasking agent's efforts. The literature has also discussed what happens in a dynamic setting with the question of desirability of long-term contracts (e.g., Malcomson and Spinnewyn, 1988) and renegotiation-proofness. One usual assertion is that the principal and welfare are not hurt by more information (measurement), but this is due to the assumed ability

[4]See, for example, Basu et al. (1985), Rao (1990), Raju and Srinivasan (1996).

[5]Some measurement can also be obtained on the market conditions by allowing lobbying by sales people on the incentive scheme (e.g., Simester and Zhang, 2014).

of the principal to put a sufficiently low weight on what is being measured. Alternatively, we consider the problem of the principal's possible inability to commit to the future contracts and show that the optimal contract in the presence of more information (more performance measures) may lead to decreased profitability and social welfare.

Cremer (1995) considers a dynamic consistency problem of the principal's commitment to incentivize effort by the threat of firing the agent. Assuming that the agent values job renewal, Cremer shows that in some cases, the principal would like to commit not to observe information about the reason the agent fails in order to increase the agent's incentive to perform. By not observing the reason for failure and therefore effectively committing to fire the agent if output is not up to a standard, the principal is able to circumvent the positive-pay restriction and increase the incentive for the agent to perform well without increasing the expected pay. In contrast to Cremer, we examine the opposite problem: when the principal would like to commit not to incentivize the agent.

Dynamic (in)consistency issues have been studied in various other contexts, such as a durable-goods monopoly setting prices over time (e.g., Coase 1972, Desai and Purohit 1998), and central bank policies on money supply (e.g., Kydland and Prescott, 1977). In this paper, we essentially consider the implications of the dynamic inconsistency issue in a principal–agent framework.

3. The Model

In the next subsection, we formulate the model setup with only a limited justification for the assumptions, and then discuss some assumptions, their consequences, and potential variations in the following subsection.

3.1. Model Setup

Consider a principal–agent model with agents having different abilities and interacting with the firm (principal) during two periods, indexed by $t = 1, 2$. In the context of this paper, the agent is a salesperson. Since we abstract away from any effects of one salesperson's behavior on other salespeople or the incentives the principal has in treating other salespeople, we consider, without loss of generality, a single agent whose type is uncertain to the principal. The principal's (management's) objective is to maximize the total expected payoff (profit) net of the compensation paid to the salesperson across the two periods by choosing a contract (compensation conditional on observables) to offer the salesperson at the beginning of each period. Let π_t denote the profit gross of the expenditure on salespeople compensation in period t, and let C_t be the compensation paid to the salesperson in period t.[6] To simplify the presentation, assume no discounting. Then, the firm's problem is

$$\max_{C_{1,2}\geq 0} \sum_{t=1}^{2} \mathrm{E}(\pi_t - C_t) \tag{1}$$

The salesperson compensation (contract) could depend on everything the principal observes prior to the offer or at the time of payment, as we assume the payment is done after the relevant period is over. However, the contract is restricted to provide non-negative pay to the salesperson for any outcome (limited liability). This could be justified, for example, because obtaining money from a salesperson who received no income in the current period may not be possible.

Period t's profit is uncertain and depends on the salesperson's choice of the effort allocation $\overline{e} \equiv (x, y) \in \mathbb{R}^{+2}$ where x and y are the two dimensions of effort. For example, x could represent the number of telephone calls and y could represent the time spent preparing for those calls. To be clear, we will call the vector $\overline{e} \equiv (x, y)$ the effort allocation, and the sum of effort components $e \equiv x + y$ the total effort. Assume that the salesperson has a per-period budget constraint on his total effort $x_{it} + y_{it} \leq e_i$, where e_i depends on the salesperson's type, which can be low, ℓ, or high, h, that is, $i \in \{\ell, h\}$. Assume that besides the budget constraint on effort, the salesperson has no intrinsic disutility of effort.

To simplify the potential contract structures, we model profit as possibly attaining one of just two possible values, one of which is normalized to 1 and the other is denoted by $-B$, with $B > 0$, and model the effect of the salesperson's effort allocation as affecting the probability of achieving the high profit level. Assume that the probability of high profit increases in the

[6] Table 1 presents the full notation.

salesperson's productivity defined as $\alpha(x, y) = x \cdot y$. Let us call the maximal productivity of a salesperson his ability a. Maximizing $\alpha(x, y)$ subject to $x + y \leq e_i$, we have $a_i = (e_i/2)^2$. Thus, the salesperson's ability is a characteristic of the salesperson, while his productivity is his choice variable constrained by his ability. To be specific, we assume the following gross profit specification as a function of the salesperson's productivity:

$$\pi_t(\alpha_{it}) = \begin{cases} 1, & \text{with probability } (b + \alpha_{it})/(b+1); \\ -B, & \text{with probability } (1 - \alpha_{it})/(b+1), \end{cases} \tag{2}$$

where $\alpha_{it} = x_{it}y_{it}$ is the salesperson's choice of productivity in period t, and $b > 0$ is a parameter which determines how much the profit is (in expectation) informative about the salesperson's ability.

To satisfy the constraint that the probabilities of the two profit outcomes are positive, we need $0 \leq e_\ell < e_h \leq 2$. (If $e_h = 2$, the efficient allocation $x = y = 1$ leads to $a_h = 1$.) To simplify derivations, assume $e_\ell = 0$ and $e_h = 2$, so that $a_\ell = 0$ and $a_h = 1$. To avoid considerations of the boundary conditions on the wage of the low-type salesperson, assume $b \geq B$ (so that the expected profit from the low type worker given no wage is non-negative).

To introduce a non-trivial employee retention problem, assume that the salesperson has an outside option z in the second period, where z is uniformly distributed on $[0, 1]$ and independent of salesperson's type. The outside option is zero in the first period so the salesperson accepts any non-negative offer in the first period.[7] The second-period outside option is known to the salesperson before his decision of whether to accept the second period offer, but may or may not be known to him before his choice of the effort allocation $\bar{e}_1$ in the first period. We first assume that the salesperson does not know z before the effort-allocation stage of the first period; he learns it just before he needs to decide whether to accept the second period's offer. We then consider a variation of the model to see how the results change if the salesperson can condition his effort allocation $\bar{e}_1$ on the second period outside option z or if he has some (but not full)

[7]To avoid potential incentives for salespeople to signal high ability through refusing a first period offer in order to gain a higher compensation in the second period, assume that the firm does not engage in hiring at all in the second period. From the equilibrium that we derive, such signaling will also not be optimal in this setting.

information about it.

In addition to choosing the contract to offer, the principal needs to decide whether to introduce measurement $m(x)$ of the x-component of the salesperson's effort. In the example above this would be measurement of the number of telephone calls made by the salesperson. We assume that the measurement of the y-component, beyond its inference from the profit realization and any available measurement of x, is not feasible. In the example above, it is not possible to measure the time spent by the salesperson in the preparation of the telephone calls. In order to reduce the complexity of the compensation structure, assume that $m(x)$ may take only one of two values, call them 0 and 1, but the probability of the high measurement realization ("1") increases in x.[8] For analytical tractability assuming linearity and that the high type can ensure $m(x) = 1$ by allocating all effort toward x, we use the following measurement specification:

$$m(x) = \begin{cases} 1, & \text{with probability } x/e_h = x/2; \\ 0, & \text{with probability } 1 - x/e_h = 1 - x/2. \end{cases} \tag{3}$$

If measurement is introduced, the compensation in each period is a function of the current and past profits and measures of x. If measurement is not introduced, the compensation can only be a function of current and past profits. There are three conceptually different possibilities of the timing when $m(x)$ is introduced: (1) after the salesperson decided on his effort allocation $\bar{e}_1$ in the first period; (2) before the salesperson decided on his effort allocation $\bar{e}_1$ in the first period, but after the first period compensation rule was set (and it was done without having the possibility of measurement in mind), and (3) before the salesperson decided on his effort allocation $\bar{e}_1$ in the first period, and the first period compensation rule was determined with the possibility of the measurement in mind. We consider each of these cases. We first analyze the case in which the measurement of the component x is only possible in the first period effort (Section 4). In Section 5.2, we consider the case in which the measurement of component x is possible in both the first and second periods.

[8]This measurement structure simplifies the analysis. In Section 5.1, we also consider the case of precise measurement $m(x) = x$, which yields similar results.

3.2. Discussion of the Model Assumptions

In the model setup we have made some specific assumptions about the functional forms of the salesperson's productivity $\alpha(x, y)$, the profit function $\pi(\alpha)$, the effort component measurement function $m(x)$, the outside option z, and some parameter values either to be specific or to simplify the analysis. We now discuss these assumptions in turn.

First, we have assumed a multiplicative production function $\alpha(x, y) = xy$. In the running example of the salesperson allocating effort between preparing for calls and making calls, this specification captures the idea that salesperson telephone calls without preparation have limited effect, and preparing telephone calls without actually making them has also zero effect on sales. The probability of high profits in this specification is maximized at $x = y = e_i/2$, i.e., the efficient effort allocation, which is also the most desired by the firm, is for the salesperson to equally split the total effort between the two effort components. This exact functional form of $\alpha(x, y)$ is not important, but what we need is that there is some optimal split of effort across two (or more) components, so that directing all effort toward one of the components reduces profits. For example, we have checked that our main results also hold if $\alpha(x, y) = \sqrt{x} + \sqrt{y}$.

Let us next consider the functional form of the profit function. The informativeness of the profit realization (whether it is "1" or "$-B$") about the salesperson's actions depends on b and tends to zero when b tends to infinity. For clearer presentation, we will focus the analysis on the case of large b and B, but discuss, when appropriate, what happens when these parameters are not too large. The case of large b and B can be seen as a case in which the positive output is highly likely (so that even the low-ability salespeople can easily sell), but the loss is important when the positive output does not occur. For example, in some types of work, a mistake (selling a dangerous item, causing an injury while demonstrating a product, making an inappropriate comment, etc.) may lead to a lawsuit or a viral consumer backlash. In terms of the model, large b means that the output obtained is not very informative of the salesperson's ability, which is realistic, for example, when an individual salesperson is a part of a large group (i.e., the profit of a large retailer is not very informative about the contribution of each individual employee). A special case $b = B$ of the above profit specification has a special meaning: in this case, the expected profit is equal to α_{it}. Therefore, considering this case allows one to consider how the results depend on the informativeness of profit realization (driven by b) keeping constant the importance of the salesperson's ability to the firm.

Note that given $a_\ell = 0$, the allocation decision of the low type (allocating 0 across the two components) is immaterial, which considerably simplifies the analysis. When $a_\ell = 0$, the probability of the profit being high (being "1") is $b/(b+1)$ and the probability of the profit being low (being "$-B$") is $1/(b+1)$. When $a_h = 1$ and the salesperson chooses the efficient effort allocation, the firm gets the high profit with probability one. If the salesperson is choosing the efficient effort allocation, a low profit implies that the salesperson has low ability for sure (this part also considerably simplifies the analysis), but a high profit does not necessarily imply that the salesperson has the high ability.

Turning to the technology of measurement (the specification of $m(x)$), it is important for our results that only one component could be possibly measured, but the exact specification is not essential. Allowing $m(x)$ to probabilistically take one of the two outcomes simplifies the analysis as it implies that the compensation is not a function of a continuous variable but of a binary one. Note that with the measurement technology as defined and given the assumptions $e_\ell = 0$ and $e_h = 2$, a measurement of "1" indicates that the salesperson has high ability (which simplifies the analysis), while a measurement of "0" does not necessarily indicate that the salesperson has low ability as long as the high ability salesperson is not expected to shift all his effort toward component x. We have also analyzed (see Section 5.1) the possibility that x can be measured precisely, and while that analysis is more complicated (in particular, because it requires the consideration of $a_\ell > 0$), our main results continue to hold. This shows some robustness to the assumptions on the measurement technology.

Finally, we have assumed that the salesperson has an outside option z in the second period. It is important for our results, because it introduces the retention problem in the firm's objective function, which leads to the second-period's compensation being correlated with the firm's expectation of the salesperson's ability. This outside option could come from the utility of working elsewhere, or staying at home (for example, given family changes), the changing cost of traveling to work due to the potential salesperson's move for family reasons, etc. For simplicity, we assume that z does not depend on the salesperson's type i. This could be potentially justified by idiosyncratic preferences by the salespeople or other employers that are not related to how the salesperson is productive in the present firm. For example, the cost of traveling to work could affect the outside option of a salesperson without affecting the productivity of the salesperson once employed, or the salesperson's desire to stay at home to raise a baby may not be dependent on the salesperson's skill at work. On the other hand, outside work offers may be positively correlated with the salesperson's ability. If we would consider z positively correlated with the ability, we would have that the probability of retention of low ability salespeople would be increased, and this could increase the incentives to discriminate between the high and the low ability salespeople and, therefore, potentially further increase distortions.

4. Model Analysis

To understand the effect of the measurement, we need to compare the outcomes when measurement is not possible, which we will call the benchmark case, with the outcomes when measurement is introduced and the agents (the salesperson and the manager) optimally react to the new information. For the latter, the outcomes could be different depending on when the measurement is introduced and, if it is not introduced at the start, whether the introduction is expected by the salesperson. We therefore consider what happens under different timings of the measurement introduction.

We first derive the optimal compensation schedule in the benchmark case when the measurement of x is not a possibility (Section 4.1). We then consider how the second period's compensation and outcomes are affected if the measurement of the first period's component x of the effort is unexpectedly introduced in period 1 after the salesperson already committed to his first period's effort allocation (Section 4.2). Next, we consider how the outcomes change if the salesperson optimally responds to the presence of the measurement of the first period effort, but assuming that the first period's compensation rule is unchanged after the measurement is unexpectedly introduced (Section 4.3). Conceptually, the last consideration is when the measurement introduction was unexpected by the firm, while the earlier one is when it was unexpected by the salesperson. Finally, we consider optimal compensation contracts in both periods given that the measurement of the first period's x component is present, or is expected to be introduced (Section 4.4). Comparing the outcomes under the last scenario with the previous ones, we show that the introduction of measurement could be detrimental to the principal and to social welfare. We discuss what one would expect in equilibrium where the introduction of the measurement is a manager's decision variable in Section 4.5. The case of measurement of x component in both periods is presented in Section 5.2.

4.1. Benchmark Case: Optimal Contract without Measurement

Given that the salesperson's outside option is zero in the first period and that he does not have an intrinsic preference of how to allocate the maximum effort e_i at his disposal, compensation $C_1 = \varepsilon \mathbf{1}_{\pi_1=1}$ with $\varepsilon > 0$ achieves efficient allocation (so far as the salesperson does not expect the second period contract to provide perverse incentives for the salesperson to lower the firm's expectation of their abilities).[9] Thus, in equilibrium, we have efficient effort allocation and $C_1 = 0$, i.e., the first-best for the principal (as contracts are not allowed to have negative pay) with the expected first-period profit of $\frac{1}{2} + \frac{1}{2}\frac{b-B}{b+1}$.

In the second period, the problem is slightly more complicated. Let P_h be the posterior probability of the salesperson having the high ability given the observables after the first period, i.e., the first-period profit in this case. Then, one can derive that the optimal second-period contract $C_2 = c_1 + c_2\mathbf{1}_{\pi_2=1}$ has (see Appendix for details):

$$c_1 = 0 \quad \text{and} \quad c_2 = \frac{P_h(1+b)^2 + (1-P_h)(b-B)b}{2(b^2 + P_h + 2P_h b)}. \tag{4}$$

If $P_h = 1$, then the firm is indifferent between any $c_1 > 0$ and $c_2 > 0$ as far as their sum is the same; this is because $\pi_2 = 1$ for sure for the high type. But if $P_h < 1$, the above solution is

[9] The term $\mathbf{1}_{condition}$ is the indicator functions and takes the value of 1 if the "condition" is true, and takes the value of 0 otherwise.

uniquely optimal. By Bayes' rule:

$$\Pr(h|\pi_1 = 1) = \frac{1+b}{1+2b}, \quad \text{and} \quad \Pr(h|\pi_1 = -B) = 0. \tag{5}$$

Substituting these in (4), we obtain the second period payment C_2 to the salesperson as a function of the first and second period profits:

$$C_2 = \left(\frac{b-B}{2b} + \widehat{c}_2 \mathbf{1}_{\pi_1=1}\right) \cdot \mathbf{1}_{\pi_2=1}, \quad \text{where} \quad \widehat{c}_2 \equiv \frac{(1+b)^3 B}{2(1+2b)(1+b+b^2)b}. \tag{6}$$

The expected net-of-compensation profit is $1 - \frac{1}{2}\frac{B+1}{b+1}$ in the first period (it is equal to 1 if the salesperson is of high ability and $\frac{b-B}{b+1}$ if he is of low ability) and

$$\mathrm{E}(\pi_2 - C_2) = \frac{((1+b)^3 + (b-B)b^2)^2}{8(1+b)^3(1+2b)(1+b+b^2)} + \frac{(b-B)^2}{8(b+1)^3} \tag{7}$$

in the second period. This follows from substituting the optimal contract, equation (6), into the expected profit, equation (16), and taking into account that the probability of $\pi_1 = 1$ is $\frac{2+b}{2(1+b)}$.

For example, for $b = B = 0$, we have $C_2 = 1/2 \cdot \mathbf{1}_{\pi_1=\pi_2=1}$, $\mathrm{E}(\pi_2 - C_2) = 1/8$, and the expected total net profit of 5/8.[10] For $b = B = 10$, we have $C_2 = 0.29 \cdot \mathbf{1}_{\pi_1=\pi_2=1}$, $\mathrm{E}(\pi_2 - C_2) = 0.07$, and the expected total net profit of 0.57. As $b = B \to \infty$, we have $C_2 \to 1/4$, $\mathrm{E}(\pi_2 - C_2) \to 1/16$, and the expected total net profit converging to 9/16.

Intuitively, the second period's and the total net profits decline with $b = B$ as the first period profit becomes less informative of the salesperson's ability when $b = B$ increases, and the resulting less efficient retention is detrimental to both the firm and, on average (across salesperson types), to the salesperson. In particular, as $b = B$ increases from zero to infinity, the high type

[10] Note that the assumption of relatively low retention rate (relatively high outside option) forces the profit to be low in the second period – both due to the low retention probability and the extra expenditure on salespeople compensations. However, reducing the upper bound on the outside option would complicate the analysis as it would require considering the boundary case of the retention probability equal to one under a potentially optimal contract. One possibility to bring the first and second period's profits closer – without changing the effects presented – would be to consider z to be a mixture of 0 and the uniform component (i.e., a mass point at zero and the rest of the distribution still uniform on $[0, 1]$).

salesperson's expected surplus (over the outside option z) from the second period compensation decreases from 1/8 to 1/32, while the low type salesperson's surplus increases from zero to 1/32, resulting in the average salesperson surplus decreasing from 1/16 to 1/32. Note that the average compensation remains the same, but the expected value of it decreases because the value of C_2 to the salesperson is $\Pr(C_2 > z) \cdot \mathrm{E}(C_2 - z \,|\, C_2 > z) = \mathrm{E}(C_2^2)/2$, i.e., convex in C_2.

Note also that the analysis and all the results of the benchmark case apply whether the salesperson knows his second period outside option at the beginning of the game or only just before his decision on whether to accept the second period offer. This is because in equilibrium, the salesperson allocates the effort efficiently in the first period regardless of whether he plans to leave or stay with the firm in the second period.

By differentiating c_2 in Equation (4) with respect to P_h, one can see that the optimal offer increases in the principal's belief about the salesperson's ability:

$$\frac{d\,c_2}{d\,P_h} = \frac{(1+b)^2 Bb}{2(b^2 + P_h + 2P_h b)^2} > 0. \tag{8}$$

This means that if there is some way for the salesperson to demonstrate high ability without incurring a significant cost, he would strictly prefer to do so. This is the key to the result that a performance measurement of one effort component would be used by the high type salesperson to convince the firm that he is of high type at a cost of the first period profit. Of course, if profit is itself very informative or if the weight on the first period profit is sufficiently high, then this first period possible distortion by the salesperson may not happen. Also note that the above inequality means that, effectively, the expectation of the second period's contract puts an implicit positive weight on the first period's profit (since all else being equal, $\pi_1 = 1$ implies a higher probability of the high type than $\pi_1 = -B$ does), which gives the salesperson a strictly positive incentive to allocate the first period's effort correctly. That is, even though the first period's equilibrium contract leaves the salesperson indifferent as to how to allocate his effort if the salesperson were myopic, the equilibrium is actually strict due to the expected-by-the-salesperson second period contract's positive dependence on the first period's profit realization.

4.2. Unexpected Measurement Introduced After First Period's Effort Allocation

Now consider a situation in which the measurement is unexpectedly introduced after the first period's effort allocation. This is an off-equilibrium case since we assume that the salesperson expects to be in the situation of the benchmark case, i.e., he does not expect performance measurement, but the firm then introduces the performance measurement. Thus, the results of this case are going to be used to understand the driving forces and incentives to introduce the performance measurement, and not as predictions in and of themselves. This case is also useful for understanding and predicting what the salespeople should expect if the firm is unable to commit to whether it would, or would not, introduce a performance measurement mid-game.

In this case, the first period contract, effort allocation, and profits are the same as in the previous case since the firm can do no better and the salespeople do not expect any measurement to occur. But the second period's contract can now be conditioned not only on π_t $(t = 1, 2)$ but also on the measurement realization $m(x_1)$.[11]

Given the efficient effort allocation in the first period, which we have since the salesperson(s) did not expect the performance measurement, Bayes' rule now gives the following probabilities of the salesperson being of the high type conditional on the realizations of π_1 and $m(x_1)$:

$$\begin{aligned} \Pr(h|\pi_1 = 1 \;\&\; m(x_1) = 1) &= 1; \quad \Pr(h|\pi_1 = 1 \;\&\; m(x_1) = 0) = \frac{1+b}{1+3b}; \\ \Pr(h|\pi_1 = -B \;\&\; m(x_1) = 1) &= 1; \quad \Pr(h|\pi_1 = -B \;\&\; m(x_1) = 0) = 0. \end{aligned} \tag{9}$$

Note that the event $\{\pi_1 = -B \;\&\; m(x_1) = 1\}$ does not occur if effort is allocated efficiently, so the Bayes' rule does not apply in that case. Therefore, $\Pr(h|\pi_1 = -B \;\&\; m(x_1) = 1)$ could take any value. We set this value at 1 because a low type salesperson could never get $m(x_1) = 1$, while a high type salesperson could get $\pi_1 = -B$ and $m(x_1) = 1$ by not choosing an efficient effort allocation. In other words, this belief assignment is required for an equilibrium to be sequential. However, the results below would also hold if any other value (between 0 and 1) is assumed.

[11] The case when the second period contract can also be conditioned on the second period's measurement realization $m(x_2)$ is considered in Section 5.2.

Since in the second period, the only new observable is π_2, equation (4) continues to hold with P_h now depending on both the first period's profit and on the first period's measurement of x according to (9) above. Thus, the optimal second period contract offer is $c_2\mathbf{1}_{\pi_2=1}$, where $c_2 = 1/2$ when $m(x_1) = 1$ or $\pi_1 = -B$ & $m(x_1) = 1$, $c_2 = (b - B)/(2b)$ when $\pi_1 = -B$ & $m(x_1) = 0$, and

$$c_2 = \widetilde{c}_2 \equiv \frac{(1+b)^3 + 2(b-B)b^2}{2[2b^3 + (1+b)^3]} \quad \text{when} \quad \pi_1 = 1 \ \& \ m(x_1) = 0. \tag{10}$$

While the first period decisions and, hence profit, are the same as in the benchmark case, the optimal contract above leads to the expected second period net profit of

$$\mathrm{E}(\pi_2 - C_2) = \frac{1}{8}\frac{(1+2b)(1+b+b^2)}{2b^3 + (1+b)^3} + (b-B)\frac{2(b+1)^2b^2 + (2b+1)(b^2+1)(b-B) - b}{8(b+1)^2(2b^3 + (1+b)^3)}, \tag{11}$$

which is higher than the second-period net profit in the benchmark case by $\frac{1}{8}\frac{b^4B^2}{(b^2+b+1)(1+2b)(1+3b+3b^2+3b^3)}$. As $b = B$ increases from zero to infinity, the above profit decreases from 1/8 to 1/12. Given that the firm now has better information about the salesperson's ability in the current case relative to the benchmark, as one would expect, the high type salesperson retention is higher, the low type salesperson retention is lower, and the profit and average salesperson surplus is increased.

4.3. Measurement Introduced Before First Period's Effort Allocation

Consider now that the measurement was introduced when the first period compensation was set to zero, as in the benchmark case (Section 4.1), but before the salesperson decides on his effort allocation. Then, the salesperson knows that the manager will observe the measure $m(x_1)$ at the end of the first period. Then, it is a dominant strategy for the high type salesperson to fully distort his first period effort allocation toward the x component. This is because demonstrating high type increases the expected payoff in the second period in some instances but in no instance decreases it.

Therefore, the expected first period profit is reduced to $(b - B)/(b + 1)$, i.e., it is as if all salespeople were of low type. The expected second period profit increases to $(1 - 1/2) \cdot 1/2 = 1/4$ if the salesperson is of high type, which is the net profit from a retained high type salesperson multiplied by the probability of retention of a high type salesperson given the optimal contract (a high-type salesperson is for sure identified through $m(x) = 1$ in this case). Likewise, it increases to $\left(\frac{b-B}{b+1} - \frac{1}{2}\frac{b-B}{b} \cdot \frac{b}{b+1}\right)\left(\frac{1}{2}\frac{b-B}{b} \cdot \frac{b}{b+1}\right) = \frac{1}{4}\left(\frac{b-B}{b+1}\right)^2$ if the salesperson is of low type (which is also for sure identified through $m(x) = 0$). The total net second-period profit is thus $\frac{1}{8}\left(1 + \frac{(b-B)^2}{(b+1)^2}\right)$. Note that relative to the benchmark case, the expected second period net profit increases due to the better identification and the better retention of the high type salesperson, but the first period profit suffers.

The negative impact of the measurement on the first period effort allocation due to the expected second period contract adjustment leads to the idea that the first period's incentive to distort effort allocation should be countered in the first period compensation package. We consider this strategy in the following subsection. But first, to illustrate the difficulty of countering the first period distortion, let us derive the high type salesperson's benefit of the maximal distortion in the first period relative to no distortion assuming that the firm does not expect a distortion.[12]

The salesperson's benefit of distortion comes from the possibility of the $\{\pi_1 = 1 \ \& \ m_1 = 0\}$ outcome in the first period. In this case, which has probability 1/2 from the high type salesperson's point of view, the expected second period offer is $c_2 = \widetilde{c}_2$, defined in (10). The salesperson will accept it with probability $\widetilde{c}_2$, achieving an average surplus, over the outside option and given acceptance, of $\widetilde{c}_2/2$. Thus the expected second period's surplus contribution of this event ($\pi_1 = 1 \ \& \ m(x_1) = 0$) to the salesperson's expected utility is $\widetilde{c}^2/4$. If the high type salesperson converts this outcome to $m(x_1) = 1$, the surplus is calculated similarly but with c_2 replaced by 1/2, which results in the expected surplus of 1/16. For example, for $b = B = 10$, the benefit is .0525, which can be seen as quite substantial when compared to the expected second period's net profit of .07, derived in Section 4.1. This example illustrates that convincing salespeople not to distort the first period's effort allocation is going to be quite costly to the firm relative to its expected second period's net profit. On the other hand, as we have seen above,

[12] The benefit would be even higher if the firm expects a distortion towards $x > 1$, since in that case the high type salesperson is better identified and, therefore, the second period contract offer following $m(x_1) = 0$ would be even lower, while the offer following $m(x_1) = 1$ would be the same and equal to $c_2 = 1/2$.

not countering the maximal distortion is quite detrimental to the first period profit.

The above analysis was performed under the assumption that the salesperson does not know the second period's outside option value z before allocating effort in the first period. Let us now consider what happens if the salesperson knows z before allocating effort in the first period. As presented above, we only need to consider the high type salesperson's effort allocation decisions, as the low type salesperson can only exert zero effort. In this case, if $z \geq 1/2$, the high type salesperson, in equilibrium, does not value the possible second period offers and thus will, in equilibrium, efficiently allocate his effort in the first period. If $z < 1/2$, the high type salesperson strictly prefers to show that he is of the high type to receive the second period offer with $c_2 = 1/2$. Therefore, he will distort his effort maximally. Thus the high type salesperson distorts his effort with probability of one half, and if he distorts it, he does so maximally. Therefore, the first period profit contribution coming from high-type salespeople decreases to $\frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$. Since all salespeople who can potentially stay distort their effort maximally in the first period, the second period's net profit is the same as in the case where salespeople did not know their outside offer, and the first period profit reduces to $\frac{1}{4} + \frac{3}{4}\frac{b-B}{b+1}$.[13] Thus, the total profit is still reduced relative to the one in the benchmark case. Note that in this case, the benefit of distortion is not the same for all salespeople who distort their effort allocation: those with z close to $1/2$ are almost indifferent between distorting and not distorting their effort allocation, while those with z below c_2 offered in the $\{\pi_1 = 1 \ \& \ m(x_1) = 0\}$ outcome have the highest incentive to distort.

4.4. Measurement Introduced Before First Period's Contract

We now turn to the main case of the contract design when all the parties know at the beginning of the game that the measurement was introduced. Since distorting x upward from the efficient allocation in the first period reduces the first period's expected profits due to the increased x_1,

[13] Note that not everything is the same in the second period: given the salesperson strategy, the firm optimally updates its belief about the outside option of the salespeople based on the profit and performance measurement realization. Effectively, $m(x_1) = 0$ signals that the salesperson expects a high outside option or is of low ability. But if the firm then decides to increase second-period offer in this case to $1/2 \cdot \mathbf{1}_{\pi_2=1}$ or above, the total profit becomes less than if the firm just committed to have the second period wage of $1/2 \cdot \mathbf{1}_{\pi_2=1}$ regardless of π_1, which obviously results in the total profit lower than in the benchmark case.

the principal may try, at least partially, to counteract this first period's distortion in the first period's contract by a combination of (a) paying for $\pi_1 = 1$ when $m(x_1) = 1$, (b) paying for $\pi_1 = 1$ when $m(x_1) = 0$, or (c) paying for $m(x_1) = 0$ when $\pi_1 = -B$. Option (c) is clearly worse than (b) as it allocates more spending toward low type salespeople and provides less incentive to efficiently allocate effort to increase profit. The relative optimality of the first two instruments is less straightforward. Still, for $b > 4$, an increase in the weight on outcome in (a) is counter-productive (increasing x increases the probability of this outcome when $b \geq 4$).[14] Therefore, for large b, the optimal contract will involve a positive pay only on the outcome in (b). This is intuitive as such a contract gives salespeople the incentive to move x down towards the efficient amount both through conditioning on π_1 being high (profit sharing incentivizes an efficient allocation) and through conditioning on $m(x_1)$ being low (an incentive to reduce x from whatever level it would be otherwise set).

Consider now the conditions under which a high type salesperson prefers no distortion to the maximal distortion given the first period incentive $C_1 = c\mathbf{1}_{\pi_1=1,m(x_1)=0}$ and assuming that the firm expects no distortion (the Appendix also shows that partial distortion is never optimal). Note that the maximal distortion redistributes the event $\{\pi_1 = 1 \text{ \& } m(x_1) = 0\}$, which for a high type salesperson with no distortion has probability 1/2, to the event $\{\pi_1 = 1 \text{ \& } m(x_1) = 1\}$ with (total additional) probability $\frac{b-1}{2(1+b)}$ and to the event $\{\pi_1 = -B \text{ \& } m(x_1) = 1\}$ with (total additional) probability $\frac{1}{1+b}$. Each of these cases leads to an increase in the second period's offer from $c_2 = \widetilde{c}_2$ derived in Subsection 4.2 to 1/2, and results in a loss of the first period's incentive c. Note that the benefit of increased pay in the second period needs to be counted net of foregoing the outside option and taking into account the probability of staying with the firm. In other words, the expected benefit of a contract with the expected pay of w is $\int_0^w (w - z)\, dz = w^2/2$. Thus, to prevent a high type salesperson deviation to the full distortion of x, we need

$$c \geq \widetilde{c} \equiv (1/2)^2/2 - \widetilde{c}_2^2/2 \equiv \frac{1}{8} - \frac{\widetilde{c}_2^2}{2}. \tag{12}$$

[14]For a high type salesperson the probability of the outcome $\{\pi_1 = 1 \text{ \& } m(x_1) = 1\}$ is $\frac{x_1}{2}\frac{b+x_1(2-x_1)}{1+b}$, which is increasing in $x_1 \in [0, 2]$ when $b \geq 4$.

Consider now if it is beneficial for the principal to incentivize no distortion in the first period. (As we established above, this needs to be compared to the maximal distortion.) Given no effort distortion in the first period, the low type salesperson will get $\widetilde{c}$ with probability $\frac{b}{1+b}$, and the high type salesperson will get $\widetilde{c}$ with probability 1/2. The expected cost for the principal of incentivizing no distortion in the first period is therefore $\frac{1+3b}{4(1+b)}\widetilde{c}$. The expected gross benefit of incentivizing no distortion (relative to full distortion) in the first period is $\frac{1}{2}\frac{B+1}{b+1}$.

The salesperson's effort allocation distortion in the first period would help the profit in the second period due to allowing a superior second period's contract (due to the salesperson's ability being fully revealed). Subtracting the second-period net profit given no first-period distortion (derived in Section 4.2) from the second-period net profit under maximal first-period distortion (derived in Section 4.3), we obtain that the second-period value of the distortion to the firm is

$$\frac{1}{8}\frac{b^3}{2b^3+(1+b)^3}+(b-B)\frac{b-2(b+1)^2b^2+(1+b)(b-B)b}{8(b+1)^2(2b^3+(1+b)^3)}. \tag{13}$$

We can then obtain that the benefit of countering the maximal distortion is higher than the total cost of doing so. Given that the principal counters the distortion, the first period's profit (net of the cost of preventing the distortion) becomes $\frac{1}{2}\left(1+\frac{b-B}{b+1}\right)-\frac{1+3b}{4(1+b)}\widetilde{c}$, the second-period profit is as in Equation (14), and therefore the total net profit is smaller that it would be in the benchmark case. For example, for $b = B \to \infty$, the net profit converges to 1/2, which is smaller than the net profit when the measurement was not possible (which tends to 9/16), but greater than the net profit would be if the firm did not counter the first period distortion ($0+1/8 = 1/8$). Summarizing the results of this subsection, we obtain the following proposition (in the above, we needed $b > 4$ to rule out strategy (a), see Appendix for the proof that the statements hold also for other values of b):

PROPOSITION 1: *If salespeople do not know their second period outside option z before allocating their first period effort. Then, if the measurement of one of the effort components is implemented at the start of the game, we have:*

1. *The firm chooses to incentivize salespeople not to distort their first period effort allocation.*

This results in a loss in the first period's profit but with no change in efficiency.

2. *The firm uses the first period's measurement for a better retention of high type salespeople in the second period. This is (on average) beneficial to the high type salespeople and detrimental to the low type salespeople, and results in increased second period's profits.*

3. *The net result of the two effects above is that the total net profit decreases, while the salespeople, on average, are better off.*

4. *Social welfare increases.*

The proposition states that there is no loss of efficiency in the first period and that the total social welfare increases. Let us now discuss the generality of these two results when productive salespeople are heterogeneous in the first period. As we show below, the incentive to distort may be too large and the firm may then choose not to counter it. This can then lead to the possibility of welfare losses due to the lower productivity. In fact, if the productive salespeople are heterogeneous in the first period, there could be no way to design an incentive to not distort effort. To show this within this model, let us get back to the possibility that the salespeople have some information about their second period's outside option before they choose their first period's effort allocation.[15]

In contrast to the previous model analysis, now assume that at the beginning of the first period, the salespeople know z precisely. In this case, if measurement is introduced, salespeople who know that they are going to leave for sure, e.g., those with $z > 1/2$, do not have an incentive to distort x_1 unless the first period contract provides them with an incentive to distort effort.

Therefore, since potentially half of the productive salespeople will leave, when designing the incentive structure to induce no (or less) distortion by the other salespeople, the firm needs to make sure that the incentive is not strong enough to fully distort the effort allocation of the salespeople who will leave. The firm can use a combination of a positive weight on the first

[15] Another possibility is due to different incentives needed for employees with different ability levels. This results in equilibrium distortion when $a_\ell > 0$. However, the analytical expressions are much more complex when $a_\ell > 0$ and numerically, we were unable to find parameter values that lead to lower equilibrium social welfare due to the measurement introduction.

period profit π_1 to incentivize no distortion of those planning to leave, and an additional incentive through a positive weight on $m(x_1) = 0$.

The optimal compensation for π_1 cannot be higher than 2, as that is the maximum possible output over two periods of a high ability salesperson. Therefore, the maximal incentive that it provides tends to zero as b tends to infinity. This implies that if any weight is put on $m(x_1) = 0$ in the first period contract, as b tends to infinity, the salespeople that are expecting to leave the firm for sure (i.e., those with $z > 1/2$) will distort their effort allocation maximally toward the lowest x_1 (i.e., towards $x_1 = 0$). As the firm benefits from identifying the type of the salespeople who are staying with the organization for better retention efficiency in the second period, it prefers those salespeople who will stay to distort their effort allocation as opposed to the salespeople who will leave. Therefore, not to have the salespeople who will leave distort their effort allocation fully, the optimal weight on $m(x_1) = 0$ must tend to zero as b tends to infinity. That is, as b tends to infinity, the first period contract approaches the form $p\mathbf{1}_{\pi_1=1}$ (for some non-negative p).

Again, as the optimal p is bounded from above (as it is by 2), the incentive it provides not to distort the effort allocation tends to zero as b tends to infinity. Therefore, as b tends to infinity, almost all salespeople who will stay distort their effort allocation maximally. (Only those with z close to $1/2$ do not care much about the second period offer and therefore do not distort their effort given a small incentive.) In turn, since p has little effect on the salespeople behavior for large b, the optimal p tends to zero as $b \to \infty$. It then follows that for large b, the first period profit approximately equals $1/4 + 3/4 \cdot (b - B)/(b + 1)$ and the second period profit approximately equals $\frac{1}{8}\left(1 + \frac{(b-B)^2}{(b+1)^2}\right)$, and we have the following proposition (see Appendix for a formal proof):

PROPOSITION 2: *Suppose the high-type salespeople know their second period outside option z before allocating effort in the first period. Then, if measurement is introduced, for sufficiently large b,*

1. *Half of the high-type salespeople maximally distort their first-period x_1 upward and stay in the second period (accept the second-period offer).*

2. *Half of the high-type salespeople do not distort their first-period effort and leave (do not accept the second-period offer).*

3. *The total net profit and the total social welfare are lower than if the measurement were not introduced.*

Note that in a more complete model, which would account for the origins of the outside option and the benefit to other firms of salespeople leaving this firm, one can argue that any information asymmetry between firms and salespeople is welfare reducing, as it results in inefficient allocation of salespeople across firms. (Essentially, compensation packages may be only reallocating surplus from firms to salespeople, but unequal compensations, due to different information that different firms have about the salesperson, means that the salesperson may stay at a less preferable firm.) Such consideration of the social desirability of retention efforts is beyond the scope of this paper.

Proposition 2 shows how some heterogeneity among high type salespeople (namely, their knowledge of different outside option values in the second period) results in the inability of the firm to prevent salesperson effort distortion in the first period by up to one half of the salespeople. The only reason most salespeople did not distort effort is that half of the salespeople were essentially homogenous: the effort allocation of those who know they will be leaving for sure is not affected by the exact value of the outside option. With more heterogeneity in the salesperson beliefs about the probability of leaving, one could obtain that, although in aggregate the firm may incentivize any proportion of x and y efforts, it may be that nearly all salespeople maximally distort their effort. The Appendix provides an example of such a model variation.

Although in all the different set-ups above, we have that measurement, on average, benefits the high type salespeople, one can now see that introducing a first period outside option or competition between organizations, so that the salespeople may share the expected welfare surplus from employment through the first period offer, could lead to the introduction of measurement decreasing the payoffs of the salespeople and the organization alike.

4.5. Equilibrium Measurement Decision

So far, we have considered the effects of measurement without an explicit consideration of the equilibrium decision of whether to introduce it. Table 2 summarizes the results of the previous subsections with respect to the effects of measurement on productivity and profits depending on the expectations and when it is introduced. If measurement can be introduced at a moment's notice (i.e., if we add measurement introduction stages after each decision possibility of the original game), then given the assumed timeline of decisions, the analysis of Section 4.2 implies that the measurement will be introduced. The salesperson will rationally expect it, and the equilibrium outcome is as if measurement is actually introduced at the beginning of the game. According to Proposition 1, this leads to lower profits. In other words, the manager would like to commit not to introduce the measurement. If such a commitment is possible at the beginning of the game, again as is clear from Proposition 1, the equilibrium outcome will be manager's commitment not to measure, and we will have outcomes as in the benchmark case.

In reality, measurement technology may not allow instant introduction. Consider for instance our example of measuring the number (or duration) of salesperson's calls. If the salesperson has an office without all-glass walls and the telephone bill does not itemize calls, introducing the measurement (requiring a call log or open-office environment) could be simultaneous with the salesperson's ability to adjust his effort allocation. In this case, the measurement technology is as if the introduction is only possible before salesperson's choice of effort, and the equilibrium outcome (given the rest of our model) will be that measurement will not be introduced.

For an example from an academic setting, consider evaluating teaching effectiveness at universities through students' evaluations. Since it practically takes months to adopt or change the survey, teachers can adjust their teaching (for example, making classes more entertaining instead of providing knowledge that students cannot immediately verify) concurrently with when the administration can start collecting the evaluations. In this case, in equilibrium we would be in the benchmark case if measurement is not introduced. For another example, consider the management decision whether to provide per-diem reimbursement of travel expenses (a set amount per day, no receipts required) vs. reimburse all reasonably necessary expenses (but require receipts).

The latter can be viewed as measurement of allocation of expenses. Clearly, if the salesperson was not told to keep meal receipts, it will be hard to collect them later.

The above examples illustrate how, depending on the measurement technology, ability to commit may not be necessary for the management to decide not to measure and for the employees to expect no measurement. On the other hand, in some situations, if past records or surveys about the past can be used for measurement, it could be that even though measurement is detrimental for the organization's objectives, it will occur in equilibrium due to the inability of management to commit not to measure.

5. Model Variations

5.1. Precise Measurement

In the main model, we assumed that the x component of effort can only be measured probabilistically. In this section, we show that the main results that an introduction of measurement could decrease profits and social welfare also hold if the measurement is precise, i.e., if after introduction of the measurement, x can be directly observed. Formally, instead of $m(x)$ defined by (3), assume $m(x) = x$. When $a_\ell = 0$, if x can be observed, then the type is revealed as far as high type salespeople choose a positive x. Therefore, measurement introduction may lead to effectively full information about types and no distortion. In this case, the first-period profit is the same as in the benchmark model, and the second-period net profit increases due to the better identification and retention of high-type salespeople. We therefore consider a more general case of $a_\ell \in [0, 1)$ and show that the main results hold for sufficiently high a_ℓ.

If $a_\ell > 1/4$, the efficient choice $x = 1$ of high type salespeople may not identify them as the high type since the low type salespeople can mimic x up to $2\sqrt{a_\ell}$. If no incentive is provided in the first-period contract, in equilibrium, high-type salespeople will choose $x > 2\sqrt{a_\ell}$ to credibly signal to the firm that they are of high type. Thus, with an infinitesimally positive weight on π_1 in the first-period contract, the unique equilibrium salespeople strategy is for the low type to choose efficient $x = \sqrt{a_\ell}$ and for the high type to choose $x = 2\sqrt{a_\ell}$. Clearly, for a_ℓ only slightly above 1/4, this results in a higher total net profit since the detriment of a slight effort distortion by the high-type salespeople in the first period is more than offset by a higher profit in the second period. However, for a_ℓ close to $a_h = 1$, the trade-off is reversed: the benefit of identifying a salesperson's type tends to 0 as a_ℓ tends to 1, but the effort distortion and therefore its detriment to the firm increase. The firm is then interested in preventing the effort distortion of the high types.

The two possibilities of reducing effort distortion of the high type salespeople are (1) to provide a sufficient incentive for the high type salespeople not to separate from the low type ones, i.e., to offer compensation for a choice of $x < 2\sqrt{a_\ell}$ (perhaps, contingent on the value of π_1) sufficient for the high type not to be willing to separate by choosing x above $2\sqrt{a_\ell}$, and (2) offer an incentive for lower x designed to make the low-type salespeople unwilling to mimic the high type's choice. The first possibility would result in a pooling equilibrium. It turns out (see Appendix) that it is always suboptimal (i.e., results in total net profits lower than either without measurement or the second strategy listed above). The second possibility would result in the separating equilibrium since a high-type salesperson values more being recognized as such (an increase in the second-period contract's weight on $\pi_2 = 1$ is valued more by the high-type salesperson since he is more likely to achieve $\pi_2 = 1$). The firm would benefit in the second period, but at the cost of the incentive paid to the low-type salespeople in the first period. It turns out that the net effect is always negative (see Appendix for details).

Let us now consider the possibility that the salespeople know their second-period outside option z before deciding on their first-period profit allocation. Similarly to the corresponding consideration in the main model, if no (or negligible) incentive is given in the first-period contract, half of the high-type salespeople (those with $z < 1/2$) would then distort their effort allocation to separate from the low type. Alternatively, if the incentive is given for the low type to not try to mimic the high type, then half of the high-type salespeople (the ones with $z > 1/2$) will take on the offer and distort their x_1 down. Therefore, the increased social welfare in the second period will come at a cost of the decreased one in the first period. Unlike in the main model, neither upward nor downward distortions are maximal. The upward distortion in the absence of a non-negligible incentive in the first-period contract is to choose $x_1 = 2\sqrt{a_\ell}$, whereas the downward distortion is under control of the firm. Which will be preferable to the firm depends on the relative magnitude of these distortions. The Appendix presents a formal analysis and establishes the following proposition:

PROPOSITION 3: *Introduction of a precise measurement of one of the components of effort decreases profits when the difference in salespeople abilities is not very high. Furthermore, introduction of the precise performance measurement may decrease the total welfare.*

5.2. Measurement of Effort in Both Periods

In this section we investigate what happens when the measurement of component x can be done in both periods. In other words, we consider the analysis of the previous section with the added possibility of $m(x_2)$ being also available. As we will see, this brings additional complexity into the analysis without affecting the main messages of the previous section. For analytical tractability, we perform this analysis focusing on large b.

5.2.1. Unexpected Measurement Introduced After First Period's Effort Allocation

Consider first the case in which there is an unexpected introduction of the measurement after the first period's effort allocation. Just as in the analysis of Section 4.2, in the second period, uncertainty about the salesperson's type remains only if $m(x_1) = 0$.

In this case, if $m(x_2)$ is also observable and absent concerns about the second period effort allocation, the principal may want to put a positive weight on the second period measurement for a better retention of the high type salespeople. This is so when $m(x_2)$ is informative of the salesperson's ability on top of the information derived from the first period observables (π_1 and $m(x_1)$) and the second period profit realization π_2. If effort is efficiently allocated in the first period, this would be when $\pi_1 = 1$ and $m(x_1) = 0$ (otherwise, first period observations reveal the type).Thus, one can expect a strictly positive weight on $\mathbf{1}_{m(x_2)=1}$ in the second period contract when (and only when) $m(x_1) = 0$.[16] The only reason that this weight may not be positive is if it could lead to a second period's effort allocation distortion and the manager finds this suboptimal. The outcome of this trade-off depends on the probability of the salesperson being a high type: if this probability is lower, the benefit of identifying the high type (and not spending on the low type) is higher.

Similarly to the proof of Proposition 2, for any positive incentive on $m(x_2)$, as $b \to \infty$, a high type salesperson will distort the effort allocation in the second period maximally. Since maximal distortion leads to zero productivity, for large b the weight on $m(x_2)$ must be small and, therefore, most of the pay comes from the weight on π_2 (clearly, a weight on π_2 is better than a constant component, so the constant will not be used). Thus, asymptotically, the second period outcomes are as if the measurement did not exist in the second period. That is, the results of Section 4.2 apply here asymptotically for large b.

For an illustration, consider the objective function of a high type salesperson who has a contract $c_2 \mathbf{1}_{\pi_2=1}(1 + w_2 m(x_2))$ for some $c_2 > 0$ and $w_2 > 0$. If the salesperson chooses effort allocation x_2, his expected compensation is:

$$\widehat{w}_{2e} = \frac{(b + x_2(2 - x_2))(1 + w_2 \frac{x_2}{2})c_2}{1 + b}. \tag{14}$$

The first order condition with respect to x_2 implies that the optimal x_2 is

$$x_2 = \min \left\{ 2, \quad \frac{2w_2 - 2 + \sqrt{(2w_2 + 1)^2 + 3(1 + w_2^2 b)}}{3w_2} \right\}. \tag{15}$$

Note that, for larger and larger b, if w_2 is small, the optimal x_2 approaches $1 + \frac{1+b}{4} w_2$. Using the decision rule given by (15), we can now write the first-order conditions with respect to c_2 and w_2 on the second period profit in the case $\{\pi_1 = 1 \ \& \ m(x_2) = 0\}$ using the probability of the high type given in (9). The analysis is presented in the Appendix. For $b = 10$ we can obtain $c_2 = .20$

[16] Note that this could look as a special treatment for apparent (i.e., as measured) substandard salespeople to "encourage improvement." This is not exactly the case because achieving $m_1(x_1) = 1$ leads to not lower expected compensation in the second period for retention reasons.

and $w_2 = .02$, resulting in $x_2 = 1.05$.

5.2.2. *Unexpected Measurement Introduced After First Period Contract but Before First Period Effort Allocation*

In this case, exactly as in the case of Section 4.3, all salespeople maximally distort their effort allocation in the first period. The first period's measurement $m(x_1)$ is then fully informative about the salesperson's type and, thus, exactly the same outcomes follow. Note that in the first period, salespeople may expect the second period offer following $m(x_1) = 0$ to have some positive weight on $m(x_2)$, but since any significant weight on $m(x_2)$ would result in the maximal distortion (for sufficiently large b) and therefore, zero expected profits, the second period offer that the salespeople may expect following $m(x_1) = 0$ is strictly worse than the second period offer that they expect following $m(x_1) = 1$, which is $1/2 \cdot \mathbf{1}_{\pi_2=1}$. Therefore, the maximal effort distortion in the first period guaranteeing $m(x_1) = 1$ is strictly optimal for the salespeople. Again, note that the first period profit loss due to this distortion in the first period is 1/2.

5.2.3. *Measurement Introduced Before the First Period Contract*

In this case, it follows from the results in the previous section that the firm will want to prevent maximal distortion. As the second period offer given no distortion is asymptotically the same as if the measurement in the second period did not exist, the results of Section 4.4 hold here asymptotically. We thus confirm all the results of Propositions 1 and 2, except that the firm uses the second period measurement for retention in a small amount.

6. Discussion

Oftentimes, an organization is concerned with multiple objectives when creating an incentive structure for salespeople, or employees more generally. Broadly speaking, one can divide them into the objectives linked to inducing the desired performance (effort level and effort allocation across various productive tasks), and those objectives linked to hiring and/or retaining the most productive salespeople.

In a multi-period employment market where relatively short-lived contracts are renewed periodically (e.g., each year), the effort, effort allocation inducement, and salespeople retention problems are inter-temporarily linked. Performance measurement in one period is used for current compensation according to the current incentive contract, but it also affects the future contract(s) offered to the salesperson due to the management's (future) inference of the salesperson's ability from the past (at the time of the inference) performance measurements. This in turn means that when deciding on the effort and its allocation, a salesperson needs to take into account not only the current contract's incentives, but the expected effect of his effort and its allocation on the value he expects from the future contracts. In other words, the full incentives will always have an implicit component coming from the value of the future contracts.

Furthermore, management decides not only how to use the available information (performance measurements) about the salespeople, but it also decides whether and how to implement performance measurement as well (i.e., which kind of information about salespeople to obtain). The number of measures, or the amount of monitoring, is a frequently considered decision (Likierman 2009). The obvious trade-off is between the costs of measurement (e.g., the salespeople's compliance costs) and the assumed benefit of the more extensive measurement which could lead, supposedly, to a better-designed incentive structure. It is well known that there could be a tendency to put too much weight on visible measures. One may hope that a rational manager would be able to optimally weigh each metric, so that the above problem is resolved, and more measurement would end up being beneficial to management. In other words, if management is fully rational, it seems intuitive that it should be able to design a better contract when more measurements are available. After all, it is within the power of management to not reward any of the metrics (see, e.g., the flat compensation result in Holmstrom and Milgrom, 1991).

However, the implicit nature of incentives discussed above implies that management is unable to commit to appropriately weigh metrics over the long haul. The contract renewal introduces a dynamic inconsistency problem of the current objective of incentivizing effort and its allocation and the future objective of retaining salespeople whose record suggests better abilities.[17] Since

[17]Even if long-term contracts are possible, unless management can commit not to give out bonuses (which are

salespeople are forward-looking and can expect this inconsistency, introduction of a new metric results not only in an enlargement of the set of the available instruments for the management, but also in a change of the salespeople's environment. While the larger set of instruments could help to achieve the management's objectives and increase social welfare, the dynamic inconsistency could be detrimental. The net effect is not immediately clear, and as we have shown, it could be negative both on the profits and on overall productivity and social welfare. In the assistant professor example we used previously, the assistant professor allocates more effort to the number of publications and teaching ratings than what could potentially be optimal for the objective of making and disseminating advancements in the field.

One may find it curious to reflect on the Finnish education achievement puzzle from the point of view offered by the above model. As demonstrated by eventual student abilities, Finland was able to improve performance by largely eliminating both the teacher and student performance measurements (see Darling-Hammond 2010).

A tenure system may also be considered as a commitment to limit the use of measurements. For example, a standard justification for life-time appointments of justices is that otherwise they could be swayed by unscrupulous decision makers. Our research puts it in a different light: even if the supervisors are fully benevolent (they only have the objective of maximizing public welfare) and are fully rational, they may not be able to not interfere and not distort the socially efficient decisions of the salespeople.

The considerations illustrated by the model easily apply to not-for-profit organizations. In such organizations, the productivity/output (a) in fact may not be observable at all in the absence of measurement, yet the organization cares about it by definition. Note that any measurement of the efforts or productivity components might have the property of not being completely unbiased between the different components of output, i.e., almost always under or over-weigh one type of the input efforts. Furthermore, both the employees and the management of such organizations may have pride in their work, which leads to the employees preferring to optimize a absent other incentives (in the model, due to a small weight on the total output of the firm). The

not fully spelled out in the contract), the implicit nature of incentives remains present.

management, while by definition may be interested in the "good deeds" a brought to the society, also prides itself in the amount of work done by its particular organization, and therefore values the retention of the productive (i.e., high-a) employees.

Note further that retention by itself is not necessarily socially desirable. In the model we formulated, it is efficient (socially desirable) to retain a salesperson if and only if the salesperson's outside option z is less then the productive output a he can generate. Therefore, it is efficient to offer a better contract (higher base pay) to those salespeople who are expected to be more productive. As we have shown, the firm may be worse off, although the salespeople and the social welfare are better off in our model when the principal is willing and able to incentivize salespeople not to distort their effort allocation in the first period (although social welfare is lower in the model variation where salespeople observe their second period's outside offer before the first period). However, another interpretation is that z comes from the idiosyncratic salesperson preferences for outside attributes such as job location plus a job offer from an organization with exactly the same production function. In this case, given equal pay (contracts), salespeople would produce the same a's but realize the best possible location choice, i.e., the location-inconvenience cost would be minimized. Salesperson retention then makes utility generated by locations inefficiently distributed while only shifting a's between organizations. Then, the management's work (introducing measurements and fine-tuning contracts) could make both the organization and, on average, the salespeople worse off (although some salespeople may be better off). The model can then be the most strikingly characterized as exploring the tug of war between the salespeople's pride in their work being a force toward the efficient society and the managerial pride in their work being a force against efficiency and toward a distortion of both the salespeople's effort allocation and the salesperson locations.

Of course, in many other situations, the problem of effort inducement and salesperson retention is the main problem an organization faces, and the problem of optimal effort allocation between unobserved (not well-measured) components is not as essential. In those cases, performance measurements could benefit the organization and an average salesperson. The point of this paper is not that performance measurement is always or even usually counter-productive, but that it could be, and hence, one should consider the implications of measurement before its introduction. In other words, one should have an idea of how to use the data before collecting it.

This paper assumes that one effort dimension can be measured costlessly while the other effort dimension cannot be measured, which can also be interpreted as being infinitely costly to measure, or that it is too noisy to provide any useful information. More broadly one could think of the problem of the firm of which effort dimensions to try to measure given their different measurement costs, and different informativeness of the noisy measure. In this setting one may expect that the firm may prefer to measure effort dimensions which are measured more precisely or are measured at a lower cost. The investigation of this general problem is left for future research.

Appendix

Second-Period Contract and Profit in the Benchmark Case:

As the outside option in the second period is positive and uniformly distributed on $[0,1]$, the salesperson has a positive chance of leaving and a positive chance of staying given any reasonable offer (since profit is either one or negative, the optimal contract cannot provide expected utility to the salesperson that exceeds the outside option with probability one). Consider now the second period's contract $C_2 = c_1 + c_2\mathbf{1}_{\pi_2=1}$, where $c_1 \geq 0$ and $c_2 \geq 0$ are the firm's decision variables and are functions of the observables after the first period (i.e., functions of π_1). Thus in this benchmark case c_1 and c_2 may be functions of the first period's profit realization and are non-negative to ensure a non-negative compensation in any possible outcome.[18]

Given this contract, the expected pay of the salesperson with ability a_i is $c_1+c_2(b+a_i)/(1+b)$ if the salesperson chooses the efficient effort allocation, which is assured by $c_2 > 0$. Since the outside option is uniformly distributed on $[0,1]$, this expected pay is also the probability that the salesperson stays. The expected gross second period profit from the type-i salesperson who is staying and allocating his effort optimally for the firm is $1-(B+1)(1-a_i)/(b+1)$, which leads to the expected net second period profit of $1-c_1-c_2-(B+1-c_2)(1-a_i)/(b+1)$. Letting the probability that the salesperson is of high type be P_h, one can then compute that the expected net second-period profit as a function of P_h is

$$\mathrm{E}(\pi_2 - C_2) = (1-c_1-c_2)(c_1+c_2)P_h - \left(c_1 + \frac{B-b+bc_2}{1+b}\right)\left(c_1 + c_2\frac{b}{1+b}\right)(1-P_h). \quad (16)$$

Maximizing the above expected profit with respect to c_1 and c_2 under the constraint that $c_1 \geq 0$, we find that the constraint $c_1 \geq 0$ is binding (FOC without the constraint leads to negative c_1), and given $c_1 = 0$, the optimal c_2 is as in Equation (4).

Sub-Optimality of Partial First Period Distortion:

[18] Although $c_2 < 0$ is technically allowed as far as $c_1 + c_2 \geq 0$, it is straightforward to check that it cannot be a part of an optimal contract – it would give an incentive to inefficiently allocate effort without the benefit of higher retention of the high type salesperson.

Consider the high type agent's first period effort allocation decision x_1 given the first period contract $c\mathbf{1}_{\pi_1=1\&m(x_1)=0}$ and the expected second period offer $\frac{1}{2}\mathbf{1}_{m(x_1)=1} + \widetilde{c}_2\mathbf{1}_{\pi_1=1\&m(x_1)=0} + \frac{b-B}{2b}\mathbf{1}_{\pi_1=-B\&m(x_1)=0}$. The salesperson's value of increasing x above $x = 1$ is coming from the increased probability of $m(x_1) = 1$ and the associated second period equilibrium pay increase from $\widetilde{c}_2$ to 1/2:

$$\text{Benefit} = \left(\frac{x}{2} - \frac{1}{2}\right)\left(\frac{(1/2)^2}{2} - \frac{\widetilde{c}_2^2}{2}\right). \tag{17}$$

This benefit comes at a cost of lower expected first period pay

$$\text{Cost}_{\text{first period}} = \left(\frac{1}{2} - \frac{b+\alpha}{1+b}\left(1 - \frac{x}{2}\right)\right)c, \quad \text{where} \quad \alpha = x(2-x), \tag{18}$$

and of the possibility that the compensation is reduced from $\widetilde{c}_2$ to $\frac{b-B}{2b}$ (event $\{\pi_1 = -B \,\&\, m(x_1) = 0\}$ becomes possible), which decreases salesperson's expected utility by

$$\text{Cost}_{\text{second period}} = \frac{1-\alpha}{1+b}\left(1 - \frac{x}{2}\right)\left(\frac{\widetilde{c}^2}{2} - \frac{(b-B)^2}{8b^2}\right). \tag{19}$$

Subtracting the two costs from the benefit, we obtain the salesperson's objective function as

$$f(x,c) = \frac{1}{16}\frac{(x-1)(1+b+4(x^2-3x-b+1)(\widetilde{c}_2^2+2c)) + (2-x)(x-1)^2(1-B/b)^2}{1+b}. \tag{20}$$

By definition, this $f(x,c) = 0$ at $x = 1$. At $c = \widetilde{c} \equiv \frac{1}{8} - \frac{\widetilde{c}_2^2}{2}$, it simplifies to

$$f(x,\widetilde{c}) = -\frac{(x-1)^2(2-x)(1-B/b)B/b}{16(1+b)}, \tag{21}$$

which clearly achieves the maximum (of 0) at $x = 1$ and $x = 2$. Thus, the incentive that is just enough to prevent the salesperson from preferring no distortion to maximal distortion, makes the salesperson strictly prefer no distortion to any intermediate distortion. Furthermore,

$$\frac{\partial f(x,c)}{\partial c} < 0 \quad \text{and} \quad \frac{\partial^2 f(x,c)}{\partial c\,\partial x} < 0 \quad \text{for} \quad x \in (1,2]. \tag{22}$$

In other words, $f(x,c)$ decreases in c and the speed (by absolute value) of this decrease increases in x. This implies that if for some $x_1 < x_2$, we have $f(x_1, c_1) \leq f(x_2, c_1)$, then for any $c < c_1$, we have $f(x_1, c) < f(x_2, c)$. Applying this to $c_1 = \widetilde{c}$ and $x_2 = 2$, we obtain that for any smaller incentive than $\widetilde{c}$ (i.e., for $c < \widetilde{c}$), we have that the effort allocation $x = 2$ is preferable to any effort allocation $x \in [1, 2)$. Conversely, any incentive larger than $\widetilde{c}$ results in $x < 1$.

Proof of Proposition 1:

We have already established that it is always optimal for the firm to incentivize no first-period effort allocation distortion through a positive weight on $\pi_1 = 1$ & $m(x_1) = 0$ instead of allowing effort distortion. This implies that it will be optimal to do so if other contract structures are allowed (remind that we have not considered allowing a contract with a positive weight on $\pi_1 = 1$ & $m(x_1) = 1$ when $b \leq 4$). This immediately implies that social welfare increases if measurement is introduced.

Let us now compare incentivizing less upward distortion of x_1 with a positive weight on $\pi_1 = 1$ & $m(x_1) = 0$ versus a positive weight on $\pi_1 = 1$ & $m(x_1) = 1$. The first strategy (per unit of the weight) provides a higher incentive to reduce x_1 from any value above the optimal $x_1 = 1$, but the second strategy is cheaper for the firm as it results in no payments to the low-type salespeople (since they can never achieve $m(x) = 1$). Therefore, although the second strategy is potentially optimal (only for low b, since it is counterproductive for $b > 4$), allocating all the weight towards the first strategy would be optimal if the firm was able to avoid payment to the low-type salespeople.

Let us then establish an upper bound on the firm's profits with measurement by using this lower bound on the cost of the first strategy above, i.e., assume that the total expenditure on the wages coming from the weight c on $\pi_1 = 1$ & $m(x_1) = 0$ in the first period will be not $\frac{1+3b}{4(1+b)}c$ but $\frac{c}{4}$. In this case, as we have argued above, it will be optimal not to use any weight on $\pi_1 = 1$ & $m(x_1) = 1$ in the first-period contract. The firm will still use the minimal c required to incentivize no distortion, i.e., $c = \widetilde{c}$. The first period profit with measurement will be $\frac{\widetilde{c}}{4}$ lower than without measurement (due to the cost of wages), and the second period profit will be higher than the one in the benchmark case by $b^4 B^2 / [8(b^2 + b + 1)(2b + 1)(b^3 + (b+1)^3)]$, which is smaller than $\frac{\tilde{c}}{4}$ for all $b > B$. Therefore, measurement always decreases profits and the proposition is proven.

Proof of Proposition 2:

If measurement is introduced and the firm provides minimal incentives not to distort effort allocation in the first period, all high-type salespeople with $z < 1/2$ will distort their x_1 maximally upward to guarantee themselves the second period offer $1/2 \cdot \mathbf{1}_{\pi_2=1}$ and the rest will choose an efficient effort allocation and leave. This salespeople strategy will result in the firm having full information about the type of salespeople who might stay but at cost of inefficiently low profit in the first period.

Let us estimate how much the firm may be willing to spend on preventing the high-type first-period effort distortion, and consequently, how many salespeople may be, in equilibrium, prevented from the maximal distortion of their effort allocation in the first period. For this purpose, we will ignore the second-period benefit of the first-period effort allocation distortion (without the distortion, some high-type employees end up with $\pi_1 = 1$ and $m(x_1) = 0$ and are pooled with the low-type salespeople), thereby deriving an upper bound on the firm's willingness to prevent effort allocation distortion and therefore, a lower bound on the mass of high-type employees with $z < 1/2$ who distort their x_1 maximally upward.

To discourage upward distortion of x_1, the firm may either put a positive weight on $\pi_1 = 1$ and additionally, possibly a negative weight on $m(x) = 1$. We therefore consider the first-period contract of the form $p\mathbf{1}_{\pi_1=1}(1 - w\,m(x_1))$. Given this contract, the expected first-period pay of a high-type salesperson for $x_1 = x$ is $(b + (2 - x)x)(1 - w\,x/2)p/(b + 1)$. Checking when the derivative of this with respect to x is negative on $x \in [0, 2]$ when $b > 4$, we observe that the optimal choice of a high-type salesperson who is sure to leave is $x = 0$ if $w \geq 4/b$.

Facing the maximal effort distortion of the high type salespeople with $z > 1/2$, the firm would prefer to instead have salespeople with $z > 1/2$ not distort their effort allocation from the optimal one ($x_1 = 1$) at all, while having the rest of the salespeople distort the effort allocation maximally upward (i.e., choose $x_1 = 2$). Since the latter outcome is achieved with the (near) zero-pay contract, a contract with $w \geq 4/b$ cannot be optimal. Thus the optimal contract must have $w < 4/b$. Furthermore, since the the benefit to the firm of a high-type salesperson's not distorting effort in the first period is at most $1-\frac{b-B}{b+1} = \frac{B+1}{b+1}$, and only half of high-type salespeople will distort with (near) zero-pay contract (which is a quarter of all salespeople), the firm will not be willing to pay more than $\frac{1}{4}\frac{B+1}{b+1}$ in the total first-period's wages to all salespeople. On the other hand, the above contract will result in the total expected expenditure on the first-period wages higher than $\frac{1}{2}\frac{b}{b+1}p + \frac{1}{2}\frac{b}{b+1}\left(1-\frac{4}{b}\right)p = \frac{b-2}{b+1}p$. Therefore, in the optimal contract, $p < \frac{B+1}{4(b-2)}$.

Now, fix arbitrary $x_1^* \in [0,2)$, and consider a high-type salesperson with the second period outside option $z^* \leq 1/2$. If $m(x_1) = 1$, the second period contract provides this salesperson the expected second period pay of $1/2$. On the other hand, in either $\{m(x_1) = 0 \ \& \ \pi_1 = -B\}$ or $\{m(x_1) = 0 \ \& \ \pi_1 = 1\}$ event, the firm has to believe that the probability that the salesperson's type is high is at most $1/2/(1/2 + b/2/(b+1)) = (b+1)/(2b+1)$. Substituting this upper bound on the probability of high type into Equation (7), we obtain that the salesperson's second-period wage increases by at least $b^2B/[2(2b+1)(b^2+b+1)]$ if $m(x_1)$ increases from 0 to 1. Therefore, increasing x_1 from x_1^* to 2 increases the expected pay (conditional on the salesperson accepting the offer) in the second period by at least $b^2B/[2(2b+1)(b^2+b+1)]$ with probability $(1-x_1^*/2)$ and never decreases it (once $x_1 = 2$, the expected pay is for sure $1/2$). The salesperson values this expected increase in the second period offer by at least

$$\left(1-\frac{x_1^*}{2}\right)\min\left\{\frac{1}{2}-z^*, \frac{b^2B}{2(2b+1)(b^2+b+1)}\right\}. \tag{23}$$

Let us now consider an upper bound on the salesperson's first-period cost of such adjustment of x_1 due to its effect on the first-period pay for $b > 4$. Given the upper bounds established above on p and w, when the salesperson chooses $x_1 = 2$ instead of $x_1 = x_1^*$, his expected first-period income decreases by

$$\frac{b+x(2-x)}{b+1}p\left(1-\frac{wx}{2}\right) - \frac{b(1-w)}{b+1} = (2-x)\frac{w\,b+2x-w\,x^2}{2(b+1)}p \leq$$
$$(2-x_1^*)\frac{4(b-2)}{(b+1)b}p \leq (2-x_1^*)\frac{B+1}{(b+1)b}, \quad \text{for } b \geq 8, \tag{24}$$

where the first inequality follows from maximizing $w\,b + 2x - w\,x^2$ over $x \in [0,2]$ and w for $b \geq 8$ and $w \leq 4/b$. Let b^* be such that

$$\frac{b^2 B}{4(2b+1)(b^2+b+1)} > \frac{B+1}{(b+1)b}, \quad \text{for } b > b^*. \tag{25}$$

Such b^* exists since the left hand side is of the order B/b and the right hand side is of the order B/b^2 as $b \to \infty$. Comparing the benefit and the cost of full distortion to the high-types salesperson with $z < 1/2$, we then obtain that for $b > b^*$, the benefit outweighs the cost if $1/2 - z > \frac{8(b-2)p}{(b+1)b}$. Thus, high-type salespeople with $z \in [0, 1/2 - \frac{8(b-2)p}{(b+1)b})$ will be maximally distorting their x_1 upward.

Let us now revisit the upper bound on the optimal p established at the beginning of the proof of this proposition. We have used estimate of the potential benefit of $p > 0$ coming from preventing the distortion of high-type salespeople with $z < 1/2$. However, for $b > b^*$, as we have shown above, the maximal distortion may only be possibly prevented of salespeople with $z \in [1/2 - \frac{8(b-2)p}{(b+1)b}, 1/2]$. The ratio of the mass of these high-type employees to all employees is $\frac{4(b-2)p}{(b+1)b}$. We thus have that for $b > b^*$, the upper bound of the benefit (to the firm, relative to the minimal incentives in the first period) is $\frac{4(b-2)p}{(b+1)b}\frac{B+1}{b+1}$ instead of $\frac{1}{4}\frac{B+1}{b+1}$. Comparing this to the lower bound on the first-period wage costs established above ($\frac{b-2}{b+1}p$), one obtains that for $b(b+1) > 4(B+1)$ (which is always true for $b > 4$), the lower bound on the cost outweighs the potential benefit. Therefore, minimal incentive in the first period is optimal for $b > b^*$ defined above. As we have already noted, minimal incentives result in all high-type employees with $z \geq 1/2$ allocating effort efficiently, and all high-type employees with $z < 1/2$ maximally distorting their x_1 upward. This establishes the first two parts of the proposition.

We now turn to the calculation of the effect of measurement on the profits and the social welfare for large b, specifically, for $b > b^*$ defined above. The net profit without measurement is derived in the main text (see Equation (7) and the sentence that precedes it). If $b > b^*$, the total expected net profit with measurement equals

$$\frac{1}{2}+\frac{1}{2}\cdot\frac{b-B}{b+1}+\frac{1}{2}\frac{1}{2}=\frac{3}{4}+\frac{b-B}{2(b+1)}, \tag{26}$$

in case of a high-type salesperson, and equals

$$\frac{b-B}{b+1}+\left(\frac{b-B}{b+1}-\text{pay}\right)\cdot\text{pay}, \tag{27}$$

in case of a low-type salesperson, where "pay" is the expected payment in the second period, which is $\frac{b}{b+1}\frac{b-B}{2b}=\frac{b-B}{2(b+1)}$. Averaging the above two expressions, we obtain the profit with measurement. Subtracting it from the profit without measurement, we obtain that measurement reduces profit by

$$\Delta\pi=\frac{4b^3B-B^2b^2+4b^3-B^2b+6b^2+6b^2B+6b+6Bb+2B+2}{8(b+1)(1+2b)(1+b+b^2)}+o(1/b)$$
$$=\frac{B+1}{4(b+1)}-\frac{bB^2}{8(2b+1)(b^2+b+1)+o(1/b),} \tag{28}$$

which is positive for large b (as far as $b>B$).

The social welfare without measurement can be calculated as the average welfare of the low-type and the high-type salesperson scenarios. If the salesperson is of low type, the welfare is

$$WL_{NoM}=\frac{b-B}{b+1}+\frac{b}{b+1}\left[\frac{b-B}{b+1}\cdot c_{21L}+\frac{1-c_{21L}^2}{2}\right]+\frac{1}{b+1}\left[\frac{b-B}{b+1}\cdot c_{20L}+\frac{1-c_{20L}^2}{2}\right], \tag{29}$$

where c_{21L} is the expected pay of the low-type salesperson in the second period if $\pi_1=1$, and c_{20L} is his expected second-period pay if $\pi_1=-B$. We have: $c_{2iL}=\frac{b}{b+1}c_{2i}$, where c_{2i} is the equilibrium wage conditional on the first period profit given by Equation (6). The above equation represents the expected first-period profit plus the probability of $\pi_1=1$ times (opening bracket) the expected second-period profit gross of wages times the probability of acceptance (which is the event $z<c_{21L}$) plus the surplus coming from the outside option in case of rejection (closing bracket), plus the similar term representing the event of $\pi_1=0$. If the salesperson is of

the high type, the social welfare is

$$WH_{NoM} = 1 + 1 \cdot \left[c_{21} + \frac{1 - c_{21}^2}{2}\right] + 0 = \frac{(1 + c_{21})(3 - c_{21})}{2}. \tag{30}$$

This is derived in the same way as the previous equation, but it is much simpler because $\text{Prob}(\pi_1 = 1 \mid \text{high type}) = 1$, and the expected second-period pay of the high type given wage $c \cdot 1_{\pi_2=1}$ is c.

If the measurement is introduced (and $b > b^*$), the salespeople with $z < 1/2$ fully distort their effort allocation in the first period and those with $z > 1/2$ allocate effort efficiently and leave in the second period. Furthermore, the high type salespeople with $z < 1/2$ are then recognized as the high types, receive second-period wage offer of $1/2$ and stay. Therefore, if the salesperson is of high type, the expected welfare is

$$WH_m = 1 \cdot \frac{1}{2} + \frac{1}{2} \cdot \frac{b - B}{b + 1} + \frac{1 - (1/2)^2}{2} + \frac{1}{2} = \frac{11}{8} + \frac{1}{2}\frac{b - B}{b + 1}, \tag{31}$$

where the first two terms represent the expected profit in the first period (coming from those with $z > 1/2$ and $z < 1/2$, respectively) and the last two terms represent the expected social welfare in the second period (coming from those with $z > 1/2$ and $z < 1/2$, respectively). The low-type salespeople are identified by the measurement precisely and, therefore, receive the second-period wage offer of $\frac{b-B}{2b} 1_{\pi_2=1}$, which they value at $c_{2L} = \frac{b-B}{2(b+1)}$. Therefore, the social welfare conditional on low type is

$$WL_m = \frac{b - B}{b + 1} + \left[\frac{b - B}{b + 1} c_{2L} + \frac{1 - c_{2L}^2}{2}\right] = \frac{(3b - B + 2)(5b - 3B + 2)}{8(b + 1)^2}. \tag{32}$$

Averaging the two social welfare equations above, we obtain

$$W_m = \frac{3}{16} + \frac{3}{16} \frac{(3b - B + 2)^2}{(b + 1)^2} \tag{33}$$

Subtracting the above from the social welfare in without measurement, we obtain that the welfare without measurement is greater by

$$\Delta W = \frac{8b^3B - 3B^2b^2 + 8b^3 - 3B^2b + 12b^2B + 12b^2 + 12Bb + 12b + 4B + 4}{16(b^2+b+1)(2b+1)(b+1)}$$
$$= \frac{B+1}{4(b+1)} - \frac{3B^2}{16(b^2+b+1)(2b+1)} > 0, \quad \text{for } b > B, \quad (34)$$

which concludes the proof of the proposition.

Case When Salespeople Have Partial Information about Second Period Outside Option:

Assume now the following information structure that the salesperson has at the beginning of the game about his second period outside option. Assume at the beginning of the game that the salesperson has a signal (k, σ) about his second period outside option, where $k \in \{0, \dots, K\}$ represents the reliability of σ and is uniformly distributed on $\{0, \dots, K\}$, while σ is equal to $\mathbf{1}_{z>1/2}$ with probability k/K and is a random draw from $\{0, 1\}$ otherwise. Each of these $2K+2$ possible (and equally likely) signals corresponds to a different value that the high type salesperson receiving such a signal places on the second period equilibrium offer.

Let $c_2^\star$ be the equilibrium second period c_2 when $\pi_1 = 1$ & $m(x_1) = 0$ and the manager expects efficient effort allocation in the first period (by the same argument as in Section 4.2, if the manager expects a distortion, c_2 will be lower). As before, by maximally distorting the x component of effort upwards, the high type salesperson can increase c_2 and, therefore, the value of the second period offer (since it can ensure $\pi_2 = 1$ by allocating effort efficiently in the second period), to $1/2$. Again, as before, the value of this action equals the probability of accepting the contract with $c_2 = 1/2$ times the expected increase in the payoff (which is $1/2 - \max\{z, c_2^\star\}$) conditional on accepting the second period offer. The difference lies in the evaluation of the probability of accepting the offer, which now is a function of the signal (k, σ).

Signal (k, σ) is uninformative with probability $1 - k/K$. In this case, z is uniformly distributed on $(0, 1)$ and therefore, the value of increasing c_2 from $c_2^\star$ to $1/2$ is $v^\star \equiv (1/2 - c_2^\star) \cdot c_2^\star + \int_{c_2^\star}^{1/2} (1/2 - z)\, dz$. With probability k/K, the signal is informative. In this case, if $\sigma = 1$, the salesperson is sure to leave even with $c_2 = 1/2$ and, therefore, has no value of increasing the second period offer. However, if $\sigma = 0$ (and the signal is true), the salesperson is sure to stay and values increasing the offer at $2v^{\star}$. Aggregating the above (taking into account the probability of the event $\{\pi_1 = 1 \ \& \ m(x_1) = 0\}$), we find the values $v_{k,\sigma}$ the high type salesperson with signal (k, σ) places on increasing the second period offer. These values are ordered as:

$$v_{K,1} < v_{K-1,1} < \cdots < v_{0,1} < v_{0,0} < \ldots v_{K,0}. \tag{35}$$

These values represent the incentive the salesperson has to maximally distort his effort allocation toward $x = 2$ given signal (k, σ). Let dv be the smallest distance between any pair of the above values. Again, if the manager believes the effort allocation is distorted, dv will be greater. Thus, $dv > 0$ represents the lower bound on how different incentives are between the high type agents with different signals.

Similar to the argument leading to Proposition 2, one can now prove that if b is large enough, a potentially optimal first period contract (e.g., one where the weight on π_1 is bounded from above by a number independent of b) may only incentivize no–effort distortion by agents with one of the signal possibilities. The rest will be distorting x either maximally upward or maximally downward. Thus, for large K, almost all high type agents distort their first period effort allocation maximally, and we have the following proposition.

Proposition 4: *If salespeople have some information about their second period outside option, it is impossible to profitably prevent maximal effort distortion by nearly all salespeople in the first period.*

Precise Measurement:

Allowing $a_\ell > 0$, which is important to achieve interesting results when $m(x) = x$, adds analytical complexity to the expressions of the benchmark model, but conceptually, the derivations are the same as in the main model. The expected profit given productivity α is $\frac{b+\alpha-(1-\alpha)B}{b+1}$. Therefore, given efficient allocation and negligible wage in the first period, the expected first-period profit is $\frac{2b+1+a_\ell+(1-a_\ell)B}{2(b+1)}$. The second-period contract given the probability P_h that the salesperson is of high-type is still of the form $c_2\mathbf{1}_{\pi_2=1}$, with

$$c_2 = c_2^{opt}(P_h) = \frac{P_h(1+b)^2 + (1-P_h)(b-B+a_\ell+a_\ell B)(b+a_\ell)}{2[(b+a_\ell)^2 + P_h(1-a_\ell)(2b+1+a_h)]}, \tag{36}$$

and the second-period net profit given probability P_h of high type is

$$f(P_h) = \frac{1}{4}\frac{[(b+a_\ell)^2 + (1-a_\ell)[(2b+1+a_\ell)P_h - (b+a_\ell)(1-P_h)B]]^2}{(b+1)^2[(b+a_\ell)^2 + (1-a_\ell)(2b+1+a_l)P_h]}. \tag{37}$$

This still means $c_2 = 1/2$ if $P_h = 1$ but $c_2 = \frac{1}{2} - \frac{(1-a_\ell)B}{2(b+a_\ell)}$ if $P_h = 0$. Likewise, (given efficient effort allocation in the first period) if $\pi_1 = 0$ we still have $P_h = 0$, but if $\pi_1 = 1$, we have $P_h = \frac{b+1}{2b+1+a_\ell}$. Furthermore, if firm's belief about the type changes from low to high, c_2 increases by $\frac{(1-a_\ell)B}{2(b+a_\ell)}$. The expected net profit in the second period with no measurement is

$$\pi_2^{nm} = \frac{1}{8}\frac{[(b+a_\ell)^2(b-B+a_\ell+a_\ell B) + (1+b)^3]^2}{(b+1)^3(2b+1+a_\ell)[(b+1)^2 - (b+a_\ell)(1-a_\ell)]} + \frac{1}{8}\frac{(1-a_\ell)(b-B+a_\ell+a_\ell B)^2}{(b+1)^3}. \tag{38}$$

With measurement, if the low and high types choose different x_1 (i.e., if the equilibrium is separating), the firm effectively has complete information in the second period and is able to condition the second-period contract (c_2) on the salesperson's type. In this case, the second-period net profit is

$$\pi_2^{sep} = \frac{1}{8}\frac{(1+a_\ell)^2(b+1)^2 + (1-a_\ell)(b-B)[(b+1)a_\ell + 2a_\ell - (1-a_\ell)B]}{(b+1)^2}, \tag{39}$$

which is an improvement over the second-period net profit in the benchmark case by

$$\Delta_2^{sep} = \frac{1}{8}\frac{(1-a_\ell)^2(b+a_\ell)B^2}{(2b+1+a_\ell)[(b+1)^2 - (b+a_\ell)(1-a_\ell)]}. \tag{40}$$

However, in a separating equilibrium, the low type salespeople will have an incentive to imitate the high type (if able) equal to $\frac{(1-a_\ell)B}{2(b+a_\ell)}\frac{b+a_\ell}{b+1}$ (the expected increase in c_2 times the probability of achieving $\pi_2 = 1$ under efficient allocation). Obviously, if $2\sqrt{a_\ell} < 1$ (i.e., $a_\ell < 1/4$), the low type salespeople are unable to imitate efficient x_1 of the high types, and the firm benefits from the introduction of the measurement. On the other hand, if $2\sqrt{a_\ell} > 1$ (i.e., $a_\ell > 1/4$), which we assume from now on, the firm has two potentially optimal strategies of ensuring type separation in the first period. The **first** strategy is to offer first-period pay c_l for $x = \sqrt{a_\ell}$ (i.e., the efficient allocation of low-type salespeople) just enough to make the low types indifferent between choosing $x_1 = \sqrt{a_\ell}$ (and receiving second-period offer corresponding to $P_h = 0$) and choosing $x_1 = 1$ (and receiving the second-period offer corresponding to $P_h = 1$). One can derive that this offer is

$$c_l = \frac{1}{8}\frac{(1-a_\ell)(2b - B + a_\ell B + 2a_\ell)B(b + 2\sqrt{a_\ell} - 1)}{(b+1)^3}. \tag{41}$$

Note that since the value of being recognized as the high type is higher for the high type (since the probability of achieving $\pi_2 = 1$ is higher for them), the high type will still choose their efficient allocation in the first period. Therefore, following this strategy leaves the firm with the same first-period gross profit, but achieves separation in exchange for the cost $c_l/2$ (division by 2 due to 1/2 probability of the low type). Thus, this strategy achieves the net total profit higher than that of the benchmark case if and only if $c_l < 2\Delta_2^{sep}$.

The **second** potentially optimal strategy is to not offer any significant incentives in the first period, in which case the high type salespeople will choose x unavailable to the low-type salespeople (i.e., choose x just above $2\sqrt{a_\ell}$) to signal their high type. Although this causes the firm losses of some profits in the first period (due to the inefficient effort allocation by the high-type salespeople), the firm saves $c_l/2$ in the first-period wages relative to the first strategy above. The first-period profit loss due to the effort allocation distortion of the high-type salespeople is

$$c_{distsep} = \frac{1}{2}\frac{(1-2\sqrt{a_\ell})^2(B+1)}{b+1}. \tag{42}$$

Again, the benefit relative to the no measurement case is the full information about the salespeople types in the second period. If the first-period effort distortion is small (i.e., $2\sqrt{a_\ell}$ is not much higher than 1), the profit with this strategy is clearly higher than the profit under no measurement.

Finally, the **third** potentially optimal strategy when measurement is introduced is to incentivize the high-type salespeople not to separate in their choice of x_1 from the low-type ones by paying for a certain choice of x_1 enough for the high types not be willing to separate. In this case, $m(x)$ will not be informative of the type, and only the profit realization will be. Since the low-type salespeople achieve $\pi_1 = 1$ with a lower probability then the high type ones (keeping their choice of x_1 fixed and equal), the least costly way to induce the high type ones not to deviate is to make pay conditional on $\pi_1 = 1$ & $m(x_1) = x$. Although relative to the first strategy, more salespeople will get paid in the first period (the low type salespeople also get paid for choosing $x_1 = x$ when $\pi_1 = 1$), the cost per salesperson is lower because under this strategy, the benefit of deviation to a high x_1 is not changing the firm's belief of the high type from 0 to 1 but from a positive number (approximately 1/2 when b is large) to 1.

Let us now consider which of the above three strategies may result in the firm's profit higher than it would have without measurement. If a_ℓ is only slightly above 1/4, so that $2\sqrt{a_\ell}$ is only slightly above 1, the second strategy results in a small cost of distortion and has the benefit of information (which decreases in a_ℓ). On the other hand, the cost (of the first-period wages) in the first and the second strategies increases as a_ℓ decreases to 1/4. Therefore, the second strategy is the best and results in the profit higher than without measurement. On the other hand, when a_ℓ approaches 1, the benefit of full information decreases to zero (see Equation (40)), but the cost of first-period distortion in the second strategy increases. Also, as $a_\ell \to 1$, the direct cost (of first-period wages) of the first strategy tends to zero (since the second-period wage conditional on $P_h = 0$ tends to the second-period wage conditional on $P_h = 1$), and its benefit of information is the same as that of the second strategy. Therefore, for a_ℓ close to 1, the second strategy results in a decrease in profits relative to both the first strategy and no measurement.

The first and the third strategies turn out to never improve profits relative to no measurement when $a_\ell \geq 1/4$ (if $a_\ell < 1/4$, the low type salespeople cannot imitate the high type ones, and therefore the second strategy achieves the first best). To prove that the first strategy never improves profits relative to no measurement, note that

$$c_l > \underline{c}_l \equiv \frac{1}{8}\frac{(1-a_\ell)(a_\ell B + 2a_\ell - B + 2b)Bb}{(b+1)^3}, \tag{43}$$

when $2\sqrt{a} > 1$ (the right hand side is obtained from the expression for c_l by replacing $2\sqrt{a_\ell}$ to 1). It is therefore sufficient to prove that $\underline{c}_l - 2\Delta_2^{sep} > 0$ for $b \geq B$ and $a_\ell > 1/4$. To do this, we rearrange $\underline{c}_l - 2\Delta_2^{sep} > 0$ as

$$\begin{aligned}\underline{c}_l - 2\Delta_2^{sep} = B(1-a_\ell)\big[&2a_\ell(ba_\ell^3 + Ba_\ell + 2b^4 B) + b(3ba_\ell + 7ba_\ell^3 - B + a_\ell^4 B + 6Ba_\ell^2)\\ &+ b^3(10ba_\ell - 3B + 4Ba_\ell + 5Ba_\ell^2) + 3b^2(2ba_\ell + 4ba_\ell^2 + Ba_\ell^2 - B + a_\ell^3 B + Ba_\ell)\big]\\ &\Big/\big[8(b+1)^3(2b+1+a_\ell)[(b+a_\ell)^2 + (1-a_\ell)(b+1)]\big], \end{aligned} \tag{44}$$

and observe that the quantities within each pair of parentheses are positive for $b > B > 0$ and $a_\ell > 1/4$.

The third strategy is also always worse than the profit without measurement. The proof is similar to the above, albeit with longer expressions: we again construct an upper bound on the net total profit under the third strategy and show that it is below the net equilibrium profit without measurement. Specifically, to construct the upper bound, consider only the informational and direct-cost effects of the third strategy relative to the equilibrium without measurement (i.e., ignore the detriment of the effort distortion). Further, use the upper bound on the information the firm has in the second period by assuming that the induced distortion leads to the probabilities of the first-period profit outcomes as if $(2\sqrt{a_\ell} - x)x = 0$ for the low type (i.e., minimal value), and $(2-x)x = 1$ for the high type (i.e., maximal value), which also implies a lower bound on the direct cost of the first-period wage (because it both minimizes the payment to the low type salespeople and the high-type salespeople's incentive to establish that they are high type with probability one). Thus, we have the following upper bound on the net second-period profit under the third strategy:

$$\pi_2^{pool} \leq \overline{\pi}_2^{pool} = f\left(\frac{b+1}{2b+1}\right)\frac{2b+1}{2b+2} + f(0)\frac{1}{2b+2}, \tag{45}$$

where $f(\cdot)$ is defined in Equation (37), and the high-type salespeople should receive at least

$$\underline{w}_1^{pool} = \frac{c_2^{opt}(1)^2}{2} - \frac{c_2^{opt}\left(\frac{b+1}{2b+1}\right)^2}{2} \tag{46}$$

expected compensation in the first period to prevent him from deviating to an unavailable for the low type salesperson level of x_1 in the first period (the function $c_2^{opt}(\cdot)$ is defined in Equation (36)). Given the above bounds and that half of the salespeople are of high type, in order to prove that the third strategy always results in a lower profit than the profit under no measurement, it suffices to show

$$\pi_2^{nm} - \left(\overline{\pi}_2^{pool} - \underline{w}_1^{pool}/2\right) > 0 \tag{47}$$

for $b \geq B \geq 0$. To show this, one can substitute $b = B + \beta$ into the left hand side and factor it to obtain a fraction with numerator being $(1 - a_\ell)$ times a polynomial in B, β and a_ℓ all coefficients of which are positive, and the denominator $((b+a_\ell)^2 b + (b+1)^3)^2((b+a_\ell)^2 + (1-a_\ell)(b+1))(2b + 1 + a_\ell)(b+1)$, which is also positive. We do not report the full expression as it is long. But to illustrate, if $a_\ell = 1/3$ and $b = B$, we have that the left hand side of Equation (47) is

$$\frac{3(18b^4 + 30b^3 + 19b^2 - 3)(27b^4 + 63b^3 + 72b^2 + 41b + 9)}{4(18b^3 + 33b^2 + 28b + 9)^2(9b^2 + 12b + 7)(3b + 2)} > 0, \quad \text{when } b > 1. \tag{48}$$

Thus, the only possibility for the total net profit to increase due to the introduction of the precise measurement of x is when the second strategy is optimal. Therefore, the total net profit decreases when a_ℓ is sufficiently high (for example, for $b = B = 1$, measurement decreases profits if $a_\ell > 0.32$ and for $b = B = 100$, measurement decreases profits when $a_\ell > 0.38$). This establishes the first claim of the proposition.

In fact, the third strategy is not only worse than not having the measurement, but is also always worse than the first strategy. To prove this, we need to use a slightly better upper bound on the profits under the third strategy, which we establish as follows. Let us still not count the detriment of the first-period's effort allocation distortion of the third strategy. But observe that under pooling on $x_1 = x \in [0, 2\sqrt{a_\ell}]$, conditional on $\pi_1 = 1$, we have that $P_h = \frac{(b+x(2-x)}{2b+x(2+x)+x(2\sqrt{a_\ell}-x)}$

increases in $x \in [0, 2\sqrt{a_l}]$, since $(P_h)'_x = \frac{1}{2}\frac{(1-\sqrt{a_\ell})(x^2+b)}{(b+x-x^2+x\sqrt{a_\ell})^2}$. Therefore, $x = 2\sqrt{a_\ell}$ leads to the best high-type identification in the event of $\pi_1 = 1$ and the least-costly incentive to prevent the high-type salespeople from separating if they do not take into account the possibility that their effort distortion may result in $\pi_0 = -B$. Thus, calculating the minimum payment required in the case $\pi_1 = 1$ to make the high-type salespeople indifferent between separating and not separating when they disregard the possibility of $\pi_1 = -B$ (a lower bound on the first-period wage cost), and calculating the second-period profits with the assumption that for $\pi_1 = 1$, the firm uses

$$P_h = P_{h1} \equiv \frac{(b + x(2-x)}{2b + x(2+x) + x(2\sqrt{a_\ell} - x)}, \quad \text{where } x = 2\sqrt{a_\ell}, \tag{49}$$

and for $\pi_1 = 0$, it uses $P_h = 0$, together with counting the probability of $\pi_1 = -B$ as being $b/2/(b+1)$ and the probability of $\pi_1 = 1$ as being $\frac{b+1}{2(b+1)}$ (i.e., as if "magically" all the high-type salespeople who should have obtained $\pi_1 = -B$ are switched back to $\pi_1 = 1$, and the firm gets the information about that only if $\pi_1 = -B$, thereby resulting in a higher second-period profit), we obtain an upper bound on the net second-period profit under the third strategy:

$$\pi_2^{pool} \leq \overline{\pi}_2^p \equiv f\left(P_{h1}\right)\frac{2b+1}{2b+2} + f(0)\frac{1}{2b+2}, \tag{50}$$

while the firm must offer the high-type salespeople first-period expected wage for choosing the pooling x_1 of at least

$$\underline{w}_1^p = \frac{c_2^{opt}(1)^2}{2} - \frac{c_2^{opt}\left(P_{h1}\right)^2}{2}. \tag{51}$$

Since the probability of $\pi_1 = 1$ is lower for the low type, the most efficient way to compensate the high-type salespeople for choosing the pooling x_1 is to condition the payment on $\pi_1 = 1$. Taking a lower bound $b/(b+1)$ on the probability that the low type achieves $\pi_1 = 1$, we obtain that the total spending on first-period wage is at least

$$\underline{c}_h \equiv \left(\frac{1}{2} + \frac{1}{2}\frac{b}{b+1}\right)\underline{w}_1^p. \tag{52}$$

We further use the lower bound on the profit under the first strategy by replacing its first-period wage cost c_l by

$$\overline{c}_l \equiv \frac{1}{8}\frac{(1-a_\ell)(a_\ell B + 2a_\ell - B + 2b)B}{(b+1)^2}, \tag{53}$$

which is obtained from c_l by replacing $2\sqrt{a_\ell}$ by its highest value 2. We then subtract the upper bound on the profit under the third strategy from the lower bound on the profit under the first strategy to obtain that the difference in total net profits is at least

$$(\pi_2^{sep} - \overline{c}_l) - (\overline{\pi}_2^p - \underline{c}_h), \tag{54}$$

which we now need to prove is positive when $b > B > 0$ and $a_\ell \in (1/4, 1)$. Do do this, it suffices to do the following: substitute $b = B + \beta$ and $a_\ell = a_L^2$ (now, we have $B > 0$, $\beta > 0$ and $a_L \in [1/2, 1]$), substitute $a_L = A + 1/2$ (now, $A \in [0, 1/2]$), and factor. By doing this, one obtains a fraction where the denominator is a complete square and the numerator is a product of $(1 - A^2)$ and a long polynomial. Thus, we need to prove that the polynomial is positive for $B > 0$, $\beta > 0$ and $A \in [0, 1/2]$. To do this, it suffices to replace all positive terms with A^i for $i \geq 1$ with 0 (decreasing the polynomial value for any positive values of B, β and A), and then substitute $A = 1/2$ (i.e., taking the maximal value of all negative terms) which turns out to be (after combining the terms with the same powers of β and B) a polynomial in B and β with only positive coefficients. Although these polynomials are too long to explicitly report here, the above procedure is technically straightforward. To illustrate (54), consider again $B = 1$ and $a_\ell = 1/3$. In this case, (54) becomes

$$\begin{aligned}
&\frac{43 - 20\sqrt{3}}{280368(b+1)^4(63b^2 + 24b\sqrt{3} + 9b + 16\sqrt{3} - 13 + 27b^3)^2} \times \\
&\quad \times \Big[(1700744 - 538712\sqrt{3}) + (10546030 - 1621576\sqrt{3})b + (23349603 + 6280212\sqrt{3})b^2 \\
&\qquad + (24599817 + 36043524\sqrt{3})b^3 + (63015408\sqrt{3} + 30127842)b^4 \\
&\qquad + (62019756 + 47135520\sqrt{3})b^5 + (12992724\sqrt{3} + 70704495)b^6 \\
&\qquad\qquad + (609444\sqrt{3} + 33671781)b^7 + 5677452b^8, \Big]
\end{aligned} \tag{55}$$

which is positive for all $b \geq 0$.

The calculation of the social welfare without measurement follows exactly the same procedure as in the main model, and therefore for the sake of brevity, we skip the details. The second-period social welfare without measurement turns out to be

$$W_{2B} = \frac{11}{16} + \frac{3}{16}\frac{(b + a_\ell - (1 - a_\ell)B)^2}{(b + 1)^2} - \frac{3}{16}\frac{(a_\ell - 1)^2(b + a_\ell)B^2}{((b + a_\ell)^2 + (1 - a_\ell)(b + 1))(2b + 1 + a_\ell)}. \tag{56}$$

The social welfare under the first strategy in the first period is exactly the same as without measurement (the only difference is the wage transfer from the firm to the high-type salespeople). In the second period, the calculation of social welfare under the first strategy is similar to that without measurement, but simpler because types are fully identified. It turns out to be

$$W_{21} = \frac{11}{16} + \frac{3}{16}\frac{(b + a_\ell - (1 - a_\ell)B)^2}{(b + 1)^2}. \tag{57}$$

Finally, under the second strategy, the total social welfare decreases relative to one under the first strategy by $c_{distsep}$ defined in Equation (42) due to the distortion of the first-period effort allocation. Clearly, the total welfare under the first strategy is higher than the total welfare without measurement. However, if the firm prefers the second strategy, total welfare ends up decreasing if

$$\frac{3(a_\ell - 1)^2(b + a_\ell)B^2}{((b + a_\ell)^2 + (1 - a_\ell)(b + 1))(2b + 1 + a_\ell)} < \frac{8(1 - 2\sqrt{a_\ell})^2(B + 1)}{b + 1}. \tag{58}$$

For example, consider $b = B = 1$ and $a_\ell = 1/3$. In this case, with measurement, the second strategy provides the highest profit and results in the second-period welfare of $293/420 = 0.708(3)$, but at a cost of welfare loss of $7/6 - 2/\sqrt{3} \approx 0.011966$ in the first period (relative to no measurement), while with no measurement, the second-period welfare is $293/420 \approx 0.697619$. Therefore, measurement reduces the total welfare by $971/840 - 2/\sqrt{3} \approx 0.00125$. At higher values of b and B, social welfare may decrease even more. For example, when $b = B = 100$ and $a_\ell = 0.43$, social welfare reduces by 0.0185. This concludes the proof of the proposition.

Unexpected Measurement of the Second Period Effort After the First Period's Effort Allocation:

The optimal x_2 maximizes (14). Using implicit differentiation of the first order condition $4 + 4w_2x_2 - 4x_2 - 3w_2x_2^2 + w_2b = 0$ on the optimal x_2, we obtain

$$\frac{\partial x_2}{\partial w_2} = \frac{4x_2 - 3x_2^2 + b}{4 - 4w_2 + 6w_2x_2}. \tag{59}$$

The firm's expected profit in the second period given $\pi_1 = 1\&m(x_1) = 0$ is

$$E(\pi_2|\pi_1 = 1\&m(x_1) = 0) = \phi_{10}\frac{a+b}{1+b}c_2(1+x_2w_2/2)[a-\frac{a+b}{1+b}c_2(1+x_2w_2/2)]+(1-\phi_{10})\frac{b}{1+b}c_2[0-\frac{b}{1+b}c_2], \tag{60}$$

where ϕ_{10} is the probability of high type conditional on $\pi_1 = 1\&m(x_1) = 0$ which is $\phi_{10} = \frac{1+b}{1+3b}$ per (9), $a = x_2(2 - x_2)$, and x_2 is a function of w_2, which is implicitly defined by the FOC on the salesperson's objective function (or explicitly by (15)).

The optimal c_2 and w_2 can then be found by taking the derivatives of (60) with respect to c_2 and w_2, using (59) for the derivative of x_2 with respect to w_2, and making those derivatives equal to zero.

Table 1: Notation

variable	description
t	period, either period 1 or period 2
π_t	profit in period t gross of payment to salesperson
C_t	payment to salesperson in period t
x	dimension of effort that can be potentially measured (x_t means x in period t)
y	dimension of effort that cannot be measured
$\overline{e}$	vector (x, y) of effort choices
e_i	total effort that can be exerted by salesperson of type i, $e_i = x_i + y_i$ assumption is that $e_h = 2$ and $e_\ell = 0$.
α	"productivity" of salesperson, affecting probability of high outcome, $\alpha = xy$
a_i	ability of salesperson of type i, maximal "productivity", $a_i = (e_i/2)^2$
b	parameter that indexes informativeness of profit realization
$\frac{b+\alpha}{1+b}$	probability of high outcome (whose payoff is set to "1")
$-B$	low profit outcome, with $B > 0$
z	outside option of salesperson in second period; distributed uniformly on $[0, 1]$.
$m(x)$	measurement of effort dimension x; It is "1" with probability x/e_h, and "0" with probability $1 - x/e_h$
P_h	posterior probability of high type salesperson given the observables in the first period
c_1	base compensation
$c_2, \widehat{c}_2, \widetilde{c}_2$	payment to salesperson if high outcome and/or high measurement

Table 2: Effects of Measurement

Timing \ Outcome	Productivity in Period 1	Productivity in Period 2	Total Net Profits
After Effort Allocation	No change	Increases	Increase
Before Effort Allocation	Decreases	Increases	Decrease
Before Contract Choice	No change	Increases	Decrease

Notes. All changes are relative to the corresponding outcomes without measurement. Timing is relative to the first-period decisions. Productivity changes are due to the effort allocation changes in Period 1 and due to changed retention in Period 2.

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