Input Price Discrimination by Resale Market*

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Abstract

This paper analyzes the rationale behind and the welfare effects of supply contracts that discriminate between resale in different markets. In a vertical-contracting game between a supplier and competing downstream firms that resell the supplier's input in multiple (independent or interdependent) markets, we show that, all else equal, the supplier wants to discriminate against resale in the market with more intense downstream competition. Unlike monopolistic third-degree price discrimination in final-goods markets, input price discrimination by resale market can have a positive allocation effect, which implies that welfare can rise with discrimination even if total output decreases. The output effect of input price discrimination by resale market, in turn, is shown to depend on the competitive pass-through rates and on the curvatures of the demand functions. Our insights are relevant for the policy treatment of vertical restraints on online sales.

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1 Introduction

Price discrimination is a core topic of industrial organization. Starting with Pigou (1920) a century ago, economists have been seeking to understand the effects on firm profits and welfare of allowing a seller to charge different prices to different buyers or buyer groups (see Robinson 1933, Schmalensee 1981, Varian 1985, Schwartz 1990, and Aguirre, Cowan and Vickers 2010, among others). Recognizing that many instances of price discrimination involve intermediate-goods markets, the literature has also looked at the effects of allowing an upstream supplier to use discriminatory supply contracts when dealing with asymmetric downstream firms rather than final consumers (Katz 1987, DeGraba 1990, Yoshida 2000, Inderst and Shaffer 2009). The common, or one may say defining, feature of price discrimination in these studies remains that the seller discriminates *across buyers*.

This paper considers a different (and novel) form of price discrimination in intermediate-goods markets: price discrimination *across resale markets*, rather than across buyers. This form of price discrimination occurs, for instance, when supply contracts specify different wholesale prices for products resold in different geographic locations by the same downstream firms. A prominent example in the pharmaceutical industry are contracts between manufacturers and wholesalers in which the wholesale price of a drug depends on the country in which the wholesaler resells the drug.[1] This form of input price discrimination also encompasses situations in which multiple outputs are produced from a common input and input prices depend on which final output is produced. For example, the licensing fees for many standards-essential patents are set as a fixed percentage of the final product's selling price, resulting in different per-unit licensing fees for different end products sold by the same downstream firms, a practice recently found in violation of the Indian Competition Act (see Teece et al. 2017).[2] Input price discrimination across resale markets also oc-

[1] For example, a series of antitrust cases in Europe involved manufacturers setting two wholesale prices when selling to wholesalers in Spain, one for reselling through the Spanish health care system, and another, higher wholesale price for exporting. See Commission Decision 2001/79 1/EC; GlaxoSmithKline Services v Comission, T- 168/01, ECR, EU:T:2006:265; ECJ Judgment of 6 October 2009, cases C-501, 513, 515, 519/06 P; Spanish Competition Authority, Pfizer/Cofares, S/DC/0546/15, 19 January 2017.

[2] An older example can be found in *CARTER-WALLACE, INC. v. The UNITED STATES* 449 F.2d 1374 (1971), where the manufacturer of the chemical compound meprobamate charged a lower wholesale price for meprobamate used in combination drugs than for meprobamate resold as a stand-alone drug. Another example is the "value-in-use" pricing system that Du Pont uses to sell its *Kevlar* fiber. Du Pont sells the identical fiber at more than a dozen different price points depending on end use. If a downstream

curs when supply contracts specify different wholesale price for products resold in different distribution channels, or to different customer segments, by the downstream firms.

An example of particular current policy relevance are input supply contracts that discriminate against resale on the internet. In the European Union, the 2010 Guidelines on Vertical Restraints prohibit suppliers from charging a higher wholesale price for products that are intended to be resold by the distributor online than for products that are intended to be resold off-line, a practice also known as 'dual pricing.' The related practice of restricting the share of online sales in the total sales made by a distributor, i.e., from "agreeing that the distributor shall limit its proportion in overall sales made over the internet," is prohibited as well.[3] Recent cases in Europe include *Dornbracht*, in which, in a three-layer distribution network, the German Federal Cartel Office prohibited contracts that required wholesalers to pay higher input prices for products resold to online retailers as opposed to traditional offline retailers.[4] In another recent case, the German Federal Cartel Office prohibited *Bosch-Siemens*'s practice of granting wholesale price discounts for household appliances that were proportional to a retailer's offline versus online sales.[5]

To analyze the rationale behind and the effects of input price discrimination across resale markets, we consider a setting with a supplier selling to competing retailers that operate in multiple (independent or interdependent) downstream markets.[6] When retailers

firm puts *Kevlar* to a different end use than the one it was initially purchased for or resells it, it must pay Du Pont an amount representing the difference between the initial purchase price and the price for the ultimate end use. See *Akzo N. V. v. USITC*, 808 Fed 2d 1471 (1986).

[3] See paragraphs 52 (c) and 52 (d) of the *Commission notice of 10 May 2010: Guidelines on vertical restraints* [SEC (2010) 411 final]. Both practices are treated as hardcore restrictions of competition according to the EU Guidelines. Hardcore restrictions give rise to the presumption that the agreement is prohibited under Article 101(1) of the Treaty on the Functioning of the European Union. Although firms have the possibility to demonstrate pro-competitive effects of such restrictions under Article 101(3), it is considered "unlikely that vertical agreements containing such hardcore restrictions fulfil the conditions of Article 101(3)" (Commission Regulation (EU) No 330/2010).

[4] For a full case summary (in German), see Bundeskartellamt, "Fallbericht: Bundeskartellamt stellt sicher: Hochwertige Sanitärarmaturen im Internet erhältlich," B5-100/10, Dec. 12, 2011. See also Bundeskartellamt, "Vertical Restraints in the Internet Economy," Background Paper – Working Group on Competition Law, October 10, 2013.

[5] For a full case summary (in German), see Bundeskartellamt, "Fallbericht: Rabattsystem der Bosch Siemens Hausgeräte GmbH, München," B7-11/13, Dec. 23, 2013. See also Bundeskartellamt, "Vertical Restraints in the Internet Economy," Background Paper – Working Group on Competition Law, October 10, 2013.

[6] With single-market downstream firms, discrimination across resale markets necessarily implies discrimination across retailers active in different markets, hence the distinction between discrimination across resale markets versus across retailers becomes somewhat blurred. While our paper focuses on symmetric

compete in multiple markets, the marginal input price that aligns the retailers' downstream pricing incentives with the firms' collective incentives in one market generally differs from the marginal input price that aligns individual and collective incentives in another market. As we show, this implies that standard supply contracts that depend on the total amount ordered by each retailer will *not* allow the supplier to achieve the integrated monopoly outcome in equilibrium – even if general non-linear supply tariffs are feasible. To align incentives and maximize total channel profits, supply contracts must discriminate between resale in different markets, e.g., by making wholesale prices depend on the resale market or by imposing restrictions on the relative quantities sold in different markets.[7]

We find that three conceptually distinct factors determine *which* downstream market the supplier wants to discriminate against: the retail prices that the supplier seeks to induce, the intensity of competitive pressure in each downstream market (which, as we will show, can be measured by a conduct parameter that depends on diversion ratios within and across markets), and the marginal production costs. The supplier wants to discriminate against resale in the more competitive downstream market if the optimal (from the firms' viewpoint) retail prices in the two market are sufficiently close, or the optimal retail price is higher in the more competitive downstream market. Conversely, if the competitive conducts in the two markets are sufficiently similar, or the intensity of competition is stronger in the market with the higher optimal retail price, the supplier wants to discriminate against the market with the higher optimal retail price.

The welfare effects of input price discrimination by resale market are ambiguous *a priori*. The total welfare effect can be divided into an allocation effect and an output effect, as in the well-known case of monopolistic third-degree price discrimination in final-goods markets, which corresponds to a limit case of our setting. However, two fundamental differences emerge between the welfare analysis of input price discrimination by resale market and that of third-degree price discrimination in final-goods markets. First, the allocation effect, which is always negative in the case of third-degree price discrimination, can be positive in the case of input price discrimination by resale market. Second, with input price

multi-market downstream firms for this reason, the analysis will also yield several new insights about input price discrimination by resale market in settings with single-market downstream firms, as will be described in more detail below.

[7] Resale price maintenance, which under some conditions can achieve similar effects as input price discrimination by resale market, is assumed to be infeasible.

discrimination by resale market, the output effect depends on the competitive pass-through rates, in addition to the curvatures of the market demand functions.

The broad takeaway from the welfare analysis can be summarized as follows: Input price discrimination by resale market has a positive welfare effect for commonly used functional forms if (i) the downstream markets are sufficiently asymmetric in terms of competitive conduct and (ii) the (unconstrained) optimal retail prices are close enough to each other. With close optimal retail prices and asymmetric conduct, the supplier wants to discriminate against resale in the more competitive market, as discussed earlier. Discrimination thus leads to a higher retail price in the more competitive market and a lower retail price in the less competitive market.

Condition (ii) guarantees that such discrimination against resale in the more competitive market either has a positive allocation effect because it makes the retail prices less discriminatory, or, if the allocation effect is negative (which could happen if the optimal retail in the more competitive market exceeds that in the less competitive market), that its size is small.

The output effect, in turn, depends crucially on the competitive pass-through rates, i.e., the rates at which the retailers pass changes in the wholesale prices through to retail prices. We find that discrimination against resale in the more competitive market has a positive output effect if the pass-through rate in the more competitive market is high enough relative to that in the less competitive market. Intuitively, this result holds because the supplier's optimal *non-discriminatory* wholesale price is high when decreasing the wholesale price (below the level that would be optimal in the more competitive market) creates a large retail price distortion in the more competitive market. The positive effect of discrimination on output in the less competitive downstream market therefore dominates the negative effect of discrimination on output in the more competitive downstream market in this case.

In the case of independent markets, the log-concavity of the market demand functions determines the direction in which competitive conduct affects the pass-through rate: if demand is log-concave (which is equivalent to a curvature below one of the inverse demand function), more intense competition implies a higher pass-through rate, otherwise more intense competition implies a lower pass-through rate. Our main finding in this case is that when the market demand functions have constant inverse demand curvatures between zero and one, discrimination against resale in the more competitive market always raises output if the intensity of competition in the more competitive market exceeds that in the less competitive market sufficiently.

Finally, in a setting with linear demands derived from the quadratic utility function of a representative consumer (allowing cross-market substitution effects), we find that input price discrimination that is motivated by asymmetries in intra-market retailer substituability between the two markets always has positive allocation and output effects, raising total welfare as well as consumer surplus. This result contrasts sharply with findings from the literature on monopolistic third-degree price discrimination in final-goods markets, where discrimination always lowers welfare in linear-demand settings (unless new markets are served).

Our findings have implications for the policy treatment of vertical restraints on online sales. The view in the EU Guidelines on Vertical Restraints is that restraints on online sales are tantamount to output restrictions because they limit distributors' ability "to reach more and different consumers than will be reached when only more traditional sales methods are used."[8] According to this view, removing restrictions on online sales, by banning practices like dual wholesale pricing and relative quantity restrictions, will necessarily result in higher sales. This view thus ignores that suppliers may adjust their input supply terms depending on whether vertical restraints on online sales are allowed or not. It also fails to consider why a manufacturer might want to specifically limit the *online* sales of its distributors. If restricting output were the goal, then presumably the manufacturer could achive this goal by imposing high wholesale prices on all of a retailer's sales, not just on the sales intended for the online market.

The explanation for vertical restraints on online sales that we propose follows from the fact that online and offline markets typically differ in their demand characteristics. In particular, consumers tend to view the online stores of competing retailers as closer substitutes than the physical stores of the same retailers, among other things because the internet decreases the importance of geographical store location. Moreover, search costs are typically lower online, which further tends to intensify competitive pressure.[9] Thus, the

[8]See Paragraph 52 of the Guidelines, which talks about the internet as a powerful sales tool.

[9]See Lieber and Syverson (2012) and the references therein for a detailed discussion of online versus off-line competition.

intensity of competition among retailers can be expected to be higher in the online market than in the offline market, which, following our results, provides an explanation for why the supplier wants to discriminate against resale in the online market.[10]

The output and welfare effects of discrimination against resale in the online market are ambiguous a priori according to our theory. While discrimination leads to higher prices in the online market, it also leads to lower prices in the offline market. And, as our analysis shows, discrimination that is motivated mainly by asymmetries in competitive conduct between markets increases welfare for commonly used demand specifications. In summary, our results suggest that suppliers may impose vertical restraints on online sales for price discrimination reasons, and that the welfare effects of such restraints may well be positive. The current policy treatment of such restraints in the European Union therefore appears to be overly aggressive.

The existing literature on vertical contracting when downstream firms operate in multiple markets or channels is sparse. Chen (1999) shows that a supplier can benefit from imposing resale price maintenance when retailers operate in two independent markets, allowing for between-retailer competition in one of the two markets. Arya and Mittendorf (2010) analyse the effects of a ban on input price discrimination across retailers in a setting with asymmetric retailers, one of whom operates in multiple markets. In contrast, the retailers in our model are symmetric, and we focus on vertical contracts that discriminate by resale market rather than on resale price restraints. Further distinguishing our work from these earlier studies, we allow retailers to compete in all, not just one, of the downstream markets and we allow demands to be interdependent across markets. Moreover, our paper uses general functional forms and offers a thorough analysis of the welfare effects of input price discrimination by resale market.

While this paper focuses on the case of multi-market retailers, we also obtain new in-

[10] An alternative explanation for vertical restrictions on internet sales is that suppliers seek to mitigate potential free-riding on the investment efforts of their retailers' brick-and-mortar stores – see Iacobucci and Winter (2015) for a nice summary of this theory and a criticism of the EU Guidelines on its basis. Our theory does not rely on investment incentives to explain vertical restraints on online sales and thus also applies to products for which free-riding on services may be less of a concern. Moreover, it also applies to markets in which the same retailers operate both online and offline, so that no clear "victim" of free-riding can be identified in the downstream market. Helfrich and Herweg (2017) and Dertwinkel-Kalt and Köster (2017) offer explanations for why a supplier may want to fully ban online distribution of its products, building on the assumption that consumer decisions are distorted by salient thinking as in Bordalo et al. (2013). Our theory assumes that consumers are fully rational and applies to hybrid retailers.

sights for settings with single-market retailers, in which price discrimination across resale markets implies price discrimination across buyers. Existing theories explain input price discrimination by market demand characteristics and costs. Allowing the supplier to deal with multiple competing retailers in each downstream market, our paper shows that asymmetries in the intensity of between-retailer competition across downstream markets offer another explanation for discriminatory input pricing. In contrast, existing work has considered situations with either a single retailer in each downstream market (e.g., Herweg and Müller 2012) or with a single downstream market (e.g., Yoshida 2000, Shaffer and Inderst 2009), thus ignoring asymmetries in competitive conduct across markets as an explanation for input price discrimination.

The remainder of this paper proceeds as follows. In the next section, we describe the framework. Section 3 derives some preliminary findings that will be useful for subsequent analyses. Section 4 explains the channel coordination problem faced by a supplier when competing retailers resell the supplier's input in multiple markets and characterizes the optimal dual wholesale prices that allow the supplier to overcome the problem. Section 5 analyzes the output, allocation, and welfare effects of input price discrimination by resale market. Formal proofs of the propositions and lemmas are relegated to Appendix A.

2 The model

We consider a setting with a single supplier and n competing downstream firms (henceforth called retailers). Each retailer operates in the same two markets. The retailers are indexed by $i = 1, 2, ..., n$, and the markets are indexed by $s = A, B$.[11] The retailers transform the supplier's inputs into final outputs at zero marginal cost using a one-to-one technology (i.e., one unit of input is used to make one unit of output). The supplier has a constant marginal cost $c \geq 0$.

Each retailer sets two prices, one for market A and one for market B. The demand for retailer i in market $s \in \{A, B\}$ is given by

$$D_{si}\left(p_{s1}, p_{s2}, ..., p_{sn}, p_{-s1}, p_{-s2}, ..., p_{-sn}\right),$$

[11] Allowing firms to compete in more than one channel distinguishes our model from much of the contracting literature on vertical control, which typically assumes that the firms compete in only one market.

where p_{sj} denotes j's price in market s and p_{-sj} denotes j's price in market $-s \neq s$. Consumers view the $n \times 2$ final products as (weak) substitutes, and own-price effects dominate cross-price effects. That is, for all $i \neq j$ and $s \neq -s$, we have that

$$\begin{aligned} \frac{\partial D_{-si}}{\partial p_{si}}, \frac{\partial D_{sj}}{\partial p_{si}}, \frac{\partial D_{-sj}}{\partial p_{si}} &\geq 0, \\ -\frac{\partial D_{si}}{\partial p_{si}} &> \sum_{j \neq i} \frac{\partial D_{sj}}{\partial p_{si}} + \sum_{k=1,2,...,n} \frac{\partial D_{-sk}}{\partial p_{si}}. \end{aligned}$$

This formulation allows for the possibility of demand-side interactions within and across markets. Note that the special case of independent markets obtains if and only if there are no cross-market price effects, i.e., if and only if $\frac{\partial D_{-si}}{\partial p_{si}} = \frac{\partial D_{-sj}}{\partial p_{si}} = 0$ for all $i \neq j$ and $s \neq -s$.

To focus on demand asymmetries between markets, we assume that the retailers are symmetric, so that in each market s,

$$D_{si}(p_s, p_s, ..., p_s, p_{-s}, p_{-s}, ..., p_{-s}) = D_{sj}(p_s, p_s, ..., p_s, p_{-s}, p_{-s}, ..., p_{-s})$$

for any (p_s, p_{-s}) and $i \neq j \in \{1, 2, ..., n\}$. This then allows us to denote the quantity demanded in market s (per-retailer) at symmetric prices by the *market demand function*[12]

$$Q_s(p_s, p_{-s}) = D_{s1}(p_s, p_s, ..., p_s, p_{-s}, p_{-s}, ..., p_{-s}).$$

The marginal change in the quantity demanded in market s when all retailers' prices move in tandem is thus

$$\begin{aligned} \frac{\partial Q_s}{\partial p_s} &= \frac{\partial D_{si}}{\partial p_{si}} + \sum\nolimits_{j \neq i} \frac{\partial D_{si}}{\partial p_{sj}} = \frac{\partial D_{si}}{\partial p_{si}} + \sum\nolimits_{j \neq i} \frac{\partial D_{sj}}{\partial p_{si}}, \\ \frac{\partial Q_s}{\partial p_{-s}} &= \frac{\partial D_{si}}{\partial p_{-si}} + \sum\nolimits_{j \neq i} \frac{\partial D_{si}}{\partial p_{-sj}} = \frac{\partial D_{si}}{\partial p_{-si}} + \sum\nolimits_{j \neq i} \frac{\partial D_{sj}}{\partial p_{-si}}. \end{aligned}$$

It will prove convenient to denote retailer i's variable profit given (hypothetical) marginal costs c_A in market A and c_B in market B by

$$\pi_i(p_{s1}, ..., p_{sn}, p_{-s1}, ..., p_{-sn}; c_A, c_B) = \sum_{s=A,B} (p_{si} - c_s) D_{si}(p_{s1}, ..., p_{sn}, p_{-s1}, ..., p_{-sn}).$$

We assume that there exists a unique retail price equilibrium (when the retail prices are chosen simultaneously and independently), and that this equilibrium is symmetric (because the retailers are symmetric) and stable for any vector (c_A, c_B) (below some choke level).

[12] With n symmetric downstream firms, Q_s represents $1/n$ of the total quantity demanded of the supplier's input in channel s at symmetric prices. In the case of independent channels ($\frac{\partial Q_s}{\partial p_{-s}} = 0$), Q_s corresponds to Chamberlin's (1933) *DD* demand curve. See also Anderson, de Palma and Kreider (2001).

Industry profits are equal to the sum of the profits across the $n \times 2$ final products:

$$\Pi\left(p_{A1},...,p_{An},p_{B1},...,p_{Bn}\right) = \sum_{i=1,2,...,n} \sum_{s=A,B} \left(p_{si} - c\right) D_{si}\left(p_{s1},...,p_{sn},p_{-s1},...,p_{-sn}\right).$$

We assume the Hessian matrix of Π is negative definite, which implies that industry profits are strictly concave in the retail prices. This and symmetry implies that there is a unique symmetric pair of prices that maximize industry profits, which we denote by (p_A^*, p_B^*). *Full channel coordination* is achieved if the retail prices that arise in equilibrium maximize industry profits, i.e., if all retailers charge p_A^* in market A and p_B^* in market B. To economize on the notation, we let $D_{si}^* = D_{si}\left(p_s^*,...,p_s^*,p_{-s}^*,...,p_{-s}^*\right)$ and $Q_s^* = Q_s\left(p_s^*, p_{-s}^*\right)$.

The game proceeds as follows:

1. The supplier makes simultaneous non-discriminatory contract offers to the retailers. We consider two types of tariffs: (i) tariffs of the form $T(q_i)$, where q_i is the total quantity ordered by retailer i, and (ii) tariffs of the form $T(q_{Ai}, q_{Bi})$, where q_{si} denotes the quantity ordered for resale in market s by retailer i. The retailers make simultaneously make acceptance decisions.[13]

2. Given the offers and acceptance decisions, the retailers simultaneously set their downstream prices (p_{Ai}, p_{Bi}) in the two markets. Each retailer orders enough quantity to serve its demands in the two markets and pays the supplier according to its contract.[14]

3 Preliminaries

Our goal in this section is to obtain measures of where competitive pricing in each of the two markets falls in the realm between perfect competition and monopoly pricing, while allowing for interdependencies between the demands in the two markets. To achive this goal, we extend the conduct parameter approach that is commonly used to measure monopoly

[13] Indifferences are broken in favor of contract acceptance.

[14] We assume that each retailer must serve its entire demand, i.e., the retailers cannot ration consumers. This assumption, which is commonly made in the vertical-contracting literature (see, for example, Rey and Vergé (2004) and Ulsaker (2015)), ensures that the downstream pricing game has a pure-strategy equilibrium. One interpretation of the assumption is that supply contracts permit retailers to order as many units as they wish, although tariffs may specify lump-sum penalities or high marginal prices for orders above a certain threshold. An alternative interpretation is that retailers have access to an outside source of supply in case their demand exceeds the limit in their contract with the supplier.

power in single-product firm oligopolies (Bresnahan 1989; Genesove and Mullin 1998; Weyl and Fabinger 2013) to our setting with multi-product firm oligopolies and asymmetries among products.

Hypothetical downstream monopolist

We begin by deriving implicit conditions for the retail prices that a hypothetical downstream monopolist would set. Because demands within each market are symmetric, the profit-maximizing prices in each market will also be symmetric (i.e., $p_{A1} = p_{A2} = ... = p_{An}$ and $p_{B1} = p_{B2} = ... = p_{Bn}$). Given marginal costs c_A and c_B for sales in markets A and B, respectively, it follows that the monopolist's first-order conditions can be reduced to the following two conditions:

$$Q_A + (p_A - c_A)\frac{\partial Q_A}{\partial p_A} + (p_B - c_B)\frac{\partial Q_B}{\partial p_A} = 0, \quad (1)$$

$$Q_B + (p_A - c_A)\frac{\partial Q_A}{\partial p_B} + (p_B - c_B)\frac{\partial Q_B}{\partial p_B} = 0. \quad (2)$$

Solving the system of equations in (1) and (2) for the markups in the two markets yields

$$\begin{bmatrix} p_A - c_A \\ p_B - c_B \end{bmatrix} = -\begin{bmatrix} \frac{\partial Q_A}{\partial p_A} & \frac{\partial Q_B}{\partial p_A} \\ \frac{\partial Q_A}{\partial p_B} & \frac{\partial Q_B}{\partial p_B} \end{bmatrix}^{-1} \begin{bmatrix} Q_A \\ Q_B \end{bmatrix}, \quad (3)$$

or, equivalently, for each $s \in \{A, B\}$,

$$p_s - c_s = \frac{\frac{Q_s}{-\partial Q_s/\partial p_s} + R_{s-s}\frac{Q_{-s}}{-\partial Q_{-s}/\partial p_{-s}}}{1 - R_{AB}R_{BA}}, \quad (4)$$

where

$$R_{s-s}(p_s, p_{-s}) \equiv \frac{\partial Q_{-s}/\partial p_s}{-\partial Q_s/\partial p_s}.$$

$R_{s-s}(p_s, p_{-s})$, which is bounded below by zero and above by one, measures the diversion of sales to market $-s$ when the price in market s changes. Specifically, it can be thought of as the share of sales lost in market s that is captured by market $-s$ when p_s increases.

Notice that with independent markets ($R_{AB} = R_{BA} = 0$), the monopoly markup in market s simplifies to the standard monopoly pricing formula, $p_s - c_s = -\frac{Q_s}{\partial Q_s/\partial p_s}$, whereas with interdependent markets, the monopoly markup in market s depends not only on the inverse semi-elasticity of demand in market s, $\frac{Q_s}{-\partial Q_s/\partial p_s}$, but also on the inverse semi-elasticity of demand in market $-s$, $\frac{Q_{-s}}{-\partial Q_{-s}/\partial p_{-s}}$. The monopoly markup in market s is proportional to a weighted sum of the inverse semi-elasticities in the two markets, where the inverse semi-elasticity in market s is weighted by one and the inverse semi-elasticity in market $-s$ is weighted by $R_{s-s}(p_s, p_{-s})$. Intuitively, the monopoly markup in market s is decreasing not only in the market's own semi-elasticity of demand but also in the semi-elasticity of demand in market $-s$ because the latter proxies for the profitability of sales in market $-s$ and the products sold in the different markets are substitutes.

Competitive pricing

Now consider the pricing decisions of n independent retailers. Symmetry between retailers again implies that $p_{A1} = p_{A2} = ... = p_{An}$ and $p_{B1} = p_{B2} = ... = p_{Bn}$ in equilibrium. It follows that, given marginal costs c_A and c_B in markets A and B, the first-order conditions of each firm i, $i \neq j \in \{1, 2, ..., n\}$ can be written as

$$Q_A + (p_A - c_A)\frac{\partial D_{Ai}}{\partial p_{Ai}} + (p_B - c_B)\frac{\partial D_{Bi}}{\partial p_{Ai}} = 0, \tag{5}$$

$$Q_B + (p_A - c_A)\frac{\partial D_{Ai}}{\partial p_{Bi}} + (p_B - c_B)\frac{\partial D_{Bi}}{\partial p_{Bi}} = 0. \tag{6}$$

Solving the system of equations in (5) and (6) for the markups in the two markets yields

$$\begin{bmatrix} p_A - c_A \\ p_B - c_B \end{bmatrix} = -\begin{bmatrix} \frac{\partial D_{Ai}}{\partial p_{Ai}} & \frac{\partial D_{Bi}}{\partial p_{Ai}} \\ \frac{\partial D_{Ai}}{\partial p_{Bi}} & \frac{\partial D_{Bi}}{\partial p_{Bi}} \end{bmatrix}^{-1} \begin{bmatrix} Q_A \\ Q_B \end{bmatrix}, \tag{7}$$

which can be rewritten as

$$\begin{bmatrix} p_A - c_A \\ p_B - c_B \end{bmatrix} = -\begin{bmatrix} \frac{1}{1-d_A}\frac{\partial Q_A}{\partial p_A} & \frac{1}{1+d_{AB}}\frac{\partial Q_B}{\partial p_A} \\ \frac{1}{1+d_{BA}}\frac{\partial Q_A}{\partial p_B} & \frac{1}{1-d_B}\frac{\partial Q_B}{\partial p_B} \end{bmatrix}^{-1} \begin{bmatrix} Q_A \\ Q_B \end{bmatrix}, \tag{8}$$

where

$$d_s(p_s, p_{-s}) \equiv \frac{\sum_{j\neq i}\frac{\partial D_{sj}}{\partial p_{si}}}{-\frac{\partial D_{si}}{\partial p_{si}}} \tag{9}$$

measures the diversion of sales to the other retailers in market s when p_{si} changes, and

$$d_{s-s}(p_s, p_{-s}) \equiv \frac{\sum_{j\neq i}\frac{\partial D_{-sj}}{\partial p_{si}}}{\frac{\partial D_{-si}}{\partial p_{si}}} \tag{10}$$

measures the diversion of sales to the rival retailers in market $-s$ versus to the same retailer (i) in market $-s$ when p_{si} changes. One can think of d_A and d_B as capturing the strength of within-market (intra-market) competitive externalities, and of d_{AB} and d_{BA} as capturing the strength of cross-market (inter-market) competitive externalities.

Writing the markups in non-matrix form, we obtain, for each $s \neq -s \in \{A, B\}$,

$$p_s - c_s = \frac{(1-d_s)\frac{Q_s}{-\partial Q_s/\partial p_s} + \frac{(1-d_A)(1-d_B)}{1+d_{s-s}} R_{s-s} \frac{Q_{-s}}{-\partial Q_{-s}/\partial p_{-s}}}{1 - \frac{(1-d_A)(1-d_B)}{(1+d_{AB})(1+d_{BA})} R_{AB} R_{BA}}. \tag{11}$$

The equilibrium markups in (11) can be seen to be weakly smaller than the monopoly markups in (4), with equality if and only if the within-market and the cross-market competitive externalities are zero for all i.[15] Intuitively, only the diversion of sales between markets for the same retailer are internalized in the pricing decisions under competition. Diversion of sales that go to the rival retailers, whether they occur within the same market or between markets, are not internalized.

Conduct parameters

We are now in a position to assess the relative intensity of competition in the two markets. Let $\xi_s(p_s, p_{-s})$ denote the right-hand side of (4), so the implicit conditions for the monopoly prices can be expressed as

$$p_s - c_s = \xi_s(p_s, p_{-s}). \tag{12}$$

We can then define the conduct parameters for the two markets under competition by rewriting (11) as follows:

$$p_s - c_s = \theta_s(p_s, p_{-s})\, \xi_s(p_s, p_{-s}), \tag{13}$$

where the conduct parameters are

$$\theta_s(p_s, p_{-s}) = \kappa(p_A, p_B) \frac{(1-d_s)\frac{Q_s}{-\partial Q_s/\partial p_s} + \frac{(1-d_A)(1-d_B)}{1+d_{AB}} R_{s-s} \frac{Q_{-s}}{-\partial Q_{-s}/\partial p_{-s}}}{\frac{Q_s}{-\partial Q_s/\partial p_s} + R_{s-s} \frac{Q_{-s}}{-\partial Q_{-s}/\partial p_{-s}}}, \tag{14}$$

with

$$\kappa(p_A, p_B) = \frac{1 - R_{AB} R_{BA}}{1 - \frac{(1-d_A)(1-d_B)}{(1+d_{AB})(1+d_{BA})} R_{AB} R_{BA}} \in (0, 1]. \tag{15}$$

Our assumption that the final products are substitutes and own-price effects dominate cross-price effects imply that $\theta_s(p_s, p_{-s}) \in (0, 1]$. The case $\theta_s = 1$, which holds if and only

[15] This follows because the denominator in (11) is weakly larger than the denominator in (4) while the weights on the semi-elasticities in the numerator in (11) are weakly smaller than the analog weights in (4).

if the markets are independent ($R_{AB} = R_{BA} = 0$) and $d_s = 0$, corresponds to monopoly conduct in market s. As θ_s falls, the divergence between competitive pricing and monopoly pricing in market s rises. The limit case $\theta_s \to 0$ corresponds to perfect competition in market s.

The following lemma summarizes the main insights so far, which will be useful for subsequent analyses:

Lemma 1 *(i) The prices that maximize industry profits given a marginal cost c_s in market s are implicitly defined by*

$$p_s - c_s = \xi_s(p_s, p_{-s}) \equiv \frac{\frac{Q_s}{-\partial Q_s/\partial p_s} + R_{s-s}\frac{Q_{-s}}{-\partial Q_{-s}/\partial p_{-s}}}{1 - R_{AB}R_{BA}}. \tag{16}$$

(ii) There exists conduct parameters $\theta_s(p_s, p_{-s}) \in (0,1]$ such that the equilibrium retail prices of competing multi-market retailers facing a marginal cost c_s in market s are implicitly defined by

$$p_s - c_s = \theta_s(p_s, p_{-s})\, \xi_s(p_s, p_{-s}). \tag{17}$$

With independent markets, the conduct parameters are simply $\theta_s = 1 - d_s$, as in settings with symmetric single-product firms (see Weyl and Fabinger 2013). With interdependent markets, however, the expressions for the conduct parameters incorporate the semi-elasticities in both markets and the demand diversions between markets, because the profit impact of a price change in one market depends on the profit margins in both markets and the latter can be asymmetric. The semi-elasticities of the market demand functions thus do not cancel out of the conduct parameter formulae, resulting in relatively long and harder-to-interpret expressions than in single-product firm settings. In spite of this, as our next result shows, the comparison of conducts between the two markets boils down to a comparison of readily interpretable *aggregate diversion ratios* between the retailers:

Lemma 2 $\theta_A(p_A, p_B) > (<) \theta_B(p_B, p_A)$ *if and only if* $DR_A(p_B, p_A) < (>) DR_B(p_B, p_A)$, *where*

$$DR_s(p_s, p_{-s}) = \frac{\sum_{j \neq i}\left(\frac{\partial D_{Aj}}{\partial p_{si}} + \frac{\partial D_{Bj}}{\partial p_{si}}\right)}{-\left(\frac{\partial D_{Ai}}{\partial p_{si}} + \frac{\partial D_{Bi}}{\partial p_{si}}\right)}$$

is the aggregate diversion ratio when one of the retailers changes its price in market s.

The aggregate diversion ratio DR_s can be thought of as the share of the *net* loss in quantity sold for i when it raises p_{si} (taking into account that i sells a substitute product in market $-s$) that goes to the rival retailers (in either market). As shown in Lemma 2, the market with the higher aggregate diversion ratio features more competitive retailer conduct.

4 The rationale for input price discrimination by resale market

4.1 The channel coordination dilemma

To see why coordinating the decisions of multi-market retailers poses a fundamental problem, suppose that the supplier offers a tariff of the form $T(q_i)$. Suppose further that the tariff induces a symmetric continuation equilibrium in which all retailers set the prices (p_A^e, p_B^e). We will show that $(p_A^e, p_B^e) \neq (p_A^*, p_B^*)$, which implies that full channel coordination is impossible with a tariff of the form $T(q_i)$.[16]

The key observation is that in equilibrium no retailer must be able to gain by adjusting its prices so as to sell more in one market and less in the other market without changing its total quantity sold. Formally, for the prices (p_A^e, p_B^e) to arise in an equilibrium of the downstream pricing game, it is necessary that[17]

$$(p_A^e, p_B^e) = \arg \max_{(p_{Ai}, p_{Bi})} p_{Ai} D_{Ai}(p_{Ai}, p_{Bi}; p_A^e, p_B^e) + p_{Bi} D_{Bi}(p_{Bi}, p_{Ai}; p_B^e, p_A^e)$$

$$\text{s.t.} \quad D_{Ai}(p_{Ai}, p_{Bi}; p_A^e, p_B^e) + D_{Bi}(p_{Bi}, p_{Ai}; p_B^e, p_A^e) = D_{Ai}(p_A^e, p_B^e; p_A^e, p_B^e) + D_{Bi}(p_B^e, p_A^e; p_B^e, p_A^e),$$

otherwise each retailer would have a profitable unilateral deviation to different retail prices.[18]

The equilibrium retail prices (p_A^e, p_B^e) must thus satisfy the following first-order condi-

[16] The same result would hold if the supplier could make (public) discriminatory offers of the form $T_i(q_i)$.

[17] In what follows, $D_{si}(p_{si}, p_{-si}; p_s^e, p_{-s}^e)$ denotes the demand of retailer i in channel s when i sets the prices (p_{Ai}, p_{Bi}) and all other retailers charge (p_A^e, p_B^e). Moreover, $D_{si}^e = D_{si}(p_{si}^e, p_{-si}^e; p_s^e, p_{-s}^e)$.

[18] No restrictions on the functional form of the tariff T are required to make this argument, because we consider deviations in which a retailer changes its allocation of quantity across the two channels without changing how much it orders in total.

tions:

$$D^e_{Ai} + (p^e_A - \lambda)\frac{\partial D_{Ai}(p^e_A, p^e_B; p^e_A, p^e_B)}{\partial p_{Ai}} + (p^e_B - \lambda)\frac{\partial D_{Bi}(p^e_B, p^e_A; p^e_B, p^e_A)}{\partial p_{Ai}} = 0, \quad (18)$$

$$D^e_{Bi} + (p^e_A - \lambda)\frac{\partial D_{Ai}(p^e_A, p^e_B; p^e_A, p^e_B)}{\partial p_{Bi}} + (p^e_B - \lambda)\frac{\partial D_{Bi}(p^e_B, p^e_A; p^e_B, p^e_A)}{\partial p_{Bi}} = 0. \quad (19)$$

Given that the constraint limits the total quantity sold across the two markets, the Lagrange multiplier λ plays the role of a "shadow marginal cost" in each market. Using Lemma 1(ii), conditions (18) and (19) can thus be expressed as

$$p^e_A - \lambda = \theta_A (p^e_A, p^e_B)\, \xi_A(p^e_A, p^e_B), \quad (20)$$

$$p^e_B - \lambda = \theta_B (p^e_B, p^e_A)\, \xi_B(p^e_B, p^e_A). \quad (21)$$

Full channel coordination requires that there exists a λ such that both conditions hold at $(p^e_A, p^e_B) = (p^*_A, p^*_B)$. If no such λ exists, then for any tariff $T(q_i)$ the equilibrium prices differ from the prices that maximize industry profits. Setting the retail prices equal to the monopoly prices $(p^e_A, p^e_B) = (p^*_A, p^*_B)$ and using Lemma 1(i), (20) and (21) become[19]

$$p^*_A - \lambda = \theta^*_A (p^*_A - c), \quad (22)$$

$$p^*_B - \lambda = \theta^*_B (p^*_B - c), \quad (23)$$

where $\theta^*_s = \theta_s (p^*_A, p^*_B)$. In the presence of demand asymmetries between markets A and B, conditions (22) and (23) are conflicting because, generically, $p^*_A \neq p^*_B$ if the market demand functions ($Q_A(\cdot)$ and $Q_B(\cdot)$) are asymmetric and $\theta^*_A \neq \theta^*_B$ if the product demand functions $D_{Ai}(\cdot)$ and $D_{Bi}(\cdot)$ are asymmetric across markets (even if the market demand functions were symmetric). Hence, the supplier will be unable to achieve full coordination with a tariff of the form $T(q_i)$. The following proposition summarizes this insight:

Proposition 1 *Tariffs that do not discriminate between resale in A and resale in B cannot achieve full channel coordination in the presence of asymmetries between the two markets.*

The channel coordination problem arises because each retailer allocates its total quantity across the two markets with the goal of maximizing its own profit rather than industry profits. Even if the supply tariff induces each retailer to order and sell a total quantity

[19] Recall that (p^*_A, p^*_B) are the retail prices that maximize total industry profit given the true marginal cost c.

of $D^*_{Ai} + D^*_{Bi}$, full channel coordination will fail because the total quantity will not be distributed optimally across the two markets (from an industry profit standpoint). Full channel coordination therefore requires a supply contract that distinguishes between resale in A and B — sales in one market must be discriminated against relative to sales in the other market.

4.2 Input price discrimination by resale market

A "dual two-part tariff" combining a fixed fee with two wholesale prices – one for resale in each market – constitutes one way for the supplier to achieve full channel coordination by discriminating between resale in the two markets:

Proposition 2 (Dual pricing) *The dual two-part tariff*

$$T\left(q_{Ai}, q_{Bi}\right) = \pi_i\left(p^*_A, ..., p^*_A, p^*_B, ..., p^*_B; w^*_A, w^*_B\right) + w^*_A q_{Ai} + w^*_B q_{Bi}, \tag{24}$$

with

$$w^*_s = \theta^*_s c + \left(1 - \theta^*_s\right) p^*_s, \tag{25}$$

achieves full channel coordination.

Before discussing the dual wholesale prices (w^*_A, w^*_B), it is worth noting that other tariffs of the form $T\left(q_{Ai}, q_{Bi}\right)$ can achieve full channel coordination as well. For example, if $w^*_A < w^*_B$, the supplier can combine a (standard) two-part tariff with wholesale price $w = \frac{Q^*_A}{Q^*_A+Q^*_B} w^*_A + \frac{Q^*_B}{Q^*_A+Q^*_B} w^*_B$ with an upper limit of $\frac{Q^*_B}{Q^*_A+Q^*_B}$ on the proportion of sales in market B in the total sales of each retailer. That is, the supplier could use a relative quantity restriction to control the share of sales in the market that it wants to discriminate against (see Appendix B for a formal proof). Alternatively, the supplier could impose a restriction on the absolute quantity sold in one of the two markets. For example, given $w^*_A < w^*_B$, the supplier can achieve the first-best outcome by combining a two-part tariff with wholesale price w^*_A with a penalty for sales above D^*_{Bi} in market B, or by combining a two-part tariff with wholesale price w^*_B with a penalty for sales below D^*_{Ai} in market A.[20]

[20] A potential drawback of restrictions on absolute quantities, as opposed to dual wholesale pricing or relative quantity restrictions, can be the associated loss of flexibility in responding to fluctuations in aggregate demand.

The common feature of all these tariffs is that the "effective" wholesale prices, taking into account the shadow prices of any constraints imposed on absolute or relative quantities, are (w_A^*, w_B^*) at the equilibrium quantities and that the market with the higher w_s^* is the one that is being discriminated against.

The key contribution of Proposition 2 lies in the characterization of the optimal dual wholesale prices in (25).[21] Three conceptually distinct factors determine the optimal dual wholesale price for each market: the retail price that the supplier seeks to induce, the competitive conduct downstream, and marginal costs. Specifically, the wholesale price for resale in market s, w_s^*, is a weighted average of the downstream monopoly price in that market and the upstream marginal cost, where the weight on the downstream monopoly price equals the conduct parameter in market s.[22]

In the limit case of monopoly conduct ($\theta_s^* = 1$), which corresponds to independent markets and local retail monopolies in market s, a wholesale price equal to upstream marginal cost (i.e., $w_s^* = c$) induces each retailer to set the integrated monopoly price in market s. The retailer's pricing decision in s has no vertical externality on the supplier when the upstream margin is zero, which, given the absence of horizontal externalities in the monopoly case, implies that each retailer's incentives are aligned with collective incentives. At the other extreme, as conduct in market s approaches perfect competition ($\theta_s^* \to 0$), the optimal dual wholesale price approaches the downstream monopoly price. In the limit, intense competition between the retailers fully erodes the downstream margin in market s, hence retail prices will equal the wholesale price and full channel coordination requires the wholesale price to be set equal to the monopoly retail price.

Between the two limit cases of monopoly and perfectly competitive conduct, the wholesale price required to offset the horizontal externalities between the retailers is increasing in the intensity of competition in market s (for a given downstream monopoly price).[23] Intuitively, as competitive pressure intensifies in a market, the supplier needs to raise its

[21] To our knowledge, no prior work has offered a similar characterization of the marginal input prices that induce the integrated monopoly prices downstream.

[22] Positive downstream costs could be incoporated easily. If the retailers face a constant marginal cost $k > 0$, then $w_s^* = \theta_s^* c + (1 - \theta_s^*)(p_s^* - k)$.

[23] For an example of a demand specification for which competitive conduct can vary independently of the monopoly prices, see Section 5.2, which considers linear demand functions derived from a representative consumer quadratic-utility model. With such a demand specification, changes in the parameters measuring retailer substitutability affect competitive conduct without affecting the monopoly prices.

wholesale price for that market to ensure that the retailers continue to set the retail price that maximizes industry profits although more intense downstream competition in the market erodes the retailers' downstream margins.

In addition to the conduct parameters, which determine where each wholesale price falls in the range between the upstream marginal cost and the market's downstream monopoly price, which market is discriminated against also depends on the levels of the monopoly prices themselves. From the formulae for the monopoly margins in (4) it follows that $p_A^* < p_B^*$ if and only if

$$\frac{-\partial Q_B^*/\partial p_B}{Q_B^*}(1 - R_{BA}) < \frac{-\partial Q_A^*/\partial p_A}{Q_A^*}(1 - R_{AB})\,. \tag{26}$$

In the absence of demand interdependencies between the two markets ($R_{AB} = R_{BA} = 0$), the monopoly price is lower in the market with the more elastic market demand, a well-known result from the literature on third-degree price discrimination. In the presence of demand interdependencies between the two markets, the demand diversion patterns between the two markets also affect the monopoly prices, and the elasticities in condition (26) are weighted by (what can be thought of as) the share of demand that would be lost *altogether* (i.e., that would not be captured by market $-s$) if the retailers raised their price in market s slightly. Intuitively, a multi-product monopolist will set a relatively high price in a market where raising price leads to a small reduction in its *total* quantity demanded, incorporating substitutions to the other market.

Taking into account competitive conducts and downstream monopoly prices, $w_A^* < w_B^*$ if and only if

$$(1 - \theta_A^*)(p_A^* - c) < (1 - \theta_B^*)(p_B^* - c)\,. \tag{27}$$

The market with the lower θ_s^* features (locally) more competitive conduct. By condition (27), if the downstream monopoly prices are close (or the downstream monopoly price is higher in the locally more competitive market), the supplier wants to discriminate against resale in the (locally) more competitive market. Moreover, if conduct in the market with the lower downstream monopoly price is sufficiently more competitive than conduct in the other market, discrimination against resale in the market with the lower downstream monopoly price can be optimal for the supplier. In the limit case of monopoly conduct in one market but some competition in the other market ($\theta_B^* < \theta_A^* = 1$),[24] the supplier always wants to discriminate against resale in the more competitive market (here, B), regardless of the downstream monopoly prices.

Remark: Single-market retailers While our analysis focuses on multi-market retailers, the characterization of the optimal wholesale prices in Proposition 2 is also relevant for settings with single-market retailers. To see this, suppose there are n symmetric retailers in each of the two downstream markets, making for a total of $n \times 2$ retailers. We first derive the conduct parameters for this setting. Given marginal costs c_A and c_B in markets A and B, the first-order conditions that determine the competitive prices in the two downstream markets can be written as

$$Q_A + (p_A - c_A)\frac{\partial D_{Ai}}{\partial p_{Ai}} = 0, \quad (28)$$

$$Q_B + (p_B - c_B)\frac{\partial D_{Bi}}{\partial p_{Bi}} = 0. \quad (29)$$

Solving for the markups in the two markets yields

$$p_s - c_s = (1 - d_s)\frac{Q_s}{-\partial Q_s/\partial p_s} \quad (30)$$

for each $s \in \{A, B\}$. The markups of a hypothetical downstream monopolist selling all $n \times 2$ final products remain unchanged, as given in (4). The conduct parameters thus become, for each $s \neq -s \in \{A, B\}$,

$$\widetilde{\theta}_s(p_s, p_{-s}) = (1 - R_{AB}R_{BA})\frac{(1 - d_s)\frac{Q_s}{-\partial Q_s/\partial p_s}}{\frac{Q_s}{-\partial Q_s/\partial p_s} + R_{s-s}\frac{Q_{-s}}{-\partial Q_{-s}/\partial p_{-s}}}. \quad (31)$$

With independent markets ($R_{AB} = R_{BA} = 0$), the conduct parameters are $\widetilde{\theta}_s = 1 - d_s$, as in the case of multi-market retailers. With interdependent markets, $\widetilde{\theta}_s(p_s, p_{-s}) < \theta_s(p_s, p_{-s})$, because single-market retailers do not take into account any of the competitive externalities on substitute products in the other market, whereas multi-market retailers internalize the effects on their own products in the other market. Comparing the conduct parameters across markets, we obtain a result analogous to Lemma 2: $\widetilde{\theta}_A(p_A, p_B) > \widetilde{\theta}_B(p_A, p_B)$ if and only if

$$\frac{\sum_{j\neq i}\frac{\partial D_{Aj}}{\partial p_{Ai}} + \sum_{j=1}^{n}\frac{\partial D_{Bj}}{\partial p_{Ai}}}{-\frac{\partial D_{Ai}}{\partial p_{Ai}}} < \frac{\sum_{j\neq i}\frac{\partial D_{Bj}}{\partial p_{Bi}} + \sum_{j=1}^{n}\frac{\partial D_{Aj}}{\partial p_{Bi}}}{-\frac{\partial D_{Bi}}{\partial p_{Bi}}}.$$

[24] This case holds if the channels are independent, $d_A = 0$, and $d_B > 0$.

The aggregate diversion ratio is now the sum of the diversion ratios to the $n \times 2 - 1$ competing retailers, as each retailer only considers the profits from the single product it sells.

Given the single-market firm conduct parameters $\widetilde{\theta}_A$ and $\widetilde{\theta}_B$, full channel coordination can then be achieved by means of discriminatory two part tariffs, with the tariff for retailers in market s featuring a wholesale price

$$w_s^* = \widetilde{\theta}_s^* c + \left(1 - \widetilde{\theta}_s^*\right) p_s^*,$$

where $\widetilde{\theta}_s^* = \widetilde{\theta}_s \left(p_s^*, p_{-s}^*\right)$, and a fixed fee $F = p_s^* Q_s \left(p_s^*, p_{-s}^*\right)$. Hence, as in the case of multi-market retailers, more intense competition in market s implies a higher optimal wholesale price in market s (for a given downstream monopoly price).

5 The welfare effects of input price discrimination by resale market

We now turn to the effects on social welfare (defined as the sum of consumer surplus and profits) of allowing tariffs that discriminate between resale in A and B. To this end, we will compare welfare when the supplier offers a dual two-part tariff to welfare when the supplier offers standard two-part tariff of the form $T(q_i) = F + wq_i$ to the retailers.[25] Given that the supplier can extract the retailers' profits through its fixed fees, it will seek to maximize total industry profit in both cases.

Before turning to the analysis, it is instructive to recap some key insights about the welfare effects of monopolistic third-degree price discrimination in final-goods markets (Pigou 1920; Robinson 1933; Schmalensee 1981; Varian 1985; Schwartz 1990; Aguirre, Cowan and Vickers 2010) and outline the main differences to our setting. There are two reasons for this. First, monopolistic third-degree price discrimination in final-goods markets corresponds to a limit case of our setting, that of independent markets and perfect competition between retailers in each market. With perfect competition downstream, retail prices are equal to wholesale prices, which implies that the supplier's choices of (wholesale and thus

[25] As shown in Appendix B, restricting attention to two-part tariffs is without loss of generality in both cases.

retail) prices will coincide with those of a monopoly seller serving two markets with demand functions $Q_A(p_A)$ and $Q_B(p_B)$. Hence, monopolistic third-degree price discrimination corresponds to the limit case of perfect competition in two independent markets of our setting, and existing results from the literature on monopolistic third-degree price discrimination apply to this case. Second, as in the literature on monopolistic price discrimination, the total welfare effect of dual wholesale pricing can be decomposed into an *allocation effect* and an *output effect*, and we can use some of the methods of analysis from that literature.

The key insights of the literature on monopolistic third-degree price discrimination in final-goods markets can be summarized as follows. First, price discrimination has a *negative allocation effect*, because when prices differ across markets output is not allocated to the consumers who value it most. This misallocation effect implies that third-degree price discrimination in final-goods markets lowers welfare unless it leads to a sufficiently large increase in the total output sold.[26] The sign of the *output effect*, in turn, has been to show to depend on the relative curvatures of the demand functions in the different markets (see Aguirre, Cowan and Vickers 2010). Notably, in the special case of linear demand (and, in the case of interdependent markets, symmetric cross-market effects), the output effect is zero, thus price discrimination decreases total welfare due to the misallocation effect.[27]

Although the welfare effect of dual wholesale pricing can also be decomposed into an allocation and an output effect, we will show that fundamental differences to monopolistic third-degree price discrimination in final-goods markets emerge with respect to both of these effects. The first part of the subsequent analysis will focus on the case of independent markets, which will facilitate comparisons to the literature on monopolistic price third-degree price discrimination and enable us to derive fairly general analytical results. We will then extend the welfare analysis to interdependent markets and illustrate that the main qualitative insights carry over.[28]

[26] Most of the literature assumes that all markets are equally costly to serve. If costs differ across markets, price discrimination also has a cost allocation effect. See Chen and Schwartz (2015) for an in-depth analysis of differential pricing when costs of service differs across groups.

[27] See Pigou 1920, Robinson 1933, and Schmalensee 1981 for the case of independent markets, and Layson 1992 for the case of interdependent markets.

[28] The literature on monopolistic price discrimination in final-goods markets has focused on independent markets, with the rare exception of Layson (1992). Unfortunately, however, Layson's analysis yields clear-cut predictions only for the case of linear demands with symmetric cross-market price effects, and the welfare analysis of dual wholesale pricing with interdependent market will face similar limitations.

5.1 Independent markets

5.1.1 Model and additional notation

With independent markets, the market demand function $Q_s(p_s, p_{-s})$ is independent of p_{-s}, so with a slight abuse of notation we will denote $Q_s(p_s) = Q_s(p_s, p_{-s})$. The price elasticity of the market demand function in s will be denoted by $\eta_s(p) = -p\frac{Q_s'(p)}{Q_s(p)}$, and the convexity (or curvature) of the inverse market demand in s by $\sigma_s(p) = Q_s\frac{Q_s''}{(Q_s')^2}$. The market demand function in s is log-concave if and only if $\sigma_s < 1$.

The equilibrium retail price in market s given the wholesale price w_s for resale in s is denoted by $p_s^e(w_s) = w_s + \theta_s\frac{-Q_s}{Q_s'}$ (see Lemma 1(ii)). By the definition of w_s^*, $p_s^e(w_s^*) = p_s^*$. Crucial roles will be played in the analysis by the competitive pass-through rates, which depend on conduct in the downstream markets as well as on demand curvatures:[29]

$$\rho_s(w_s) = \frac{\partial p_s^e(w_s)}{\partial w_s} = \frac{1}{1 + \theta_s(1-\sigma_s) + \frac{\partial \theta_s}{\partial p_s}\frac{-Q_s}{Q_s'}} > 0,$$

where $\theta_s = 1 - d_s$. In the case of monopoly conduct ($\theta_s = 1$), $\rho_s = \frac{1}{2-\sigma_s}$, and in the case of perfect competition ($\theta_s \to 0$), $\rho_s = 1$. Our discussion will focus on constant conduct parameters, so that

$$\rho_s = \frac{1}{1 + \theta_s(1-\sigma_s)},$$

but the results hold more generally as long as $\left|\frac{\partial \theta_s}{\partial p_s}\right|$ is not too large. We also assume that that the inverse market demand functions have constant curvature, so that the pass-through rates are constant in price. Many common functional forms in industrial organization, including the linear, exponential, and constant-elasticity forms in Bulow and Pfleiderer (1983)'s seminal paper about pass-through in monopoly, satisfy this assumption.[30]

The monopoly pass-through rate, $\rho_s = \frac{1}{2-\sigma_s}$, is less than 1 if and only if the market demand function is log-concave, i.e., $\sigma_s < 1$. Log-concavity of demand also determines how the competitive pass-through rate varies with the intensity of downstream competition, as captured by the conduct parameter. Given σ_s, the competitive pass-through rate ρ_s

[29] Applying the implicit function theorem to $p_s^e(w_s) = w_s + \theta_s\frac{-Q_s}{Q_s'}$ yields the expression for the pass-through rate. Positivity of the pass-through rate is implied by the stability condition for the downstream pricing game (see also Anderson, de Palma and Kreider 2001).

[30] See Weyl and Fabinger (2009, working paper) for a detailed discussion of the properties of pass-through rates for a wider range of functional forms.

is greater for lower θ_s (more intense competition) when the market demand function in s is log-concave, but greater for higher θ_s (less intense competition) if the market demand function is s is log-convex.

The industry profit generated in market s given a common price p_s is denoted by $\Pi_s(p_s) = n(p_s - c)\, Q_s(p_s)$. Symmetry between retailers and negative definiteness of the Hessian matrix of $\Pi(p_{A1}, p_{A2}, p_{B1}, p_{B2})$ imply that $\Pi_s'' < 0$. Note that $\Pi_s'' < 0$ implies that $2 - \sigma_s > 0$. Profits as a function of wholesale rather than retail prices will be denoted by

$$\pi_s(w_s) = \Pi_s(p_s^e(w_s)).$$

Given constant pass-through rates, $\Pi_s'' < 0$ implies that also $\pi_s'' = \rho_s^2 \Pi_s'' < 0$.

When the supplier can discriminate between resale in A versus B, it sets the optimal dual wholesale prices (w_A^*, w_B^*) so that $\pi_s'(w_s^*) = 0$ for each s. For the remainder of the analysis, suppose that

$$w_A^* < w_B^*,$$

which holds if and only if $(1 - \theta_A)(p_A^* - c) < (1 - \theta_B)(p_B^* - c)$. A sufficient condition for $w_A^* < w_B^*$ is monopoly conduct in A ($\theta_A = 1$) and some competition in B ($\theta_B < 1$).

When the supplier is constrained to setting a uniform wholesale price, it chooses the wholesale price $\overline{w}$ such that[31]

$$\pi_A'(\overline{w}) + \pi_B'(\overline{w}) = \rho_A \Pi_A'(p_A^e(\overline{w})) + \rho_B \Pi_B'(p_B^e(\overline{w})) = 0.$$

Concavity of each profit function and the first-order condition imply that $\overline{w} \in (w_A^*, w_B^*)$, $\pi_A'(\overline{w}) = \rho_A Q_A(p_A^e(\overline{w}))\left[1 - L_A(p_A^e(\overline{w}))\, \eta_A(p_A^e(\overline{w}))\right] < 0$, and $\pi_B'(\overline{w}) = \rho_B Q_B(p_B^e(\overline{w}))\left[1 - L_B(p_B^e(\overline{w}))\, \eta_B\right.$ 0. Hence, $L_B(p_B^e(\overline{w}))\, \eta_B(p_B^e(\overline{w})) < 1 < L_A(p_A^e(\overline{w}))\, \eta_A(p_A^e(\overline{w}))$.

5.1.2 Analysis

Adopting a method commonly used in the literature on third-degree price discrimination in final-goods markets,[32] we assume that the supplier chooses its wholesale prices subject to the constraint that $w_B - w_A \leq r$ where $r \geq 0$ is the degree of discrimination allowed. The objective function is $\pi_A(w_A) + \pi_B(w_A + r)$ and the first-order condition is $\pi_A'(w_A) +$

[31] We assume that both channels are served at the uniform wholesale price.

[32] See Schmalensee (1981), Holmes (1989), and Aguirre et al. (2010), among others.

$\pi'_B(w_A + r) = 0$ when the constraint binds. As r rises between 0 and $r^* = w_B^* - w_A^*$, the wholesale price in market A falls and the wholesale price in market B rises:

$$w'_A(r) = \frac{-\pi''_B}{\pi''_A + \pi''_B} < 0; w'_B(r) = \frac{\pi''_A}{\pi''_A + \pi''_B} > 0. \tag{32}$$

It follows that, as r rises, the retail price in market A falls and the retail price in market B rises:

$$\frac{dp_A^e}{dr} = \rho_A w'_A(r) < 0; \frac{dp_B^e}{dr} = \rho_B w'_B(r) > 0.$$

The marginal change in total social welfare as more differential wholesale pricing is allowed is

$$W'(r) = (p_A^e(w_A(r)) - c) Q'_A(p_A^e(w_A(r))) \rho_A w'_A(r) + (p_B^e(w_B(r)) - c) Q'_B(p_B^e(w_B(r))) \rho_B w'_B(r) \tag{33}$$

Denoting the marginal effect on total output by $Q'(r) = Q'_A(p_A^e(w_A(r))) \rho_A w'_A(r) + Q'_B(p_B^e(w_B(r))) \rho_B w'_B(r)$, (33) can be rewritten as

$$W'(r) = \underbrace{(p_A^e - p_B^e) Q'_A \frac{dp_A^e}{dr}}_{\text{allocation effect}} + \underbrace{(p_B^e - c) Q'(r)}_{\text{(value of) output effect}} \tag{34}$$

The first term represents the marginal allocation effect, and the second term the marginal output effect. Integrating (34) over $r \in [0, r^*]$ gives the total welfare effect of dual wholesale pricing. In what follows, we will first analyze the allocation and the output effect separately and then turn to the total welfare impact of dual wholesale pricing.

Allocation effect It is evident from (34) that dual wholesale pricing has a positive marginal allocation effect if and only if $p_A^e(w_A(r)) > p_B^e(w_B(r))$. Since the retail price in market A falls with r and the retail price in market B rises with r, we have $p_A^e(w_A(r)) \geq p_A^e(w_A^*) = p_A^*$ and $p_B^e(w_B(r)) \leq p_B^e(w_B^*) = p_B^*$ for all $r \in [0, r^*]$. It follows that the marginal allocation effect is positive for all r if $p_A^* \geq p_B^*$, because a rise in r brings the retail prices closer together. Hence, $p_A^* \geq p_B^*$ ensures that the total allocation effect, obtained by integrating the first term in (34) over $r \in [0, r^*]$, is positive. If $p_A^e(\overline{w}) > p_B^e(\overline{w})$ but $p_A^* < p_B^*$, then the marginal allocation effect is first positive and then negative as r rises, making the sign of the total allocation effect ambiguous. If $p_A^e(\overline{w}) \leq p_B^e(\overline{w})$, then $p_A^e(w_A(r)) \leq p_B^e(w_B(r))$ for all $r \in [0, r^*]$, hence the marginal allocation effect is negative throughout and the total allocation effect is negative. The next lemma summarizes these observations:

Lemma 3 *Suppose $w_A^* < w_B^*$. (i) Dual wholesale pricing has a positive allocation effect if $p_A^* \geq p_B^*$. (ii) Dual wholesale pricing has a negative allocation effect if $p_A^e(\overline{w}) \leq p_B^e(\overline{w})$. (iii) Dual wholesale pricing can have a positive or a negative allocation effect if $p_A^e(\overline{w}) > p_B^e(\overline{w})$ and $p_A^* < p_B^*$.*

The relative competitive conducts in the two markets play important roles for the ranking of the retail prices under a uniform wholesale price $\overline{w}$. Given the market demand function, as conduct in s varies between perfect competition and monopoly, the retail price $p_s^e(\overline{w})$ varies between $\overline{w}$ and $\arg\max\left(p_s - \overline{w}\right) Q_s\left(p_s\right) > p_s^*$.[33] Hence, if conduct in market B is close to perfect competition and conduct in market A is close to monopoly, then $p_A^e(\overline{w}) > p_B^e(\overline{w})$. Since conduct in B close to perfect competition and conduct in A close to monopoly also guarantees that $w_A^* < w_B^*$, it follows that either case (i) or case (iii) of Lemma 3 applies when the two markets are sufficiently asymmetric in terms of competitive conduct. Thus, either the marginal allocation effect is positive throughout, or the marginal allocation effect is first positive and then negative as more discrimination is allowed, if the markets are sufficiently asymmetric in terms of competitive conduct.

Combining the allocation and the output effect to obtain the total welfare effect of dual wholesale pricing, part (i) of Lemma 3 implies the following:

Remark 1 *If $p_A^* \geq p_B^*$ and dual wholesale pricing increases output, then dual wholesale pricing increases welfare.*

Output effect Using the comparative statics formulae for the wholesale prices in (32), the marginal output effect becomes

$$\begin{aligned} Q'\left(r\right) &= Q_A'\left(p_A^e\left(w_A\left(r\right)\right)\right)\rho_A \frac{-\pi_B''}{\pi_A''+\pi_B''} + Q_B'\left(p_B^e\left(w_B\left(r\right)\right)\right)\rho_B \frac{\pi_A''}{\pi_A''+\pi_B''} \\ &= \frac{-\rho_A\rho_B Q_A' Q_B'}{\pi_A''+\pi_B''}\left(\rho_B\left[2 - L_B\eta_B\sigma_B\right] - \rho_A\left[2 - L_A\eta_A\sigma_A\right]\right), \end{aligned}$$

which has the sign of

$$\rho_B\left[2 - L_B\eta_B\sigma_B\right] - \rho_A\left[2 - L_A\eta_A\sigma_A\right] = \frac{2 - L_B\eta_B\sigma_B}{1+\theta_B\left(1-\sigma_B\right)} - \frac{2 - L_A\eta_A\sigma_A}{1+\theta_A\left(1-\sigma_A\right)}. \tag{35}$$

[33] The latter inequality is implied by $\overline{w} > c$ and $p_s^* = \arg\max\left(p_s - c\right) Q_s\left(p_s\right)$.

Because $p_A^e(w_A(r)) > p_A^*$ and $p_B^e(w_B(r)) < p_B^*$ for all $r \in [0, r^*)$, concavity of the market profit functions implies that $L_A\eta_A > 1 > L_B\eta_B$ for all $r \in [0, r^*)$. Hence, a sufficient condition for $L_B\eta_B\sigma_B \leq L_A\eta_A\sigma_A$ throughout the range of relevant prices is that $\sigma_A \geq \sigma_B \geq 0$, and a sufficient condition for $L_B\eta_B\sigma_B \geq L_A\eta_A\sigma_A$ throughout the range of relevant prices is that $0 \geq \sigma_B \geq \sigma_A$. Given this, it is evident from the first expression in (35) that output increases with dual wholesale pricing if $\sigma_A \geq \sigma_B \geq 0$ and $\rho_B \geq \rho_A$ (strictly if at least one inequality is strict). Similarly, output decreases with dual wholesale pricing if $0 \geq \sigma_B \geq \sigma_A$ and $\rho_B \leq \rho_A$ (strictly if at least one inequality is strict). The next lemma summarizes these observations:

Lemma 4 *Suppose $w_A^* < w_B^*$.*

(a) *Output increases with dual wholesale pricing if (i) inverse market demands are convex and more so in market A ($\sigma_A \geq \sigma_B \geq 0$), and (ii) the competitive pass-through rate is higher in market B than in market A ($\rho_B \geq \rho_A$).*

(b) *Output decreases with dual wholesale pricing if (i) inverse market demands are concave and more so in market A ($0 \geq \sigma_B \geq \sigma_A$), and (ii) the competitive pass-through rate is lower in market B than in market A ($\rho_B \leq \rho_A$).*

As discussed earlier, in the limit case of perfect competition in each market, dual wholesale pricing is equivalent to monopolistic third-degree price discrimination where A is the 'weak' market featuring a monpopoly price $p_A^* = w_A^*$ and B the 'strong' market featuring a monopoly price $p_B^* = w_B^* > p_A^*$. The pass-through rates are $\rho_A = \rho_B = 1$ in this limit case, so condition (ii) is satisfied (with equality) in part (a) as well as in part (b) of the lemma. Thus, $\sigma_A \geq \sigma_B \geq 0$ is a sufficient condition for a positive output effect, and $0 \geq \sigma_B \geq \sigma_A$ a sufficient condition for a negative output effect, in this case, which replicates the insights in Proposition 4(iii) and (iv) in Aguirre et al..[34]

[34]The remaining parts of Proposition 4 in Aguirre et al. (2010) contain conditions about the curvature of the direct demand functions. To derive analog conditions in our setting, denote the convexity (or curvature) of the direct channel demand in s as $\alpha_s(p) = -p\frac{Q_s''(p)}{Q_s'(p)}$. The curvature of the direct demand function is related to that of the indirect demand function by $\alpha_s = \eta_s\sigma_s$. Given this, it follows from (35) that output increases with dual wholesale pricing if the channel demand is concave in B but convex in A ($\alpha_A \geq 0 \geq \alpha_B$) and $\rho_B \geq \rho_A$, and that output decreases with dual wholesale pricing if the channel demand is convex in B but concave in A ($\alpha_A \leq 0 \leq \alpha_B$) and $\rho_B \leq \rho_A$. The two additional cases in Proposition 4

With less than perfect competition in one or both markets, the competitive pass-through rates are generally asymmetric across markets and the relative pass-through rates play an important role for the output effect. From the first expression in (35), it can be seen that $Q'(r) > 0$ if and only if the ratio of the pass-through rates

$$\frac{\rho_B}{\rho_A} > \frac{2 - L_A \eta_A \sigma_A}{2 - L_B \eta_B \sigma_B}, \tag{36}$$

where the right-hand-side is evaluated at $(p_A^e(w_A(r)), p_B^e(w_B(r)))$. Intuitively, a high pass-through rate in market B relative to market A implies that the optimal uniform wholesale price $\overline{w}$ is close to w_B^*, as moving away from w_B^* implies a greater distortion in retail prices away from the integrated monopoly outcome than moving away from w_A^*. A high $\overline{w}$, in turn, depresses the total output under a uniform wholesale price, against which output under dual wholesale pricing is compared to compute the output effect.

The competitive pass-through rates depend on the curvatures of the market demand functions and on the intensity of downstream competition in each of the two markets, with

$$\rho_B \geq \rho_A \text{ if and only if } \theta_B (1 - \sigma_B) \leq \theta_A (1 - \sigma_A).$$

Whether the market with more intense downstream competition has a higher or a lower pass-through depends critically on whether the market demand functions are log-concave or log-convex. For $\sigma_A = \sigma_B < 1$, market B has a higher pass-through rate than A if and only if competition is more intense in B than in A $(\theta_A \geq \theta_B)$. Conversely, for $\sigma_A = \sigma_B > 1$, market B has a higher pass-through rate than A if and only if competition is less intense in B than in A $(\theta_A \leq \theta_B)$. Part (a) of Lemma 4 thus implies that output increases with dual wholesale pricing if $0 \leq \sigma_A = \sigma_B < 1$ and $\theta_A \geq \theta_B$ (strictly if at least one of the weak inequalities is strict). Conversely, for $\sigma_A = \sigma_B > 1$, part (a) of Lemma 4 implies that output increases with dual wholesale pricing if $\theta_A \leq \theta_B$. And for $\sigma_A = \sigma_B \leq 0$, part (b) of Lemma 4 implies that output decreases with dual wholesale pricing if $\theta_A \leq \theta_B$ (strictly if at least one of the weak inequalities is strict).[35]

of Aguirre et al. (2010) are direct demand that is concave in both markets but less so in the weak market, which implies a positive output effect; and demand that is convex in both markets but less so in the weak market, which implies a negative output effect. The results for these two cases can be obtained by noting that $p_A^e(\overline{w}) = p_B^e(\overline{w}) = \overline{w}$ in the special case of perfect competition in each channel, which implies that $L_A(r) \leq L_B(r)$ for all $r \in [0, r^*]$.

[35] Recall, however, that $w_A^* < w_B^*$ if and only if $(1 - \theta_A)(p_A^* - c) < (1 - \theta_B)(p_B^* - c)$, which restricts the extent to which θ_B can exceed θ_A, if at all. In contrast, $w_A^* < w_B^*$ always holds if θ_A is close enough to 1, i.e., if conduct in channel A is close enough to monopoly, and $\theta_B \neq 1$.

In the case of linear demands, which satisfies log-concavity and lies at the boundary of the two cases in Lemma 4, output strictly increases with dual wholesale pricing if and only if downstream competition is more intense in market B (i.e., $\theta_A > \theta_B$). This insight contrasts sharply with the well-known result that monopolistic third-degree price discrimination in final-goods markets has no impact on total output when demands are linear (Pigou 1920).

Further progress in cases where $\sigma_A \neq \sigma_B$ can be made by noting that if the inverse market demand functions are convex ($\sigma_s \geq 0$), then, for any $r \in [0, r^*]$, the expression in (35), which determines the sign of $Q'(r)$, is bounded below by

$$\frac{2 - \sigma_B}{1 + \theta_B (1 - \sigma_B)} - \frac{2 - \sigma_A}{1 + \theta_A (1 - \sigma_A)}, \tag{37}$$

because $L_A \eta_A \geq 1 \geq L_B \eta_B$ for $r \in [0, r^*]$. The sum in (37) is decreasing in σ_B and increasing in σ_A. Moreover, it is decreasing in θ_B if and only if $\sigma_B < 1$ and increasing in θ_A if and only if $\sigma_A < 1$. In the limit case of monopoly conduct in A ($\theta_A = 1$) and perfect competition in B ($\theta_B = 0$), (37) simplifies to $1 - \sigma_B$, which exceeds zero if $\sigma_B < 1$. Together these observations imply the following sufficient conditions for a positive output effect:[36]

Proposition 3 *Suppose that $w_A^* < w_B^*$. (a) Output increases with dual wholesale pricing if $0 \leq \sigma_B \leq \sigma_A < 1$ and competition is more intense in market B than in market A ($\theta_B \leq \theta_A$). (b) Output increases with dual wholesale pricing if $0 \leq \sigma_A < \sigma_B < 1$ and competition is sufficiently more intense in market B than in market A (i.e., $\theta_A - \theta_B$ is close enough to 1).*

Proposition 3 implies that, given $\sigma_s \in [0, 1)$ in both markets, dual wholesale pricing raises output if the two markets are sufficiently asymmetric in terms of downstream competitive conduct.[37] With sufficiently asymmetric conduct, the supplier wants to dis-

[36] Using a similar reasoning, it can also be shown that (i) output increases with dual wholesale pricing if $1 \leq \sigma_B \leq \sigma_A$, $\theta_B \geq \theta_A$, and $w_A^* < w_B^*$, and (ii) output decreases with dual wholesale pricing if $0 \geq \sigma_B \geq \sigma_A$, $\theta_B \geq \theta_A$, and $w_A^* < w_B^*$. Proposition 3 focuses instead on *discrimination against resale in the more competitive channel*, i.e., on cases in which $\theta_B \leq \theta_A$ and $w_A^* < w_B^*$. There are two reasons for this. First, the condition $w_A^* < w_B^*$, which is equivalent to $(1 - \theta_A) p_A^* < (1 - \theta_B) p_A^*$, imposes limitations on the extent to which θ_B can exceed θ_A, if at all. Second, in our lead example of discrimination against resale in the online channel, we propose that discrimination is driven by the greater intensity of between-retailer competition in the online channel, which corresponds to a lower conduct parameter in the channel with the higher dual wholesale price.

[37] It is worth emphasizing that the results in Proposition 3 are *not* implied by those in Lemma 4(a). In part (a) of Proposition 3, condition (i) of Lemma 4(a) is satisfied, but condition (ii) of Lemma 4(a) can be

criminate against the more competitive market and such discrimination raises total output for $\sigma_s \in [0,1)$.

Total welfare effect Using the comparative statics formulae for the wholesale prices in (32), the marginal welfare effect can be written as

$$W'(r) = \underbrace{\frac{-\pi''_A\pi''_B}{\pi''_A+\pi''_B}}_{>0}\left[\rho_A\frac{(p^e_A(w_A(r))-c)\,Q'_A(p^e_A(w_A(r)))}{\pi''_A} - \rho_B\frac{(p^e_B(w_B(r))-c)\,Q'_B(p^e_B(w_B(r)))}{\pi''_B}\right], \tag{38}$$

which has the same sign as

$$\frac{\rho_A(p^e_A-c)\,Q'_A}{\rho_A^2(2Q'_A+(p^e_A-c)\,Q''_A)} - \frac{\rho_B(p^e_B-c)\,Q'_B}{\rho_B^2(2Q'_B+(p^e_B-c)\,Q''_B)} = \frac{p^e_A-c}{\rho_A(2-L_A\eta_A\sigma_A)} - \frac{p^e_B-c}{\rho_B(2-L_B\eta_B\sigma_B)}. \tag{39}$$

The second expression in (39) relates readily to our discussions of the allocation effect and the output effect. The numerator in the first term exceeds that in the second term if and only if the marginal allocation effect is positive. Conversely, the denominator of the first term lies below that of the second if and only if the marginal output effect is positive.

Our next two proposition make use of the following assumption, which we borrow from Aguirre et al.'s (2010) analysis of monopolistic third-degree price discrimination:

Increasing ratio condition (Aguirre et al. 2010) $\frac{p_s-c}{2-L(p_s)\eta_s(p_s)\sigma_s}$ is increasing in p_s.

As discussed in Aguirre et al. (2010), the increasing ratio condition (henceforth IRC) holds for a large set of demand functions. Specifically, it holds for all inverse market demand functions with constant $\sigma_s \geq 0$, which includes linear, exponential, and constant-elasticity functions. It also holds for $\sigma_s < 0$ if prices are close enough to costs.

The increasing ratio is useful because it implies that $W'(r) \geq 0$ for all $r \in [0, r^*]$ if $W'(r^*) \geq 0$, which leads to the following sufficient condition for dual wholesale pricing to raise welfare:

Proposition 4 *Suppose $w^*_A < w^*_B$. Given IRC, dual wholesale pricing raises welfare if*

$$\frac{p^*_A-c}{\rho_A(2-\sigma_A)} \geq \frac{p^*_B-c}{\rho_B(2-\sigma_B)}. \tag{40}$$

violated because $\rho_B < \rho_A$ for $\theta_B = \theta_A$ when $\sigma_B < \sigma_A < 1$. In part (b) of Proposition 3, condition (i) of Lemma 4(a) is violated because the ranking of the convexities of the inverse channel demand functions is reversed.

As shown by Aguirre et al. (2010) (Proposition 2), when $p_A^* < p_B^*$ and IRC holds, monopolistic price discrimination in the final-goods markets raises welfare if

$$\frac{p_A^* - c}{2 - \sigma_A} \geq \frac{p_B^* - c}{2 - \sigma_B}. \tag{41}$$

The sufficient condition for dual wholesale pricing to raise welfare when $w_A^* < w_B^*$ is similar to (41) except that each side of the inequality gets divided by the competitive pass-through rate in the respective market. Echoing the analysis of the output effect, if the pass-through rate is higher in the market that is being discriminated against than in the other market, the sufficient condition for dual wholesale pricing to raises welfare, (40), is less stringent than the sufficient condition for monopolistic price discrimination to raise welfare, (41).

To link the sufficient condition for a welfare increase to competitive conducts, we use the expressions for the pass-through rates to rewrite (40) as

$$\frac{\frac{2-\sigma_B}{1+\theta_B(1-\sigma_B)}}{\frac{2-\sigma_A}{1+\theta_A(1-\sigma_A)}} \geq \frac{p_B^* - c}{p_A^* - c}. \tag{42}$$

If the market demand functions are log-concave, so that more intense competition implies a higher pass-through rate, the left-hand side of this condition is decreasing in θ_B and increasing in θ_A. Evaluating (42) in the limit case $(\theta_A = 1, \theta_B = 0)$ yields the following result:

Proposition 5 *Suppose $w_A^* < w_B^*$. Given IRC and log-concave market demand functions, dual wholesale pricing raises welfare if (i) the intensity of competition in market B is sufficiently high relative to the intensity of competition in market A, and (ii) $(2 - \sigma_B)(p_A^* - c) > p_B^* - c$ (i.e., p_A^* does not fall too far below p_B^*).*

The result in Proposition 5 can be summarized as follows. With constant inverse market demand curvatures between zero and one (so that IRC holds and market demands are log-concave), dual wholesale pricing raises social welfare if

- the markets are sufficiently asymmetric in terms of competitive conduct (so that the supplier wants to discriminate against resale in the more competitive market and condition (i) in Proposition 5 holds), and

- the monopoly retail price in the less competitive downstream market does not fall too far below the monopoly retail price in the more competitive downstream market.

Remark: Single-market retailers The results in this section are also relevant for settings with symmetric single-market retailers in each downstream market. With independent markets, the conduct parameter is $1-d_s$ and the retail price $p_s^e(w_s)$ in market s, regardless of whether the $n \times 2$ final products are sold by multi-market or single-market retailers. Interpreting a ban on discrimination as a ban on discriminatory wholesale prices (but not fixed fees) in two-part tariffs, the supplier would choose the same uniform wholesale price $\overline{w}$ and the same discriminatory wholesale prices (w_A^*, w_B^*) in a setting with $n \times 2$ single-market retailers as in our setting with n multi-market retailers. Hence, the effects of allowing discriminatory wholesale prices in two-part tariffs when selling to retailers in different markets depend on the relative competitive conducts in the two markets and on the market demand functions as shown above.

5.2 Interdependent markets

With interdependent markets, we denote the equilibrium retail prices given the pair of wholesale prices (w_A, w_B) by $p_A^e(w_A, w_B)$ and $p_B^e(w_A, w_B)$. The pass-through rate from the wholesale price w_t $(t = A, B)$ on the equilibrium retail price in market s is denoted by ρ_t^s:

$$\rho_A^A = \frac{\partial p_A^e}{\partial w_A}, \rho_B^A = \frac{\partial p_A^e}{\partial w_B}, \rho_A^B = \frac{\partial p_B^e}{\partial w_A}, \rho_B^B = \frac{\partial p_B^e}{\partial w_B}.$$

Total industry profits as a function of the wholesale prices are denoted by

$$\pi^w(w_A, w_B) = \Pi\left(p_A^e(w_A, w_B), ..., p_A^e(w_A, w_B), p_B^e(w_A, w_B), ..., p_B^e(w_A, w_B)\right).$$

Subscripts will be used to denote derivatives, e.g., $\pi_A^w = \frac{\partial \pi}{\partial w_A}$ and $\pi_{AB}^w = \frac{\partial^2 \pi}{\partial w_A \partial w_B}$.

In what follows, continue to assume that $w_A^* < w_B^*$. Adopting the same method of analyzing marginal effects as in the case of independent markets, we assume that the supplier chooses its wholesale prices subject to the constraint that $w_B - w_A \leq r$ where $r \geq 0$ is the degree of discrimination allowed. The objective function is $\pi^w(w_A, w_A + r)$ and the first-order condition is $\pi_A^w(w_A, w_A + r) + \pi_B^w(w_A, w_A + r) = 0$ when the constraint is binding. We assume that the second-order condition, $\pi_{AA}^w + 2\pi_{AB}^w + \pi_{BB}^w < 0$, is satisfied.[38] For

[38] The second-order condition holds if and only if

$$\left(\rho_A^A + \rho_B^A\right)^2 \Pi_{AA} + 2\left(\rho_A^A + \rho_B^A\right)\left(\rho_B^B + \rho_A^B\right)\Pi_{AB} + \left(\rho_B^B + \rho_A^B\right)^2 \Pi_{BB} < 0.$$

It is satisfied in the linear-demand example used later on.

$r \geq r^* = w_B^* - w_A^*$, the constraint does not bind and the supplier can achieve full market coordination. For $r \in (0, r^*)$,[39]

$$w_A'(r) = -\frac{\pi_{AB}^w + \pi_{BB}^w}{\pi_{AA}^w + 2\pi_{AB}^w + \pi_{BB}^w}, w_B'(r) = \frac{\pi_{AA}^w + \pi_{AB}^w}{\pi_{AA}^w + 2\pi_{AB}^w + \pi_{BB}^w}.$$

The marginal effects on the retail prices follow from the marginal effects on the wholesale prices and the pass-through rates:

$$\begin{aligned} \frac{dp_A^e}{dr} &= \rho_A^A w_A'(r) + \rho_B^A w_B'(r), \\ \frac{dp_B^e}{dr} &= \rho_A^B w_A'(r) + \rho_B^B w_B'(r). \end{aligned}$$

The marginal change in social welfare as more wholesale price discrimination is allowed is

$$W'(r) = (p_A^e(w_A(r), w_B(r)) - c) Q_A'(r) + (p_B^e(w_A(r), w_B(r)) - c) Q_B'(r),$$

where $Q_s'(r)$ is the marginal effect on total output in market $s \in \{A, B\}$. Denoting the marginal effect on total output by $Q'(r) = Q_A'(r) + Q_B'(r)$, the marginal effect on social welfare can again be decomposed into an allocation and an output effect:

$$W'(r) = \underbrace{(p_A^e - p_B^e) Q_A'(r)}_{\text{allocation effect}} + \underbrace{(p_B^e - c) Q'(r)}_{\text{(value of) output effect}}$$

The sign of the first term is determined by the change in output in market A and the relative retail prices. If $\pi_{AB}^w + \pi_{BB}^w < 0$ and $\pi_{AA}^w + \pi_{AB}^w < 0$, so that $\frac{dp_A^e}{dr} < 0 < \frac{dp_B^e}{dr}$ for $r \in (0, r^*)$, then $Q_A'(r) > 0$. As in the case of independent markets, the marginal consumption allocation effect is then positive if and only if $p_A^e > p_B^e$ and the insights from Lemma 3 about the sign of the allocation effect apply.

To gain further insights into the sign of the output effect, we write the marginal output

[39] It is worth noting that $w_A^* < w_B^*$ does not necessarily imply that $w_A'(r) < 0 < w_B'(r)$. The wholesale price in market A decreases and that in B increases as r rises if and only if $\pi_{AB}^w + \pi_{BB}^w < 0$ and $\pi_{AA}^w + \pi_{AB}^w < 0$. If the markets are asymmetric and $\pi_{AB}^w > 0$, then one of these conditions may be violated, in which case either both wholesale prices increase or both wholesale prices decrease. (See Layson 1992 for a related discussion of the direction of price changes in the context of third-degree price discrimination in final-goods markets with interdependent markets.) Our analysis will focus on the more common case in which $w_A'(r) < 0 < w_B'(r)$ for $r \in (0, r^*)$.

effect as follows:[40]

$$Q'(r) = \left[\left(\frac{\partial Q_A}{\partial p_A}+\frac{\partial Q_B}{\partial p_A}\right)\rho_A^A+\left(\frac{\partial Q_A}{\partial p_B}+\frac{\partial Q_B}{\partial p_B}\right)\rho_A^B\right]w_A'(r) + \left[\left(\frac{\partial Q_A}{\partial p_A}+\frac{\partial Q_B}{\partial p_A}\right)\rho_B^A+\left(\frac{\partial Q_A}{\partial p_B}+\frac{\partial Q_B}{\partial p_B}\right)\rho_B^B\right]w_B'(r). \tag{43}$$

Using the expression for $w_A'(r)$ and $w_B'(r)$ and assuming that demands are linear with symmetric cross-market effects ($\frac{\partial Q_A}{\partial p_B}=\frac{\partial Q_B}{\partial p_A}$), the marginal output effect can be simplified to the following expression:

$$Q'(r) = \underbrace{\frac{\left(\rho_A^A\rho_B^B-\rho_A^B\rho_B^A\right)\left(\frac{\partial Q_A}{\partial p_A}\frac{\partial Q_B}{\partial p_B}-\frac{\partial Q_A}{\partial p_B}\frac{\partial Q_B}{\partial p_A}\right)}{\Pi_{AA}^w+2\Pi_{AB}^w+\Pi_{BB}^w}}_{<0}\left[\left(\rho_A^A+\rho_B^A\right)-\left(\rho_B^B+\rho_A^B\right)\right]. \tag{44}$$

Since the first term in (44) is negative, $Q'(r)>0$ in the case of linear demands and symmetric cross-market effects if and only if

$$-\left(\rho_A^B-\rho_A^A\right)<\rho_B^B-\rho_B^A. \tag{45}$$

This condition can be interpreted as requiring that an increase in w_B has a greater positive impact on $p_B^e-p_A^e$ than a decrease in w_A of the same magnitude. As in the case of independent markets, the intuition is that the optimal *uniform* wholesale price for the supplier is close to w_B^* (and thus high) if $\rho_B^B-\rho_B^A$ is large relative to $\rho_A^A-\rho_A^B$, which (all else equal) implies a low total output with a uniform wholesale price.

The marginal welfare effect can be written as

$$W'(r) = \left[(p_A^e-c)\left(\frac{\partial Q_A}{\partial p_A}\rho_A^A+\frac{\partial Q_A}{\partial p_B}\rho_A^B\right)+(p_B^e-c)\left(\frac{\partial Q_B}{\partial p_A}\rho_A^A+\frac{\partial Q_B}{\partial p_B}\rho_A^B\right)\right]w_A'(r) + \left[(p_A^e-c)\left(\frac{\partial Q_A}{\partial p_A}\rho_B^A+\frac{\partial Q_A}{\partial p_B}\rho_B^B\right)+(p_B^e-c)\left(\frac{\partial Q_B}{\partial p_A}\rho_B^A+\frac{\partial Q_B}{\partial p_B}\rho_B^B\right)\right]w_B'(r).$$

In the case of linear demand with symmetric cross-market effects ($\frac{\partial Q_A}{\partial p_B}=\frac{\partial Q_B}{\partial p_A}$), the marginal welfare effect simplifies to

$$W'(r) = \underbrace{\frac{\left(\rho_A^A\rho_B^B-\rho_A^B\rho_B^A\right)\left(\frac{\partial Q_A}{\partial p_A}\frac{\partial Q_B}{\partial p_B}-\frac{\partial Q_A}{\partial p_B}\frac{\partial Q_B}{\partial p_A}\right)}{\pi_{AA}^w+2\pi_{AB}^w+\pi_{BB}^w}}_{<0}\left[\left(\rho_A^A+\rho_B^A\right)(p_B^e-c)-\left(\rho_B^B+\rho_A^B\right)(p_A^e-c)\right], \tag{46}$$

[40] To simplify, we assume again that the pass-through rates are constant in price.

which has the sign of

$$\frac{p_A^e - c}{\rho_A^A + \rho_B^A} - \frac{p_B^e - c}{\rho_B^B + \rho_A^B}. \tag{47}$$

The numerator in the first terms exceeds that in the second term if and only if the marginal allocation is positive, while the denominator of the first term lies below that of the second terms of and only if the marginal output effect is postitive. Moreover, if (47) holds for r^*, that is, if[41]

$$\frac{p_A^* - c}{\rho_A^A + \rho_B^A} > \frac{p_B^* - c}{\rho_B^B + \rho_A^B}, \tag{48}$$

then $W'(r) > 0$ for all r. (48) generalizes the sufficient condition for a welfare increase in the case of independent markets and linear demands, $\frac{p_A^* - c}{\rho_A} > \frac{p_B^* - c}{\rho_B}$, to the case of interdependent markets where each wholesale price affects the retail prices in both markets.

Quadratic-utility linear-demand example To illustrate how asymmetries in the intensity of competition between the two markets can affect the pass-through rates and the welfare effects of dual wholesale pricing in the case of interdependent markets, suppose that there are two retailers and that the representative consumer has the following quadratic utility function:

$$\begin{aligned} U(q_{A1}, q_{A2}, q_{B1}, q_{B2}) &= \alpha_A (q_{A1} + q_{A2}) - \frac{\beta_A}{2}\left(q_{A1}^2 + q_{A2}^2\right) - \gamma_A q_{A1} q_{A2} \\ &\quad + \alpha_B (q_{B1} + q_{B2}) - \frac{\beta_B}{2}\left(q_{B1}^2 + q_{B2}^2\right) - \gamma_B q_{B1} q_{B2} \\ &\quad - \gamma_C (q_{A1} q_{B1} + q_{A2} q_{B2}) - \gamma_D (q_{A1} q_{B2} + q_{A2} q_{B1}). \end{aligned}$$

The parameter γ_s measures the degree of intra-market retailer substitutability in market s, γ_C measures the degree of intra-retailer cross-market substitutability, and γ_D measures the degree of cross-retailer cross-market substitutability. We assume that $\beta_A > \gamma_A > \gamma_C$, $\beta_B > \gamma_B > \gamma_C$, $\gamma_C \geq \gamma_D \geq 0$, and $(\beta_A - \gamma_A)(\beta_B - \gamma_B) > (\gamma_C - \gamma_D)^2$. These assumptions are sufficient to ensure that the Hessian of the utility functions is (strictly) negative definite. The special case of independent markets corresponds to $\gamma_C = \gamma_D = 0$.

Maxmizing the utility function subject to a budget constraint yields the inverse demand functions:

$$P_{si}(q_{si}, q_{s-i}, q_{-si}, q_{-s-i}) = \frac{\partial U}{\partial q_{si}} = \alpha_s - \beta_s q_{si} - \gamma_s q_{s-i} - \gamma_C q_{-si} - \gamma_D q_{-s-i}.$$

[41] This follows because with linear demand functions p_A^e decreases and p_B^e increases with r.

Direct demands in the region where all four demands are positive are given by

$$D_{si} = a_s - b_s p_{si} + c_s p_{s-i} + h p_{-si} + g p_{-s-i},$$

where $a_s = \frac{\alpha_s(\beta_{-s}+\gamma_{-s})}{\Delta_-} - \frac{\alpha_{-s}(\beta_s+\gamma_s)}{\Delta_+}$, $b_s = \frac{1}{2}\left(\frac{\beta_{-s}-\gamma_{-s}}{\Delta_-} + \frac{\beta_{-s}+\gamma_{-s}}{\Delta_+}\right)$, $c_s = \frac{1}{2}\left(\frac{\beta_{-s}-\gamma_{-s}}{\Delta_-} - \frac{\beta_{-s}+\gamma_{-s}}{\Delta_+}\right)$, $h = \frac{1}{2}\left(\frac{\gamma_C-\gamma_D}{\Delta_-} + \frac{\gamma_C+\gamma_D}{\Delta_+}\right)$, $g = \frac{1}{2}\left(-\frac{\gamma_C-\gamma_D}{\Delta_-} + \frac{\gamma_C+\gamma_D}{\Delta_+}\right)$, $\Delta_- = (\beta_A - \gamma_A)(\beta_B - \gamma_B) - (\gamma_C - \gamma_D)^2$, and $\Delta_+ = (\beta_A + \gamma_A)(\beta_B + \gamma_B) - (\gamma_C + \gamma_D)^2$. Our parameter assumptions imply that $b_s > c_s > 0$ and $h > 0$. Moreover, there exists some $\widehat{\gamma} \in (0, \gamma_C)$ such that $g > 0$ if and only if $\gamma_D > \widehat{\gamma}$.

Solving for the prices that maximize total industry profits and for the conduct parameters yields

$$p_s^* = \frac{\alpha_s + c}{2}, \tag{49}$$

and

$$\theta_s = \frac{(\alpha_s - c)\left(\beta_{-s}(\beta_s - \gamma_s) - \gamma_C(\gamma_C - \gamma_D)\right) + (\alpha_{-s} - c)(\beta_s\gamma_D - \gamma_s\gamma_C)}{(\alpha_s - c)(\beta_A\beta_B - \gamma_C^2)}. \tag{50}$$

To isolate the role played by differences in between-retailer substitutability between the two markets, suppose from now onwards that

$$\gamma_B > \gamma_A,$$

but that the markets are symmetric otherwise: $\alpha_A = \alpha_B$ and $\beta_A = \beta_B$. We then have that $p_A^* = p_B^*$ and

$$\theta_A = \frac{\beta + \gamma_C - (\gamma_A + \gamma_D)}{\beta + \gamma_C} > \theta_B = \frac{\beta + \gamma_C - (\gamma_B + \gamma_D)}{\beta + \gamma_C},$$

which together imply that

$$w_A^* < w_B^*.$$

Moreover, w_A and p_A decrease while w_B and p_B increase when the supplier adopts dual wholesale prices, as is easy to check. Together, these observations imply that dual wholesale pricing motivated by asymmetries in the intra-market substitution parameters has a *positive allocation effect*.

To evaluate the output effect, we compute the pass-through rates:

$$\rho_s^s = \frac{\beta\left(2\beta - \gamma_{-s}\right) - \gamma_C(2\gamma_C - \gamma_D)}{(2\beta - \gamma_A)(2\beta - \gamma_B) - (2\gamma_C - \gamma_D)^2}, \tag{51}$$

$$\rho_t^s = \frac{\gamma_s\gamma_C - \beta\gamma_D}{(2\beta - \gamma_A)(2\beta - \gamma_B) - (2\gamma_C - \gamma_D)^2}. \tag{52}$$

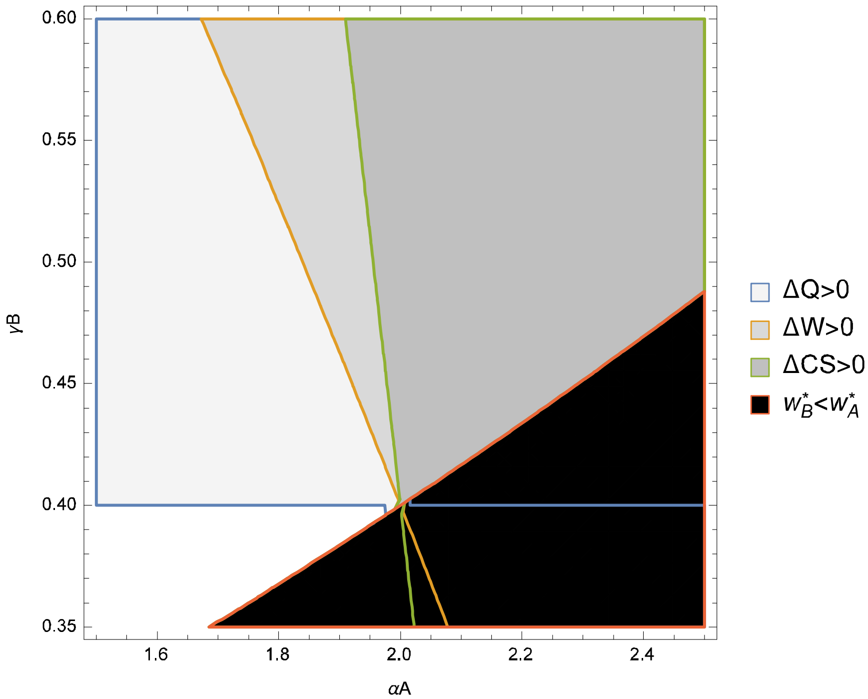

Figure 1: Output, welfare and consumer surplus effects of dual pricing ($\alpha_B = 2, \gamma_A = 0.4, \gamma_C = 0.2, \gamma_D = 0.18, c = 0$)

It is easy to check that $\gamma_B > \gamma_A$ implies $\rho_B^B + \rho_A^B > \rho_A^A + \rho_B^A$. Hence, using our earlier insights linking the sign of the output effect to pass-through rates (see condition (45)), we can conclude that dual wholesale pricing motivated by differences in intra-market substitutability has a *positive output effect*. Given that the allocation effect is positive as well, dual wholesale pricing raises total welfare. The next proposition summarizes these insights:

Proposition 6 *In the quadratic-utility linear-demand setting, dual wholesale pricing motivated by differences in intra-market substitutability has positive allocation and output effects and thus raises total welfare.*

Figure 1 provides a graphical illustration of the output effect, total welfare effect, and consumer surplus effect of dual pricing in our linear-demand specification. The parameter γ_B measures the degree of between-retailer substitutability in market B. market B features greater intra-market substitutabiliy than market A if and only if $\gamma_B > \gamma_A = 0.4$. The parameter α_A captures the "size" of market A, which affects the monopoly retail price: $p_A^* > p_B^*$ if and only if $\alpha_A > \alpha_B = 2$. The region of interest lies above the black area (otherwise

$w_A^* \geq w_B^*$). In the dark-shaded region in the north-east corner, dual pricing has positive effects on output, welfare, and consumer surplus. In the medium-shaded region, dual pricing raises output and total welfare, but decreases consumer surplus. In the light-shaded region, the output effect is positive, but dual pricing lowers total welfare and consumer surplus. In the unshaded region, all three effects are negative.

When the markets differ only in the intra-market between-retailer substitutability, i.e., for $\alpha_A = \alpha_B = 2$, the output effect and the welfare effect are both positive, confirming the insight of Proposition 6. Dual pricing also has a positive effect on consumers surplus in this case. For $\alpha_A > \alpha_B = 2$, dual pricing raises output and welfare provided that γ_B is high enough to ensure $w_B^* > w_A^*$ although $p_A^* > p_B^*$. In this region, dual pricing has a positive allocation effect (because $p_A^* > p_B^*$) and a positive output effect (because $\gamma_B > \gamma_A$). If $\alpha_A < \alpha_B$, the welfare effect can be negative due to a negative allocation effect. In particular, there exists an area in which the output effect is positive but welfare nonetheless falls because dual pricing worsens the allocation of output across markets. However, if γ_B exceeds γ_A sufficiently, the positive output effect again compensates for the negative allocation effect and welfare rises as a result of dual wholesale pricing.

6 Conclusion

We have shown that restriction on online sales can arise as part of a manufacturer's profit-maximizing strategy to align the incentives of its retailers in order to achieve full channel coordination. Simply offering supply contracts that are based on each retailer's total purchases is not enough – because the marginal input that aligns retailers' incentives with the firms' overall collective incentives in one market (e.g., sales in the brick-and-mortar market) will not generally coincide with the marginal input price needed to align fully the incentives in the other market (e.g., sales online). Thus, just as one might think of two-part tariffs (or more general non-linear tariffs) as necessary to solve double marginalization concerns in the traditional single market retailer, one can think of online restrictions (or, equivalently, the use of dual wholesale pricing) as necessary to solve for the inefficiencies that can arise across distribution markets (such as when the same retailer sells the manufacturer's product both online and through its brick and mortar outlets).

Our theory contrasts sharply with the conventional wisdom of policymakers that online sale restrictions are nothing more than a means of dampening competition between markets and/or of allowing competing manufacturers to engage in tacit collusion. Moreover, it suggests that the restrictions should not be prohibited per se. All else equal, we find that when online sales restrictions, (or, equivalently, dual wholesale pricing) are made to control for differences in intra-market substitutability, total output and welfare cam increase (and always do increase in the standard quadratic-utlity, linear-demand setting).

In addition to contributing on the policy front, our analysis also contributes to the literatures on conduct parameters and third-degree price discrimination. With respect to the former, it extends the seminal works of Bresnahan (1989), Genesove and Mullin (1998), and others to allow parameterization of conduct across multiple distribution markets, where there is some substitutability of consumers between markets. With respect to the latter, we have shown that forbidding dual pricing can have adverse welfare consequences – and indeed might be expected to have adverse consequences when markets differ mainly in terms of intensity of competition.

7 Appendix A: Proofs

Proof of Lemma 2. Given the definition of $\xi_s(\cdot)$, $\xi_A(p_A, p_B) < \xi_B(p_B, p_A)$ if and only if

$$\frac{Q_A}{-\partial Q_A/\partial p_A}(1 - R_{BA}) < \frac{Q_B}{-\partial Q_B/\partial p_B}(1 - R_{AB}),$$

which can be rewritten as

$$e_B \frac{1 - d_B - d_B^O - d_B^C}{1 - d_B} < e_A \frac{1 - d_A - d_A^O - d_A^C}{1 - d_A},$$

where $e_s = \frac{-\partial Q_s/\partial p_s}{Q_s}$ and

$$d_B^O \equiv \frac{\frac{\partial D_{Ai}}{\partial p_{Bi}}}{-\frac{\partial D_{Bi}}{\partial p_{Bi}}}, d_B^C \equiv \frac{\sum_{j \neq i} \frac{\partial D_{Aj}}{\partial p_{Bi}}}{-\frac{\partial D_{Bi}}{\partial p_{Bi}}}.$$

d_A^O and d_A^C are defined analogously. Note that $DR_s = \frac{d_s + d_s^C}{1 - d_s^O}$. Further simpliying and using DR_s yields the following result:

$$\begin{aligned} \xi_A(p_A, p_B) &< \xi_B(p_B, p_A) \\ &\Longleftrightarrow \\ e_B(1 - d_A)\left(1 - d_B^O\right)(1 - DR_B) &< e_A(1 - d_B)\left(1 - d_A^O\right)(1 - DR_A). \end{aligned}$$

Next, using the definitions of θ_s and ξ_s and the new notation for d_s^O, we obtain that

$$\begin{aligned} \theta_A(p_A, p_B)\,\xi_A(p_A, p_B) &< \theta_B(p_B, p_A)\,\xi_B(p_B, p_A) \\ &\Longleftrightarrow \\ e_B(1 - d_A)\left(1 - d_B^O\right) &< e_A(1 - d_B)\left(1 - d_A^O\right). \end{aligned}$$

We can rewrite these two insights as follows. First,

$$0 < \xi_B(p_B, p_A) - \xi_A(p_A, p_B) \tag{53}$$

$$\Longleftrightarrow$$

$$e_B(1 - d_A)\left(1 - d_B^O\right)\left(\frac{1 - DR_B}{1 - DR_A} - 1\right) < e_A(1 - d_B)\left(1 - d_A^O\right) - e_B(1 - d_A)\left(1 - d_B^O\right) \tag{54}$$

Second,

$$\left(\frac{\theta_A(p_A, p_B)}{\theta_B(p_B, p_A)} - 1\right)\xi_A(p_A, p_B) < \xi_B(p_B, p_A) - \xi_A(p_A, p_B) \tag{55}$$

$$\Longleftrightarrow$$

$$0 < e_A(1 - d_B)\left(1 - d_A^O\right) - e_B(1 - d_A)\left(1 - d_B^O\right). \tag{56}$$

If $\theta_A = \theta_B$, then (55) and (53) coincide, hence (56) and (54) must be equivalent, which implies that $DR_B = DR_A$. Similarly, if $DR_B = DR_A$, then (56) and (54) coincide, hence (55) and (53) must be equivalent, which implies that $\theta_A = \theta_B$. In summary, $\theta_A = \theta_B$ if and only if $DR_B = DR_A$.

Next, we show that $\theta_A > \theta_B$ implies $DR_B > DR_A$. Suppose first that $\theta_A > \theta_B$ and (55) holds. Given $\theta_A > \theta_B$, (55) implies (53). Since (53) is equivalent to (54), this means that (55) implies (54). Since (55) and (56) are equivalent, this in turn means that (56) implies (54); i.e., if

$$0 < e_A (1 - d_B) \left(1 - d_A^O\right) - e_B (1 - d_A) \left(1 - d_B^O\right),$$

then

$$e_B (1 - d_A) \left(1 - d_B^O\right) \left(\frac{1 - DR_B}{1 - DR_A} - 1\right) < e_A (1 - d_B) \left(1 - d_A^O\right) - e_B (1 - d_A) \left(1 - d_B^O\right).$$

For this to hold, the left-hand side of (54) must be non-positive, which, together with $DR_B = DR_A$ if and only if $\theta_A = \theta_B$, implies that $DR_B > DR_A$.

Second, suppose that $\theta_A > \theta_B$ and (53) is reversed. Given $\theta_A > \theta_B$, (53) reversed implies (55) reversed. Since (53) reversed is equivalent to (54) reversed, this means that (54) reversed implies (55) reversed. Since (55) and (56) are equivalent, this in turn means that (54) reversed implies (56) reversed; i.e., if

$$e_B (1 - d_A) \left(1 - d_B^O\right) \left(\frac{1 - DR_B}{1 - DR_A} - 1\right) > e_A (1 - d_B) \left(1 - d_A^O\right) - e_B (1 - d_A) \left(1 - d_B^O\right),$$

then

$$0 > e_A (1 - d_B) \left(1 - d_A^O\right) - e_B (1 - d_A) \left(1 - d_B^O\right).$$

For this to be true, it must be that the left-hand side of the first condition is smaller than the left-hand side of the second condition, which, together with $DR_B = DR_A$ if and only if $\theta_A = \theta_B$, implies that $DR_B > DR_A$.

The only remaining case in which $\theta_A > \theta_B$ is that (53) holds but (55) is reversed:[42]

$$0 < \xi_B(p_B, p_A) - \xi_A(p_A, p_B) < \left(\frac{\theta_A (p_A, p_B)}{\theta_B (p_B, p_A)} - 1\right) \xi_A(p_A, p_B)$$

[42] If $\theta_A > \theta_B$ and (55) holds with equality, then (56) also holds with equality and (53) holds with a strict inequality. Hence, (54) implies $DR_B > DR_A$. Similarly, if $\theta_A > \theta_B$ and (53) holds with equality, then (55) is reversed, which implies that the right-hand side of (54) is strictly negative. Given that (54) has to hold with equality when (53) holds with equality, this implies that $DR_B > DR_A$. The analysis in the text therefore focuses on cases with strict inequalities.

In this case, (56) is reversed, because it is equivalent to (55) reversed. (56) reversed means that the right-hand side of (54) is negative. This means that for (54) to hold, it must be that $DR_B > DR_A$.

In summary, $\theta_A > \theta_B$ implies $DR_B > DR_A$. The proofs of the "only if" part of the statement and the case $\theta_A < \theta_B$ proceed in the same manner and are therefore omitted. ■

Proof of Proposition 2. Suppose the supplier offers each retailer i the same "dual" two-part tariff of the form $T(q_{Ai}, q_{Bi}) = F + w_A q_{Ai} + w_B q_{Bi}$. If all retailers accept this tariff, downstream competition will yield the prices (p_A^*, p_B^*) that maximize industry profits if and only if the wholesale prices w_A and w_B are such that

$$\frac{\partial \pi_i}{\partial p_{Ai}}(p_A^*, ..., p_A^*, p_B^*, ..., p_B^*; w_A, w_B) = 0, \quad (57)$$

$$\frac{\partial \pi_i}{\partial p_{Bi}}(p_A^*, ..., p_A^*, p_B^*, ..., p_B^*; w_A, w_B) = 0. \quad (58)$$

Using Lemma 1, we obtain that the retailers' first-order conditions are satisfied at the industry profit-maximizing prices $(p_A^*, .., p_A^*, p_B^*, ..., p_B^*)$ if and only if $w_A = w_A^* \equiv \theta_A^* c + (1 - \theta_A^*) p_A^*$ and $w_B = w_B^* \equiv \theta_B^* c + (1 - \theta_B^*) p_B^*$. If the supplier offers a tariff of the form $T(q_{Ai}, q_{Bi}) = F + w_A^* q_{Ai} + w_B^* q_{Bi}$ and then retailers accept the offer, the retailers thus set the prices (p_A^*, p_B^*) in the downstream pricing game. The supplier can then obtain the integrated monopoly profit $\Pi(p_A^*, ..., p_A^*, p_B^*, ..., p_B^*)$ by asking for a fixed fee $F = \pi_i(p_A^*, ..., p_A^*, p_B^*, ..., p_B^*; w_A^*, w_B^*)$ equal to each retailer's downstream variable profit. ■

Proof of Proposition 4. The proof follow the steps of the proofs of Lemma and Proposition 2 in Aguirre et al. (2010). Assume that IRC holds and denote $z_s = \frac{p_s - c}{\rho_s(2 - L(p_s)\eta_s(p_s)\sigma_s)}$. By IRC, z_s is increasing in p_s. From (38),

$$W''(r) = \underbrace{\frac{-\pi_A''\pi_B''}{\pi_A'' + \pi_B''}}_{>0}(z_A'\rho_A w_A' - z_B'\rho_A w_B') + (z_A - z_B)\frac{d}{dr}\left(\frac{-\pi_A''\pi_B''}{\pi_A'' + \pi_B''}\right),$$

which is negative if $W' = 0$ because $z_A = z_B$ when $W' = 0$ and $z_A'\rho_A w_A' < 0 < z_B'\rho_A w_B'$. It follows that $W(r)$ is either monotonic in r or has a single interior peak. Thus, if $W'(r^*) \geq 0$, then $W'(r) \geq 0$ for all $r \in [0, r^*)$ and the total welfare effect is positive. From (39) and $L_s(p_s^*)\eta_s(p_s^*) = 1$ for all s, $W'(r^*) \geq 0$ if and only if

$$\frac{p_A^* - c}{\rho_A(2 - \sigma_A)} \geq \frac{p_B^* - c}{\rho_B(2 - \sigma_B)}.$$

■

8 Appendix B

Proposition 7 (Proposition B1) *Given $w_A^* < w_B^*$, a tariff of the form*

$$T(q_{Ai}, q_{Bi}) = \begin{cases} F + wq_i & \text{if } q_{Bi} \leq \frac{Q_B^*}{Q_A^* + Q_B^*} q_i, \\ S + F + wq_i & \text{if } q_{Bi} > \frac{Q_B^*}{Q_A^* + Q_B^*} q_i, \end{cases},$$

where

$$w = \frac{Q_A^*}{Q_A^* + Q_B^*} w_A^* + \frac{Q_B^*}{Q_A^* + Q_B^*} w_B^*,$$

can achieve full channel coordination.

Proof. Let $w_A^* < w_B^*$. Suppose the supplier offers a tariff of the form specified in the proposition, with S large enough so that violating the constraint is unprofitable for the retailers. If both retailers accept the supplier's offer, both retailers setting the integrated monopoly prices is an equilibrium of the downstream pricing game if and only if

$$(p_A^*, p_B^*) = \max_{(p_{Ai}, p_{Bi})} (p_{Ai} - w) D_{Ai}(p_{Ai}, p_{Bi}; p_A^*, p_B^*) + (p_{Bi} - w) D_{Bi}(p_{Ai}, p_{Bi}; p_A^*, p_B^*)$$
$$\text{s.t. } D_{Bi}(p_{Ai}, p_{Bi}; p_A^*, p_B^*) \leq \frac{Q_B^*}{Q_A^* + Q_B^*} \left(D_{Ai}(p_{Ai}, p_{Bi}; p_A^*, p_B^*) + D_{Bi}(p_{Ai}, p_{Bi}; p_A^*, p_B^*) \right).$$

The Kuhn-Tucker first-order conditions, evaluated at the monopoly retail prices, are

$$\frac{\partial \pi_i}{\partial p_{Ai}} \left(p_A^*, ..., p_A^*, p_B^*, ..., p_B^*; w - \lambda \frac{Q_B^*}{Q_A^* + Q_B^*}, w + \lambda \left(1 - \frac{Q_B^*}{Q_A^* + Q_B^*} \right) \right) = 0, \quad (59)$$
$$\frac{\partial \pi_i}{\partial p_{Bi}} \left(p_A^*, ..., p_A^*, p_B^*, ..., p_B^*; w - \lambda \frac{Q_B^*}{Q_A^* + Q_B^*}, w + \lambda \left(1 - \frac{Q_B^*}{Q_A^* + Q_B^*} \right) \right) = 0, \quad (60)$$
$$\lambda \geq 0, \quad (61)$$

where λ is the Lagrange multiplier of the constraint. By the definition of (w_A^*, w_B^*), (59) and (60) hold if and only if

$$w - \lambda \frac{Q_B^*}{Q_A^* + Q_B^*} = w_A^*, \quad (62)$$
$$w + \lambda \left(1 - \frac{Q_B^*}{Q_A^* + Q_B^*} \right) = w_B^*. \quad (63)$$

Substituting $w = \left(1 - \frac{Q_B^*}{Q_A^* + Q_B^*} \right) w_A^* + \frac{Q_B^*}{Q_A^* + Q_B^*} w_B^*$ into (62) and (63) and solving for λ yields

$$\lambda = w_B^* - w_A^*.$$

Hence, $w_A^* < w_B^*$ implies that (61) holds. The supplier can thus obtain the integrated monopoly profit $\Pi(p_A^*, ..., p_A^*, p_B^*, ..., p_B^*)$ by offering both retailers a tariff of the form specified in the proposition, with $F = \pi_i(p_A^*, ..., p_A^*, p_B^*, ..., p_B^*; w_A^*, w_B^*)$ and S large enough to make violating the constraint on relative quantities unprofitable. ■

Two-part tariffs are without loss of generality If tariff of the form $T(q_{Ai}, q_{Bi})$ are feasible, the supplier can achieve full channel coordination by means of a dual two-part tariff with a fixed fee $\pi_i(p_A^*, ..., p_A^*, p_B^*, ..., p_B^*; w_A^*, w_B^*)$ and wholesale prices w_A^* and w_B^* (see Proposition 2), hence there is no loss of gerenality in restricting attention to two-part tariffs.

If only tariffs of the form $T(q_i)$ are allowed, two-part tariffs are without loss of generality as well. Given a contract of the form $T(q_i)$, there must exist a λ such that the equilibrium retail prices (p_A^e, p_B^e) satisfy (see Section 4.1):

$$\frac{\partial \pi_i(p_A^e, ..., p_A^e, p_B^e, ..., p_B^e; \lambda, \lambda)}{\partial p_{Ai}} = \frac{\partial \pi_i(p_A^e, ..., p_A^e, p_B^e, ..., p_B^e; \lambda, \lambda)}{\partial p_{Bi}} = 0. \tag{64}$$

Now suppose that instead of $T(q_i)$, the supplier offers the retailers a two-part tariff with wholesale price $w = \lambda$ and fixed fee

$$F = T(q_i^e) - w\left(Q_A(p_A^e, p_B^e) + Q_B(p_A^e, p_B^e)\right).$$

The retailers are willing to accept this two-part tariff if they are willing to accept the tariff $T(q_i)$, and each will order exactly $Q_A(p_A^e, p_B^e) + Q_B(p_A^e, p_B^e)$ units if the supplier uses the two-part tariff. The supplier's profit with the two-part tariff becomes $2\left(T(q_i^e) - c\left(Q_A(p_A^e, p_B^e) + Q_B(p_A^e, p_B^e)\right)\right)$, which is (at least) as much as the profit $2\left(T(q_i^e) - cq^e\right)$ it would have earned by offering $T(q_i)$.[43]

[43] The two profits are the same if the quantity ordered $q^e = Q_A(p_A^e, p_B^e) + Q_B(p_B^e, p_A^e)$. If $q^e > Q_A(p_A^e, p_B^e) + Q_B(p_B^e, p_A^e)$, the two-part tariff yields a higher profit for the supplier because it implies lower upstream varibale costs.

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