INFORMATIONAL COMPLEMENTARITY

Very Preliminary. Do Not Circulate.

T. TONY KE
MIT
kete@mit.edu

SONG LIN
Hong Kong University of Science and Technology
mksonglin@ust.hk

January 2018
INFORMATIONAL COMPLEMENTARITY

ABSTRACT

Many products are correlated because they share some similar or common attributes. We show that when these attributes are uncertain to consumers, a complementary effect can arise among competing products, in the sense that a lower price of one product may increase the demands of others. The effect occurs when consumers optimally search for information about both common and idiosyncratic product attributes prior to purchase. When firms compete in price, although product correlation can weaken differentiation between firms, the complementary effect due to correlated search can raise equilibrium price and profit.

Keywords: search theory, information, complementarity, price competition
In many markets, different products under a consumer’s purchase consideration share common or similar attributes. For example, houses in the same neighborhood share similar characteristics in transportation accessibility, quality of schools and crime statistics. Take cars as another example. BMW is well known for performance, Mercedes for luxury, Volvo for safety, and Toyota for reliability, etc. All BMW cars share the brand image, as well as similar technologies, design, and after-sales service. Even products of competing brands can share common attributes. For example, electric cars from BMW and Mercedes share the same country of origin and similar engine technologies. Among products with common or similar attributes, consumers’ preference will be correlated. For example, when BMW improves its service, consumers will in general have a stronger preference for all its models.

Consumers’ preference for different products may be not only correlated but also uncertain. We live in an age of information explosion. We, as consumers, spend considerable time gathering, processing, and understanding information before making many choices and purchasing decisions. A 2010 survey by Zillow.com find that an average US consumer spent 40 hours in searching for information before purchasing a new home, 10 hours for a major home improvement or a car, 5 hours for a vacation or a mortgage, 4 hours for a computer, and 2 hours for a television set. In the search process, new information about one product may update a consumer’s preference for other products which share similar attributes. For example, as a consumer researches a BMW electric car, she learns the cost and benefit of an electric motor. This information will update her preference for all other electric cars.

In this research, we consider multiple products with correlated uncertain information, and show that although different alternatives in a choice set are naturally substitutes, demands for them can surprisingly exhibit complementary effects, if consumers need to search for information about these alternatives before making a choice. Specifically, we show that lowering the price of one alternative can increase the demand for others that share common attributes. To fix the idea, consider a consumer who wants to buy an electric car from either BMW or Mercedes, with an outside option of not buying anything (or maybe instead buying a gasoline car). The two electric cars share similar technologies—notably electric propulsion, the exact benefits of which are unknown to her a priori. Suppose initially, the prices of both cars are so high that the consumer will neither search nor buy any of the two cars. Now suppose the BMW lowers the price for its car, attracting the consumer to start searching for information about the car. As she learns more about the BMW electric car, it is possible that she starts to like electric cars but not so much about BMW. Thus, she decides to explore more electric cars and continue to search for information about the Mercedes electric car.
Eventually, she may like the Mercedes better and purchase it. Therefore, a lower price of BMW may increase the demand of Mercedes. We term this effect “informational complementarity”, which is the central phenomenon of this paper.

We develop a stylized sequential search model that allows us to explore the conditions under which informational complementarity can arise and its implications. Consumers are interested in buying one of two competing products which are correlated through a common attribute. The benefits of the common attribute, and of other idiosyncratic attributes, are unknown to consumers, who then need to gather information about these attributes in order to make an informed decision. Searching for information is costly and consumers need to optimally determine whether they should stop or continuing searching, and make a purchase or not. Although in general there is no clean form solution for the optimal decision rule in such set up, we characterize the conditions under which informational complementarity can arise. Specifically, when the search cost is not too large, and/or the correlation between the products is sufficiently strong, reducing the price of one product can increase the demand of the other. We also show that this effect not only arises when the products are positively correlated, but also when they are negatively correlated.

The informational complementarity has important implications for competitive pricing. In a simple duopoly model, we illustrate that product correlation introduces two opposing effects. On one hand, the informational complementarity effect expands the search region for competing products. This effect brings in new consumers who would not consider any of the products in the first place were they independent. On the other hand, with correlated features, products are less differentiated. If consumers find out the common attribute to be negative by searching one product, then the negative impression spills over to the correlated product, lowering the purchase likelihood for both. Both products enjoy less market power and are faced with fiercer price competition. When the search cost is sufficiently small, price competition is intensified to the degree that it outweighs the benefit of informational complementarity. Equilibrium price becomes lower when products are correlated, resulting in lower equilibrium profit.

Related Literature

The research is closely related to a large body of literature on optimal sequential search (notably, McCall 1970, and Weitzman 1979), and its applications on pricing (e.g., Diamond 1971, Wolinsky 1986, Stahl 1989, Anderson and Renault 1999), advertising (e.g., Anderson and Renault 2006, Mayzlin and Shin 2011), and product design (e.g. Kuksov and Villas-Boas 2010, Bar-Isaac et al. 2012). The basic premise of this literature is that consumers are uncertain about some attributes or their overall utility of a product. They can acquire information through costly search, before
they make the decision on which product to purchase. These papers assume that all available information of a product will be revealed after one search action, and information are independent across products. We relax both assumptions in this paper. In this sense, this paper is in line with a recent stream of literature that allows information of a product to be revealed gradually over multiple search actions (e.g., Branco et al. 2012, Ke et al. 2016, Ke and Villas-Boas 2017). However, different from this literature, we allow for correlated product information by incorporating both common attribute and idiosyncratic attribute for each product. Ke et al. (2016) has also considered correlated product information but under a different setup. They build a continuous-time model and assume every attribute is correlated across products. Moreover, their framework, by construction, has ruled out the possibility of informational complementarity due to the assumption of constant informativeness. To summarize, this paper contributes to the large literature of search theory by incorporating information correlation among alternatives and exploring its implications on multi-product demand functions.

The proposed research may have implications for several areas in marketing and economics, including measuring consumer demand and price elasticities, product line design, and competitive pricing. First, the fact that consumers often seek information (actively or passively) when they are uncertain about a product’s attributes has promoted empirical research to measure its impact on product demands (e.g., Erdem and Keane 1996; Kim et al. 2010; Lin et al. 2015). However, much of the work either rules out complementarity by assuming independent learning, or considers information spillover under myopic information acquisition (e.g., Erdem 1998). Our research not only considers information spillover, but also examines how it can arise under optimal sequential search without the assumption of consumers making multiple purchases. Furthermore, the way that the complementarity effect arises is not through enhancing the correlated features (a better electric motor increases the demand for both BMW and Mercedes electric cars), but instead through price reduction (reduced price of BMW increases demand for both BMW and Mercedes). This mechanism has far reaching implications on product and pricing strategies, but it is overlooked in the literature of information spillover.

Another stream of empirical literature of demand estimation incorporates complementarity by allowing consumers to choose multiple goods on one purchase occasion (e.g., Manski and Sherman 1980; Train et al. 1987; Hendel 1999; Gentzkow 2007). Consumers are assumed to possess perfect information about products and are allowed to purchase multiple products at the same time so that complementarity among products is a very common effect. Our research instead uncovers the possibility of complementarity in a single-choice problem. Second, many firms sell a line of varying products, trying to segment consumers, who then often face the choice among these products. It is quite common that different products made by the same firm share some features, resulting in
correlated preferences. This research can shed light on how correlated search and thus informational complementarity can impact firms’ product line policies, which have received extensive research effort (e.g., Mussa and Rosen 1978; Moorthy 1984; Desai 2001). Third, when correlated products are offered by different firms instead of one firm, competitive pricing becomes an important issue. While the literature on competitive pricing has mainly focused on the case when goods are substitutes, our research can allow us to examine whether complementarity among goods can soften price competition and its implications for consumer and social welfare.

2. A Model of Correlated Search

There are two products in the market indexed by \( i = 1, 2 \). A representative consumer’s utility of product \( i \) is given by

\[
U_i = \alpha_i + X - p_i + \varepsilon_i,
\]

where \( p_i \) is the price, \( X \) is the common attribute shared by the two products, both \( \alpha_i \) and \( \varepsilon_i \) are idiosyncratic utilities for product \( i \). The consumer knows \( p_i \) and \( \alpha_i \) a priori, but do not know \( X \) and \( \varepsilon_i \) \((i = 1, 2)\). Particularly, it is assumed that \( X \) follows distribution \( G \) with support in \([X, \bar{X}]\), and \( \varepsilon_i \) follows distribution \( F \) with support in \([\varepsilon, \bar{\varepsilon}]\), where \( X, \bar{X}, \varepsilon \) and \( \bar{\varepsilon} \) can be finite or infinite. \( X, \varepsilon_1 \) and \( \varepsilon_2 \) are assumed to be independent. We further assume that \( G \) and \( F \) do not depend on the prices \( p_1 \) and \( p_2 \). Basically, we do not allow the firms to use price to signal product quality. Without loss of generality, we can assume that \( E[X] = E[\varepsilon_i] = 0 \) \((i = 1, 2)\). The outside option is assumed to be deterministic and known, and is normalized as zero. Because both \( p_i \) and \( \alpha_i \) are known ex ante, it is convenient to define the consumer’s ex ante expected utility \( u_i \equiv E[U_i] = \alpha_i - p_i \), and work with \( u_i \) instead of \( p_i \) below.

A consumer searches sequentially over the two products before making a purchase decision. We assume that each time the consumer searches a product, she pays a search cost \( c > 0 \) and uncovers all available information about the product. Therefore, if she searches product \( i \) first, she will discover both \( X \) and \( \varepsilon_i \). At this time point, she remains unknown about \( \varepsilon_j \), the idiosyncratic attribute of project \( j \). If she continues to search product \( j \), she will further discover \( \varepsilon_j \). It is worthwhile to notice that in the setup above, we have allowed the information of product \( j \) to be revealed gradually over time (Ke et al. 2016, Ke and Villas-Boas 2017). Following the majority of the literature (e.g., Weitzman 1979, Wolinsky 1986), we assume that a consumer has to search a product before purchasing it. This is a reasonable assumption if we consider cases where a consumer has to pay travel costs to visit a store before making a purchase. We relax this assumption in Section
5.4 and find that our main finding is robust to the assumption.\footnote{When a consumer is allowed to purchase a product before searching it, the consumer’s optimal search problem is very complicated. In general, it is not a multi-arm bandit problem, and the index policies such as Weitzman’s Pandora rule is not optimal. See Doval (2014) and Ke and Villas-Boas (2017) for discussions.} A consumer conducts optimal sequential search to maximize her expected utility. This means that she can optimally decide which product to search and when to stop searching at any time.

Obviously, the consumer’s decision process lasts for at most three periods. We can therefore formulate her optimal search problem as a dynamic decision problem in three periods, and solve it by backward induction. In the last period, she has searched both products and discovered $X$, $\varepsilon_1$ and $\varepsilon_2$. She needs to decide which product to buy or to take the outside option. Her maximum expected utility is then,

$$V_2(X, \varepsilon_1, \varepsilon_2) = \max\{u_1 + X + \varepsilon_1, u_2 + X + \varepsilon_2, 0\},$$

where $V_2(X, \varepsilon_1, \varepsilon_2)$ is the value function, and $X$, $\varepsilon_1$ and $\varepsilon_2$ together are the state variables that characterize the consumer’s current information. Obviously, besides $X$, $\varepsilon_1$ and $\varepsilon_2$, the consumer’s decision will also depend on $u_1$ and $u_2$, which are assumed to be known initially. Therefore, we do not explicitly express the value function as depending on them.

Going back one period, the consumer has already searched one product, and decides whether to search the other product, or to make a purchase decision immediately. Conditioning that she has searched product $i$ and discovered $X$ and $\varepsilon_i$, the consumer’s maximum expected utility is,

$$V_{1i}(X, \varepsilon_i) = \max\{U_i, 0, -c + E[V_2(X, \varepsilon_1, \varepsilon_2)|X, \varepsilon_i]\}$$

$$= \max\{u_i + X + \varepsilon_i, 0, -c + E[V_2(X, \varepsilon_1, \varepsilon_2)|X, \varepsilon_i]\}, \text{ for } i \neq j = 1, 2,$$

where subscript $i$ in the value function $V_{1i}(\cdot, \cdot)$ stands for “conditioning on that the consumer has searched product $i$”. In the first equation above, there are three terms in the maximization on the righthand side of the equation. They are respectively, the utility of purchasing product $i$, the utility from outside option, and lastly the conditional expected utility of continuing to search product $j$. Notice that the information revealed by product $i$ has implications on the expected utility of continuing to search product $j$, because there is a common attribute $X$ for the two products.

Finally, Let’s consider the decision in the first period. At this time point, the consumer has not searched any product. She chooses which product to search first or to take the outside option, so
as to maximize her expected utility:

\[ V = \max \{0, -c + E[V_{11}(X, \varepsilon_1)] , -c + E[V_{12}(X, \varepsilon_2)] \}. \]

(3)

where the three terms in the maximization on the righthand side of the equation above correspond to the outside option, the consumer’s expected utility from searching product 1, and her expected utility from searching product 2.

So far, we have laid out a consumer’s sequential search problem with correlated product information, and formulated it as a dynamic decision problem. We will analyze the consumer’s optimal search strategy and characterize her demand functions in the next section. We conclude this section by considering the benchmark case where a consumer needs to make a purchase decision immediately without information search. In this case, the demand function of product \( i \) is that,

\[ D^B_i(u_i, u_j) = \Pr (U_i > U_j \text{ and } U_i > 0) \]

\[ = \Pr (u_i + \varepsilon_i > u_j + \varepsilon_j \text{ and } u_i + X + \varepsilon_i > 0), \]

(4)

where the superscript “B” stands for “under the benchmark case without information search”. Obviously, as \( u_j \) increases, the condition in equation (4) is less likely to be satisfied. Therefore, \( D^B_i(u_i, u_j) \) is a non-decreasing function of \( u_j \), i.e., \( \partial D^B_i / \partial u_j \leq 0 \) for \( j \neq i \). This is very intuitive—product \( i \) and \( j \) are substitutes, so when product \( j \) becomes more attractive, the demand of product \( i \) gets lower, despite of correlated product information. In contrast, we will show below that when consumers are allowed to search for information on the two products, this is no longer true, and we may have that \( \partial D_i / \partial u_j > 0 \).

3. Characterization of Informational Complementarity

Let’s consider a consumer’s optimal search decision in the first period. Equation (3) implies that the consumer prefers to search product \( i \) if and only if \( E[V_{1i}(X, \varepsilon_i)] \geq E[V_{1j}(X, \varepsilon_j)] \) and \( E[V_{1i}(X, \varepsilon_i)] \geq c \). This condition is complicated and not easy to work with. Below, we will first characterize the consumer’s optimal search strategy in simple forms, and then characterize the condition for informational complementarity to emerge.

Let’s start with the following definitions. Define \( u_0 \) by the following equation,

\[ E[V_{1i}(X, \varepsilon_i)]|_{u_1 = u_2 = u_0} = c. \]

(5)
In the appendix (proof of Proposition 1 below), we have shown that $u_0$ is well defined given $c > 0$. By definition, given $u_1 = u_2 = u_0$, a consumer will be indifferent among searching product 1, searching product 2 and taking the outside option in the first period.

Define $\bar{u}$ by the following equation,

$$E[\max\{\bar{u} + X + \varepsilon_i, 0\}] = c.$$  \hspace{1cm} (6)

To understand $\bar{u}$, consider a different consumer search problem, where a consumer has only two options—either to search product $i$ or to take the outside option. Product $j$ is absent from the consumer’s consideration for search or purchase. In this case, the consumer’s optimal search strategy is very simple—she will prefer to search product $i$ if and only if $u_i \geq \bar{u}$. In another word, $\bar{u}$ is the threshold utility at which, the consumer is indifferent between searching product $i$ and taking the outside option, given product $j$ is absent. This can be inferred directly from equation (6), where the left hand side is the expected gain from searching product $i$, and the right hand side is the search cost.

By the definitions of $u_0$ and $\bar{u}$ above, it is straightforward to show that,

$$\bar{u} \geq u_0.$$\hspace{1cm} (7)

In fact, $u_0$ is the utility, at which point, the consumer is indifferent between searching $i$ and taking the outside option, in presence of product $j$ with $u_j = u_0$; while, $\bar{u}$ is the utility, at which point, the consumer is indifferent between searching $i$ and taking the outside option, given product $j$ is absent, or equivalently, $u_j = -\infty$. Obviously, the presence of product $j$ makes product $i$ more attractive relative to the outside option, because the consumer has the option to continue to search product $j$ after searching product $i$. Therefore, the consumer’s indifference point should be lower with the presence of product $j$, i.e., $u_0 \leq \bar{u}$.

Now, we are ready to present the following proposition, with proof in the appendix.

**Proposition 1 (Search Order):**

- Between the two products, a consumer prefers to first search product $i$ if and only if $u_i \geq u_j$ for $i \neq j = 1, 2$.
- When $u_j < u_0$, the consumer will never search product $j$ first for any $u_i$; on the other hand, when $u_j \geq u_0$, the consumer will search product $j$ first for some $u_i$.
- When $u_j \geq \max\{u_i, \bar{u}\}$, it is optimal for the consumer to search product $j$ in the first period.
The first statement in Proposition 1 implies that a consumer’s optimal search order between the two products does not depend on the correlation between the two products—a consumer always searches the product with higher expected utility first. The second statement implies that when a consumer’s expected utility of a product is below a tight threshold $u_0$, she will never search it first—she will choose to either search the other product or take the outside option first. The third statement looks at the other extreme when a consumer’s expected utility of a product is sufficiently high, in which case, it is optimal to search this product in the first period. The three statements in Proposition 1 together partially characterize a consumer’s optimal search strategy, as shown in Figure 1. We have not fully determined the consumer’s optimal search strategy in the gray area.

![Figure 1: Illustration of a consumer’s optimal search strategy inferred by Proposition 1.](image)

To further understand the consumer’s optimal search strategy in the gray area in Figure 1, we notice that Proposition 1 implies that when $u_j < u_0$, the consumer will not search product $j$ first. However, this does not mean that the consumer will never search product $j$. In fact, it is still possible that she will continue to search product $j$ after searching product $i$ first. Let’s analyze this possibility below.

After a consumer first searches product $i$, she discovers the common and idiosyncratic attributes, $X$ and $\varepsilon_i$. Conditioning on the common attribute $X$, there is no dependence between the two products, so the consumer’s subsequent optimal search problem can be characterized by Weitzman’s Pandora rule. Particularly, the consumer continues to search project $j$ if and only if her maximum utility from product $i$ and the outside option is less than some threshold $U_j^*$, where $U_j^*$ is the
reservation utility that makes the consumer indifferent between continuing searching product $j$ and adopting an option with utility $U_j^*$. We can write down $U_j^* = u_j + X + \varepsilon^*$, where $\varepsilon^*$ is defined by the following equation.

$$c = \int_{\varepsilon^*}^{+\infty} (\varepsilon - \varepsilon^*)dF(\varepsilon).$$ \hfill (7)

We assume that $c$ is not very large such that $\varepsilon^*$ is well defined by equation (7). To summarize, after searching product $i$ and discovering $X$ and $\varepsilon_i$, a consumer continues to search product $j$ if and only if $u_j + X + \varepsilon^* \geq \max\{u_i + X + \varepsilon_i, 0\}$, or equivalently,

$$u_j \geq \max\{u_i + \varepsilon_i, -X\} - \varepsilon^*.$$ \hfill (8)

Notice that the righthand side of inequality (8) increases with $\varepsilon_i$ and decreases with $X$, and thus takes the minimum value of $\max\{u_i + \varepsilon_i, -X\} - \varepsilon^*$, when $\varepsilon_i = \overline{\varepsilon}$ and $X = \overline{X}$. This implies that, given $u_i$, when $u_j < \max\{u_i + \overline{\varepsilon}, -\overline{X}\} - \varepsilon^*$, a consumer will never continue to search product $j$, regardless of what values $X$ and $\varepsilon_i$ realized to be in her first search. Combining with the results in Proposition 1, we know that given $u_i$, when $u_j < \min\{u_0, \max\{u_i + \overline{\varepsilon}, -\overline{X}\} - \varepsilon^*\}$, the consumer will never search product $j$ in either first or second period, and therefore, product $j$ is out of the consumer’s consideration set. Notice that the threshold for product $j$, $\min\{u_0, \max\{u_i + \overline{\varepsilon}, -\overline{X}\} - \varepsilon^*\}$, depends on the expected utility of product $i$, $u_i$. In fact, we can prove a stronger and cleaner statement by the following proposition, where we have a constant and tight threshold for the consideration set. The proof is in the appendix.

**Proposition 2 (Consideration Set):** Define $\underline{u}$ by the following equation,

$$\underline{u} \equiv \max\{\overline{u} + \overline{\varepsilon}, -\overline{X}\} - \varepsilon^*.$$ \hfill (9)

- When $u_j > \min\{u, \overline{u}\}$, a consumer will search product $j$ in the first period, or there is a positive probability that a consumer will search and buy product $j$ in the second period.

- When $u_j < \min\{u, \overline{u}\}$, a consumer will never search or buy product $j$. In this case, the consumer’s sequential search problem is reduced to the decision of whether to search product $i$ or to take the outside option. She prefers to search product $i$ if and only if $u_i \geq \overline{u}$.

Proposition 2 completely characterizes the threshold structure of the consumer’s consideration set. It implies there exists a constant threshold such that when a consumer’s expected utility of a product is below this threshold, she will never consider this product for search or purchase; on the other hand, when her expected utility of the product is above this threshold, she will consider the
product for search and purchase with some positive probability. At the threshold, the consumer will be indifferent between considering the product or not. The threshold has a simple form of \( \min\{u, \overline{u}\} \), which is finite when \( u \) is finite, or equivalently when either \( \varepsilon \) or \( X \) is finite.

Now, based on Proposition 1 and Proposition 2, we are ready to characterize a consumer’s optimal search strategy. There are two cases to consider.

First, we notice that if \( u \geq \overline{u} \), we can completely characterize a consumer’s optimal search strategy in simple forms by the following corollary. The proof is straightforward and thus omitted (it is also an intermediate step in the proof of Proposition 2).

**Corollary 1:** If \( u \geq \overline{u} \), it is optimal to search product \( i \) if and only if \( u_i \geq \max\{u_j, \overline{u}\} \) for \( i \neq j = 1, 2 \); it is optimal to take the outside option if and only if \( u_1 < \overline{u} \) and \( u_2 < \overline{u} \).

The left panel in Figure 2 illustrates a consumer’s optimal search strategy, as characterized by Corollary 1. It implies that if \( u \geq \overline{u} \), the consumer’s optimal search strategy will be the same with that in the case without information correlation between the two products.

In the second case, if \( u < \overline{u} \), we have the following corollary, with proof in the appendix.

**Corollary 2:** If \( u < \overline{u} \), the indifference curve between searching \( j \) and taking the outside option decreases with \( u_i \), and \( \overline{u} \leq u_0 \leq \overline{u} \).

The right panel in Figure 2 illustrates a consumer’s optimal search strategy, as inferred by Corollary 2. In this case, the consumer never searches product 1 in either period when \( u_1 < u \); on the other hand, when \( u_0 \geq u_1 \geq u \), there is a positive probability that a consumer will search product 1 in the second period, which makes searching product 2 more attractive in the first period relative to the outside option. Therefore, as we increase \( u_1 \) above \( u \), we can see from Figure 2 that the indifference curve between searching 2 and the outside option falls below \( \overline{u} \).

Now, based on the optimal search strategy, we are ready to characterize a consumer’s demand functions of the two products, and more particularly the cross-elasticity of the demand functions. According to equation (3) and Proposition 1, we can write down the demand function for product \( i \) as the following.

\[
D_i(u_i, u_j) = \mathbb{1}_{\{E[V_i(X,\varepsilon_i)] \geq c\}} \mathbb{1}_{\{u_i \geq u_j\}} \Pr (u_i + X + \varepsilon_i \geq \max\{u_j + X + \min\{\varepsilon_j, \varepsilon^*\}, 0\}) \\
+ \mathbb{1}_{\{E[V_j(X,\varepsilon_j)] \geq c\}} \mathbb{1}_{\{u_j \geq u_i\}} \Pr (u_i + X + \min\{\varepsilon_i, \varepsilon^*\} \geq \max\{u_j + X + \varepsilon_j, 0\}). \tag{10}
\]

To understand how we calculate the demand function in equation (10), one notices that there are two cases under which a consumer will purchase product \( i \) eventually—either a consumer searches
Let’s look at these two cases one by one. A consumer searches product $i$ first, if and only if $E[V_{1i}(X,\varepsilon_i)] \geq c$ and $u_i \geq u_j$. After a consumer searches product $i$, $X$ and $\varepsilon_i$ realize, and she decides whether to continue to search product $j$. This continuation problem is a standard sequential search problem without inter-product correlations. Armstrong (2017) and Choi et al. (2016) show that when consumers conduct a sequential search before purchase a la Weitzman (1979), the resulting demand system is equivalent to a static discrete choice model if one defines the “equivalent utility” for product $j$ as $\min\{U_j, U_j^*\} = u_j + X + \min\{\varepsilon_j, \varepsilon^*\}$. Therefore, given a consumer has searched product $i$ and discovered $X$ and $\varepsilon_i$, the probability that she will purchase product $i$ is $\Pr(u_i + X + \varepsilon_i \geq \max\{u_j + X + \min\{\varepsilon_j, \varepsilon^*\}, 0\} | X, \varepsilon_i)$, which includes both cases when the consumer purchases product $i$ immediately without searching product $j$, and when the consumer returns to purchase product $i$ after searching product $j$. Based on these observations, we can see that the first line on the righthand side of equation (10) is exactly a consumer’s demand for product $i$ given that she first searches product $i$. Similarly, on the other
hand, a consumer first searches product \( j \) if and only if \( \mathbb{E}[V_{ij}(X, \varepsilon_j)] \geq c \) and \( u_j > u_i \). After searching product \( j \) and discovering \( X \) and \( \varepsilon_j \), the consumer purchases product \( j \) with probability
\[
\Pr (u_i + X + \min\{\varepsilon_i, \varepsilon^*\} \geq \max\{u_j + X + \varepsilon_j, 0\} | X, \varepsilon_j).
\]
Therefore, the second line on the right-hand side of equation (10) represents the consumer’s demand for product \( i \) when she first searches product \( j \).

Based on equation (10) and the consumer’s optimal search strategy characterized by Proposition 1 and 2, we can derive our main result on the informational complementarity as follows. The proof is in the appendix.

**Theorem 1 (Informational Complementarity):**

- If \( \bar{u} \leq u \), then there is no complementarity effect: \( D_i(u_i, u_j) \) always weakly decreases with \( u_j \).
- If \( \bar{u} > u \), then,
  - complementarity effect arises when \( u_i \in (u, \bar{u}) \): \( D_i(u_i, u_j) \) first increases and then decreases with \( u_j \);
  - there is no complementarity for \( u_i \notin (u, \bar{u}) \): \( D_i(u_i, u_j) \) always weakly decreases with \( u_j \).

Theorem 1 completely characterizes the condition for the informational complementarity to arise. The demand of product \( i \) will first decrease and then increase with the price of product \( j \) if and only if \( \bar{u} > u_i > u \). When a consumer’s expected utility of product \( i \) is very low, i.e., \( u_i \leq u \), product \( i \) is out of the consumer’s consideration set, and thus the demand of product \( i \) is always zero. In the other extreme, when a consumer’s expected utility of product \( i \) is very high, i.e., \( u_i \geq \bar{u} \), it is optimal for the consumer to search either product \( i \) or product \( j \). As the expected utility of product \( j \), \( u_j \) gets higher, it makes it more attractive to search and buy product \( j \). Consequently, the demand of product \( i \) always decreases with \( u_j \), or equivalently, increases with the price of product \( j \). Finally, when a consumer’s expected utility of product \( i \) is in an intermediate range, i.e., \( \bar{u} > u_i > u \), the consumer will take the outside option right away when her expected utility of product \( j \), \( u_j \) is relatively low. As \( u_j \) increases, it becomes optimal for the consumer to either search product \( j \) or \( i \) first. Particularly, as shown by Figure 2, when \( \bar{u} > u_i \geq u_0 \), the consumer will search product \( j \) first as \( u_j \) increases; when \( u_0 > u_i > u \), the consumer will search product \( i \) first as \( u_j \) increases. Both cases will eventually lead the consumer to purchase product \( i \) with a positive probability. This explains why the demand function of product \( i \) will first increase with \( u_j \). As \( u_j \)

\[\text{Here we have assumed the tie-breaking rule that when a consumer is indifferent between searching product } i \text{ and } j, \text{ she will prefer to search product } i. \text{ We will get similar results under other tie-breaking rules.}\]
increases further, product \( j \) becomes very attractive, and the demand function of product \( i \) will eventually decrease with \( u_j \) again.

Theorem 1 implies that for informational complementarity to arise, it is necessary that \( \bar{u} > u \). The following proposition translates this condition in terms of model primitives.

**Proposition 3:** \( u < \bar{u} \) if and only if \( \text{Var}[X] > 0 \) and \( 0 < c < E[\varepsilon] - \varepsilon \).

Proposition 3 implies that the condition for informational complementarity to arise is very weak. In fact, \( 0 < c < E[\varepsilon] - \varepsilon \) is equivalent to \( \varepsilon^* \in (\varepsilon, \overline{\varepsilon}) \), which ensures that after searching one product, the consumer is willing to continue to search the other product under some circumstances.

### 3.1. An Example: Two-Point Distribution

To illustrate the main results, let’s consider a simple example where both \( X \) and \( \varepsilon_i \) follow two point distributions. Specifically, we assume that \( X \) takes the value of either \( -\beta \) or \( \beta \) with equal probability, and that \( \varepsilon_i \) equals either \( -1 \) or \( 1 \) with equal probability. According to Proposition 3, to ensure \( \bar{u} > u \), we need \( c < 1 \). We can calculate \( \bar{u} \) and \( u \) by equations (6) and (9) respectively.

If \( \beta \leq 1 \), \( \bar{u} = \begin{cases} 4c - \beta - 1, & c \leq \frac{1}{2}\beta \\ 2c - 1, & \frac{1}{2}\beta < c \leq 1 - \frac{1}{2}\beta \\ \frac{1}{3}(4c - \beta - 1), & 1 - \frac{1}{2}\beta < c < 1 \end{cases} \) and \( u = \begin{cases} 2c - \beta - 1, & c \leq 1 - \frac{1}{2}\beta \\ \frac{1}{3}(10c - \beta - 7), & 1 - \frac{1}{2}\beta < c < 1 \end{cases} \).

If \( \beta > 1 \), \( \bar{u} = \begin{cases} 4c - \beta - 1, & c \leq \frac{1}{2} \\ 2c - \beta, & \frac{1}{2} < c \leq \beta - \frac{1}{2} \\ \frac{1}{3}(4c - \beta - 1), & \beta - \frac{1}{2} < c < 1 \end{cases} \) and \( u = \begin{cases} 2c - \beta - 1, & c \leq \frac{1}{2} \\ 4c - \beta - 2, & \frac{1}{2} < c \leq \beta - \frac{1}{2} \\ \frac{1}{3}(10c - \beta - 7), & \beta - \frac{1}{2} < c < 1 \end{cases} \).

Figure 3 illustrates the demand function \( D_1(u_i, u_j) \) under \( c = 0.5 \) and \( \beta = 1 \). We can see that informational complementarity arises in Figure 3 when \( u_2 \) is in the intermediate range \( (u_2 = -0.5 \text{ and } u_2 = 0) \). As the consumer’s expected utility of product 1, \( u_1 \) increases, the purchase likelihood of product 2 first increases and then decreases.

### 4. Competitive Pricing under Correlated Search

We have shown that competing products may exhibit informational complementarity arising from consumers’ correlated search. A natural question to ask is how this mechanism will impact firms’ pricing behaviors under competition. Will competing firms always prefer a higher correlation between their products, so that they can charge a higher price due to informational complementarity?
Figure 3: Demand of two products as a function of $u_1$ (for $c = 0.5$ and $\beta = 1$).
To answer this question, we first examine the benchmark case with independent products, and then analyze the equilibrium price under the correlated products.

We assume that there is a set of consumers with size one in the market, and each has a unit demand. The two products available in the market are owned by two firms, who compete by setting publicly observable prices, \( p_1 \) and \( p_2 \). Therefore, consumers are informed about the prices prior to search and the role of searching is to acquire information about product attributes. This assumption is consistent with our consumer search model in the previous section, and fits well with the setting of online shopping, where consumers can easily browse the prices of many products with little cost first, before clicking on each one to learn more about the product information (Choi et al. 2016). The marginal costs of production for both products are normalized to zero.

Even with independent products, it is in general impossible to obtain closed-form solutions for the equilibrium prices under the consumer search model (Choi et al. 2016). To make our analysis tractable, we use the discrete setup in Section 3.1 and assume that \( \beta = 1 \) and \( c \leq \beta/2 = 1/2 \). To produce relatively smooth demands, we assume that consumers are \textit{ex ante} heterogeneous in \((\alpha_1, \alpha_2)\). Particularly, \( \alpha_1 \) and \( \alpha_2 \) follow independent uniform distributions in \([\underline{\alpha}, \overline{\alpha}]\). The lower bound on \( \alpha_i \) is assumed to be sufficiently small, \( \underline{\alpha} \leq -2 \), so that even with positive signals on both \( X \) and \( \varepsilon_i \), some consumers are not interested in buying product \( i \). The upper bound on \( \alpha_i \) is assumed to be zero, \( \overline{\alpha} = 0 \), an assumption made to simplify the analysis. The assumptions on the bounds imply that we are interested in a market where consumers have relative pessimistic beliefs about both products a priori. This is to ensure that a sufficient large segment of consumers are subject to the informational complementarity effect. Under these assumptions, \( u_i = \alpha_i - p_i \leq 0 \) for any \( \alpha_i \in [\underline{\alpha}, \overline{\alpha}] \) if firm \( i \) does not price below the marginal cost. Therefore, for the product to be chosen, it is necessary that the realizations of both \( X \) and \( \varepsilon_i \) are positive. That is, \( U_i = \alpha_i - p_i + X + \varepsilon_i > 0 \) only if \( X = \varepsilon_i = 1 \). This observation simplifies the calculation of firms’ demand functions below substantially.

4.1. Independent Case

In the model with independent products, we assume that product \( i \) has two idiosyncratic attributes, \( X_i \) and \( \varepsilon_i \), where, \( X_1, X_2, \varepsilon_1 \) and \( \varepsilon_2 \) are independent. It is well known that the optimal search rule follows an index strategy, or Weitzman (1979)’s Pandora’s rule. Specifically, the reservation utility of

\[ u_i = \alpha_i - p_i + X + \varepsilon_i \]

A different approach to modeling competitive pricing in the consumer search market is to assume that consumers do not observe the firms’ prices a priori—instead, they form rational expectations of the prices. Upon visiting a firm, a consumer discovers both the idiosyncratic product characteristics and the price, which, in equilibrium, coincides with the consumer’s expectation (see, e.g., Wolinsky 1986, Anderson and Renault 1999, Armstrong et al. 2009).
product \( i \) is \( \alpha_i - p_i - p, \) where \( p \) is defined by equation (6). A consumer with \( \alpha_1 \) and \( \alpha_2 \) first searches product \( i \) if and only if \( \alpha_i - p_i - p \geq \max\{\alpha_j - p_j - p, 0\} \), or equivalently, \( \alpha_i - p_i \geq \max\{\alpha_j - p_j, p\} \).

Given our distributional assumption, it is straightforward that \( p = 4c - 2 \leq 0 \). Consider a consumer who first searches product \( i \). With probability 1/4, the outcome is \( (X_i, \varepsilon_i) = (1, 1) \), under which case, the consumer will purchase product \( i \) right away, because \( U_i = \alpha_i - p_i + X_i + \varepsilon_i = \alpha_i - p_i + 2 \geq \max\{\alpha_j - p_j, p\} + 2 \geq \max\{\alpha_j - p_j - p, 0\} \). With the remaining probability 3/4, any other realizations will discourage the consumer from buying the product, because now \( U_i < 0 \). The question then is whether to search product \( j \) or take the outside option. Applying the search rule, continuing to search product \( j \) is optimal as long as \( \alpha_j - p_j > p = 4c - 2 \). Conditional on the second search, product \( j \) will be purchased only if \( (X_j, \varepsilon_j) = (1, 1) \), which occurs with probability 1/4. In sum, the demand of product \( i \) comes from two sources. One is from first searching \( i \) that yields positive signals on both \( X_i \) and \( \varepsilon_i \). The other comes from first searching competitor \( j \) that yields a negative signal on either \( X_j \) or \( \varepsilon_j \) or both, followed by a second search on product \( i \) that yields positive signals on both \( X_i \) and \( \varepsilon_i \).

A symmetric equilibrium price \( p_{\text{ind}}^* \), if exists, must lie within \([0, -p]\); otherwise, both products will be dominated by the outside option, where we have used the subscript \( \text{ind} \) to denote the case of independent products. To derive the equilibrium price, let’s fix the price of firm 2 at the equilibrium price \( p_2 = p_{\text{ind}}^* \) and solve for firm 1’s pricing problem. Recall the notations that \( u_i = \alpha_i - p_i \) and \( u_j = \alpha_j - p_j \). As discussed above, the demand of product 1 comes from two sources.

First, let \( S'_{\text{ind}}(p_1) \) denote the size of the consumer segment with \( u_1 \geq u_2 \) and \( u_1 \geq p \). These consumers will search product 1 first, and 1/4 of them will eventually purchase product 1. We have that,

\[
S'_{\text{ind}}(p_1) = \frac{1}{\alpha^2} \left[ \frac{1}{2}(p_1 + p)^2 + (\alpha - p - p_{\text{ind}}^*)(p_1 + p) - \frac{1}{2} \min\{p_{\text{ind}}^* - p_1, 0\}^2 \right].
\]

Second, let \( S''_{\text{ind}}(p_1) \) denote the size of the consumer segment with \( u_2 > u_1 \geq p \). These consumers will search product 2 first, and \( 3/4 \times 1/4 = 3/16 \) of them will eventually purchase product 1. We have that,

\[
S''_{\text{ind}}(p_1) = \frac{1}{\alpha^2} \left[ \frac{1}{2}(p_{\text{ind}}^* + p)^2 - \frac{1}{2} \min\{p_1 - p_{\text{ind}}^*, 0\}^2 \right].
\]

Figure 4 illustrates the two consumer segments, \( S'_{\text{ind}}(p_1) \) and \( S''_{\text{ind}}(p_1) \).

Combining the two consumer segments together, firm 1’s profit is,

\[
\Pi_{\text{ind}}(p_1) = p_1 \left[ \frac{1}{4} S'_{\text{ind}}(p_1) + \frac{3}{16} S''_{\text{ind}}(p_1) \right].
\]

\(^4\)The plot is made in the case with \( p_1 > p_{\text{ind}}^* \). The other case with \( p_1 \leq p_{\text{ind}}^* \) can be plotted similarly.
Firm 1’s objective of profit maximization implies the following optimality condition,

\[ p_{\text{ind}}^* = \arg \max_{p_1} \Pi_{\text{ind}}(p_1). \]

We can solve the equilibrium price \( p_{\text{ind}}^* \) by the first and second order optimality condition.

\[ p_{\text{ind}}^* = 2 - 4c + 8\alpha + 4\sqrt{4\alpha^2 + (1 - 2c)\alpha}. \]

The equilibrium profit is then given by,

\[ \Pi_{\text{ind}}^* = \Pi_{\text{ind}}(p_{\text{ind}}^*) = p_{\text{ind}}^* \left[ \frac{1}{4} S'_{\text{ind}}(p_{\text{ind}}^*) + \frac{3}{16} S''_{\text{ind}}(p_{\text{ind}}^*) \right] = -\frac{1}{4\alpha} p_{\text{ind}}^{*2}. \]

4.2. Correlated Case

When the products are correlated, we can derive the search thresholds following the derivation in Section 3.1, with \( \varepsilon^* = 1 - 2c, u_0 = \frac{10}{3}c - 2, \pi = 4c - 2 \) and \( \underline{u} = 2c - 2 \). Consider a consumer who first searches product \( i \). By Proposition 1, we must have that \( \alpha_i - p_i \geq \max\{\alpha_j - p_j, u_0\} \). With probability \( 1/4 \), the outcome is \((X, \varepsilon_i) = (1, 1)\), under which case, the consumer will purchase the product right away, because \( U_i = \alpha_i - p_i + X + \varepsilon_i = \alpha_i - p_i + 2 \geq \max\{\alpha_j - p_j, u_0\} + 2 \geq \max\{\alpha_j - p_j + X + \varepsilon^*, 0\} \). With the remaining probability \( 3/4 \), any other realizations will discourage the consumer from buying...
the product, because \( U_i < 0 \). The question then again is whether to search product \( j \) or leave the market. It is optimal for the consumer to continue to search product \( j \) if and only if two conditions are satisfied. First, \((X, \varepsilon_i) = (1, -1)\) after the first search, which occurs with probability \(1/4\). Second, \( \alpha_j - p_j \geq -1 - \varepsilon^* = u \). Conditional on the second search, product \( j \) will be purchased if \( \varepsilon_j = 1 \), which occurs with probability \(1/2\).

We can also explicitly write down the conditions that characterize a consumer’s optimal search strategy by the following lemma. The proof, provided in the appendix, is based on explicit calculation of the value function of searching product \( i \) first, \(-c + E[V_i(X, \varepsilon_i)]\).

**Lemma 1:** Assume that \( X \) and \( \varepsilon_i \) follow independent two-point distributions that take values of \( 1 \) and \(-1\) with equal probability, and \( c \leq 1/2 \). It is optimal to search product \( i \) in the first period if and only if \( u_i \geq u_j \) and \( u_i \geq \min \{ \bar{u}, \frac{3}{2}u_0 - \frac{1}{2}u_j \} \).

We again look for symmetric price equilibrium, and denote the equilibrium price as \( p_{\text{cor}}^* \). This includes two possibilities. First, the equilibrium price \( p_{\text{cor}}^* \) could lie within \([0, -\bar{u}]\) similar to the independent case. Second, it is also possible that \( p_{\text{cor}}^* > -\bar{u} \), resulting in a smaller search region but a higher margin. Next, we derive the equilibrium under the first case, which exists for any \( c \leq 1/2 \), and leave the details of the second case and its existence condition to the appendix.

Like in the independent case, given \( p_2 = p_{\text{cor}}^* \), the demand of product 1 comes from two sources.

First, let \( S'_{\text{cor}}(p_1) \) denote the size of the consumer segment with \( u_1 \geq u_2 \) and \( u_1 \geq \min \{ \bar{u}, \frac{3}{2}u_0 - \frac{1}{2}u_2 \} \). These consumers will search product 1 first, and \(1/4\) of them will eventually purchase product 1. We have that,

\[
S'_{\text{cor}}(p_1) = \frac{1}{\alpha^2} \left[ \frac{1}{2} (p_1 + \bar{u})^2 + (\alpha - \bar{u} - p_{\text{cor}}^*)(p_1 + \bar{u}) - \frac{1}{2} \min \{ p_{\text{cor}}^* - p_1, 0 \}^2 + \frac{1}{2} (\bar{u} - u)(\bar{u} - u_0) \right].
\]

Comparing \( S'_{\text{cor}}(p_1) \) with \( S'_{\text{ind}}(p_1) \), we find that there is an extra term \( \frac{1}{2} (\bar{u} - u)(\bar{u} - u_0) \) in \( S'_{\text{cor}}(p_1) \), which captures the expanded consumer search region for product 1, as a result of informational complementarity.

Second, let \( S''_{\text{cor}}(p_1) \) denote the size of the consumer segment with \( u_2 > u_1 \geq u \) and \( u_2 \geq \min \{ \bar{u}, \frac{3}{2}u_0 - \frac{1}{2}u_1 \} \). These consumers will search product 2 first, and \(1/4 \times 1/2 = 1/8\) of them will eventually purchase product 1. We have that,

\[
S''_{\text{cor}}(p_1) = \frac{1}{\alpha^2} \left[ \frac{1}{2} (p_{\text{cor}}^* + \bar{u})^2 - (\bar{u} - u)(p_{\text{cor}}^* + \bar{u}) - \frac{1}{2} \min \{ p_1 - p_{\text{cor}}^*, 0 \}^2 + \frac{1}{2} (\bar{u} - u)(\bar{u} - u_0) \right].
\]

\(^5\)If \( X = -1 \), then \( U_j < 0 \) for any realizations of \( \varepsilon_j \). This implies that the consumer will never purchase product \( j \) and thus she will never continue to search \( j \).
Figure 5 illustrates the two consumer segments, $S'_{\text{cor}}(p_1)$ and $S''_{\text{cor}}(p_1)$. Compared with Figure 4, we can see that both consumer segments have expanded due to informational complementarity in the case of correlated products.

Figure 5: Illustration of firm 1’s demand from two consumer segments, in the case of correlated products. The solid lines represent the boundaries that correspond to a consumer’s optimal search strategy.

Firm 1’s objective of profit maximization implies the following optimality condition,

$$p_{\text{cor}}^* = \arg \max_{p_1} \Pi_{\text{cor}}(p_1) = \arg \max_{p_1} p_1 \left[ \frac{1}{4} S'_{\text{cor}}(p_1) + \frac{1}{8} S''_{\text{cor}}(p_1) \right].$$

We can solve the equilibrium price $p_{\text{cor}}^*$ by the first and second order optimality condition.

$$p_{\text{cor}}^* = 2 - 6c + 4\alpha + 2\sqrt{(c - 2\alpha)^2 + 2(1 - 2c)\alpha + c^2}.$$ 

The equilibrium profit is then given by,

$$\Pi_{\text{ind}}^* = \Pi_{\text{cor}}(p_{\text{cor}}^*) = p_{\text{cor}}^* \left[ \frac{1}{4} S'_{\text{cor}}(p_{\text{cor}}^*) + \frac{1}{8} S''_{\text{cor}}(p_{\text{cor}}^*) \right] = -\frac{1}{4\alpha} p_{\text{cor}}^{*2}.$$ 

We can then compare the equilibrium prices and profits between the two cases. The following theorem summarizes the results with the proof in the appendix.
Theorem 2: There exists a threshold
\[ \hat{c} \equiv -6\alpha + 2\sqrt{\alpha(1 + 6\alpha)} - \sqrt{6}\alpha \left(1 + 10\alpha - 4\sqrt{\alpha(1 + 6\alpha)}\right), \]
such that if \( c \leq \hat{c} \), then \( p_{\text{ind}}^* \geq p_{\text{cor}}^* \) and \( \Pi_{\text{ind}}^* \geq \Pi_{\text{cor}}^* \); on the other hand, if \( c > \hat{c} \), then \( p_{\text{ind}}^* < p_{\text{cor}}^* \) and \( \Pi_{\text{ind}}^* < \Pi_{\text{cor}}^* \).

Figure 6 compares the equilibrium prices and profits under the two cases as the search cost \( c \) varies when \( \alpha = -2 \). The result illustrates a trade-off between complementarity and differentiation. Sharing the common feature \( X \) introduces two opposing effects. On one hand, it increases consumers’ incentive to inspect a product and results in an expanded search region. This effect brings in new consumers who would not consider any of the products in the first place were they independent. On the other hand, with correlated features, products are less differentiated. A negative realization of the common attribute spills over to the correlated product, lowering the purchase likelihood for both. Consequently, both products have less market power and are faced with fiercer price competition. Which effect dominates depends on the size of the search cost. When the search cost is sufficiently small, price competition is intensified to the degree that it outweighs the benefit of informational complementarity. As a result, the equilibrium price becomes lower when products are correlated, resulting in lower equilibrium profit. With larger search cost, however, the complementarity effect dominates the competition effect, leading to higher equilibrium price and profit in the case of correlated products.
5. Extensions

5.1. Negative Correlation

In some cases, it is possible that positive information from one product speaks negatively of the other. For example, a brand can highlight some superior features of its own product using its competitors as inferior benchmarks. Even products under the same brand can share negative information about each other. For example, when searching for information on electric cars, consumers may get reviews of disadvantages of traditional gasoline cars. In short, information correlation between products may be negative. We expect informational complementarity can also occur with negative information correlation—in fact, even stronger. Intuitively, if a consumer searches for information about one product and learns that it is not attractive on the common attribute, then she will become more fond of the other product that is negatively correlated on the common attribute. Consequently, she may get more likely to purchase the second product. Therefore, a lower price of the first product may attract a consumer to start searching, and eventually make a purchase of the second product.

To formulate the idea, consider the following setup for consumers’ utility:

\[
\begin{align*}
U_1 &= u_1 + X + \varepsilon_1, \\
U_2 &= u_2 - X + \varepsilon_2.
\end{align*}
\]  
(11)

To make the two products symmetric, we also need to make the assumption that the distribution of \( X \) is symmetric with respect to 0, and thus \( X = -X \). In this setup, if the consumer discovers a positive \( X \) for product 1, then it implies that the effect of \( X \) on her utility of product 2 is negative.

We can obtain a similar result to Theorem 1. The proof is similar and thus omitted. First, we can derive the same threshold \( \bar{u} \) following equation (6). In deriving the second threshold \( u_n \), notice that after searching product \( i \) and discovering \( X \) and \( \varepsilon_i \), a consumer continues to search product \( j \) if and only if \( u_j - X + \varepsilon^* \geq \max \{u_i + X + \varepsilon_i, 0\} \). or equivalently, \( u_j \geq \max \{u_i + \varepsilon_i + 2X, X\} - \varepsilon^* \), which takes the minimum value of \( \max \{u_i + \varepsilon_i + 2X, -X\} - \varepsilon^* \), when \( \varepsilon_i = \varepsilon \) and \( X = -X \). Following a similar analysis, we can replace equation (9) with the following definition, and prove the following theorem.

\[
u_n \equiv \max \{\bar{u} + \varepsilon - 2X, -X\} - \varepsilon^*.
\]  
(12)

Theorem 3:

- If \( \pi \leq \nu_n \), then there is no complementarity effect: \( D_i(u_i, u_j) \) always weakly decreases with \( u_j \).
• If $\pi > u_n$, then,
  
  - complementarity effect arises when $u_i \in (u_n, \pi)$: $D_i(u_i, u_j)$ first increases and then decreases with $u_j$;
  
  - there is no complementarity for $u_i \notin (u_n, \pi)$: $D_i(u_i, u_j)$ always weakly decreases with $u_j$.

Notice that by definition, $u_n < u$. This implies that compared with positive correlation in our main model, information complementarity is more likely to arise under negative correlation, in the sense of a wider range of $u_i$, in which, $D_i(u_i, u_j)$ is non-monotonic with respect to $u_j$. This is consistent with our intuition above.

To illustrate, consider the two-point distributional assumption that $X$ takes the value of either $-\beta$ or $\beta$ with equal probability, and that $\varepsilon_i$ equals either $-1$ or $1$ with equal probability. Figure 7 illustrates the demand of product 2 as a function of $u_1$ and $u_2$, under the parameter setting that $\beta = 1$ and $c = 0.5$. We can see that informational complementarity arises in Figure 7 when $u_1$ is in the intermediate range.

![Figure 7: Demands of two products as a function of $u_1$ with negative information correlation (for $c = 0.5$ and $\beta = 1$).](image-url)
5.2. Exogenous Search Order

In our main model, the decision maker endogenously determines the order of products to search. In many real-world settings, however, consumers may search in an exogenously given order. For examples, a car dealer may present a lower-end (or higher-end) model to a customer and induce her to inspect it first before other models of the same make. Many online retailers frequently recommend products to targeted consumers, inducing them to learn about information on these recommended products first before browsing the retailer’s other products. To model these situations, we assume that a consumer has to search product 1 first before searching product 2.

Given exogenous search order, we notice that the consumer’s decision problems in the second and last periods are exactly the same as that in the case with endogenous search order. We still have the value functions \( V_2(X, \varepsilon_1, \varepsilon_2) \) and \( V_{11}(X, \varepsilon_1) \) defined by equation (1) and (2). Notice that we do not have \( V_{12}(X, \varepsilon_1) \) here, because the consumer always searches product 1 first. In the first period, the consumer decides between the outside option and searching product 1, and her maximum expected utility is,

\[
V = \max \{0, -c + E[V_{11}(X, \varepsilon_1)]\}. \quad (13)
\]

By equations (6) and (9), we can similarly define \( \underline{u} \) and \( \overline{u} \) in the setting with an exogenous search order. Similar with Proposition 2, we can prove the following proposition, where the proof is similar and thus omitted.

**Proposition 4:**

- When \( u_2 > \underline{u} \), after searching product 1 in the first period, there is a positive probability that a consumer will search product 2 in the second period.

- When \( u_2 < \underline{u} \), a consumer will never search or buy product 2. In this case, the consumer’s sequential search problem is reduced to the decision of whether to search product 1 or to take the outside option. She prefers to search product 1 if and only if \( u_1 \geq \overline{u} \).

Proposition 4 characterizes the threshold condition for when a consumer will consider searching or purchasing product 2. We are also interested in when a consumer will consider product 1. Intuitively, if \( u_2 \) is very high and \( u_1 \) is very low, a consumer may want to search or purchase product 2, but before that, she has to search product 1 first, by assumption. It is possible that in this case, the consumer will never purchase product 1—the sole purpose for her to search product 1 is to continue to search product 2. Formally, when \( u_1 < -\overline{X} + \xi \), product 1 is dominated by...
the outside option, so the consumer will never purchase product 1. In this case, we can simplify
\[ E[V_{11}(X, \varepsilon_1)] = E[\max\{-c + E[\max\{u_2 + X + \varepsilon_2, 0\}|X], 0\}] \]. Define \( \pi_e \) by the following equation,
\[
E[\max\{-c + E[\max\{\pi_e + X + \varepsilon_2, 0\}|X], 0\}] = c. \quad (14)
\]

Then, by definition, we know that when \( u_1 < -X + \xi \), a consumer will search product 1 in the first
period if and only if \( u_2 \geq \pi_e \). On the other hand, for \( u_1 \geq -X + \xi \), searching product 1 becomes
even more preferable than taking the outside option. Therefore, we have that when \( u_2 \geq \pi_e \), it is
optimal to search product 1 in the first period. In the appendix, we prove the following lemma.

**Lemma 2:** For any parameter setting, \( \pi_e \geq u \).

Based on Proposition 4, Lemma 2, and the discussion above, we can sketch a consumer’s optimal
search strategy in the first period in Figure 8. When \( u_2 \leq \bar{u} \), the consumer will never consider
product 2, and will search product 1 if and only if \( u_1 \geq \bar{u} \); when \( u_2 \geq \pi_e \), the consumer will
optimally search product 1; lastly, when \( \bar{u} < u_2 < \pi_e \), the consumer’s indifference curve between
searching product 1 and taking the outside option should decrease with \( u_2 \), because intuitively, a
higher expected utility of product 2 makes searching product 1 more preferable.

Based on the optimal search strategy in Figure 8, we are ready to derive a consumer’s demand
of product 2. Similar with equation (10), we can write down the consumer’s demand function of

![Figure 8: Illustration of a consumer’s optimal search strategy under an exogenous search order.](image)
product 2 as the following,

\[ D_2(u_2, u_1) = \mathbb{1}_{\{E[V_{11}(X, \varepsilon_1)] \geq c\}} \Pr (u_2 + X + \min\{\varepsilon_2, \varepsilon^*\} \geq \max\{u_1 + X + \varepsilon_1, 0\}). \]  \tag{15}

The following theorem characterize the condition for the complementarity effect to arise. The proof is very similar to that of Theorem 1, and thus is omitted.

**Theorem 4:** Complementarity effect arises when \( u_2 \in (u, \overline{u}_e) \): \( D_2(u_2, u_1) \) first increases and then decreases with \( u_1 \); there is no complementarity for \( u_2 \notin (u, \overline{u}_e) \): \( D_2(u_2, u_1) \) always weakly decreases with \( u_1 \).

It is worthwhile noticing that with an exogenous search order, the complementarity effect can arise even with independent products. This is implied by noticing that Lemma 2 does not rely on the assumption that \( \text{Var}[X] > 0 \). The reason is that, in order for a consumer to purchase product 2, she has to search product 1 first. A lower price of product 1 makes searching product 1 more attractive, which can increase the demand of product 2. Figure 9 illustrates the demand functions \( D_i(u_1, u_2) \) under the assumption of two-point distributions of \( X \) and \( \varepsilon_i \). We can see that informational complementarity arises in Figure 9 when \( u_2 \) is in the intermediate range.

### 5.3. Repeated Purchases

In the main model, we focus on the single-purchase context where consumers make only one single purchase and acquire information prior to purchase. In this section, we extend the idea of informational complementarity to the repeated purchase context where consumers can acquire information through past choices and consumptions. Such scenario naturally occurs for many consumer packaged goods such as food and beverage, health care, and personal care products. Making one product attractive may induce consumers to try a different yet correlated product, resulting in complementarity between the two products.

Following the main model, we assume a finite horizon with three periods without time discounting. Unlike the main model, we assume that, in each period, consumers can purchase either one of the two products and learn its quality, or take the outside option. For simplicity, it is assumed that consumers do not acquire information about the product quality prior to purchase; instead, the product information only reveals after a consumer makes a purchase. This assumption applies for experience goods.

Products are correlated in the same manner as in the main model: when consumers try product \( i \), they completely uncover the common attribute \( X \) shared by both products and the idiosyncratic
Figure 9: Demands of two products as a function of $u_1$ under exogenous search order (for $c = 0.5$ and $\beta = 1$).
attribute $\varepsilon_i$ unique to $i$. Under this setup, consumers’ purchase behaviors will become steady after at most three periods. That’s the reason why we only need to consider a three-period model. We are interested in the steady-state demand, $D_i$ for product $i$, in the last period.

We can formulate a consumer’s decision problem as a dynamic program in three periods. In the last period, the consumer’s uncertainty about both products have been completely revealed, and the value function $W_2(X, \varepsilon_1, \varepsilon_2)$ is the same as $V_2(X, \varepsilon_1, \varepsilon_2)$ in equation (1). In the second period, the consumer has purchased product $i$ before, and is deciding whether to continue to purchase product $i$ in this period, or to purchase product $j$ ($j \neq i$), or to take the outside option. We have the value function as the following,

$$W_{1i}(X, \varepsilon_i) = \max \{2U_i, 0, u_j + X + E[W_2(X, \varepsilon_1, \varepsilon_2)|X, \varepsilon_i]\}$$

for $i \neq j = 1, 2$. (16)

The first term in the maximization in equation (16) represents the expected utility from purchasing product $i$, which is equal to $2U_i$, because if the consumer chooses to purchase product $i$ in the current period, she learns nothing new, and she will continue to purchase product $i$ in the next period. The second term represents the expected utility from taking the outside option. The third term represents the expected utility from purchasing product $j$, where the consumer gets the expected utility of $u_j + X$ in the current period, and expects $E[W_2(X, \varepsilon_1, \varepsilon_2)|X, \varepsilon_i]$ in the next period. Finally, in the first period, the consumer’s value function is that,

$$W = \max \{0, u_1 + E[W_{11}(X, \varepsilon_1)], u_2 + E[W_{12}(X, \varepsilon_2)]\}.$$ (17)

To illustrate the result, we again focus on the two-point distributions. Let’s assume $X$ takes the value of either $-\beta$ or $\beta$ with equal probability, and that $\varepsilon_i$ equals either $-1$ or $1$ with equal probability. Without loss of generality, assume that $\beta \leq 1$.

The derivation is very similar to the search problem. Let us first derive the threshold, $\overline{u}$, for trying product $i$ when product $j$ is dominated in every period. The threshold is a solution to the following Bellman equation:

$$\overline{u} + 2E[\max \{\overline{u} + X + \varepsilon_i, 0\}] = 0..$$ (18)

Under the distributional assumption, we have $\overline{u} = -1/2$ if $\beta < 1/2$, and $\overline{u} = -(1+\beta)/3$ if $\beta > 1/2$. To derive the second threshold that the presence of product $j$ makes trying product $i$ more attractive, we set $u_i = \overline{u}$ and search for the smallest $u_j$ that can encourage the consumers to try product $j$. Similar to the argument in the search model, the smallest $u_j$ is attained when the realization of product $i$ provides the strongest incentive to try product $j$, that is, when $(X, \varepsilon_i) = (\overline{X}, \varepsilon) = (\beta, -1)$. 27
The threshold, \( u \), is then the solution to the following Bellman equation:

\[
 u + \bar{X} + \mathbb{E}[\max\{u + \bar{X} + \varepsilon_j, U_0\}] = 2U_0, \tag{19}
\]

where the left-hand side is the option value of trying product \( j \), and the right-hand side is the value of choosing either product \( i \) or the outside option. Note that

\[
 U_0 = \max\{\bar{u} + \bar{X} - \varepsilon, 0\} = \max\{\bar{u} + \beta - 1, 0\} = 0, \tag{20}
\]

under either \( \bar{u} = -1/2 \) or \( \bar{u} = -(1 + \beta)/3 \). It then follows that \( u = -(1 + 3\beta)/3 \). Finally, to ensure \( \bar{u} > u \), it is necessary that \( \beta > 1/6 \).

**Proposition 5:** Assume that both \( X \) and \( \varepsilon \) follow the two-point distributions.

- If \( \beta \leq 1/6 \), then there is no complementarity effect: \( D_i(u_i, u_j) \) always weakly decreases with \( u_j \).
- If \( \beta > 1/6 \), then,
  - complementarity effect arises when \( u_i \in (u, \bar{u}) \): \( D_i(u_i, u_j) \) first increases and then decreases with \( u_j \);
  - there is no complementarity for \( u_i \notin (u, \bar{u}) \): \( D_i(u_i, u_j) \) always weakly decreases with \( u_j \).

Figure 10 illustrates how product demands vary with \( u_1 \). We can see that informational complementarity arises when \( u_2 \) is in the intermediate range.

### 5.4. Purchase Before Search

A consumer’s optimal sequential search problem becomes much more complicated when she is allowed to purchase a product before searching it first. In this case, the optimal search problem is no longer a multi-arm bandit problem, and the index policy (i.e., Weitzman (1979)’s Pandora rule) is suboptimal (Doval 2014, Ke and Villas-Boas 2017). This is because, each time a consumer pulls an arm (i.e., select a product), she could do one of the two things—either to search it first or to purchase it right away. Mathematically, the consumer’s search problem becomes a *stoppable bandit problem* (Glazebrook 1979, Gittins et al. 2011), and there is no closed-form characterization of the optimal search strategy under a general setting. Nevertheless, we will characterize the consumer’s optimal search strategy and the resulting demand function under some distributional assumptions.
Figure 10: Demands of two products as a function of $u_1$ under repeated choice problem ($\beta = 1$).
When the consumer is allowed to purchase a product before searching it first, her value functions in the second and first periods can be written down as follows:

\[
V_{1i}(X, \varepsilon_i) = \max \{U_i, E[U_j|X], 0, -c + E[V_2(X, \varepsilon_1, \varepsilon_2)|X, \varepsilon_i]\}
\]

\[
= \max \{\alpha_i + \beta X - p_i + \varepsilon_i, \alpha_j + \beta X - p_j, 0, -c + E[V_2(X, \varepsilon_1, \varepsilon_2)|X, \varepsilon_i]\}, \quad (21)
\]

\[
V = \max \{E[U_1], E[U_2], 0, -c + E[V_{11}(X, \varepsilon_i)], -c + E[V_{12}(X, \varepsilon_2)]\}
\]

\[
= \max \{\alpha_1 - p_1, \alpha_2 - p_2, 0, -c + E[V_{11}(X, \varepsilon_1)], -c + E[V_{12}(X, \varepsilon_2)]\}. \quad (22)
\]

Compared with the expression of \(V_{1i}(X, \varepsilon_i)\) in equation (2) in our main model, we can see that now in equation (21), there is an extra term, \(E[U_j|X]\), which represents the option of purchasing product \(j\) before searching it. Similarly, compared with the expression of \(V\) in equation (3) in our main model, we can see that now in equation (22), there are two extra terms, \(E[U_1]\) and \(E[U_2]\), which represent the options of purchasing product 1 and 2 respectively before searching them.

Under the two-point distributions, the following theorem completely characterizes the condition for informational complementarity to arise (with proof in the appendix).

**Theorem 5:** Assume that \(X\) takes the value of either \(-\beta\) or \(\beta\) with equal probability, and that \(\varepsilon_i\) takes the value of either \(-1\) or \(1\) with equal probability, with \(\beta \leq 1\).

1. With small search cost, \(c < \frac{1}{4}(\beta + 1)\),
   - if \(u_i \geq 4c - \beta - 1\), \(D_i(u_i, u_j)\) weakly decreases with \(u_j\);
   - if \(2c - \beta - 1 \leq u_i < 4c - \beta - 1\), \(D_i(u_i, u_j)\) first weakly increases and then weakly decreases with \(u_j\) (complementarity);
   - if \(u_i \leq 2c - \beta - 1\), \(D_i(u_i, u_j) = 0\).

2. With moderate search cost, \(\frac{1}{4}(\beta + 1) \leq c < \frac{1}{4}(2\beta + 1)\),
   - if \(u_i \geq 0\), \(D_i(u_i, u_j)\) weakly decreases with \(u_j\);
   - if \(4c - 2\beta - 1 \leq u_i < 0\), \(D_i(u_i, u_j)\) first weakly increases and then weakly decreases with \(u_j\) (complementarity);
   - if \(u_i \leq 4c - 2\beta - 1\), \(D_i(u_i, u_j) = 0\).

3. With large search cost, \(c \geq \frac{1}{4}(2\beta + 1)\), there is no complementarity, and
   - if \(u_i \geq 0\), \(D_i(u_i, u_j)\) weakly decreases with \(u_j\);
   - if \(u_i < 0\), \(D_i(u_i, u_j) = 0\).
Theorem 5 is consistent with Theorem 1. The complementarity effect arises only when the search cost is not too large. Figure 11 illustrates the demand functions under the parameter setting with $c = 0.5$ and $\beta = 1$. Similar with the main model, we can see that informational complementarity arises in Figure 11 when $u_2$ is in the intermediate range.

Figure 11: Demands of two products as a function of $u_1$ when purchase before search is allowed (for $c = 0.5$ and $\beta = 1$).

6. Conclusion

Based on a consumer sequential search model, we show that correlated uncertain information among products can lead to demand complementary effect for substitutes. This effect of informational complementarity arises when the search cost is not very large, and when a consumer’s expected utility of the alternative is close to the outside option. We also show that this effect is robust to negative information correlation as well as the assumption of whether consumers can purchase without search.

One important implication of the informational complementarity for competitive pricing. We take a first step to investigate this issue and illustrate the trade-off between complementarity and
differentiation. Being correlated can lead to less differentiation and intensified price competition. However, informational complementarity provides room for firms to collectively raise price. This benefit can be sufficiently strong that helps firms raise equilibrium price and profit.

We are also exploring several extensions and applications of the theory. First, the idea of informational complementarity is not necessarily restricted to search goods. It applies to search goods where consumers acquire product information by experience instead of search. It will be interesting to extend the framework to consider learning instead of information search through repeated purchases. Second, consider the two products under a consumer’s consideration come from two competing firms. By collusively manipulating the information correlation between the two products, the two firms may be able to shift their demands from substitutes to complements. Therefore, there is a possibility for horizontal collusion via product information instead of price. Third, consider the two products under a consumer’s consideration come from the same firm. We can study the online product line design problem. The optimal level of product commonality balances demand cannibalization and informational complementarity.
APPENDIX

PROOF OF PROPOSITION 1:

Proof. Let’s first prove the three statements one by one.

First statement: According the law of iterated expectations, we have that \( E[V_{i1}(X, \varepsilon_i)] = E[E[V_{i1}(X, \varepsilon_i)|X]] \). Notice that, we can construct another consumer search problem, in which \( X \) is given and consumers conduct a sequential search to discover \( \varepsilon_i \) for product \( i \). Then, \(-c + E[V_{i1}(X, \varepsilon_i)|X] \) is the expected utility of first searching product \( i \) in this problem. By applying Weitzman’s Pandora rule to this problem, we can get that \(-c + E[V_{i1}(X, \varepsilon_i)|X] \geq -c + E[V_{i2}(X, \varepsilon_i)|X] \) if and only if \( u_i \geq u_j \). This implies that \( E[V_{i1}(X, \varepsilon_i)] = E[E[V_{i1}(X, \varepsilon_i)|X]] \geq E[E[V_{i2}(X, \varepsilon_i)|X]] = E[V_{i2}(X, \varepsilon_i)] \) if and only if \( u_1 \geq u_2 \).

Second statement: We first prove that when \( u_j < u_0 \), it is never optimal to search product \( j \) first. According to equations (1) and (2), we can write down the expression of \( E[V_{ij}(X, \varepsilon_j)] \) as the following,

\[
E[V_{ij}(X, \varepsilon_j)] = E\left[ \max\left\{ u_j + X + \varepsilon_j, 0, -c + E[\max\{u_i + X + \varepsilon_i, u_j + X + \varepsilon_j, 0\}|X, \varepsilon_j]\right\} \right].
\]

To explicitly signify the dependence relationship of \( E[V_{ij}(X, \varepsilon_j)] \) on \( u_1 \) and \( u_2 \), let’s define the function \( EV_{ij}(u_j, u_i) \equiv E[V_{ij}(X, \varepsilon_j)] \). Notice that \( EV_{ij}(u_j, u_i) \geq 0 \) and weakly increases with \( u_i \). This implies that,

\[
1\{u_i \leq u_j\} EV_{ij}(u_j, u_i) \leq EV_{ij}(u_j, u_j),
\]

where \( 1\{u_i \leq u_j\} \) is the indicator function that is equal to one when \( u_i \leq u_j \) and zero otherwise. Further notice that \( EV_{ij}(u_j, u_j) \) is a continuous and increasing function of \( u_j \), and converges to zero as \( u_j \) goes to \(-\infty\). Also, as \( u_j \) decreases, \( EV_{ij}(u_j, u_j) \) strictly decreases with \( u_j \) before reaching zero. This implies that \( u_0 \) is well defined, and when \( u_j < u_0, EV_{ij}(u_j, u_j) < c \). Then, by inequality (i), we know that when \( u_j < u_0 \), \( 1\{u_i \leq u_j\} EV_{ij}(X, \varepsilon_j) = 1\{u_i \leq u_j\} EV_{ij}(u_j, u_i) \leq EV_{ij}(u_j, u_j) < c \) for any \( u_i \). Lastly, notice that \( 1\{u_i \leq u_j\} EV_{ij}(X, \varepsilon_j) < c \) implies that either \( u_j < u_i \) or \( E[V_{ij}(X, \varepsilon_j)] < c \). Therefore, when \( u_j < u_0 \), it is never optimal to search product \( j \) first, by equation (3).

Now let’s prove that when \( u_j \geq u_0 \), it is optimal to search product \( j \) given some \( u_i \). In fact, given \( u_i = u_0 \), we know that \( u_j \geq u_0 = u_i \), and also \( EV_{ij}(u_j, u_i) \geq EV_{ij}(u_0, u_0) = c \). Therefore, according to equation (3), it is optimal to search product \( j \).

Third statement: First, let’s consider a consumer search problem with only product \( j \) and an outside option, where the consumer decides whether to search product \( j \) or to take the outside
option. If she decides to search product \( j \), her expected utility is 
\[-c + E[\max \{u_j + X + \varepsilon_j, 0\}]\]; otherwise, if she decides to take the outside option, her expected utility is 0. The consumer prefers to search product \( j \) if and only if 
\[-c + E[\max \{u_j + X + \varepsilon_j, 0\}] \geq 0, \]
or equivalently, 
\[u_j \geq \bar{u}.\]

Now, consider our original problem with two products and an outside option. Obviously, the option of searching product \( i \) after searching product \( j \) makes searching product \( j \) more attractive than taking the outside option immediately. Mathematically, 
\[E[V_{1j}(X, \varepsilon_j)] \geq E[\max \{u_j + X + \varepsilon_j, 0\}].\]
Therefore, we must have that when \( u_j \geq \bar{u} \) and \( u_j \geq u_i \), it is optimal for the consumer to search product \( j \) first. ■

PROOF OF PROPOSITION 2:

Proof. Let’s first prove the two statements one by one.

First statement: Let’s first prove the statement that when \( u_j > \min\{u, \bar{u}\} \), a consumer will search product \( j \) in the first period, or there is a positive probability that a consumer will search and buy product \( j \) in the second period. We will prove by contradiction.

Suppose when \( u_j > \min\{u, \bar{u}\} \), a consumer will never search or buy product \( j \) for any \( u_i \). In this case, the consumer’s search problem is reduced to a decision between searching product \( i \) and taking the outside option. If she decides to search product \( i \), her expected utility is 
\[-c + E[\max \{u_i + X + \varepsilon_i, 0\}]\]; otherwise, if she decides to take the outside option, her expected utility is 0. The consumer prefers to search product \( i \) if and only if 
\[-c + E[\max \{u_i + X + \varepsilon_i, 0\}] \geq 0, \]
or equivalently, \( u_i \geq \bar{u} \). Let’s fix \( u_i = \bar{u} \). If \( u_j \geq \bar{u} = u_i \), according to the third statement in Proposition 1, we know that it is optimal for the consumer to search product \( j \), which contradicts to our assumption that a consumer will never search product \( j \) for any \( u_i \). Therefore, we must have that \( u_j < u = u_i \). Applying the third statement in Proposition 1 again, we know that it is optimal for the consumer to search product \( i \) in the first period. Moreover, combining \( u_j < u \) with the assumption that \( u_j > \min\{u, \bar{u}\} \), we must have
\[u_j > u = \max\{\bar{u} + \varepsilon, -X\} - \varepsilon^* = \max\{u_1 + \varepsilon, -X\} - \varepsilon^*. \quad (ii)\]
After searching product \( i \), the consumer will discover \( X \) in the neighborhood of \( X \) and \( \varepsilon_i \) in the neighborhood of \( \varepsilon \) with some positive probability. By comparing inequality (ii) with equation (8), we know that there is a positive probability that the consumer will search product \( j \) after searching product \( i \). This is a contradiction. Therefore, we have proved the original proposition.

Second statement: Now, Let’s prove the second statement that when \( u_j < \min\{u, \bar{u}\} \), a consumer will never search or buy product \( j \). First, notice that when \( X = \infty \) and \( \varepsilon = -\infty \), we
have that $u = -\infty$, under which case, $u_j < \min\{u, \bar{u}\}$ results in an empty set. Therefore, we only need to consider the case where $u$ is finite below. Our proof consists of two steps. In the first step, we will prove that there exists a constant threshold such that when $u_j$ is below the threshold, a consumer will never search or buy product $j$. In the second step, we will determine the threshold as $\min\{u, \bar{u}\}$.

**First step:** According to the second statement in Proposition 1, we know that $u_j < u_0$, it is never optimal to search product $j$ in the first period. Next, we will show that there exists a constant threshold such that when $u_j$ is below the threshold, it is never optimal to search product $j$ in the second period. By applying the second statement in Proposition 1 again, we know that when $u_i < u_0$, it is never optimal to search product $i$ in first. Conditioning that a consumer has searched product $i$ first, we must have that $u_i \geq u_0$. Equation (8) states that the consumer will not search product $j$ in the second period if $u_j \geq \max\{u_i + \varepsilon_i, -X\} - \varepsilon^*$. Meanwhile, we know that

$$\max\{u_0 + \varepsilon, -X\} - \varepsilon^* \leq \max\{u_i + \varepsilon_i, -X\} - \varepsilon^*.$$

for any $u_i \geq u_0$ and any realizations of $X$ and $\varepsilon_i$. That means that when $u_j < \max\{u_0, u_0 + \varepsilon, -X\}$, a consumer will never search or buy product $j$ in either first or second period.

**Second step:** In the first step, we have proved that when $u_j < \max\{u_0, u_0 + \varepsilon, -X\}$, a consumer will never search product $j$ for any $u_i$. In this case, she decides only between searching product $i$ and taking the outside option, and she prefers to search product $i$ if and only if $u_i \geq \bar{u}$.

Now if we increase $u_j$ above $\max\{u_0, u_0 + \varepsilon, -X\}$ to some point, it will become optimal to search product $j$ again for some $u_i$, because we know that when $u_j \geq \max\{\bar{u}, u_i\}$, it is optimal to search product $j$, according to the third statement in Proposition 1. Therefore, we can define $u^*$ as the infimum of $u_j$, under which it is optimal to search product $j$ for some $u_i$. Obviously, $u^*$ is well defined and $\bar{u} \geq u^* \geq \max\{u_0, u_0 + \varepsilon, -X\}$. At $u_j = u^*$, there are two possibilities—either it is optimal to search product $j$ in the first period, or with a positive probability it is optimal to search product $j$ in the second period.

- Let’s consider the second case first. By definition, at $u_j = u^*$, the consumer will be indifferent between searching product $j$ or not in the second period. This means that, in the first period, the consumer’s optimal search problem is same with the case when product $j$ is out of the consumer’s consideration set. That is, the consumer will search product $i$ if and only if $u_i \geq \bar{u}$ in the first period. Conditioning on that the consumer has searched product $i$ first and discovered $X$ and $\varepsilon_i$, and then she will continue to search product $j$ if and only if $u_j \geq \max\{u_i + \varepsilon_i, -X\} - \varepsilon^*$, according to equation (8). Meanwhile, given $u_i \geq \bar{u}$ and for any $X$ and $\varepsilon_i$, we know that $\max\{u_i + \varepsilon_i, -X\} - \varepsilon^* \geq \max\{\bar{u} + \varepsilon, -X\} - \varepsilon^* \equiv u$, which is indeed the infimum of $u_j$ such that it is optimal to search
product \( j \) in the second period. The infimum of \( u \) is also attainable. In fact, given \( u_i = \bar{u} \), the consumer will search product \( i \) in the first period, and given \( u_j = u \), the consumer will search product \( j \) with positive probability in the second period, when \( X \) and \( \varepsilon_i \) realized to be \( \overline{X} \) and \( \varepsilon_j \) respectively. Therefore, by the definition of \( u^* \), we have that \( u^* = u \).

- Now, let’s consider the other case, where at \( u_j = u^* \), it is optimal for the consumer to search product \( j \) in the first period for some \( u_i \). By definition, at \( u_j = u^* \), it is never optimal for the consumer to search product \( j \) in the second period for any \( u_i \). This implies that \( u^* < u \). We are going to prove that \( u \geq \bar{u} \). We prove by contradiction. Suppose \( u < \bar{u} \). We are going to show that for \( u_j = \frac{1}{2}(u + u^*) \), it is never optimal to search product \( j \) in either period.

In fact, consider \( u_j = \frac{1}{2}(u + u^*) \) and \( u_i = \bar{u} \). First, because \( u_i > u_j \), it is not optimal to search product \( j \) in the first period, according to the first statement in Proposition 1. Moreover, given \( u_i = \bar{u} \) and \( u_j < u \), we know that it is never optimal to search product \( j \) in the second period either, according to the analysis above. Therefore, given \( u_j = \frac{1}{2}(u + u^*) \) and \( u_i = \bar{u} \), the consumer will not search product \( j \) in either period, and the consumer’s search problem is reduced to the decision between searching product \( i \) and the taking the outside option. Given \( u_i = \bar{u} \), the consumer will be indifferent between searching product \( i \) and taking the outside option, and thus expects zero utility. Next, consider \( u_j = \frac{1}{2}(u + u^*) \) and \( u_i < \bar{u} \). Obviously it will be optimal to take the outside option, because the consumer’s expected utility from searching the two products increases with \( u_i \) and \( u_j \). Finally, consider \( u_j = \frac{1}{2}(u + u^*) \) and \( u_i > \bar{u} \). By the third statement in Proposition 1, we know it is optimal to search product \( i \) first, and by the analysis above, we know that it is never optimal to search product \( j \) in the second period. To summarize, we have shown that given \( u_j = \frac{1}{2}(u + u^*) \), it is never optimal to search product \( j \) in either period. By the definition of \( u^* \), this means that \( u^* \geq \frac{1}{2}(u + u^*) \), or \( u^* \geq u \), which is a contradiction. Therefore, we proved our original proposition that \( u \geq \bar{u} \).

Now, we are going to show that \( u \geq \bar{u} \) implies that \( u^* = \bar{u} \). In fact, we only need to prove that it is optimal to search product \( i \) in the first period, if and only if \( u_i \geq \max\{u_j, \bar{u}\} \). That is, under the condition that \( u \geq \bar{u} \), the consumer’s optimal search strategy can be completely characterized. First, the third statement in Proposition 1 proves that \( u_i \geq \max\{u_j, \bar{u}\} \) is the necessary condition. To prove the sufficiency, we only need to show that given \( u_j \leq u_i < \bar{u} \), it is optimal to take the outside option. In fact, given \( u \geq \bar{u} \), we know that \( u_j \leq u_i < \bar{u} \leq u \). Following similar procedure above as we prove \( u \geq \bar{u} \), we can show that in this case, it is never optimal to search product \( j \) in the second period. Then, the consumer’s search problem is reduced to the decision between searching product \( i \) and taking the outside option. Given \( u_i < \bar{u} \), it is optimal to take the outside option. This proves that \( u^* = \bar{u} \).
**Proof of Corollary 2:**

**Proof.** Similar with the proof of Proposition 1, we explicitly signify the dependence relationship of \( \text{EV}_{ij}(X, \varepsilon_j) \) on \( u_1 \) and \( u_2 \) by defining the function \( \text{EV}_{ij}(u_j, u_i) \equiv \text{E}[\text{V}_{ij}(X, \varepsilon_j)] \). Obviously, \( \text{EV}_{ij}(u_j, u_i) \) increases with \( u_j \) and \( u_i \). This implies that the indifference curve between searching \( j \) and taking the outside option, \( \overline{u}(u_i) \), defined by \( \text{EV}_{ij}(\overline{u}(u_i), u_i) = 0 \), will decrease with \( u_i \).

Moreover, given that \( u < \overline{u} \) and \( \text{EV}_{ij}(\overline{u}, u) = 0 = \text{EV}_{ij}(u, u_0) \), we must have \( u \leq u_0 \leq \overline{u} \). ■

**Proof of Theorem 1:**

Let’s first consider the case with \( u < \overline{u} \). There are four circumstances to consider.

- Given \( u_i \leq u \), product \( i \) is out of the consumer’s consideration set, \( D_i(u_i, u_j) = 0 \) for any \( u_j \). There is no complementary effect.
- Given \( u_i \geq \overline{u} \), we know that it is optimal to search either product \( i \) or product \( j \) first, according to Proposition 1. In this case, we can simplify \( D_i(u_i, u_j) \) in equation (10) as the following,

\[
D_i(u_i, u_j) = \mathbb{1}_{\{u_i \geq u_j\}} \text{Pr}(u_i + X + \varepsilon_i \geq \max \{u_j + X + \min \{\varepsilon_j, \varepsilon^*\}, 0\}) \\
+ \mathbb{1}_{\{u_j > u_i\}} \text{Pr}(u_i + X + \min \{\varepsilon_i, \varepsilon^*\} \geq \max \{u_j + X + \varepsilon_j, 0\}).
\]

Notice that both \( \text{Pr}(u_i + X + \varepsilon_i \geq \max \{u_j + X + \min \{\varepsilon_j, \varepsilon^*\}, 0\}) \) and \( \text{Pr}(u_i + X + \min \{\varepsilon_i, \varepsilon^*\} \geq \max \{u_j + X + \varepsilon_j, 0\}) \) decrease with \( u_j \) and

\[
\text{Pr}(u_i + X + \varepsilon_i \geq \max \{u_j + X + \min \{\varepsilon_j, \varepsilon^*\}, 0\}) \geq \text{Pr}(u_i + X + \min \{\varepsilon_i, \varepsilon^*\} \geq \max \{u_j + X + \varepsilon_j, 0\}),
\]

for any \( u_i \) and \( u_j \). This implies that \( D_i(u_i, u_j) \) decreases with \( u_j \). There is no complementary effect.

- Given \( \underline{u} < u_i \leq u_0 \), we know that it is optimal to take the outside option when \( u_j < \underline{u} \). More precisely, it is optimal to take the outside option when \( u_j \) is below the indifference curve between searching product \( j \) and taking the outside option. In this case, \( D_i(u_i, u_j) = 0 \). As \( u_j \) increases above the indifference curve between searching product \( j \) and taking the outside option, we have the demand function for product \( i \) as

\[
D_i(u_i, u_j) = \text{Pr}(u_i + X + \min \{\varepsilon_i, \varepsilon^*\} \geq \max \{u_j + X + \varepsilon_j, 0\}),
\]

which decreases with \( u_j \). To summarize, as \( u_j \) increases, \( D_i(u_i, u_j) \) first keeps at zero, and then jumps at the indifference curve between searching product \( j \) and taking the outside option, and
subsequently decreases with \( u_j \). To complete the analysis, we only need to show that there is a positive jump at the indifference curve. In fact, we only need to show that \( D_i(u_i, \bar{u}) > 0 \), because the indifference curve is below \( u_j = \bar{u} \).

\[
D_i(u_i, \bar{u}) = \Pr (u_i + X + \min\{\varepsilon_i, \varepsilon^*\} \geq \max\{\bar{u} + X + \varepsilon_j, 0\})
\]

\[
= \Pr (u_i \geq \max\{\bar{u} + \varepsilon_j, -X\} - \min\{\varepsilon_i, \varepsilon^*\})
\]

\[
\geq \Pr (\varepsilon_i \geq \varepsilon^*) \Pr (u_i \geq \max\{\bar{u} + \varepsilon_j, -X\} - \varepsilon^*).
\]

Notice that \( u_i > u \equiv \max\{\bar{u} + \varepsilon, -X\} - \varepsilon^* \) implies that \( \Pr (u_i \geq \max\{\bar{u} + \varepsilon_j, -X\} - \varepsilon^*) > 0 \), which in turn, implies that \( D_i(u_i, \bar{u}) > 0 \).

- Given \( u_0 < u_i < \bar{u} \), we know that it is optimal to take the outside option when \( u_j < u \). More precisely, it is optimal to take the outside option when \( u_j \) is below the indifference curve between searching product \( i \) and taking the outside option. In this case, \( D_i(u_i, u_j) = 0 \). As \( u_j \) increases above the indifference curve between searching product \( i \) and taking the outside option, we have the demand function for product \( i \) as

\[
D_i(u_i, u_j) = \mathbb{1}_{\{u_i \geq u_j\}} \Pr (u_i + X + \varepsilon_i \geq \max\{u_j + X + \min\{\varepsilon_j, \varepsilon^*\}, 0\})
\]

\[
+ \mathbb{1}_{\{u_j > u_i\}} \Pr (u_i + X + \min\{\varepsilon_i, \varepsilon^*\} \geq \max\{u_j + X + \varepsilon_j, 0\}).
\]

which, as shown above, decreases with \( u_j \). To summarize, as \( u_j \) increases, \( D_i(u_i, u_j) \) first keeps at zero, and then jumps at the indifference curve between searching product \( i \) and taking the outside option, and subsequently decreases with \( u_j \). To complete the analysis, we only need to show that there is a positive jump at the indifference curve. In fact, when \( u_j \) is above the indifference curve and \( u_j \leq u_i \), it is optimal to search product \( i \) first. There must be a positive probability with which the consumer purchases product \( i \) eventually; otherwise, the consumer will never search product \( i \).

Now, let’s consider the other case with \( u \geq \bar{u} \). Following the exactly same proof above, we can show that there is no complementarity effect for any \( u_i \leq \bar{u} \) and \( u_i > \bar{u} \).

**Proof of Proposition 3:**

**First step:** Let’s first prove that the condition that \( \text{Var}[X] > 0 \) and \( 0 < c < E[\varepsilon] - \varepsilon \) is sufficient for \( u < \bar{u} \).

Notice that \( u = \max\{\bar{u} + \varepsilon, -X\} - \varepsilon^* \) from equation (9). Using the definition of \( \varepsilon^* \) in equation
(7), we can rewrite $u$ as the following,

$$-c + E[\max \{ u + \varepsilon_j + \min \{ -\bar{u} - \varepsilon, X \}, 0 \}] = 0. \quad \text{(iii)}$$

We will consider two cases: $\bar{u} + \varepsilon \leq -X$ and $\bar{u} + \varepsilon > -X$. From equation (6), $E[\max \{ \bar{u} + X + \varepsilon_i, 0 \}] = c$, we know that $\bar{u}$ increases with $c$, and therefore, $\bar{u} + \varepsilon \leq -X$ is equivalent to

$$E[\max \{ -\varepsilon - X + \varepsilon_i, 0 \}] \geq c,$$

$$\Leftrightarrow E[\varepsilon_i - \varepsilon + \max \{ -X + X, -\varepsilon_i + \varepsilon \}] \geq c,$$

$$\Leftrightarrow c \leq E[\varepsilon_i] - \varepsilon - E[\min \{ X - X, \varepsilon_i - \varepsilon \}].$$

- Consider the first case with $\bar{u} + \varepsilon \leq -X$, or equivalently $c \leq E[\varepsilon_i] - \varepsilon - E[\min \{ X - X, \varepsilon_i - \varepsilon \}]$. Equation (iii) reduces to $c = E[\max \{ \bar{u} + X + \varepsilon_i, 0 \}]$. By the definition of $\bar{u}$, we have that

$$E[\max \{ \bar{u} + X + \varepsilon_i, 0 \}] = c = E[\max \{ \bar{u} + X + \varepsilon_j, 0 \}] > E[\max \{ \bar{u} + X + \varepsilon_i, 0 \}],$$

where the last inequality is due to $\text{Var}[X] > 0$. This implies that $\bar{u} > u$.

- Consider the other case with $\bar{u} + \varepsilon > -X$, or equivalently $c > E[\varepsilon_i] - \varepsilon - E[\min \{ X - X, \varepsilon_i - \varepsilon \}]$. Equation (iii) reduces to $c = E[\max \{ u - \bar{u} + \varepsilon_j - \varepsilon, 0 \}]$. Given $c < E[\varepsilon_i] - \varepsilon = E[\max \{ \varepsilon_j - \varepsilon, 0 \}]$, we have that

$$E[\max \{ u - \bar{u} + \varepsilon_j - \varepsilon, 0 \}] = c < E[\max \{ \varepsilon_j - \varepsilon, 0 \}]$$

This implies that $\bar{u} > u$.

**Second step:** Now, let’s prove that $\text{Var}[X] > 0$ and $0 < c < E[\varepsilon] - \varepsilon$ are necessary conditions for $u < \bar{u}$. We prove by contradiction. In fact, suppose $\text{Var}[X] = 0$ or $c \geq E[\varepsilon] - \varepsilon$ (when $c = 0$, $u$ and $\bar{u}$ are not well defined). Obviously, if $\text{Var}[x] = 0$, then $\bar{u} = u$. Now consider the implication of $c \geq E[\varepsilon] - \varepsilon$. Notice that $E[\min \{ X - X, \varepsilon_i - \varepsilon \}] \geq 0$, so $c \geq E[\varepsilon] - \varepsilon \geq E[\varepsilon_i] - \varepsilon - E[\min \{ X - X, \varepsilon_i - \varepsilon \}]$. This implies that $\bar{u} + \varepsilon \geq -X$. Equation (iii) reduces to

$$E[\max \{ u - \bar{u} + \varepsilon_j - \varepsilon, 0 \}] = c \geq E[\max \{ \varepsilon_j - \varepsilon, 0 \}].$$

This implies $u \geq \bar{u}$, which is a contradiction. Therefore, the original statement is valid. ■
Proof of Lemma 1:

Proof: Note that the value function of first searching product \( i \) is given by,

\[-c + E[V_{1i}(X, \varepsilon_i)] = -c + \frac{1}{4}(2 + u_i) + \frac{1}{4} \max \left\{ -c + \frac{1}{2}(2 + u_j), 0 \right\}.
\]

The above equation holds because a consumer will purchase product \( i \) only if \( X = \varepsilon_i = 1 \), and will continue to search product \( j \) only if \( X = 1 \) and \( \varepsilon_i = -1 \).

It is optimal for a consumer to search product \( i \) in the first period if and only if

\[ E[V_{1i}(X, \varepsilon_i)] \geq E[V_{1j}(X, \varepsilon_j)] \]

and

\[ -c + E[V_{1i}(X, \varepsilon_i)] \geq 0. \]

In the proof of Proposition 1, we have shown that

\[ E[V_{1i}(X, \varepsilon_i)] \geq E[V_{1j}(X, \varepsilon_j)] \]

is equivalent to \( u_i \geq u_j \). Moreover,

\[ -c + E[V_{1i}(X, \varepsilon_i)] \geq 0 \]

is equivalent to

\[ u_i \geq \min\{4c - 2, 5c - 3 - \frac{1}{2} u_j\} = \min\{\bar{u}, \frac{3}{2} u_0 - \frac{1}{2} u_j\}. \]

Multiple Equilibria and Proof of Theorem 2:

Multiple Equilibria

We next show that there exists an alternative equilibrium when \( c > 4/9 \). This occurs when the equilibrium price lies within \([2 - 4c, 2 - 2c]\). The total size of consumers who start searching product 1 is given by

\[ S_1 = \frac{1}{2}(\bar{u} - u)h \left( \frac{h - (\bar{u} - p_1)}{h} \right)^2, \]

where \( h = \bar{u} - u_0 = 2c/3 \). Similarly, the segment of consumers who start searching product 2 can be calculated as follows:

\[ S'_2 = \frac{1}{2}(\bar{u} - u)h \left( \frac{h - (\bar{u} - p_2)}{h} \right)^2 - \frac{1}{2}(p_1 - p_2)^2, \]

Using the first-order condition and setting \( p_1 = p_2 = p^* \) we obtain the solution,

\[ p^*_{(cor, 2)} = \frac{6 - 10c}{7}. \]

This equilibrium satisfies the condition \( p^* \in [2 - 4c, 2 - 2c] \) if \( c > 4/9 \).

Profit Comparison

Next, we compare the equilibrium profits under independent and correlated cases. We focus on the equilibrium with \( p^* \in [0, 2 - 4c] \) for the correlated case. Under both cases, the equilibrium profit
can be written in the form of
\[ \Pi^* = -\frac{1}{4\alpha}p^{*2}, \]
which increases with \( p^* \). Comparing the profits amounts to comparing the equilibrium prices. Define the difference of the equilibrium prices:
\[ \Delta p \equiv p_{cor}^* - p_{ind}^* = -2c - 4\alpha + 2\sqrt{(c - 2\alpha)^2 + 2(1 - 2c)\alpha + c^2} - 4\sqrt{4\alpha^2 + (1 - 2c)\alpha}. \]
We can solve \( \hat{c} = -6\alpha + 2\sqrt{\alpha(1 + 6\alpha)} - \sqrt{6\alpha(1 + 10\alpha - 4\sqrt{\alpha(1 + 6\alpha)})} \) by the equation of \( \Delta p = 0 \). Then, it is straightforward to show that \( \Delta p < 0 \) if and only if \( c < \hat{c} \). Moreover, \( \hat{c} \in [0, 1/2] \). In fact, at \( c = 0 \), \( \Delta p = -4\alpha + 2\sqrt{4\alpha^2 + 2\alpha} - 4\sqrt{4\alpha^2 + \alpha} \). With some algebra, this quantity can be shown to be negative. At \( c = 1/2 \), \( \Delta p = -1 + 4\alpha + 2\sqrt{(1/2 - 2\alpha)^2 + 1/4} \), which can be shown to be positive.

**Proof of Theorem 5**

**Proof.** Let us first prove the following result:

**Lemma 3:** In the first period, if the outside option is dominated by searching product 1, i.e., \(-c + E[V_{11}(X,\varepsilon_1)] > 0\), then \( D_1(u_1, u_2) \) is non-increasing in \( u_2 \).

Since the outside option is dominated, there are four possibilities of the initial decision: (1) buying 2 without search; (2) searching 2 first; (3) searching 1 first; (4) buying 1 without search. We shall show that whichever decision is made, the probability of purchasing product 1 eventually is non-increasing in \( u_2 \).

*Buying 2 first.* Clearly, this occurs when \( u_2 \) is sufficiently large. Increasing \( u_2 \) will only increase the value of buying 2 immediately. Hence the demand for product 1 remains zero.

*Searching 2 first.* This arises when \(-c + E[V_{12}(X,\varepsilon_2)] > \max\{-c + E[V_{11}(X,\varepsilon_1)], u_1, u_2, 0\}\). After searching 2, \((X,\varepsilon_2)\) is realized and DM needs to decide whether to search for the value of \( \varepsilon_1 \). Conditional on the first search, let us again define a composite product \( 2' \) by bundling product 2 and the outside option. The remaining problem reduces to a standard search problem with two competing products, 1 and \( 2' \). Increasing \( u_2 \) can weakly increase the demand for product \( 2' \), thereby weakly decreasing the demand for product 1. This holds for every possible realization of \((X,\varepsilon_2)\) and thus also holds by taking the expectation over \((X,\varepsilon_2)\). Note also that increasing \( u_2 \) can also increase the value of buying 2 immediately. If buying 2 becomes optimal, then applying the argument of the previous case we can retain the non-increasing result.
Searching 1 first. This arises when \(-c + \mathbb{E}[V_{11}(X, \varepsilon_1)] > \max\{-c + \mathbb{E}[V_{12}(X, \varepsilon_2)], u_1, u_2, 0\}\). After searching 1, \((X, \varepsilon_1)\) is realized. The only remaining uncertainty is \(\varepsilon_2\) and DM decides whether to continue to search and find out its value. Let us define a composite product \(1'\) which is the bundle between product 1 and the outside option, and denote \(u_1' = \max\{u_1 + X + \varepsilon_1, 0\}\). Then the problem from \(t = 1\) onward reduces to a search problem between the uncertain product 2 and the certain bundle \(1'\). If bundle \(1'\) is eventually chosen, then the demand for product 1 is simply either zero or one, depending on the size of \(u_1 + X + \varepsilon_1\) and independent of \(u_2\). It is therefore sufficient to just show that the demand for \(1'\) is nonincreasing in \(u_2\). Clearly, given that product 2 and \(1'\) are substitute, increasing \(u_2\) will only (weakly) increase the demand for product 2 and thus (weakly) decrease the demand for \(1'\). This holds for every possible realization of \((X, \varepsilon_1)\) and thus also holds by taking the expectation over \((X, \varepsilon_1)\). Note also that increasing \(u_2\) can also increase the value of buying 2 or searching 2 immediately. If either becomes optimal, we can apply the last two cases to obtain the non-increasing result.

Buying 1 first. This occurs when \(u_1 > \max\{-c + \mathbb{E}[V_{11}(X, \varepsilon_1)], -c + \mathbb{E}[V_{12}(X, \varepsilon_2)], u_2, 0\}\). Clearly, as long as buying 1 is optimal, increasing \(u_2\) does not change the demand. If \(u_2\) becomes large enough to make either searching or buying 2 optimal, we can apply results in the previous cases and conclude that \(D_1\) is non-increasing in \(u_2\).

Next, we turn to the formal proof of the proposition.

1. Proof of Case 1: \(c < \frac{1}{4}(\beta + 1)\)

1.1. \(u_1 > 4c - \beta - 1\)

If the consumer searches product 1, then with probability \(1/4\) he obtains a good draw \((X, \varepsilon_1) = (\beta, 1)\), under which case he can secure a minimum payoff of \(u_1 + \beta + 1\) by buying product 1 immediately after search. Then,

\[\mathbb{E}[V_{11}(X, \varepsilon_1)] \geq \frac{1}{4}(u_1 + \beta + 1) > \frac{1}{4}(4c - \beta - 1 + \beta + 1) = c.\]

Hence, the outside option is dominated by starting searching product 1. Applying Lemma 3 completes the proof.

1.2. \(u_1 < 2c - \beta - 1\)

We show the results in this case with two steps: (1) if \(u_2 < 4c - \beta - 1\), then the outside option dominates; (2) if \(u_2 > 4c - \beta - 1\), then it is never optimal to buy product 1.
Step 1. Consider searching product 2 first:

\[
E[V_{12}(X, \varepsilon_2)] = \frac{1}{4} \max\{-c + E[V_2(\beta, \varepsilon_1, 1)], u_1 + \beta, u_2 + \beta + 1, 0\} + \frac{1}{4} \max\{-c + E[V_2(\beta, \varepsilon_1, -1)], u_1 + \beta, u_2 + \beta - 1, 0\} + \frac{1}{4} \max\{-c + E[V_2(-\beta, \varepsilon_1, 1)], u_1 - \beta, u_2 - \beta + 1, 0\} + \frac{1}{4} \max\{-c + E[V_2(-\beta, \varepsilon_1, -1)], u_1 - \beta, u_2 - \beta - 1, 0\}.
\]

Given that \(u_1 < 2c - \beta - 1\) and \(u_2 < 4c - \beta - 1\), it is straightforward to verify that the outside option dominates conditional on the realizations \((X, \varepsilon_2) = (\beta, -1), (-\beta, 1),\) and \((-\beta, -1)\). It remains to check when \((X, \varepsilon_2) = (\beta, 1)\). If buying product 1 after search is optimal, then

\[
E[V_{12}(X, \varepsilon_2)] = \frac{1}{4}(u_1 + \beta) < \frac{1}{4}(2c - 1) < c.
\]

If buying product 2 after search is optimal (i.e., \(u_2 + \beta + 1 > v_1^s(-1, 1)\)), then

\[
E[V_{12}(X, \varepsilon_2)] = \frac{1}{4}(u_2 + \beta + 1) < c.
\]

If continuing to search product 1 is optimal, then

\[
E[V_{12}(X, \varepsilon_2)] = \frac{1}{4}(-c + E[V_2(\beta, \varepsilon_1, 1)]) = \frac{1}{4}(-c + \frac{1}{2} \max\{u_1 + \beta + 1, u_2 + \beta + 1, 0\} + \frac{1}{2} \max\{u_1 + \beta - 1, u_2 + \beta + 1, 0\}),
\]

which is bounded by either \(u_1 + \beta + 1\) or \(u_2 + \beta + 1\). Under either case, \(E[V_{12}(X, \varepsilon_2)] < c\).

Next, consider searching product 1 first:

\[
E[V_{11}(X, \varepsilon_1)] = \frac{1}{4} \max\{-c + E[V_2(\beta, 1, \varepsilon_2)], u_1 + \beta + 1, u_2 + \beta, 0\} + \frac{1}{4} \max\{-c + E[V_2(\beta, -1, \varepsilon_2)], u_1 + \beta - 1, u_2 + \beta, 0\} + \frac{1}{4} \max\{-c + E[V_2(-\beta, 1, \varepsilon_2)], u_1 - \beta + 1, u_2 - \beta, 0\} + \frac{1}{4} \max\{-c + E[V_2(-\beta, -1, \varepsilon_2)], u_1 - \beta - 1, u_2 - \beta, 0\}.
\]

Given that \(u_1 < 2c - \beta - 1\) and \(u_2 < 4c - \beta - 1\), it is straightforward to verify that the outside option dominates conditional on the realizations \((X, \varepsilon_1) = (-\beta, 1),\) and \((-\beta, -1)\). It remains to check when \((X, \varepsilon_1) = (\beta, 1)\) and \((-\beta, -1)\). Since \(E[V_{11}(X, \varepsilon_1)]\) weakly increases with \(u_1\) and \(u_2\), we shall show that \(E[V_{11}(X, \varepsilon_1)] < c\) even when \(u_1 = 2c - \beta - 1\) and \(u_2 = 4c - \beta - 1\). Note that

\[
-c + E[V_2(\beta, 1, \varepsilon_2)] = -c + \frac{1}{2} \max\{2c, 4c, 0\} + \frac{1}{2} \max\{2c, 4c - 2, 0\} = 2c,
\]

\[
-c + E[V_2(\beta, -1, \varepsilon_2)] = -c + \frac{1}{2} \max\{2c - 2, 4c, 0\} + \frac{1}{2} \max\{2c - 2, 4c - 2, 0\} = c.
\]
It follows that,
\[
E[V_{11}(X, \varepsilon_1)] = \frac{1}{4} \max\{2c, 4c - 1\} + \frac{1}{4} \max\{c, 4c - 1\} < c.
\]

**Step 2.** We shall show that the consumer never buys product 1 even if he starts searching. Consider searching product 2 first. Conditional on \((X, \varepsilon_2) = (\beta, 1)\), buying product 1 is dominated by buying product 2 since \(u_1 < u_2\). Conditional on \((X, \varepsilon_2) = (\beta, -1)\), buying product 1 immediately is never optimal \((u_1 + \beta < 0)\). If the consumer continues searching, then the expected payoff is
\[
-c + E[V_2(\beta, \varepsilon_1, -1)] = -c + \frac{1}{2} \max\{u_1 + \beta + 1, u_2 + \beta - 1, 0\} + \frac{1}{2} \max\{u_1 + \beta - 1, u_2 + \beta - 1, 0\}.
\]
Product 1 will be bought only when \(u_1 + \beta + 1 > \max\{u_2 + \beta - 1, 0\}\). Then
\[
-c + E[V_2(\beta, \varepsilon_1, -1)] = -c + \frac{1}{2}(u_1 + \beta + 1) + \frac{1}{2} \max\{u_2 + \beta - 1, 0\} < \max\{u_2 + \beta - 1, 0\},
\]
where the inequality follows from \(u_1 < 2c - \beta - 1\). Finally, conditional on \(X = -\beta\), buying product 1 is always dominated by the outside option. Hence, there is no positive demand for product 1 when the consumer searches product 2 first.

Next consider searching product 1 first. Conditional on \((X, \varepsilon_1) = (\beta, 1)\),
\[
-c + E[V_2(\beta, 1, \varepsilon_2)] = -c + \frac{1}{2} \max\{u_1 + \beta + 1, u_2 + \beta + 1, 0\} + \frac{1}{2} \max\{u_1 + \beta + 1, u_2 + \beta - 1, 0\}
\]
\[
= -c + \frac{1}{2}(u_2 + \beta + 1) + \frac{1}{2} \max\{u_1 + \beta + 1, u_2 + \beta - 1, 0\}.
\]
Conditional on \((X, \varepsilon_1) = (\beta, -1)\),
\[
-c + E[V_2(\beta, -1, \varepsilon_2)] = -c + \frac{1}{2} \max\{u_1 + \beta - 1, u_2 + \beta + 1, 0\} + \frac{1}{2} \max\{u_1 + \beta - 1, u_2 + \beta - 1, 0\}
\]
\[
= -c + \frac{1}{2} \max\{u_2 + \beta + 1, 0\} + \frac{1}{2} \max\{u_2, 0\}.
\]
Conditional on \((X, \varepsilon_1) = (-\beta, 1)\),
\[
-c + E[V_2(-\beta, 1, \varepsilon_2)] = -c + \frac{1}{2} \max\{u_1 - \beta + 1, u_2 - \beta + 1, 0\} + \frac{1}{2} \max\{u_1 - \beta + 1, u_2 - \beta - 1, 0\}
\]
\[
= -c + \frac{1}{2} \max\{u_2 - \beta + 1, 0\} + \frac{1}{2} \max\{u_2 - \beta - 1, 0\}.
\]
Conditional on \((X, \varepsilon_1) = (-\beta, -1)\),
\[-c + E[V_2(-\beta, -1, \varepsilon_2)] = -c + \frac{1}{2} \max\{u_1 - \beta - 1, u_2 - \beta + 1, 0\} + \frac{1}{2} \max\{u_1 - \beta - 1, u_2 - \beta - 1, 0\} \]
\[= -c + \frac{1}{2} \max\{u_2 - \beta + 1, 0\} + \frac{1}{2} \max\{u_2 - \beta - 1, 0\}.\]

Note that the only case where product 1 will be chosen is \((X, \varepsilon_1) = (\beta, 1)\), and thus \(E[V_{11}(X, \varepsilon_1)]\) weakly increases with \(u_1\). With the highest value of \(u_1 = 2c - \beta - 1\), we shall show that the demand for product 1 remains zero. Indeed, even if \(u_1 + \beta > u_2\), we have
\[E[V_{11}(X, \varepsilon_1)] = \frac{1}{4} \max\{\frac{1}{2}(u_2 + 2), 2c\} + \frac{1}{4} \max\{-c + \frac{1}{2}(u_2 + 2), u_2 + 1\} < c,\]
where the inequality follows from \(u_2 < u_1 + \beta\) and \(c < 1/4(1 + \beta)\).

1.3. \(2c - \beta - 1 \leq u_1 \leq 4c - \beta - 1\)

We can prove the results in this case with three steps: (1) there exists \(u \in (2c - \beta - 1, 4c - \beta - 1)\) such that \(\forall u_2 < u\), the outside option dominates and \(D_1 = 0\); (2) there exists \(\bar{u} \geq 4c - \beta - 1\) such that \(\forall u_2 > \bar{u}\), buying product 1 in any stage is never optimal and \(D_1 = 0\); (3), \(\forall u_2 \in [u, \bar{u}]\), \(D_1 > 0\) and weakly decreases with \(u_2\).

**Step 1.** It suffices to consider \(u_2 \in (2c - \beta - 1, 4c - \beta - 1)\), and show that \(\forall u_2 \leq \min\{2c - 2\beta - u_1, 4c - 2\beta - 1 - u_1\}\) the outside option dominates searching any product: \(0 > \max\{-c + E[V_{11}(X, \varepsilon_1)], -c + E[V_{12}(X, \varepsilon_2)]\}\). Consider first \(u_1 \geq u_2\). Starting searching product 1 yields:
\[E[V_{11}(X, \varepsilon_1)] = \frac{1}{4} \max\{-c + E[V_2(\beta, 1, \varepsilon_2)], u_1 + \beta + 1, u_2 + \beta, 0\} \]
\[+ \frac{1}{4} \max\{-c + E[V_2(\beta, -1, \varepsilon_2)], u_1 + \beta - 1, u_2 + \beta, 0\} \]
\[+ \frac{1}{4} \max\{-c + E[V_2(-\beta, 1, \varepsilon_2)], u_1 - \beta + 1, u_2 - \beta, 0\} \]
\[+ \frac{1}{4} \max\{-c + E[V_2(-\beta, -1, \varepsilon_2)], u_1 - \beta - 1, u_2 - \beta, 0\}.\]

Conditional on \((X, \varepsilon_1) = (\beta, 1)\), continuing searching product 2 yields
\[E[V_2(\beta, 1, \varepsilon_2)] = \frac{1}{2} \max\{u_1 + \beta + 1, u_2 + \beta + 1, 0\} + \frac{1}{2} \max\{u_1 + \beta + 1, u_2 + \beta - 1, 0\} = u_1 + \beta + 1 < c + u_1 + \beta + 1.\]

Then the optimal decision is buying product 1 immediately (note \(u_1 + \beta + 1 > \max\{u_2 + \beta, 0\}\)).
Conditional on \((X, \varepsilon_1) = (\beta, -1)\),

\[
E[V_2(\beta, -1, \varepsilon_2)] = \frac{1}{2} \max\{u_1 + \beta - 1, u_2 + \beta + 1, 0\} + \frac{1}{2} \max\{u_1 + \beta - 1, u_2 + \beta - 1, 0\} = \frac{1}{2}(u_2 + \beta + 1).
\]

Conditional on \((X, \varepsilon_1) = (-\beta, 1)\) or \((X, \varepsilon_1) = (-\beta, -1)\), buying either 1 or 2 yields maximum payoff of \(u_1\) or \(u_2\), which is less than zero. Leaving the market is optimal. Thus,

\[
E[V_{11}(X, \varepsilon_1)] = \frac{1}{4}(u_1 + \beta + 1) + \frac{1}{4}(u_2 + \beta < c
\]

where the inequality follows from the assumption \(u_2 \leq 4c-2\beta-1-u_1\). Thus, \(-c+E[V_{11}(X, \varepsilon_1)] < 0\).

Next, we can similarly derive the value of searching product 2:

\[
E[V_{12}(X, \varepsilon_2)] = \frac{1}{4} \max\{-c + \frac{1}{2}u_1 + \frac{1}{2}u_2 + \beta + 1, u_2 + \beta + 1\} + \frac{1}{4} \max\{-c + \frac{1}{2}(u_1 + \beta + 1), u_1 + \beta, 0\}.
\]

Again, since \(E[V_{12}(X, \varepsilon_2)]\) increases with \(u_1\), we only need to consider \(u_1 + \beta > 0\) to check if it could be positive. Note that if \(-c + \frac{1}{2}u_1 + \frac{1}{2}u_2 > u_2\), or equivalently \(u_2 < u_1 - 2c\), then

\[
E[V_{12}(X, \varepsilon_2)] = \frac{1}{4}(-c + \frac{1}{2}u_1 + \frac{1}{2}u_2 + \beta + 1) + \frac{1}{4}(u_1 + \beta) < \frac{1}{4} + \frac{1}{4}(4c - 1) < c,
\]

where the first inequality follows \(u_1+u_2 \leq 2c-2\beta\) and \(u_1 \leq 4c-\beta-1\). If, conversely, \(-c+\frac{1}{2}u_1+\frac{1}{2}u_2 < u_2\), then

\[
E[V_{12}(X, \varepsilon_2)] = \frac{1}{4}(u_2 + \beta + 1) + \frac{1}{4}(u_1 + \beta) < c,
\]

where the inequality follows from the assumption \(u_2 \leq 4c-2\beta-1-u_1\). Therefore, in any case we have \(E[V_{12}(X, \varepsilon_2)] < c\).
Step 2. Given that $u_2 > \bar{u} \geq 4c - \beta - 1$, we have $u_1 < u_2$. Then

$$E[V_{12}(X, \varepsilon_2)] = \frac{1}{4} \max\{-c + E[V_2(\beta, \varepsilon_1, 1)], u_1 + \beta, u_2 + \beta + 1, 0\} + \frac{1}{4} \max\{-c + E[V_2(\beta, \varepsilon_1, -1)], u_1 + \beta, u_2 + \beta - 1, 0\} + \frac{1}{4} \max\{-c + E[V_2(-\beta, \varepsilon_1, -1)], u_1 - \beta, u_2 - \beta + 1, 0\} + \frac{1}{4} \max\{-c + E[V_2(-\beta, \varepsilon_1, -1)], u_1 - \beta, u_2 - \beta - 1, 0\}.$$

Conditional on $(X, \varepsilon_2) = (\beta, 1)$, the consumer never buys product 1 since

$$-c + E[V_2(\beta, \varepsilon_1, 1)] = -c + \frac{1}{2} \max\{u_1 + \beta + 1, u_2 + \beta + 1, 0\} + \frac{1}{2} \max\{u_1 + \beta - 1, u_2 + \beta + 1, 0\} = -c + u_2 + \beta + 1 < u_2 + \beta + 1.$$

Conditional on $(X, \varepsilon_2) = (\beta, -1)$,

$$-c + E[V_2(\beta, \varepsilon_1, -1)] = -c + \frac{1}{2} \max\{u_1 + \beta + 1, u_2 + \beta - 1, 0\} + \frac{1}{2} \max\{u_1 + \beta - 1, u_2 + \beta - 1, 0\}.$$

For sufficiently large $u_2$ such that $u_2 > u_1 + 2$, then buying product 1 is dominated even under a bad draw of $\varepsilon_2$. Conditional on $(X, \varepsilon_1) = (-\beta, 1)$ or $(X, \varepsilon_1) = (-\beta, -1)$, buying 1 yields maximum of $u_1$ or $u_2$, which is less than zero. Hence, for a sufficiently large $u_2$, there is no demand for product 1 if DM searches product 2 first. Next we shall verify that when $u_2 > u_1 + 2$, there is no demand for product 1 when searching 1 first. Indeed, even if $(X, \varepsilon_1) = (\beta, 1)$, buying 1 yields less profit $(u_1 + 2)$ than buying 2 ($u_2$).

Step 3. To prove the first part, note that searching 1 first can lead to a draw of $(X, \varepsilon_1) = (\beta, 1)$ with probability 1/4. Conditional on this realization, it is optimal to buy product 1. Searching 2 first can lead to a draw of $(X, \varepsilon_2) = (\beta, -1)$ with probability 1/4. Then the consumer faces a choice between buying 1 immediately with payoff $u_1 + \beta$, buying 2 with payoff $u_2 + \beta - 1$, taking the outside option with zero payoff, and continuing searching 1 with payoff

$$-c + E[V_2(\beta, \varepsilon_1, -1)] = -c + \frac{1}{2} \max\{u_1 + \beta + 1, u_2 + \beta - 1, 0\} + \frac{1}{2} \max\{u_1 + \beta - 1, u_2 + \beta - 1, 0\}.$$

Note that as long as $u_1 + \beta + 1 > \max\{u_2 + \beta - 1, 0\}$, or equivalently $u_2 < u_1 + 2$, then $-c + E[V_2(\beta, \varepsilon_1, -1)] = -c + \frac{1}{2}(u_1 + \beta + 1) + \frac{1}{2} \max\{u_2 + \beta - 1, 0\}$. the consumer either buys 1 immediately or continues searching 1. In either case, the purchase probability is strictly positive.

To prove the second part, note that when $u_2 > \bar{u}$, the outside option is dominated by searching
product 1. Applying Lemma 3 completes the proof.

2. Proof of Case 2: \( \frac{1}{4}(\beta + 1) \leq c < \frac{1}{4}(2\beta + 1) \)

The proof follows the same line of argument as in Case 1.

3. Proof of Case 3: \( c \geq \frac{1}{4}(2\beta + 1) \)

This part can be proved with two steps: (1) if \( u_1 < 0 \), then the outside option dominates when \( u_2 < 0 \) and buying 2 without search dominates when \( u_2 > 0 \), in either case, \( D_1 = 0 \); (2) if \( u_1 > 0 \), then searching product 1 first is dominated by buying 1 without search, and \( D_1 \) weakly decreases with \( u_2 \).

The first step follows the logic of the first step in the proof of Case 1 when \( 2c - \beta - 1 \leq u_1 \leq 4c - \beta - 1 \). The second step follows the logic of Lemma 3.

**Proof of Lemma 2:**

First, notice that,

\[
\bar{u} + X = \max\{\bar{u} + X + \varepsilon, X - \bar{X}\} - \varepsilon^*
\]

\[
\leq \max\{\bar{u} + X + \varepsilon, 0\} - \varepsilon^*
\]

\[
\leq E\{\max\{\bar{u} + X + \varepsilon_i, 0\}\} - \varepsilon^*
\]

\[
= c - \varepsilon^*,
\]

where the last equality is due to the definition of \( \bar{u} \) in equation (6). Based on the inequality above, we have that,

\[
E\{\max\{-c + E\max\{\bar{u} + X + \varepsilon_2, 0\}|X\}, 0\}\] \( \leq \) \( E\{\max\{-c + E\max\{c - \varepsilon^* + \varepsilon_2, 0\}|X\}, 0\}\]

\[
\leq E\{\max\{-c + E\max\{c - \varepsilon^* + \varepsilon_2, c\}|X\}, 0\}\]

\[
= E\{E\max\{\varepsilon_2 - \varepsilon^*, 0\}|X\}, 0\}\]

\[
= E\{\max\{c, 0\}\}
\]

\[
= c
\]

\[
= E\max\{-c + E\max\{\bar{u}_e + X + \varepsilon_2, 0\}|X\}, 0\},
\]

which implies that \( \bar{u} \leq \bar{u}_e \).
REFERENCES


