Paid vs Independent Product Recommendation By Bloggers

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1 Introduction

Sponsored product reviews are also common in the blogosphere, where skilled bloggers make thousands of dollars daily by simply offering information and advice in their own area of expertise, including beauty, health, travel, food, and technology, etc. For example, Youtube star and makeup guru Michelle Phan has a personal blog where she shares with her fans makeup ideas and beauty product reviews. She used products from French makeup brand Lancome in many of her blogs and videos, and she disclosed that she received them for free from the company. My Subscription Addiction (http://www.mysubscriptionaddiction.com) is a blog that reviews a extensive list of subscription boxes currently available in the market. The blog provides detailed reviews of the contents of each box per delivery period, including the brand, quality, item, and variety of the boxes. Each review includes a link to the subscription provider's website and coupons for subscriptions. Most of the posts are sponsored by the respective subscription box providers. The blogger gets request from firms to review their box and will add new boxes to their list. People with a popular blog, Instgram account, Youtube channel, or any other social media presence are referred to as influences in the online word-of-mouth marketing industry. Influencers who have built up a significant following can monetize their work through ad revenues or content sponsorship. Influencer marketing intermediaries, such as BzzAgent, Swaggable, Tomoson, create a marketplace for online word-of-mouth, where brands can find suitable influencers to promote their product through the influencers' own social media channel.

Furthermore, some intermediaries specialize in incentivized consumer reviews. Consumers who sign up with platforms such as Review Stream, Software Judge,and Shared Reviews and make money by posting a review on various third-party review sites. However, recently Amazon uncovered and banned sellers incentivized reviews (The Guardian, 2016) after receiving complaints about sellers diving heavily discounts on their products in exchange for positive reviews. It appeared that consumers believe the reviews would not have been genuine if the reviewers received incentives or compensation. This belief is at odds with the phenomenon that popular social media influencers often get hired by companies to promote their products. And if sponsored content is immediately discredited, then firms would not be paying influencers to write about their products.

The revenue of blogs come from pay-per-view and pay-per-click ads. Therefore, in order to be profitable, blogs must attract visitors. One of the best ways to grow readership is to provide useful information that helps consumers make informed purchase decisions. Not only blogs, but more generally any kind of recommendation and reviews sites, guides, and ratings systems (e.g. Consumer Reports, Lonely Planet, Angies List, etc) profit from their readership by providing product recommendations. Reviews are also a form of recommendation in that a positive review is essentially a "buy"- recommendation. However, these review and recommendation systems are not retailers, nor are they usually affiliated with the sellers of the products. Consumers are willing to follow the recommendation of a blogger known to give good advice, as this could reduce their search cost. However, not all bloggers give good advice, and consumers cannot always tell whether a blogger is good, especially if the blogger is new and has not yet established a reputation.

Although a blogger may recommend a product without interacting with the firm that sells it, it may be in the firm's interest to retain the positive recommendation of a reputable blogger, as recommendations are a form of online word of mouth. However, recommendations are persuasive if and only if they are impartial. The credibility of the blogger relies on her impartiality. Firm-sponsored reviews will not be perceived as impartial. Therefore, incentivizing recommendations not only invalidates them but also hurts the blogger's reputation. In blogosphere, we rarely observe blogger-firm affiliations whereby the blogger would make product recommendations exclusively for a firm. On the contrary, we often find that bloggers don't work for any particular firm and make independent recommendations for many different products. Should firms solicit positive reviews by providing incentives? Given that sponsored reviews must be disclosed, should blogger-reviewers accept incentives from the firms? In this paper, we derive conditions under which firms should solicit sponsored reviews and conditions under which firms should refrain from doing so.

2 Literature Review

The effects of recommendations on consumers' purchase decision have been studied in prior literature. Mayzlin (2006) investigated a setting in which consumers receive a mixture of unbiased recommendations from other consumers and biased promotional messages that are indistinguishable. However, in our model the consumers can actually observe the source of the recommendations – whether they come from independent or sponsored bloggers.

Bloggers also act as media through which firms inform the consumers about the product. Similar to an advertising medium such as a TV station, bloggers offer information to consumers and attract advertisers. However, making recommendations is not the primary way for the bloggers to generate income. Instead, recommendations are the "programming" that attract visitors to the blogs. Therefore, the blogger's recommendations are different from the "advertisement" in Gal-Or and Dukes (2003). Also, unlike other advertising media and platforms, the bloggers do not receive a share of the profits on product sales; hence they do not care about the sales of the products but improving their reputation. This distinguishes the issue we address from the standard adverting problem.

The current topic is also related to product endorsement. Very well-known bloggers are effectively cyber-celebrities who enjoy a certain level of fame and recognition that make them comparable to conventional celebrities. When a blogger recommends a product, it is as if she "endorsed" the product, especially when she is paid to make the recommendation. But a blogger's reputation is tied to her product recommendations to a greater extent relative to a celebrity. A movie star cares about how others perceive her skills as an actress, which would be not compromised by her act of endorsement. But a blogger's reputation is based on how others perceive her ability to recommend good products. Therefore, a blogger's recommendation directly affects her reputation. It is not uncommon for individuals to engage in word of mouth as a means to improve others perception of them (Campbell, Mayzlin, and Shin, 2013).

Similar to conventional celebrities, bloggers could be sponsored by firms to recommend products. Product endorsement and promotion by conventional celebrities has been a broadly studied topic in the consumer behavior literature. The majority of these studies explain the often observed exclusivity in endorsement relationships from a credibility angle; they generally agree that an effective endorsement requires a credible endorser, and non-exclusivity hurts the credibility of the endorser. Tripp, Jensen, and Carlson (1994) finds that a higher number of products endorsed decreases the perceived credibility of the celebrity, as well as the likability of and attitude toward that celebrity. When a celebrity endorses a large volume of different products, consumers infer that the celebrity is motivated by monetary gains only and not a genuine belief in the quality of the products (Carroll, 2009). Furthermore, Spry, Pappu, and Cornwell (2009) suggests that endorser credibility indirectly affects brand equity. While conventional wisdom says that endorsing multiple products hurts the endorser's reputation as well as the consumers' perception of the products, the current paper finds the opposite. We find that if a blogger makes an exclusive recommendation, it actually hurts her reputation.

3 Model

Suppose there is a firm selling a product at price p. A product may or may not be a good fit for the consumer, and neither the firm nor the consumer observes the product's fit ex ante. There is a consumer whose prior belief on the product fit be $Prob(G) = \rho_0 \in [0, 1]$. Let F denote the actual fit of the product, $F \in \{G, B\}$, where G stands for "good fit" and B stands for "bad fit". The firm's prior on product fit is $\rho \in [0, 1]$, which is public information. The consumer receives utility v from the product if it is a good fit, and 0 otherwise: $U(G) = v$ and $U(B) = 0$.

The blogger makes money by writing interesting or informative content on her blog, where she can post a product review or some other content. If she chooses to post a product review, she can either independently write a review about a product of her choice or get paid to write a sponsored review. By examining the product, the blogger can costlessly receive a noisy binary signal (s) on the product's fit to the consumer: $s \in \{g, b\}$, where g stands for "observed good fit" and b stands for "observed bad fit." The blogger receives a signal with probability $Prob(g \mid G) = Prob(b \mid B) = \gamma$. And let $\gamma > 0.5$ capture the fact that the blogger's review is useful to the consumers. After receiving the signal, the blogger can send a message $m = \{\mathcal{G}, \mathcal{B}\}$, where $\mathcal{G}$ denotes a positive message (that she receives a good fit signal), and $\mathcal{B}$ denotes a negative message (that she receives a bad fit signal). If the blogger posts no review, $m = \emptyset$.

The probability that the blogger receives the correct signal (g if the product is a fit and b if the product is not a fit) is γ. Conversely, the probability that she receives the wrong signal (g if the product is not a fit and b if the product is a fit) is $1 - \gamma$. The firm may make a take-it-or-leave-it offer to the blogger in exchange for a (favorable) product review. The blogger's relationship with the firm is measured by $a \in [0, 1]$, where $a = 0$ denotes the blogger is independent, $0 < a < 1$ denotes the blogger is semi-independent, and $a = 1$ denotes the blogger is fully affiliated with the firm. The blogger's affiliation with the firm is public knowledge, as by law she must publicly disclose the firm's sponsorship.

If the blogger accepts the offer ($a > 0$), she is obliged to write a review for that particular firm's product; that is, she will send a message about the product fit. However, her message will be biased — she may send a positive message even when she receives a bad fit signal, but she will always send a positive message if she receives a positive signal. Let $Prob(\mathcal{G} \mid b) = a$ and $Prob(\mathcal{G} \mid g) = 1$. Conversely, $Prob(\mathcal{B} \mid b) = 1 - a$ and $Prob(\mathcal{B} \mid g) = 0$.

If $a = 0$, there is no distortion: $Prob(\mathcal{G} \mid b) = 0$, and $Prob(\mathcal{G} \mid B) = Prob(g \mid B) = 1 - \gamma$

If $a \in (0, 1)$, the probability that blogger sends a positive message when the product is a good fit is

$$\begin{aligned} Prob(\mathcal{G} \mid G) =& Prob(\mathcal{G} \mid g) \cdot Prob(g \mid G) + Prob(\mathcal{G} \mid b) \cdot Prob(b \mid G) \\ =& \gamma + a(1 - \gamma) \end{aligned} \tag{1}$$

And the probability that the blogger sends a positive message when the product is a bad fit is

$$\begin{aligned} Prob(\mathcal{G} \mid B) =& Prob(\mathcal{G} \mid g) \cdot Prob(g \mid B) + Prob(\mathcal{G} \mid b) \cdot Prob(b \mid B) \\ =& (1 - \gamma) + a\gamma \end{aligned} \tag{2}$$

The probability that the blogger sends negative message when the product is a good fit is $Prob(\mathcal{B} \mid G) = (1 - a)(1 - \gamma)$, and the probability that she sends a negative message when the product is a bad fit is: $Prob(\mathcal{B} \mid B) = (1 - a)\gamma$.

The blogger's message is distorted by her affiliation a. The amount of message distortion of depends on the extent of her relationship with the firm. The closer the relationship (higher a), the more likely the blogger is required to send a positive message about the product (despite her getting a negative product fit signal). If $a = 1$, the blogger must send $m = g$ with probability 1 regardless of her signal, such that $Prob(\mathcal{G} \mid G) = Prob(\mathcal{G} \mid B) = 1$ and $Prob(\mathcal{B} \mid B) = Prob(\mathcal{B} \mid B) = 0$.

The consumer updates his belief about the product fit after observing the blogger's relationship with the firm (a) and her message (m), and then he makes a purchase decision. We assume that subsequently the actual fit of the product is always revealed. The timing is the following. At $t = 1$, the blogger decides whether to post a product review or some other content. At $t = 2$, the firm may make a take-it-or-leave-it offer to the blogger in exchange for a product review, which the blogger can accept or reject. At $t = 3$, the blogger receives a signal on the product fit. If the blogger accepted the firm's offer, at $t = 4$ she publicly posts a review of the product fit and reveals her affiliation with the firm. If the blogger did not accept the firm's offer, at $t = 4$ she either posts an independent review or some other content. At $t = 5$, the consumer observes whether the blogger posted a review, whether the review is positive or negative, and whether the blogger is independent or affiliated with the firm. He then makes a purchase decision based on his posterior belief on the product fit. At $t = 6$ the true fit of the product is revealed and the firm's profit is realized.

INSERT THE FIGURE HERE

3.1 The Consumer's Inference

Let's examine the consumer inference on product fit at $t = 5$ after he observes the blogger's affiliation and product review. At this stage the consumer's information set is, $\psi = [a \in [0, 1], m \in \{\emptyset, \mathcal{G}, \mathcal{B}]\}$, and the consumer updates his belief about the product fit to $\omega(\phi)$. The consumer will purchase the product if and only if the expected utility is (weakly) positive: $EU(\psi) = \omega(\psi)v - p \geq 0$. Consumers with belief $k \equiv \omega(\psi) \geq \dfrac{p}{v}$ will purchase the product.

When $a < 1$, the blogger's message is informative, and the consumer's posterior beliefs on the product fit after seeing the blogger's review are the following

$$Prob(G \mid \mathcal{G}) \equiv \omega^{+}_{a<1} = \frac{Prob(\mathcal{G} \mid G) \cdot Prob(G)}{Prob(\mathcal{G} \mid G) \cdot Prob(G) + Prob(\mathcal{G} \mid B) \cdot Prob(B)} \tag{3}$$

$$Prob(G \mid \mathcal{B}) \equiv \omega^{-}_{a<1} = \frac{Prob(\mathcal{B} \mid G) \cdot Prob(G)}{Prob(\mathcal{B} \mid G) \cdot Prob(G) + Prob(\mathcal{B} \mid B) \cdot Prob(B)} \tag{4}$$

The consumer updates her belief upwards after seeing $m = \mathcal{G}$ and lowers it after seeing $m = \mathcal{B}$:

$\omega^-_{a<1} < \rho < \omega^+_{a<1}$. Note that the posterior belief on product fit depends on the accuracy of the blogger's signal — $Prob(\mathcal{G} \mid G)$ and $Prob(\mathcal{B} \mid G)$, which were given by equation 1 and 2. Combining this with Equations 3 and 4 yields the following expressions for the posterior beliefs following a positive and a negative recommendation, respectively:

$$\omega^+_{a<1} = \frac{[\gamma + a(1-\gamma)]\rho_0}{[\gamma + a(1-\gamma)]\rho_0 + [(1-\gamma) + a\gamma](1-\rho_0)} \tag{5}$$

$$\omega^-_{a<1} = \frac{(1-\gamma)\rho_0}{(1-\gamma)\rho_0 + \gamma(1-\rho_0)} \tag{6}$$

ω^- is not a function of a because the blogger sends a negative message if and only if she receives a negative signal. Therefore, the blogger's negative signal is not affected by affiliation.

The probability that the blogger sends a positive message about the product is given by $Prob(\mathcal{G}) = [\gamma + a(1-\gamma)]\rho_0 + [1-\gamma + a\gamma](1-\rho_0)$. Let $Prob(g) = \gamma\rho_0 + (1-\gamma)(1-\rho_0)$ and $Prob(b) = \gamma(1-\rho_0) + (1-\gamma)\rho_0$, then $Prob(\mathcal{G})$ can be rewritten as $Prob(g) + aProb(b)$.

Firm sponsorship affects the blogger's message asymmetrically: it increases the probability that the blogger sends a positive message and decreases the probability that she sends a negative message. That is, for $a > 0$, $Prob(\mathcal{G}) > Prob(g)$, and $Prob(\mathcal{B}) = (1-a)Prob(b) < Prob(b)$. If $a = 1$, $Prob(\mathcal{G}) = 1$ and $Prob(\mathcal{B}) = 0$. The blogger's message becomes uninformative, as $\omega^+_{a=1} = \rho_0$. If the blogger makes no review, the consumer's belief will not update either.

3.2 The Value of Blogger's Information

As long as the blogger's message is informative ($a < 1$), consumers update their belief on the product to $\omega^+_{a<1}$ upon seeing $\mathcal{G}$, and to $\omega^-_{a<1}$ upon seeing $\mathcal{B}$. The blogger's message is persuasive when it changes a consumer's action. That is, a posterior belief of $\omega^+_{a<1}$ makes him switch from not buying to buying, and a posterior belief of $\omega^-_{a<1}$ makes him switch from buying to not buying. However, if a consumer's prior is extreme, the blogger's message will not affect his purchase decision. Without any updates, a consumer with prior belief $\rho_0 = k$ is indifferent between buying and not buying. Only a consumers who has a prior slightly higher or below k will change his purchase decision upon seeing the blogger's message. Such a consumer must have posterior beliefs given as follows:

$$\omega^{+}_{a<1} = \frac{[\gamma + a(1-\gamma)]\rho_0}{[\gamma + a(1-\gamma)]\rho_0 + [(1-\gamma) + a\gamma](1-\rho_0)} > k \tag{7}$$

$$\omega^{-}_{a<1} = \frac{(1-\gamma)\rho_0}{(1-\gamma)\rho_0 + \gamma(1-\rho_0)} < k \tag{8}$$

For simplicity of notation, we denote $\omega^{-}_{a<1}$ as ω^{-} and $\omega^{+}_{a<1}$ as ω^{+}. Note that ω^{+} is a function of a, but ω^{-} is not. When $a = 1$, the blogger's message is uninformative, the consumer's belief will not be updated: $\omega^{+} = \rho_0$. Therefore, there must be an upper bound on a below which $\omega^{+} > k$. Equation 7 implies that

$$a < \frac{(1-k)\gamma\rho_0 - k(1-\gamma)(1-\rho_0)}{k(1-\rho_0) - (1-k)(1-\gamma)\rho_0} \equiv \bar{a} \tag{9}$$

If the consumer's prior belief is high enough, even with a negative update, he will still buy the product. Therefore, there must be an upper bound on his prior belief below which his purchase decision will change following a negative update. Equation 8 implies that

$$\rho_0 < \frac{k\gamma}{(1-k)(1-\gamma) + k\gamma} \equiv \bar{\rho}_0 \tag{10}$$

The blogger's utility depends on the value of the information that she provides to the consumer. The blogger's information is valuable only to a consumer whose purchase decision will be changed by that information. In other words, a consumer whose prior is slightly below or above k. Let $c(a)$ be the blogger's information value. Note that ω^{+} and ω^{-} are also the probabilities that the product is a good fit given $\mathcal{G}$ and $\mathcal{B}$, respectively. If the consumer has a prior $\rho_0 < k$, he would not buy the product without the blogger's information. If he follows the blogger's message, he gains an expected utility of $\omega^+(v-p)$. If the consumer has a prior $\rho_0 > k$, he would buy the product without the blogger's information. If he follows the blogger's message, he can avoid an expected utility loss of $(1-\omega^-)p$. The blogger's information value is the consumer's expected utility from following her message given a. Therefore, the blogger's information value is given as follows:

$$c(a) = \begin{cases} Prob(\mathcal{G})\omega^+ \cdot (v-p) & \text{if } \rho_0 < k \\ \\ Prob(\mathcal{B})(1-\omega^-) \cdot p & \text{if } \rho_0 > k \end{cases} \tag{11}$$

If $a = 0$, the blogger is independent. Suppose with some exogenous probability δ. the blogger writes about the firm's product. With probability $1-\delta$ she writes about something else rather than about the firm's product, which in case the consumer remains unaware of the product and will not be able to purchase it. The information value of an independent blogger is as follows:

$$c(0) = \begin{cases} \delta\rho_0\gamma(v-p) & \text{if } \rho_0 < k \\ \\ \delta\gamma(1-\rho_0)p & \text{if } \rho_0 > k_0 \end{cases} \tag{12}$$

(11) can be simplified as:

$$c(a) = \begin{cases} [\gamma + a(1-\gamma)]\rho_0(v-p) & \text{if } \rho_0 < k \\ \\ (1-a)\gamma(1-\rho_0)p & \text{if } \rho_0 > k_0 \end{cases} \tag{13}$$

Lemma 1. *$c(a)$ is increasing in a if $\rho_0 < k$, whereas it is decreasing in a if $\rho_0 > k$.*

It may seem counterintuitive that $c(a)$ is increasing in a, given a is noise to the blogger's message.

Suppose there is no noise. First consider $a = 0$. Then the consumer would buy the product with probability $Prob(g)$, and his expected utility is $\omega^+(0)(v-p)$. Next consider $0 < a < 1$, then the consumer would buy the product with probability $Prob(\mathcal{G}) = Prob(g) + aProb(b) > Prob(g)$ and expected utility $\omega^+(a)(p-v)$. $\omega^+(0) > \omega^+(a)$ but $Prob(\mathcal{G}(a)) > Prob(g)$. Lastly, consider $a = 1$, then $Prob(\mathcal{G}(1)) = 1$ and $\omega^+(1) = \rho_0$. There is a trade-off between increasing the probability of a positive update and the magnitude of a positive update. $\rho_0 < Prob(g)\omega^+(0) < Prob(\mathcal{G})\omega^+(a)$ as long as $\omega^+(a) > k$.

4 Firm's Profits

If the firm does not hire the blogger, it makes a profit only when the blogger independently write about the product, with probability δ. The firm makes zero profit otherwise. Alternatively, the firm can hire the blogger to ensure a product review that has information value $c(a)$. The blogger's outside option is writing an independent product review that has information value $c(0)$. The firm's payment to the blogger must compensate the blogger for the change in her utility due to affiliation. Let the blogger's change in utility due to affiliation be $\Delta U^{\text{blogger}}$, which is $c(0) - c(a)$. The firm's payment to the blogger is given as follows:

$$\Delta U^{\text{blogger}} = \begin{cases} \{\delta\rho_0\gamma - [\gamma + a(1-\gamma)]\rho_0\}(v-p) & \text{if } \rho_0 < k \\ \delta\gamma(1-\rho_0) - (1-a)\gamma(1-\rho_0)p & \text{if } \rho_0 > k \end{cases} \tag{14}$$

which can be simplified as

$$\Delta U^{\text{blogger}} = \begin{cases} -(1-\delta)\rho_0\gamma - a(1-\gamma)\rho_0(v-p) & \text{if } \rho_0 < k \\ [a - (1-\delta)]\gamma(1-\rho_0)p & \text{if } \rho_0 > k \end{cases} \tag{15}$$

The firm's profit is given as follows when it pays the blogger to write a sponsored review:

$$\Pi^{\text{firm}}_{\text{pay}}(a) = p \cdot Prob(\mathcal{G}) - \Delta U^{\text{blogger}} \tag{16}$$

If the firm does not pay the blogger, the firm's profit is given as follows:

$$\Pi^{\text{firm}}_{\text{no pay}} = \delta \cdot p \cdot Prob(g) \tag{17}$$

If $\rho_0 > k$, then the firms pays the blogger as long as

$$\begin{aligned}
\Delta\Pi = \Pi^{\text{firm}}_{\text{pay}} - \Pi^{\text{firm}}_{\text{no pay}} \geq 0 \\
p(Prob(g) + aProb(b)) - [a - (1-\delta)]\gamma(1-\rho_0)p - \delta pProb(g) > 0 \\
(1-\delta)pProb(g) + apProb(b) - a\gamma(1-\rho_0)p + (1-\delta)\gamma(1-\rho_0)p > 0 \\
(1-\delta)pProb(g) + ap(Prob(b) - \gamma(1-\rho_0)) + (1-\delta)\gamma(1-\rho_0)p > 0 \\
(1-\delta)pProb(g) + ap(1-\gamma)\rho_0 + (1-\delta)\gamma(1-\rho_0)p > 0
\end{aligned}$$

$\Delta\Pi$ is increasing in a. Hence $a^* = 1$.

If $\rho_0 < k$, then the firms pays the blogger as long as

$$\begin{aligned}
\Delta\Pi = p(Prob(g) + aProb(b)) + a(1-\gamma)\rho_0(v-p) + (1-\delta)\gamma\rho_0 - \delta pProb(g) \geq 0 \\
(1-\delta)pProb(g) + a[pProb(b) + (1-\gamma)\rho_0(v-p)] + (1-\delta)\gamma\rho_0 \geq 0
\end{aligned}$$

$\Delta\Pi$ is increasing in a. But a is constrained by $a < \bar{a}$. Hence, $a^* = \bar{a}$. Since $a \in [0,1]$, (9) must satisfy the following conditions:

$$(1-k)\gamma\rho_0 - k(1-\gamma)(1-\rho_0) > 0 \;\Rightarrow \rho_0 < \frac{k}{k + (1-k)(1-\gamma)} \equiv \tilde{\rho}_0 \tag{18}$$

$$k(1-\rho_0) - (1-k)(1-\gamma)\rho_0 > 0 \;\Rightarrow \rho_0 > \frac{k(1-\gamma)}{(1-k)\gamma + k(1-\gamma)} \equiv \underline{\tilde{\rho}}_0 \tag{19}$$

If $\rho_0 > \bar{\rho}$, consumers will buy the product regardless of his belief update. The firm only needs to set $a = \epsilon \approx 0$ to ensure that the blogger writes about its product in order to gain awareness. If

$\underline{\tilde{\rho}}_0 < \rho_0 < k$. The firm needs both awareness and information, so it sets $a = \bar{a}$. If $k < \rho_0 < \bar{\rho}_0$ the firms needs to needs to prevent the consumer from leaving, hence it sets $a = 1$.

Proposition 1. *The firm's strategy is as follows:*

1 *For $\rho_0 < \underline{\tilde{\rho}}_0$, don't hire the blogger. $a = 0$ (intuition: prior is too low. awareness is useless)*

2 *For $\underline{\tilde{\rho}}_0 < \rho_0 < k$, hire the blogger with semi-affiliation: $a = \dfrac{(1-k)\gamma\rho_0 - k(1-\gamma)(1-\rho_0)}{k(1-\rho_0) - (1-k)(1-\gamma)\rho_0}$. (intuition: need both awareness and information)*

3 *For $k < \rho_0 < \bar{\rho}_0$, hire the blogger with full affiliation: $a = 1$. (intuition: need awareness and keep the consumer)*

4 *If $\rho_0 > \bar{\rho}_0$, hire the blogger with minimal affiliation. $a = \epsilon$.(intuition: only need awareness)*

$$
\begin{aligned}
&\frac{k\gamma}{(1-k)(1-\gamma) + k\gamma} - \frac{k}{k + (1-k)(1-\gamma)} \\
&\Rightarrow k\gamma + (1-k)(1-\gamma)\gamma - (1-k)(1-\gamma) - k\gamma \\
&\Rightarrow (1-k)(1-\gamma)(\gamma - 1) < 0 \\
&\Rightarrow \frac{k}{k + (1-k)(1-\gamma)} > \frac{k\gamma}{(1-k)(1-\gamma) + k\gamma}
\end{aligned}
$$

$$
\begin{aligned}
&\frac{k(1-\gamma)}{(1-k)\gamma + k(1-\gamma)} < \frac{k\gamma}{(1-k)(1-\gamma) + k\gamma} \\
&\Rightarrow (1-k)(1-\gamma)^2 + k\gamma(1-\gamma) < (1-k)\gamma^2 + k\gamma(1-\gamma) \\
&\Rightarrow (1-\gamma)^2 < \gamma^2, \text{ since } \gamma > \frac{1}{2}
\end{aligned}
$$

$$\frac{k}{k+(1-k)(1-\gamma)} > \frac{k(1-\gamma)}{(1-k)\gamma + k(1-\gamma)}$$

$$(1-\gamma)k + (1-k)\gamma > (1-\gamma)k + (1-k)(1-\gamma)^2$$

$$\gamma > (1-\gamma)^2 \text{ since } \gamma > \frac{1}{2}$$

Therefore, we have $\underline{\tilde{\rho}}_0 < \rho_0 < k < \bar{\rho}_0 < \tilde{\rho}_0$.

Appendix

$$\begin{aligned} Prob(\mathcal{G}) =& Prob(\mathcal{G} \mid G) \cdot Prob(G) + Prob(\mathcal{G}|B) \cdot Prob(B) \\ &= [\gamma + a(1-\gamma)]\rho_0 + [1-(1-a)\gamma](1-\rho_0) \\ Prob(\mathcal{B}) =& Prob(\mathcal{B} \mid B) \cdot Prob(B) + Prob(\mathcal{B} \mid G) \cdot Prob(G) \\ &= (1-a)[\gamma(1-\rho_0) + (1-\gamma)\rho_0] \end{aligned}$$

$$\omega^+_{a<1} = \frac{[\gamma + a(1-\gamma)]\rho_0}{[\gamma + a(1-\gamma)]\rho_0 + [(1-\gamma) + a\gamma](1-\rho_0)} > k$$

$$(1-k)[\gamma + a(1-\gamma)] > k(1-\rho_0)[(1-\gamma) + a\gamma]$$

$$(1-k)[\gamma + a(1-\gamma)]\rho_0 > k[(1-\gamma) + a\gamma] - k\rho_0[(1-\gamma) + a\gamma]$$

$$\rho_0[(1-k)[\gamma + a(1-\gamma)] + k[(1-\gamma) + a\gamma]] > k[(1-\gamma) + a\gamma]$$

$$\rho_0 > \frac{k[(1-\gamma) + a\gamma]}{(1-k)[\gamma + a(1-\gamma)] + k[(1-\gamma) + a\gamma]} \equiv \underline{\rho}_0(a)$$

$$\underline{\rho}_0(1) = k$$

$$\underline{\rho}_0(0) = \frac{k(1-\gamma)}{(1-k)\gamma + k(1-\gamma)}$$

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