The Effects of Asymmetric Disclosure on Price Informativeness and Firm Performance*

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Abstract

We examine how asymmetry in firms’ disclosure policy with respect to good vs. bad news affects investors’ incentive to trade on private information when price performs a feedback role in that firm managers condition their investment based on information conveyed in prices. Prior literature has shown that the feedback effect introduces an asymmetry in trader’s trading behavior: it reduces (increases) the profit of selling on bad news (buying on good news), as a result, bad news is incorporated more slowly into prices than good news, potentially leading to over-investment. We show that this negative impact can be mitigated when firms commit to a disclosure policy that recognizes bad news more timely than good news when the firm receives news. Timely loss recognition improves firms’ expected performance because firms can learn more useful information from stock price. Our analysis suggests an important role for disclosure policies to improve price informativeness and enhance financial market’s resource allocation role, and provides a new rationale for some long-standing accounting principles. It also suggests an alternative interpretation for empirical associations between timely loss recognition and firm performance.

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1 Introduction

One key feature of accounting is timely loss recognition (TLR), also known as conditional conservatism. It refers to the practice that financial accounting usually recognizes bad news more quickly than good news. Timely loss recognition manifests in several accounting standards, such as as lower-of-cost-or-market accounting for inventories, the impairment for long-lived assets, and the impairment of unrealized losses for held-to-maturity financial instruments. While few would dispute that accounting reporting features conservative characteristics such as TLR, there is an on-going debate on what purposes they serve. A commonly held view is that conservatism serves the stewardship role, demanded by investors (e.g., creditors and shareholders) to better contract with firm insiders with misaligned incentives (Watts (2002), Kothari, Ramana and Skinner (2010)). However, the theoretical support for this view is tenuous (e.g., Gigler, et al. (2009)). Further, Lambert (2010) notes that the stewardship view is not the consensus among standard setters and calls for more research to understand the role of conservative reporting in market setting.

In this paper, we explore a novel channel of how timely loss recognition affects firm performance via the feedback channel of financial markets. The feedback channel refers to the situation when firm managers maximize firm values and condition their investment decisions on the information revealed in prices. Prices can contain such decision-useful information because of their unique role in aggregating the diverse private information possessed by investors. The private information is used by traders to maximize their trading profits and is difficult to directly communicate with firm managers. However, the information reflects investors’ assessments of firms’ future prospects, and therefore can be useful to guide firm investment decisions and improve firm performance.\footnote{The insight that price provides useful information for guiding resource allocation decisions in the economy can be traced to Hayek (1945). See Dow and Gorton (1997) and Subramanya and Titman (1999) for early theoretical studies of the feedback effects, and see Bond, Edman, and Goldstein (2012) for a review of the related literature.} A growing body of empirical studies document evidence that firm managers do condition their investment decisions on the information revealed in stock prices, consistent with the existence of feedback channel (e.g., Luo (2005), Chen, et al. (2007)). The feedback channel highlights the importance of price informativeness, i.e., price’s ability to incorporate decision-useful private information from the trading process because under the feedback channel, more informative prices not only improve how efficiently investors allocate capital across firms, but also how efficiently managers utilize capital within firms.

We show that in the presence of the feedback effect, timely loss recognition plays a crucial role in improving price informativeness. In doing so, we adopt a new modeling approach that directly captures the asymmetric timeliness in loss recognition. Specifically, we model reporting timeliness as the probability that accounting recognizes managers’ forward-looking information about the underlying state before managers
make investment decisions; and we model asymmetrically timely loss recognition as accounting recognizing bad news with higher probability than it does good news. Our approach remedies a discrepancy between the existing theoretical and empirical studies on accounting conservatism. Whereas empirical studies focus on the *more timely* recognition of bad news relative to good news, most theory papers formulate conservatism as introducing a downward *bias* in the reported signal, i.e., the reported signal is more likely to understate firm value rather than overstate it.\(^2\) The discrepancy makes it difficult to interpret empirical evidence in rigorous theoretical frameworks. Our modeling approach captures better the notion of asymmetric timeliness in empirical literature, that is accounting recognition is conditional on the nature of the news, and upon recognition, the disclosed accounting information is not biased.

Our analysis provides a fresh perspective on how TLR can directly affect market informational efficiency which in turn affects firm performance. It reveals that by preempting more bad news, timely loss recognition changes the market dynamics when public disclosure is absent, and helps mitigate the distortion the feedback channel has on price informativeness identified in Edmans, Goldstein and Jiang (2015). Specifically, Edmans, et al. (2015) show that the feedback effect affects speculators’ incentive to trade on good and bad news *asymmetrically*: it increases (reduces) their incentive to trade on positive (negative) news. This is because when an informed trader\(^2\) trades on her private information, the firm uses information information contained in price to make better investment actions, which increases the firm value. Higher firm value increases the speculator’s profit when she buys on good news, but reduces her profit when she sells on bad news, lowering her incentive to trade on bad news to begin with. As a result, bad news faces an endogenous limit to arbitrage and is less likely to be reflected in price than good news. This reduces prices’ effectiveness in informing firms about upcoming downturns and adds to the instability of the real economy.\(^4\)

We show that timely loss recognition (TLR) can increase speculators’ incentives to trade on negative news. In our model, a firm decides whether to increase, decrease, or maintain the status quo of, its investment scale. The value of the firm depends on the state of nature as well as the decision taken: when the state is high (low), it is optimal to increase (decrease) investment. The state is observed by the firm with some positive probability but not always, which leaves the firm the chance to learn from the secondary market.

We model the secondary market as in Kyle (1985) with three parties: a speculator who may observe the state and chooses to trade on her private information, a noise trader who submits order randomly, and a

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\(^2\)Under the view that timely loss recognition is implemented by applying a lower verifiability threshold when recognizing negative news [Kothari, et al. (2010)], accounting losses are conditionally less informative than profits. However, as we discuss later, the empirical proxies for timely loss recognition puts more emphasis on the timeliness of news recognition than on the conditional informativeness of losses vs. profits. In our model, conditional on accounting recognition, both good and bad news are equally informative; timely loss recognition implies that accounting is more likely to recognize bad news than good news.

\(^3\)In this paper we use informed trader and speculator interchangeably.

\(^4\)In a related study, Dow, Goldstein and Guembel (2016) show that the feedback channel generates strategic complementarities in speculators’ incentives to acquire information about weak firms. This can result in a market breakdown when speculators stop producing information about firms experiencing small negative shocks to their fundamentals, further magnifying the shocks.
market maker who sets price to the expected firm value based on the information contained in the order flow. The firm observes the trading process and also uses the information contained in order flow to update its belief about the state and choose the corresponding investment action to maximize firm value.

We show that a reporting system featuring timely loss recognition (TLR, thereafter referred to as the conservative system) reduces the asymmetry in speculator’s trading incentives via two effects: a price impact effect and a corrective action effect. The price impact effect affects the information asymmetry between the market maker and the speculator. Specifically, under a conservative system, bad news is more likely to be pre-empted, and thus no disclosure implies that the state is more likely to be good and the market maker sets the price higher accordingly. This changes an informed trader’s incentive to trade: the profit from selling is higher if the speculator’s private information indicates the state is low, and the profit from buying is lower if her private information indicates the state is high. In contrast, the corrective action effect affects investors’ assessment of firm value because of the uncertainty caused by the feedback effect. Specifically, when there is no disclosure, the market is uncertain both about the firm’s underlying state and about whether the firm manager observes the state and therefore takes corrective actions. Higher uncertainty in general increases the speculator’s trading profit, but asymmetrically depending on the reporting system. Under the conservative system, conditional on no disclosure, a firm is more likely to have taken corrective action when the state is high than when the state is low, which leads to a larger difference in firm value between two states and thus higher trading profit for the speculator, especially when she observes the low state.

We further show that in addition to reducing the speculator’s asymmetric incentive in trading good news vs. bad news, timely loss recognition can also increase the speculator’s ex ante incentives to acquire information in the first place. This is due to the corrective action effect, which always increases the speculator’s ex ante trading profit under a conservative system but reduces it under an aggressive system (i.e., a system featuring asymmetrically timely profit recognition). In contrast, the price impact effect affects the speculator’s ex post trading profits, conditional on the news, but does not affect the speculator’s expected trading profit at the information acquisition stage (by law of iterated expectations).

In summary, timely loss recognition has two effects on the firm’s ex ante value: on the positive side, it increases the price informativeness, both ex ante (by increasing the speculator’s incentive to acquire information) and ex post (by improving price’s ability to reflect bad news). As a result, price provides more useful information to guide firm decisions when managers do not observe such information on their own. This effect increases the firm value, due to the increased investment efficiency. On the other hand, it also leads to a larger ex ante information asymmetry among traders (because it increases the speculator’s incentive to become informed) and therefore increases the noise trader’s expected trading loss. This decreases the firm’s
price as the noise trader demands a larger liquidity discount.\footnote{This confirms Lambert (2010)'s conjecture that conservative reporting may reduce market liquidity.} These two effects imply that firms would not prefer a conservative system with TLR in the absence of the feedback effect. However, the price discount is simply a wealth transfer among investors, and does not directly affect total production frontier of the economy. Therefore, from the social optimal perspective, timely loss recognition dominates either a neutral system or an aggressive system because it achieves higher production efficiency.

Our analysis identifies an alternative channel for timely loss recognition to be desirable other than via the stewardship channel. Assuming we can accurately quantify the degree of timely loss recognition, our analysis suggests it can be positively associated with firm performance even when managers always choose the optimal investment decisions. Some advocates of the stewardship view believe that the sole benefit from conservative reporting is to contract and discipline self-interested firm managers to take value-maximizing actions. In contrast, in our framework, conservative reporting motivates self-interested investors to take actions that can improve firm performance.

Our analysis endogenizes price informativeness as a function of the informational properties of accounting reports. This generates a framework to interpret estimates from price-earnings regressions. Specifically, we show that even when the accounting reports are neutral (that is, neither conservative or aggressive), we can still observe an asymmetry in earnings’ correlation with positive stock returns vs. with negative stock returns, as long as the feedback effect is present. This is because of the asymmetric effect the feedback channel has on price’s ability to reflect negative vs. positive news. Timely loss recognition mitigates the feedback channel’s asymmetric effect on price informativeness. This implies that timely loss recognition would generate less asymmetric correlations than under the neutral system, whereas the aggressive reporting would exacerbate the asymmetry. These factors imply that the commonly used Basu measure may not properly capture cross-sectional differences in firms’ timely loss recognitions: it can also differ cross-sectionally depending on the strength of the feedback effect. Since the feedback effect benefits firm performance and generates asymmetric slope estimates in earnings-stock return regressions, it can also produce positive associations between firm performance and the Basu-like measures even with neutral accounting.

Another interesting implication from our analysis is the difference in reporting properties that firms would choose versus what a benevolent social planner (say, standard setters) may choose. As discussed earlier, when firms choose their reporting property, they weigh the tradeoff between investment efficiency and liquidity discount. Our model predicts that only firms that can benefit more from learning from stock prices to make the right decisions have stronger desire to adopt timely loss recognition. Other firms may find the cost of liquidity discount to be too high and therefore may prefer neutral or aggressive reporting (out of benign managers’ desire to maximize firms’ ex ante price and not out of self-interest to inflate earnings
as commonly believed). On the other hand, a social planner aiming at maximizing real efficiency would always prefer a conservative reporting regime, because the liquidity discount is a simple wealth transfer among investors. It is interesting to note the seemingly discrepancy between advocates of conservatism among accounting researchers and standard setters in the U.S.. The advocates call for conservatism and stewardship role as a key element in FASB’s conceptual framework. The FASB explicitly rejects stewardship but nonetheless maintain key conservative features in many standards. Our analysis offers a theoretical foundation for FASB’s practice without forcing FASB to concede to the stewardship view.

Our paper contributes to the growing theoretical literature how the feedback channel affects stock market efficiency. It is closely related to Edmans, Goldstein and Jiang (2015) and Gao and Liang (2013). Edmans, et al. (2015) identify the insight that the feedback channel asymmetrically reduces price’s ability to reflect negative news and therefore generates an endogenous limit to arbitrage on negative news. We build on their insight and further examine how timely loss recognition, a long-standing feature in financial reporting, may mitigate this endogenous frictions. Our analysis alleviates the concern expressed in Edmans, et al. (2015) that more developed markets may not necessarily be better at dealing with the endogenous friction generated by the feedback channel. Our analysis supports a commonly held view that more developed markets usually have more sophisticated financial reporting systems, many of feature conservative characteristics such as TLR. Gao and Liang (2013) also examine how disclosure policy can be used to motivate speculators’ information acquisition when the feedback channel is present. In their model, there is no trading cost so there is no asymmetry in speculators’ incentive to trade good vs. bad news. They show that partial disclosure may be desirable to motivate information acquisition. We extent their line of inquiry and focus how to structure the partial disclosure. We show fine-tuning the partial disclosure policy with asymmetric timeliness can further enhance price informativeness and facilitate firms’ learning from the market.

Our paper also contributes to the large literature in accounting on the role of conservatism. As discussed earlier, we adopt a novel modeling approach to better capture the notion of timely loss recognition, thus bringing theoretical insights closer to empirical studies. More importantly, we also identify a non-governance channel for timely loss recognition to benefit firm performance: in our framework, how firms structure their disclosure policy can shape the market dynamics when there is no disclosure, and specifically, timely loss recognition can improve both investors’ general incentive to acquire information in the first place as well as their incentive to trade on negative news. The channel highlights the intricate endogenous relation between public information and private information in the financial markets, especially when information in financial markets can affect resource allocation in the real economy.

The remaining of the paper is organized as follows. Section 2 describes the model. In Section 3, we analyze the equilibria under different reporting systems, and show that with the presence of feedback effect,
2 Model

In this section we present the model and derive the equilibria at the trading stage.

2.1 Setup

The model has four dates, \( t \in \{0, 1, 2, 3\} \), and four players: a firm, a speculator, a noise trader, and a market maker. All players are assumed to be risk-neutral with zero discount rate across dates.

**Investment Decision and Firm Value** The firm value, realized at \( t = 3 \) and denoted by \( v(\theta, d) \), depends on both the underlying state and the firm’s decisions. There are two possible states, denoted by \( \theta \in \{H, L\} \). The prior probability of \( \theta \) being \( H \) is \( \Pr(\theta = H) = \frac{1}{2} \) and it is common knowledge.\(^6\) The firm takes an investment decision denoted by \( d \in \{-1, 0, 1\} \), where \( d = 1 \) means increasing investment, \( d = -1 \) means decreasing investment, and \( d = 0 \) means maintaining the current level of investment. Changing the level (i.e., \( d = -1 \) or \( 1 \)) requires an adjustment cost \( c > 0 \).

We assume that when the firm chooses to maintain the status quo (\( d = 0 \)), its value \( R_\theta \) is determined by its asset in place, and \( R_H > R_L \). When the firm takes an action, its value depends on both the asset in place \( R_\theta \) and the outcome of the investment decision. When the state is \( H \), the correct decision is to invest, which increases the gross firm value by \( x \) and thus the net firm value is \( v(H, 1) = R_H + x - c \). Decreasing investment (\( d = -1 \)) is the wrong action and it reduces firm value by \( x \): \( v(H, -1) = R_H - x - c \). When the state is \( L \), reducing investment (\( d = -1 \)) is the right decision and \( v(L, -1) = R_L + x - c \), while investing is the wrong decision and \( v(L, 1) = R_L - x - c \).

**Asymmetric timeliness in reporting** The firm needs to decide whether to keep the current level of investment, increase it, or reduce it, and it aims to maximize firm value. As in Gao and Liang (2013), the firm privately learns a signal \( y \) that fully reveals \( \theta \) with probability \( f \) and is completely uninformative with probability \( 1 - f \). One can view \( f \) as a parameter that characterizes the quality of the firm’s internal accounting information system. The firm commits to a policy that reveals information with probability \( \beta \); however, the probability of disclosure can depend on the nature of information, i.e., whether the firm learns

\(^6\) As in Edmans et al. (2015), we assume symmetry for all of the parameters other than disclosure bias to ensure that our asymmetry result is not due to asymmetry in other parameters.
the state is $H$ or $L$: $\beta_H = \beta - \xi$ is the probability that the firm discloses when it learns that the underlying state is $H$, and $\beta_L = \beta + \xi$ is the probability that the firm discloses when it learns the state is $L$. When the firm does not receive any signal ($\delta = \emptyset$), it does not disclose. Thus, $\beta = \frac{1}{2}(\beta_H + \beta_L)$ captures the overall timeliness of the reporting system, while $\xi$ captures the degree of asymmetric timeliness in reporting good and bad news: a positive $\xi$ implies that $\beta_L > \beta_H$, thus the system is more timely to disclose bad news than good news, and thus the system is conservative. A negative $\xi$ implies that $\beta_L < \beta_H$, and the system is aggressive. In the main model we assume $\beta \in (0, 1)$ is exogenously fixed and focus on the effect of asymmetric timeliness.

**Trading** After the signal $y$ is disclosed, trading in the financial market occurs at $t = 2$. A speculator is informed about the state $\theta$ with probability $\lambda$ and uninformed with probability $1 - \lambda$. In addition to the speculator, two other agents participate in the financial market: a noise trader whose trades are unrelated to the realization of $\theta$, and a risk-neutral market maker who collects the orders from the speculator and noise trader, sets a price at the expected firm value, and executes the orders out of his inventory. The noise trader submits an order $z \in \{-1, 0, 1\}$, each with probability $\frac{1}{3}$. If the speculator is informed, she makes an endogenous trading choice $s \in \{-1, 0, 1\}$. If she is not informed, she does not trade. Trading either $-1$ or $1$ costs the speculator $\kappa > 0$, and the cost is randomly drawn from a distribution $\kappa \in [0, \bar{\kappa}]$ with probability density function $g(\kappa)$ and cumulative distribution function $G(\kappa)$. $\kappa$ is realized and becomes common knowledge before trading begins. The trading cost $\kappa$ should be interpreted broadly: While direct trading costs from commissions are typically small, other indirect costs can be large. These include borrowing costs (for short sales) and the opportunity costs of capital commitment (for purchases).

Following Kyle (1985), market orders are submitted simultaneously to a competitive market maker who absorbs orders out of his inventory and sets the price equal to the expected firm value, given the information contained in the order flow. The market maker can only observe the total order flow $X = s + z$, but not its individual components $s$ and $z$. Possible order flows are $X \in \{-2, -1, 0, 1, 2\}$ and the pricing function is $p(X) = E(v(\theta, d) | X)$.

Since the firm does not always know the state $\theta$ and the speculator may know it, the firm can potentially use the information contained in stock price to infer the state and improve the investment efficiency. The firm makes investment decision after the stock price forms, and the firm can observe the order flow $X$, use it to update its posterior belief $\mu_F$, and make better investment decision. That is, $d(\delta, X)$. The stock price rationally anticipates the firm’s investment decisions and the firm value. Following prior literature on feedback effect (Edmans et al., 2015), we assume that the firm has access to information about total order

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1In Section 5, we relax this assumption and allow the firm to choose both $\beta$ and $\xi$, and the key results are robust.
flow. Next we describe the timeline and discuss the equilibria in the trading stage.

**Choice of Asymmetric timeliness at \( t = 0 \) (Section 4)** At \( t = 0 \), a reporting policy with \( \xi \) is chosen. To discuss the optimality, we need to identify an objective to maximize. We study the optimal reporting system from two perspectives: the social planner’s, who aims to maximize social welfare, and the firm’s, who aims to maximize the price at which it can sell its shares in the primary market. We leave this to Section 4. Before we proceed, we summarize the timeline in the table below, and define the Perfect Bayesian Equilibrium under any given reporting system.

| \( t = 0 \) | A reporting system with asymmetric timeliness \( \xi \) is chosen.
| - The firm receives a signal \( \delta = \theta \) (that reveals the state) with probability \( f \) and \( \delta = \emptyset \) (that reveals nothing) with probability \( 1 - f \).
| - If the firm receives the signal \( \delta \), it discloses \( r \in \{ \delta, \emptyset \} \) according to the reporting policy.
| \( t = 1 \) | With probability \( \lambda \) the speculator learns a signal \( \eta \) that reveals the state \( \theta \). Trading follows in the financial market.
| - The noise trader submits market order \( z \in \{-1, 0, 1\} \) with equal probability.
| - The speculator observes the trading cost \( \kappa \) and chooses to submit market order \( s \in \{-1, 0, 1\} \). Trading costs the speculator \( |s|\kappa \).
| - The market maker observes total order flow \( X = s + z \), and sets price equal to the expected asset value: \( P(x) = E(v|X,r) \).
| \( t = 2 \) | The firm makes investment decision \( (d) \) that maximizes the firm value \( E(v(\theta,d)|\delta,X) \).
| \( t = 3 \) | The firm value and payoffs are realized.

**The equilibrium** The equilibrium concept is *Perfect Bayesian Equilibrium*.

1. Given the price setting rule, the reporting policy \( \xi \), and the private information, the speculator’s trading strategy \( s \in \{-1, 0, 1\} \) maximizes his expected payoff: \( s \in \arg \max_s s(v-p) - |s|\kappa \).

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8Under the alternative assumption that the firm observes only \( p \) but not \( X \), an alternative equilibrium can arise, in which the firm’s investment decision is suboptimal given the information in \( X \). The assumption of observing \( X \) is reasonable, since in practice order flow information can be provided by microstructure databases at a short lag.
2. Given the speculator’s and the firm’s strategy, the market maker sets price to the expected firm value \( p = E(v(\theta, d(\delta)) | r, X). \)

3. Given the firm’s signal \( \delta \) and the order flow \( X \), the firm chooses \( d \in \{-1, 0, 1\} \) to maximize expected firm value: \( d(\delta, X) \in \arg\max_d E(v(\theta, d) | \delta, X). \)

4. The firm and the market maker use the Bayes’ rule to update their beliefs. Beliefs on outcomes not observed on the equilibrium path satisfy the Cho and Kreps (1987) Intuitive criterion.

3 The firm’s investment decision and the speculator’s trading decision

In this section we will solve for the model through backward induction. We first solve for the firm’s investment decision at \( t = 2 \), followed by the speculator’s trading decisions at \( t = 1 \). In the next section, we compare reporting systems with different level of asymmetric timeliness and identify the optimal system.

3.1 Firm’s investment decision

After observing the market order flow, the firm updates its posterior belief \( \mu_F \) about state being \( \theta = H \), and makes investment decision. Without the information from trade, the firm’s default action is to do nothing. To see this, note that the prior probability is \( \Pr(\theta = H) = \frac{1}{2} \), thus the expected firm value from doing nothing is \( \frac{1}{2} (R_H + R_L) \), while choosing \( d = \pm 1 \) leads to a lower expected value: \( \frac{1}{2}(R_H + x - c) + \frac{1}{2}(R_L - x + c) = \frac{1}{2} (R_H + R_L) - c \). This is because taking an action entails an adjustment cost, thus the firm will deviate from its default action and invest only if the signal gleaned from trade is sufficiently informative. Let \( \gamma_1 \) denote the posterior belief under which the firm is indifferent between investing and doing nothing, then

\[
\gamma_1 R_H + (1 - \gamma_1) R_L = \gamma_1 (R_H + x) + (1 - \gamma_1) (R_L - x) - c
\]

and \( \gamma_1 = \frac{1}{2} + \frac{c}{2x} \). Similarly, let \( \gamma_{-1} \) denote the firm’s posterior under which it is indifferent between deinvesting and doing nothing, then

\[
\gamma_{-1} (R_H) + (1 - \gamma_{-1}) R_L = \gamma_{-1} (R_H - x) + (1 - \gamma_{-1}) (R_L + x) - c
\]

and \( \gamma_{-1} = \frac{1}{2} - \frac{c}{2x} \). For completeness, when the firm is indifferent between doing nothing or changing the investment, we assume that the firm chooses to do nothing. Thus, the firm will invest if \( \mu_F > \gamma_1 \), deinvest
if \( \mu_F < \gamma - 1 \), and do thing if \( \gamma - 1 \leq \mu_F \leq \gamma \).

As will be shown below, \( X = \pm 2 \) is always fully revealing of \( \theta \) and the manager will always learn from it if uninformed whereas \( X = 0 \) is always completely uninformative. Whether the firm will change investment level when observing \( X = \pm 1 \) depends on how informative the speculator’s signal is. As will be shown in the proof, when \( \lambda \) is sufficiently high such that \( \frac{1}{2 - \lambda} > \frac{1}{2} + \frac{c_2}{2x} \), \( X = \pm 1 \) can be sufficiently informative so that the firm will learn from the market and choose \( d = \pm 1 \) when \( X = \pm 1 \). We will refer to this case as the “feedback case.” On the other hand, when \( \frac{1}{2 - \lambda} < \frac{1}{2} + \frac{c_2}{2x} \), i.e. when the probability that the speculator being present is sufficiently small, \( X = \pm 1 \) will be not sufficiently informative so that the firm will learn from the market and choose \( d = 0 \) when \( X = \pm 1 \). We will refer to this case as the “weak-feedback case”. The feedback case is our main focus in this paper, and we briefly discuss the weak feedback case in Section 4.

3.2 Equilibria of the trading game at \( t = 2 \) with feedback

We now solve for the equilibria of the trading game. Before proceeding, we briefly discuss the trading game and introduce some notations that will be useful in subsequent analysis. First, the speculator will never trade when the firm discloses since the speculator will then have no information advantage. Thus, trading generates profit for the speculator only when the firm does not disclose, i.e., \( r = \emptyset \). In this case, since \( R_H - x > R_L + x \), the speculator will never buy when he learns \( \theta = L \) and will never sell when he learns \( \theta = H \). This leads to only four types of pure-strategy equilibria:

(i) \( NT \) (no-trading equilibrium), when \( s(\theta) = 0 \ \forall \theta \), i.e. the speculator doesn’t trade.

(ii) \( BNS \) (buy-not-sell equilibrium), when \( s(H) = 1 \) and \( s(L) = 0 \), i.e. the speculator buys when observing \( \theta = H \) but does not trade when observing \( \theta = L \).

(iii) \( SNB \) (sell-not-buy equilibrium), when \( s(H) = 0 \) and \( s(L) = -1 \), i.e. the speculator sells when observing \( \theta = L \) but does not trade when observing \( \theta = H \).

(iv) \( T \) (trading) equilibrium, when \( s(H) = 1 \) and \( s(L) = -1 \), i.e. the speculator buys when observing \( \theta = H \) and sells when observing \( \theta = L \).

We now solve for the equilibria of the trading game. We will solve for the case without feedback, followed by the case with feedback.

3.2.1 Neutral reporting system benchmark

This subsection characterizes the subgame equilibrium in the feedback case at the trading stage under a given reporting policy \( \xi \). Depending on the trading cost \( \kappa \), different pure strategy equilibrium can arise: \( Q \in \{ T, NT, BNS, SNB \} \). As a benchmark to our main analysis on asymmetry reporting, we start from
the case when the reporting policy is symmetric ($\xi = 0$).

**Proposition 1. (Subgame equilibrium in the feedback case with symmetric reporting)** When $\xi = 0$, the equilibria depend on the trading cost $\kappa$:

1. **No-trading equilibrium** sustains when $\kappa > \kappa_{NT}^{n} = \frac{1}{3} (R_H - R_L);$

2. **Trading equilibrium** sustains when $\kappa < \kappa_{T}^{n} = \frac{1}{3} (R_H - R_L) \left( \frac{1}{2} + \frac{1-\lambda}{2-\lambda} \right) - \frac{1}{2-\lambda} \left( \frac{1-f}{1-f^\beta} \right) x;$

3. **Buy-No-Sell equilibrium** sustains when $\kappa_{BNS}^{n} < \kappa < \kappa_{NT}^{n},$ where $\kappa_{BNS}^{n} = \kappa_{T}^{n}$ and $\kappa_{BNS}^{n} = \kappa_{NT}^{n};$

4. **Sell-No-Buy equilibrium** sustains when $\kappa_{SNB}^{n} < \kappa < \bar{\kappa}_{BNS}^{n},$ where $\kappa_{SNB}^{n} = \frac{1}{3} (R_H - R_L) \left( \frac{1}{2} + \frac{1-\lambda}{2-\lambda} \right) + \frac{2}{3} \frac{1-\lambda}{1-f^\beta} x$ and $\bar{\kappa}_{BNS}^{n} = \kappa_{NT}^{n}.$

When $\kappa$ is sufficiently small ($\kappa < \kappa_{T}^{n}$), the profit from both buying and selling exceeds the trading cost and the speculator always trades on his private information, and thus $T$ is the only equilibrium. When $\kappa$ is sufficiently large ($\kappa > \kappa_{NT}^{n}$), the profit from both buying and selling is smaller than the trading cost and $NT$ is the only equilibrium. As $\kappa$ increases, the speculator trades less. Knowing that trading might move the price against them, speculators might refrain from trading. For $\kappa \in (\kappa_{T}^{n}, \kappa_{NT}^{n})$, neither trading nor no-trading can sustain as an equilibrium. However, two partial-trading equilibria may exist, in which the speculator trades on one type of information but not another. The speculator can either buy on observing good news (BNS), or sell upon observing bad news (SNB).

The intuition for the one-sided trading equilibrium is as follows. Take the BNS equilibrium as an example. In this equilibrium, the market maker believes that the speculator buys on good news and does not trade on bad news. Given that the market maker believes that the speculator buys on good news, a negative order flow is informative on that the speculator is negatively informed and the price moves sharply to reflect this. Specifically, $X = -1$ is inconsistent with the speculator having positive information (otherwise the speculator would buy and the order flow cannot be -1), and thus the market maker set the price lower, and the speculator makes little profit from selling on bad news. Knowing this, the speculator chooses not to trade on bad news. On the other hand, compared to the trading equilibrium, observing $X = 1$ is a less strong indicator of good news, since even if the speculator observes $\theta = L$, he refrains from trading and the bad news is not reflected in the order flow. Thus the price is lower and the profit from buying is higher. Taken together, this constitutes an equilibrium.

However, the BNS and SNB equilibria are not symmetric. While Buy-No-Sell equilibrium exists for any $\kappa \in (\kappa_{T}^{n}, \kappa_{NT}^{n})$, Sell-No-Buy exists only for the range $(\bar{\kappa}_{BNS}^{n}, \kappa_{NT}^{n})$, which exists if and only if $\kappa_{BNS}^{n} < \kappa_{NT}^{n}.$ Since $\kappa_{T}^{n} < \kappa_{SNB}^{n}$, the range of $\kappa$ for BNS equilibrium is strictly larger than that for SNB. This asymmetry is identified by Edmans et al. (2015) and is due to the feedback effect. That is, since the firm takes the investment decision after observing the order flow, it can use the revealed information of the speculator’s to
make better investment decision. In particular, when \( X = 2 \) (-2), the firm learns that the state is \( H \) (\( L \)) for sure and thus invests (reduces investment). When \( X = 1 \) (-1) and the speculator’s private information is sufficiently informative, the manager invests (reduces investment). Such feedback increases firm value, increases the speculator’s profit of buying on good news, and reduces his profit of selling on bad news. To see this, we first look at the profit in the trading equilibrium under neutral reporting:

\[
\begin{align*}
\pi_{T,n}^{\text{buy}} &= \frac{1}{3} \left( \frac{1}{2} + \frac{1 - \lambda}{2 - \lambda} \right) (R_H - R_L) + \frac{2}{3} \frac{1 - \lambda}{2 - \lambda} \frac{1 - f}{1 - f \beta} x \\
\pi_{T,n}^{\text{sell}} &= \frac{1}{3} \left( \frac{1}{2} + \frac{1 - \lambda}{2 - \lambda} \right) (R_H - R_L) - \frac{2}{3} \frac{1 - \lambda}{2 - \lambda} \frac{1 - f}{1 - f \beta} x
\end{align*}
\]

As a result, trading equilibrium sustains if \( \kappa < \frac{1}{3} \left( \frac{1}{2} + \frac{1 - \lambda}{2 - \lambda} \right) (R_H - R_L) - \frac{2}{3} \frac{1 - \lambda}{2 - \lambda} \frac{1 - f}{1 - f \beta} x \), and arbitrage is limited because the firm value is endogenous to the act of arbitrage.

Proposition 1 generalizes the results in Edmans et al. (2015) by allowing the firm to have internal information and report following a pre-specified stochastic rule. Edmans et al. (2015) is a special case here with \( f = 0 \), i.e., when the firm has no internal information (\( \delta = 0 \)). The upper cutoff for trading equilibrium decreases with \( \beta \), which implies that the limit to arbitrage due to feedback effect is more severe when the firm discloses more. This is because with higher \( \beta \), no disclosure is more likely to imply that the firm does not have internal information, and thus firm’s real investment decision is more likely to rely on feedback from market. Since feedback reduces the profit of selling on bad news, the trading equilibrium is harder to sustain.

Gao and Liang (2013) show that a firm can choose no disclosure to induce maximal feedback from market. Nevertheless, in our setting with trading costs, the information loss stemming from the feedback effect still persists even if the firm chooses \( \beta = 0 \) because the speculator trades less on bad news and the market price is less likely to incorporate bad news. As a consequence, changing only the absolute level of disclosure (i.e. \( \beta \)) under a neutral system is insufficient in achieving maximal feedback from market; to further facilitate its learning from market, the firm may want to condition its disclosure on the nature of the news (i.e., \( \xi \)) and thus potentially adopt an asymmetric reporting system.

### 3.2.2 Main analysis on asymmetric reporting system

This section studies how asymmetric timeliness affects the speculator’s trading behavior and price informativeness. Recall that parameter \( \xi \) captures the degree of asymmetric reporting: when \( \xi > 0 \) we have \( \beta_H > \beta_L \) and the system is conservative; when \( \xi < 0 \) we have \( \beta_H < \beta_L \) and the system is aggressive.

Note that when the firm reports, its report truthfully reveals the state, and the market understands that
the firm invests in $H$ state and deinvests in $L$ state and prices it accordingly. What is more interesting is the market dynamics when there is no disclosure. Denote $t$ as the probability that the state is $H$ when there is no disclosure. Following Bayes’ rule,

$$
t = \Pr(\theta = H|r = \emptyset) = \frac{\Pr(r = \emptyset|\theta = H)\Pr(\theta = H)}{\Pr(r = \emptyset|\theta = H)\Pr(\theta = H) + \Pr(r = \emptyset|\theta = L)\Pr(\theta = L)} = \frac{1 - f\beta_H}{1 - f\beta_H + 1 - f\beta_L} = \frac{1}{2} + \frac{1}{2}\cdot \frac{f}{1 - f\beta_L}.
\tag{1}
$$

No disclosure contains two cases: either the firm learns about the state but withholds the information, or the firm does not know about the state, and can potentially learn from the market. Thus the firm knows its information endowment while the market does not. As a result, their posterior belief conditional on observing the order flow can be different. We denote the market maker’s posterior belief upon observing order flow $X = i$ as $\mu_{M,i}$, and the firm’s posterior belief upon having no internal information and observing order flow $X = i$ as $\mu_{F,i}$. That is, $\mu_{M,i} = \Pr(\theta = H|r = \emptyset, X = i)$, and $\mu_{F,i} = \Pr(\theta = H|\delta = \emptyset, X = i)$. These beliefs depend on trading behavior in the equilibrium. The next proposition summarizes the equilibria at the trading stage ($t = 2$):

**Proposition 2.** When there is feedback effect, the equilibria are as follows:

1. No-trading equilibrium sustains if $\kappa > \kappa_{NT} = \frac{2}{3} (R_H - R_L + (\tau_H - \tau_L)(x-c)) \max(1 - \mu_0, \mu_0)$.
2. Trading equilibrium sustains if $\kappa < \kappa^T = \min\left(\pi_{buy}^T, \pi_{sell}^T\right)$, where

   $$
   \pi_{buy}^T = \frac{1}{3} (1 - t) (R_H - R_L + (\tau_H - \tau_L)(x-c)) + \frac{1}{3} (1 - \mu^T_L) (R_H - R_L + 2 (1 - \tau_L) x)
   $$$$\pi_{sell}^T = \frac{1}{3} t (R_H - R_L + (\tau_H - \tau_L)(x-c)) + \frac{1}{3} \mu^T_{-1} (R_H - R_L - 2 (1 - \tau_H) x)
   $$

3. Buy-no-sell equilibrium sustains if $\pi_{sell}^T < \kappa < \frac{2}{3} (1 - \mu_0) (R_H - R_L + (\tau_H - \tau_L)(x-c))$.
4. Sell-no-buy equilibrium sustains if $\pi_{buy}^T < \kappa < \frac{2}{3} \mu_0 ((R_H - R_L + (\tau_H - \tau_L)(x-c)))$, where

The following graph illustrates the equilibria in the feedback case.
To see this more clearly, next we discuss the four equilibria separately.

**No-trading equilibrium** In the no-trading equilibrium, the speculator does not trade on her private information and thus the order flow is not informative about state. Since the firm may have taken the correct action when it learn from internal information, the expected firm value is higher than the value of asset in place; when $\theta = H$, $R'_H = E^{NF}(v|\theta = H) = R_H + \tau_H(x - c)$ and $R'_L = E^{NF}(v|\theta = L) = R_L + \tau_L(x - c)$, where $\tau_\theta$ capture the probability that the firm learns about state $\theta$ from its internal information and has taken the correct action to improve the firm value:

\[
\tau_\theta = \frac{\Pr(\delta = \theta| r = 0, \theta)}{\Pr(r = 0|\theta)} = \frac{f(1 - \beta_0)}{1 - f\beta_0} \quad \text{when } \theta = H, L. \quad (2)
\]

The following table shows for any order flow $X$, the market maker’s posterior $\mu_M$, the firm’s posterior (if it does not have internal information) $\mu_F$, and the market price $p$.

No-trading equilibrium sustains when the profit from buying and selling are lower than the trading cost. The lower bound of the NT equilibrium, denoted as $\kappa^{NT}$, depends on the (before trading cost) profit of the most profitable deviation. When the order flow $X = 2$, it must be the case that both the noise trader and the speculator submit $s = z = 1$, and the state is revealed $\mu_2 = 1$. Similarly, when $X = -2$, it must be the
Table 1: No-trading equilibrium with feedback

<table>
<thead>
<tr>
<th>X</th>
<th>$\mu_M (r = \emptyset, X)$</th>
<th>$\mu_F (\delta = \emptyset, X)$</th>
<th>$p$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>0</td>
<td>0</td>
<td>$R_L + (x - c)$</td>
</tr>
<tr>
<td>-1</td>
<td>$t$</td>
<td>$\frac{1}{2}$</td>
<td>$tR_H' + (1 - t)R_L'$</td>
</tr>
<tr>
<td>0</td>
<td>$t$</td>
<td>$\frac{1}{2}$</td>
<td>$tR_H' + (1 - t)R_L'$</td>
</tr>
<tr>
<td>1</td>
<td>$t$</td>
<td>$\frac{1}{2}$</td>
<td>$tR_H' + (1 - t)R_L'$</td>
</tr>
<tr>
<td>2</td>
<td>$t$</td>
<td>$\frac{1}{2}$</td>
<td>$R_H + (x - c)$</td>
</tr>
</tbody>
</table>

case that both the noise trader and the speculator sell $s = z = -1$, and the state is revealed ($\mu_{-2} = 0$).\(^9\)

Deviating to buying in the NT equilibrium generates a profit when $X = 0$ and $+1$. For the NT equilibrium, order flows are completely uninformative. Thus $p(X = 0) = p(X = +1) = tR_H' + (1 - t)R_L'$. The profit before trading cost is thus

$$
\pi_{NT}^{\text{buy}} = \frac{2}{3} \{ R_H' - [tR_H' + (1 - t)R_L'] \} \\
= \frac{2}{3} (1 - t)(R_H' - R_L')
$$

Deviating to selling in the NT equilibrium generates a profit when $X = 0$ and $-1$, resulting in a profit before trading cost of

$$
\pi_{NT}^{\text{sell}} = \frac{1}{3} (p(X = 0) + p(X = -1)) - \frac{2}{3} E (v(L, d)|ND, L) \\
= \frac{2}{3} [tR_H' + (1 - t)R_L' - R_L'] \\
= \frac{2}{3} t(R_H' - R_L')
$$

Asymmetric reporting brings in two new factors (non-existent in the neutral system) that affect speculator’s trading profit. The first is that it changes the market maker’s assessment about state $\theta$ when there is no disclosure. Under neutral system, without disclosure the market maker’s posterior is $\Pr (\theta = H) = \frac{1}{2}$. However, when the system is conservative (aggressive), bad (good) news are more likely to be preempted, thus in the absence of disclosure, the market maker’s posterior about state being $H$ is higher (lower) than $\frac{1}{2}$ and thus the price is higher (lower) than $\frac{1}{2} (E (v|\theta = H) + E (v|\theta = L))$. This in turn affects stock price and the trading profit: Under a conservative system, the profit is higher if selling on bad news than buying on good news. Under an aggressive system, the profit is higher if buying on good news than selling on and news. We label this price effect.

\(^9\)Under no-trading equilibrium and partial-trading equilibrium, the case $X = +2$ (or $-2$) can be off the equilibrium path. In such case, we assume that the market maker and the firm believe that the speculator knows that the state is $H$ (or $L$). Since speculators always lose if they trade against their information, this is the only belief that is consistent with the intuitive criterion.
The trading profit through price effect and corrective action effect. We first turn to conservative reporting:

\[
\pi_{sell} = \frac{1}{3} t (R_H - R_L + (\tau_H - \tau_L) (x - c)) + \frac{1}{3} \mu_{T-1}^T (R_H - R_L - 2 (1 - \tau_H) x)
\]

\[
= \frac{1}{3} \left( 1 + \frac{1 - \lambda}{2 - \lambda} \right) (R_H - R_L) + \frac{1}{3} \left( t - \frac{1}{2} + \mu_{T-1}^T - \frac{1 - \lambda}{2 - \lambda} \right) (R_H - R_L)
\]

\[
+ \frac{1}{3} t (\tau_H - \tau_L) (x - c) - \frac{2}{3} \mu_{T-1}^T (1 - \tau_H) x
\]

The second factor is due to the firm’s withholding of internal information. In the no-trading equilibrium there is no feedback. Nonetheless, the firm has internal information to make investment decision. Thus, the firm value in \( \theta = H \), denoted as \( R'_H = R_H + \Pr (s_F = \theta | r = \emptyset) (x - c) = R_H + \tau_H (x - c) \), while the firm value in \( \theta = L \) is \( R'_L = R_L + \tau_L (x - c) \). While \( \tau_H - \tau_L = 0 \) when \( \xi = 0 \), implying that under neutral system this corrective action does not affect trading profit. When the system is conservative (aggressive), without disclosure the firm is more (less) likely to take corrective action based on internal information when \( \theta = H \) than when \( \theta = L \), thus the value difference between the two states is larger (smaller), which leads to higher (lower) trading profit for the speculator. We label this correct action effect.

Trading equilibrium Next we look at trading equilibrium. In this case, order flows of \( X = +1 \) or \(-1\) is informative, and the market maker updates his belief about \( \theta = H \). Using Bayes’ Rule, when \( X = 1 \), his belief is \( \mu_1^T = \frac{t}{t+(1-t)(1-\lambda)} \). Similarly, when \( X = -1 \), he updates his posterior \( \mu_{T-1}^T = \frac{t(1-\lambda)}{t(1-\lambda) + (1-t)\lambda} \). We assume that the speculator’s information is sufficiently informative such that \( \mu_1^T > \gamma_1 \) and the firm learns from market when \( X = \pm 1 \).

While feedback effect reduces the trading profit for selling on bad news, asymmetric timeliness affects the trading profit through price effect and corrective action effect. We first turn to conservative reporting:

\[
\pi_{sell} = \frac{1}{3} t (R_H - R_L + (\tau_H - \tau_L) (x - c)) + \frac{1}{3} \mu_{T-1}^T (R_H - R_L - 2 (1 - \tau_H) x)
\]

\[
= \frac{1}{3} \left( 1 + \frac{1 - \lambda}{2 - \lambda} \right) (R_H - R_L) + \frac{1}{3} \left( t - \frac{1}{2} + \mu_{T-1}^T - \frac{1 - \lambda}{2 - \lambda} \right) (R_H - R_L)
\]

\[
+ \frac{1}{3} t (\tau_H - \tau_L) (x - c) - \frac{2}{3} \mu_{T-1}^T (1 - \tau_H) x
\]
Table 3: The impact on trading profit

<table>
<thead>
<tr>
<th></th>
<th>Conservative</th>
<th>Neutral</th>
<th>Aggressive</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Buying</td>
<td>Selling</td>
<td>Buying</td>
</tr>
<tr>
<td>Price Effect</td>
<td>-</td>
<td>+</td>
<td>0</td>
</tr>
<tr>
<td>Corrective Action Effect</td>
<td>+</td>
<td>+</td>
<td>0</td>
</tr>
<tr>
<td>Feedback Effect</td>
<td>+</td>
<td>-</td>
<td>+</td>
</tr>
</tbody>
</table>

Table 4: Buy-no-sell in the feedback case

<table>
<thead>
<tr>
<th>X</th>
<th>μ_F</th>
<th>μ_M</th>
<th>d</th>
<th>Feedback</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>0</td>
<td>0</td>
<td>-1</td>
<td>R_L + x - c</td>
</tr>
<tr>
<td>-1</td>
<td>(1-λ)/2</td>
<td>(1-λ)/2</td>
<td>(1-t)</td>
<td>μ^2_H (R_H + τ_H x - (1 - τ_H) x - c) + (1 - μ^2_H) (R_L + x - c)</td>
</tr>
<tr>
<td>0</td>
<td>1/2</td>
<td>t</td>
<td>0</td>
<td>tR'_H + (1 - t) R'_L</td>
</tr>
<tr>
<td>1</td>
<td>1/2</td>
<td>t</td>
<td>0</td>
<td>tR'_H + (1 - t) R'_L</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>R_H + x - c</td>
</tr>
</tbody>
</table>

Similarly, we can decompose the profit from buying on good news:

\[ \pi^T_{buy} = \frac{1}{3} (1 - t) (R_H - R_L + (\tau_H - \tau_L) (x - c)) + \frac{1}{3} (1 - \mu^T_1) (R_H - R_L + 2 (1 - \tau_L) x) \]

\[ = \pi^T_{buy} - \frac{1}{3} \left( t - \frac{1}{2} + \mu^T_1 - \frac{1}{2 - \lambda} \right) (R_H - R_L) + \frac{1}{3} \left( 1 - t \right) (\tau_H - \tau_L) (x - c) + \frac{2}{3} (1 - \mu^T_1) (1 - \tau_L) x \]

Thus, when the system is conservative, the price effect and the corrective action effect increase the speculator’s profit on selling on bad news, and decreases its profit on buying on good news. Since the upper bound of the trading equilibrium is determined by the lower of the profit on buying and selling, they mitigate the impact of feedback effect. On the other hand, if the system is aggressive, the profit of selling on bad news would be even lower, and thus trading equilibrium is less likely to sustain. The following table summarizes the impact of the three forces on trading profit.

**Buy-no-sell equilibrium** Next we look at the Buy-no-sale equilibrium. When the trading cost is intermediate, the speculator finds her net profit to be negative if she trades on both good and bad news, but net profit to be positive if she refrains from trading. Thus in equilibrium she trades only on good news or bad news. We first look at the Buy-no-sell equilibrium. In this case, since she does not sell on bad news, a low order flow \( X = -2 \) becomes rare and thus very indicative of low state, thus the market maker sets price lower. As a result, the profit from selling is smaller, and she finds selling not profitable, which sustains the equilibrium.
Table 5: Sell-no-buy in the feedback case

<table>
<thead>
<tr>
<th>X</th>
<th>μ_F</th>
<th>μ_M</th>
<th>d</th>
<th>Feedback</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>0</td>
<td>0</td>
<td>-1</td>
<td>RL + x - c</td>
</tr>
<tr>
<td>-1</td>
<td>1/2</td>
<td>t</td>
<td>0</td>
<td>tR_H + (1 - t) R_L</td>
</tr>
<tr>
<td>0</td>
<td>1/2</td>
<td>t</td>
<td>0</td>
<td>tR_H + (1 - t) R_L</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1/2</td>
<td>1</td>
<td>μ_T^T (R_H + x - c) + (1 - μ_T^T) (R_L + τ_L x - (1 - τ_L) x - c)</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>RL + x - c</td>
</tr>
</tbody>
</table>

When the system is conservative, the BNS equilibrium holds if and only if

$$κ^{T,c} ≡ π_{sell}^{T,c} < κ < \bar{κ}^{BNS,c}$$

$$≡ \frac{2}{3} (1 - μ_0) (R_H - R_L + (τ_H - τ_L) (x - c))$$

$$< κ^{NT,c} = \frac{2}{3} μ_0 (R_H - R_L + (τ_H - τ_L) (x - c))$$

When the system is aggressive, the BNS equilibrium holds if

$$κ^{T,a} = κ^{BNS,a} < κ < \bar{κ}^{BNS,a} = κ^{NT,a}$$

**Sell-no-buy equilibrium** For the BNS equilibrium to sustain, the trading profit from buying on good news needs to be greater than trading cost κ while the profit from selling on bad news needs to be smaller than κ.

For the sell-no-buy equilibrium, when the system is aggressive, SNB sustains if κ satisfies $κ^T_c < κ^{SNB} < κ < \bar{κ}^{SNB} = κ^{NT}_c$. Note that there exists a range $κ ∈ (κ_c^T, κ^{SNB}_c)$ under which only BNS equilibrium exists but SNB does not. When the system is aggressive, SNB sustains if κ satisfies $κ^T_\alpha < κ^{SNB}_\alpha < κ < \bar{κ}^{SNB}_\alpha < κ^{NT}_\alpha$. Note that as discussed above, aggressive reporting system shrinks the region for SNB equilibrium as the profit from selling gets further reduced by the feedback effect.

**Corollary 1. (The impact of asymmetric timeliness on equilibrium)** Consider a neutral system, an aggressive system and a conservative system with the same degree of asymmetric timeliness but on opposite side.

1. $κ^{NT}_n < κ^{NT}_a < κ^{NT}_c$. Compared to neutral reporting or aggressive reporting, conservative reporting leads to a smaller no-trading region.

2. $κ^T_n < κ^T_a < κ^T_c$. $π_{sell}^{T}$ increases in ξ. Compared to neutral reporting or aggressive reporting, conservative reporting leads to a larger trading region and a smaller no-trading region.
4 Reporting policy of the firm at $t = 0$

Section 2 has characterized the equilibrium in the trading stage under any given reporting system and any trading cost $\kappa$. In this section, we compare reporting systems with different levels of asymmetric timeliness $\xi$, and identify the optimal asymmetric timeliness. The optimality depends on the objective to maximize.

We consider two perspectives. The first is from the firm’s perspective. The firm aims to maximize the price at which it offers shares to initial investors. These investors will subsequently suffer the liquidity shock and thus lose money on average to the speculators. Thus, they demand the offering price to contain a discount equal to their expected trading loss. At date 0, the firm chooses $\xi$ to maximize its selling price in the primary market, $p_0 = E_\xi (v(\theta, d) - \pi)$. The firm’s problem is:

$$\max_{\xi} E_\xi (v(\theta, d(\delta, X)) - \pi)$$
$$s.t. 0 \leq \beta + \xi \leq 1$$
$$0 \leq \beta - \xi \leq 1$$

The second perspective is from the social planner’s. From a social welfare perspective, the speculator’s trading profit (which is also the noise trader’s loss), and the price discount are wealth transfer and does not affect total welfare. Thus, the social planner’s objective function is:

$$\max_{\xi} E_\xi (v(\theta, d(\delta, X)))$$
$$s.t. 0 \leq \beta + \xi \leq 1$$
$$0 \leq \beta - \xi \leq 1$$

Before proceeding to the results, we add a comment on the trading cost $\kappa$, which plays a crucial role in determining the equilibrium. The trading cost $\kappa$ should be interpreted broadly: while direct trading costs from commissions and bid-ask spreads are typically small, other indirect costs, such as borrowing costs (for short sales) and the opportunity costs of capital commitment (for purchases), can be large. Furthermore, there is risk that the speculator cannot hold on to her position till arbitrage works out. To the extent that such risk requires the speculator to demand a higher profit for trading, it works similarly as a barrier to trade. Since a priori we do not know how large such opportunity cost of trading is, we assume that the trading cost is randomly drawn from a distribution $\kappa \in [0, \bar{\kappa}]$ with probability density function $g(\kappa)$ and cumulative distribution function $G(\kappa)$. $\kappa$ is realized and becomes common knowledge before trading begins.
Proposition 3. (Conservatism and investment efficiency) Suppose $\kappa < \kappa_{NT}^{n}$. Then the optimal system that maximizes $E(v(\theta,d))$ is conservative.

Proposition 3 is intuitive. Investment efficiency is maximized when informed trading is maximized. From the discussion of section, with feedback, conservative reporting results in the largest region for $T$. As a result, conservative reporting results in most informed trading and the largest investment efficiency. Note that this will also be the optimal solution if disclosure policy is not chosen by the firm and dictated by a social planner. Since the noisy trader’s loss and informed speculator’s gain cancel out with each other, the social welfare is $E(v)$, i.e. investment efficiency.

Proposition 4. Suppose $\bar{\kappa} < \kappa_{n}^{T}$. Then the optimal system that maximizes $E(v(\theta,d) - \pi)$ is aggressive.

When $\bar{\kappa}$ is below the cutoff under the trading equilibrium in a liberal system and the firm’s objective is to maximize its own value minus the adverse selection discount, then an aggressive bias is preferred. The intuition of the proposition is quite straightforward and the proof is thus omitted. When trading cost is sufficiently low, trading equilibrium prevails under all reporting regimes. In this case, the speculator always trades and the firm always learns from market, and investment efficiency is not affected by reporting asymmetry, i.e. $E(v(\theta,d))$ is the same for all reporting regimes. Compared to aggressive regime, a conservative regime gives the speculator higher expected trading profit, thus lowers the firm’s ex ante price by increasing the discount due to adverse selection.

Proposition 5. Suppose $\kappa > \kappa_{n}^{T}$ and $|\kappa - \kappa_{n}^{T}|$ is sufficiently small. Then the optimal system that maximizes $E(v(\theta,d) - \pi)$ is conservative if $(1 - f) (\lambda x - c) \geq (1 - f \beta) \frac{1}{2} \lambda (R_{H} - R_{L})$.  

When $\kappa_{n}^{T} < \kappa < \kappa_{c}^{T}$, the equilibrium at the trading stage is trading under a conservative system and BNS under a neutral or aggressive system. As $\xi > 0$ increases and the reporting system become more conservative, there is a jump in gross firm value because the firm learns more when trading equilibrium incorporates more information, and also there is a discontinuity in the trading profit since the speculator earns higher gross profits under a trading equilibrium. Whether conservatism enhances the firm price in primary market depends on the tradeoff between learning from feedback effect and adverse selection discount. Comparative statistics shows that conservatism is more beneficial when:

1. $x$ is larger: i.e., when the firm value relies more on taking the correct action, the more beneficial to learn from market;

2. $R_{H} - R_{L}$ is smaller: i.e., when firm value depends more on asset in place, which is exogenous;

---

10This is under the assumption that the impact on expected trading profit due to $\xi$ is small relative to the impact of changed equilibrium (more trading and higher profit under the conservative regime).
3. \(c\) is smaller: i.e., when the adjustment cost is lower, the firm is more likely to take the corrective action and increase its value. \(x\) and \(c\) speaks to the value of real options. The firm benefits from learning from market when \(x - c\) is larger, that is, when the change in value due to real option is larger.

4. \(f\) is lower: i.e., when the internal information is less informative, the firm benefits more from external information from market;

5. \(\lambda\) is higher: when the speculator’s information is more informative.

Note that Proposition 5 is based on the assumption that \(\kappa\) is known publicly. When \(\kappa\) is not known publicly, what is the optimal reporting bias when firm maximizes its own value, i.e. \(E(v(\theta,d) - \pi)\)? Corollary 2 offers some insight into this question.

**Corollary 2.** When \(\lambda \to 1\) and \(G\) is uniform, conservative bias maximizes \(E(v(\theta,d) - \pi)\) if \(\frac{1-f}{1-f^2} [1 + \frac{4}{3} (x-c) + \kappa] > \frac{1}{3} (R_H - R_L)\) and \(\xi\) is sufficiently small, which is more likely to hold when \(x\) is larger, \(c\) is smaller and \(f\) is lower, i.e. qualitatively the same condition as that in Proposition 5 that requires learning benefit to be larger.

The intuition of corollary 2 is essentially the same as that of Proposition 5. Conservative bias results in a larger trading equilibrium region, increasing expected firm value because of feedback effect but also increasing the liquidity discount. When the benefit of feedback is large, the first effect dominates, resulting in conservative bias being optimal. To some extent, corollary 2 shows the robustness of the results in Proposition 5 when the firm has to choose its reporting bias before knowing about the speculator’s trading cost \(\kappa\).

5 Extensions and empirical implications

5.1 Extension: Endogenize both overall and asymmetric timeliness

In the main analysis, we assume that the firm only chooses the degree of asymmetric timeliness, while the overall timeliness \((\beta)\) is fixed. In this subsection, we allow the firm to endogenize both \(\beta\) and \(\xi\). Next we show that, when learning from feedback channel is important, even if the firm can choose both \(\beta\) and \(\xi\), the firm’s optimal reporting system is conservative.

**Proposition 6.** Suppose a social planner can choose both \(\beta\) and \(\xi\) to maximize firm value, subject to ensuring that the speculator always trades on her private information. There exists a range of \(\kappa \in (\kappa, \bar{\kappa})\), where \(\bar{\kappa} = \left( \frac{1}{3} + \frac{1}{3} \left( \frac{1-\lambda}{1-\lambda} \right) \right) (R_H - R_L) - \frac{2}{3} \left( \frac{1-\lambda}{1-\lambda} \right) x\) and \(\bar{\kappa} < \left( \frac{1}{3} + \frac{1}{3} \left( \frac{1-\lambda}{1-\lambda} \right) \right) (R_H - R_L) + \frac{2}{3} \left( \frac{1-\lambda}{1-\lambda} \right) x\), such that when \(\kappa \in (\kappa, \bar{\kappa})\), the optimal reporting system has \(\beta < 1\) and \(\xi > 0\).
Proposition 6 considers the endogenous choice of both overall timeliness and asymmetric timeliness ($\beta$ and $\xi$). When learning from feedback is valuable, there exists a range of trading cost under which the optimal reporting system is conservative. To understand this result, recall that conservatism improves firm value through enabling trading equilibrium. That is, it increases the speculator’s profit from selling on bad news and thus ensures that bad private information is incorporated into price and can guide firm investment. Intuitively, when $\kappa$ is sufficiently small so that the trading equilibrium always prevails, thus a conservative system only leads to larger liquidity discount and is suboptimal. When $\kappa$ is in this intermediate range, conservatism reporting enables the speculator to always trade on her private information and improves firm value. Under the optimal system, the speculator who observes $\theta = L$ just breaks even while the speculator who observes $\theta = H$ makes a positive profit. Thus timely loss recognition ($\xi > 0$) allows the firm to maintain investment efficiency but reduce overall timeliness $\beta$ to minimize the liquidity discount, and thus is optimal.

5.2 Empirical implications

**Feedback effect and timely loss recognition** Our analysis shows that timely loss recognition improves firm value via facilitating feedback effect, that is, through affecting price’s ability to incorporate trader’s private information. In the absence of feedback effect, a conservative system is not preferred compared to an aggressive system, since conservatism leads to higher information asymmetry between informed and uninformed investors, and thus to a higher liquidity discount. Our analysis shows that other things equal, conservatism is more beneficial, and thus more likely to be adopted by a firm, when (1) the firm value depends more on the subsequent decision taken and thus can potentially benefit from feedback effect; (2) when it is less costly to adjust the firm’s investment, (3) when the firm’s internal information is of lower quality, or when the speculator’s information is of higher quality.

Also, from a social welfare perspective, the price discount from the firm to noise trader, or form noise traders to speculators, is merely a wealth transfer and does not affect the overall welfare. If a social planner wants to design the optimal reporting standards to maxis social welfare, then the socially optimal system would maximize the firm’s investment efficiency and thus would be a conservative system. Furthermore, our analysis shows that there is an optimal amount of conservatism: more conservatism is beneficial only to the extent that it increases price informativeness.

**Short-selling constraint** In practice, the cost of selling on bad news often is higher than the cost of buying on good news, due to short selling constraints. When $\kappa_{sell} > \kappa_{buy}$, this amplifies the asymmetry in trading behavior caused by the feedback effect. As a result, speculator’s private information about bad news is further less likely to get incorporated stock price. In this case, timely loss recognition would be more
useful at increasing the profit from selling on bad news, increase overall price informativeness and ultimately improve real investment efficiency.

**Implication on return-earning relation** Our analysis shows that in the presence of feedback effect, even when the accounting reports are neutral, we can still observe an asymmetry in earnings’ correlation with positive stock returns vs. with negative stock returns. This is because of the asymmetric effect the feedback channel has on price’s ability to reflect negative vs. positive news. Thus, the commonly used Basu measure may not properly capture cross-sectional differences in firms’ timely loss recognitions: it can also differ cross-sectionally depending on the differences in returns, i.e., the strength of the feedback effect. Since the feedback effect benefits firm performance and generates asymmetric slope estimates in earnings-stock return regressions, it can also produce positive associations between firm performance and the Basu-like measures even with neutral accounting. This provides a potential explanation for prior studies that finds supposedly neutral measures also exhibit asymmetric correlation with positive versus negative returns. For example, Collins et al. (2014) find asymmetry in cash flow’s correlation with positive vs. with negative stock returns, and Patatoulas and Thomas (2011) show lagged earnings also exhibits similar asymmetry. We also show that timely loss recognition mitigates the feedback channel’s asymmetric effect on price informativeness. This implies that timely loss recognition would generate less asymmetric correlations than under the neutral system, whereas the aggressive reporting would exacerbate the asymmetry.

### 6 Conclusion

This paper analyzes how timely loss recognition affects firm performance via the feedback channel of financial markets. To do so we first adopt a new modeling approach that directly captures the *asymmetric timeliness* in loss recognition. Our analysis reveals a novel channel through which conservatism affects firm’s real decisions and firm value. By preempting more bad news, timely loss recognition changes the market dynamics when public disclosure is absent, and helps mitigate the distortion the feedback channel has on price informativeness identified in Edmans, Goldstein and Jiang (2015). Specifically, Edmans, et al. (2015) show that feedback effect affects speculators’ incentive to trade on good and bad news *asymmetrically*: it increases (reduces) their incentive to trade on positive (negative) news. Thus bad news faces an endogenous limit to arbitrage and is less likely to be reflected in price than good news, which reduces firms’ investment efficiency and add to the instability of the real economy. We show that timely loss recognition in accounting reporting can mitigate the distortions the feedback channel has on speculators’ incentives to trade on negative news. This is because under a conservative system, the asymmetric withholding of firm’ internal information enlarges
the gap in firm value between two states, which increases the speculator’s trading profit (corrective action effect). Furthermore, since bad news is more likely to have already been pre-empted, no disclosure implies the firm is more likely to be in a good state. Thus the market sets price higher, which further increases the profit of selling on bad news and counteracts the asymmetry in trading behavior by feedback effects.

We show that the desirability of conservatism critically hinges on its impact on feedback effect. With feedback effect, timely loss recognition improves the firm value through enhancing price informativeness and enabling the firm to make better investment decisions. Without feedback effect, however, timely loss recognition leads to higher information asymmetry among traders and thus a larger liquidity discount.

Appendix: Proof

Proof of Proposition 1 & 2

Feedback effect affects the equilibria under different reporting regimes.

1. No-trading equilibrium.

Under this equilibrium, the speculator does not trade even when he has private information about the state. As a result, order flow contains no information, and the price is set as the expected firm value
\[ p = tR'_H + (1 - t) R'_L \]
and not moved by order flow. Note that \( X = +2 \) or \(-2\) is out-of-equilibrium path; we assume that if \( X = 2 \) (-2), the firm invests (deinvests), and the market maker sets price to \( R_H + x - c \) \((R_L + x - c)\). To sustain no-trading equilibrium, the speculator’s payoff should be lower than the trading cost. Denote \( \pi_H \) and \( \pi_L \) to be the profit from trading on good and bad news respectively.

\[
\begin{align*}
\pi_{NT,NF}^{buy} &= \frac{2}{3} \left( 1 - t \right) \left( R'_H - R'_L \right) \\
\pi_{NT,NF}^{sell} &= \frac{2}{3} t \left( R'_H - R'_L \right)
\end{align*}
\]

Thus the no-trading equilibrium sustains if \( \kappa > \max \left\{ \pi_{NT,NF}^{buy}, \pi_{NT,NF}^{sell} \right\} \) \[ = \max \left\{ \frac{2}{3} \left( 1 - t \right) \left( R'_H - R'_L \right), \frac{2}{3} t \left( R'_H - R'_L \right) \right\} \]

When the system is neutral, \( t_n = \frac{1}{2} \), and \( R'_H - R'_L = R_H - R_L \), thus \( \kappa_{NT,NF}^{c} = \frac{1}{3} \left( R_H - R_L \right) \).

When the system is conservative, \( \kappa_{NT,NF}^{c} = \frac{2}{3} t \left( R'_H - R'_L \right) \).

When the system is aggressive, \( \kappa_{NT,NF}^{a} = \frac{2}{3} \left( 1 - t \right) \left( R'_H - R'_L \right) \).

2. Trading equilibrium:

When feedback exists and speculator trades, \( \mu_F \) and \( \mu_M \) can be different. \( \mu_F \) is the firm’s belief when he does not observe the state, while \( \mu_M \) is the market maker’s belief. Trading equilibrium sustains iff
\[
\kappa < k^{T,F} = \min \left( \pi_{T,F}^{buy}, \pi_{T,F}^{sell} \right).
\]
\[
\pi_{buy}^{T,F} = \frac{1}{3} (1 - t) (R_H' - R_L') + \frac{1}{3} (1 - \mu_1^T) (R_H' + (1 - \tau_H) (x - c) - R_L' + (1 - \tau_L) (x + c)) \\
= \frac{1}{3} (1 - t) (R_H' - R_L') + \frac{1}{3} (1 - \mu_1^T) (R_H' - R_L') + \frac{1}{3} (1 - \mu_1^T) ((1 - \tau_H) (x - c) + (1 - \tau_L) (x + c)) \\
= \frac{1}{3} (1 - t) (R_H' - R_L') + \frac{1}{3} (1 - \mu_1^T) (R_H - R_L + 2 (1 - \tau_L) x) \\
\]

\[
\pi_{sell}^{T,F} = \frac{1}{3} t (R_H' - R_L') + \frac{1}{3} \mu_{-1}^T (R_H' - R_L') - \frac{1}{3} \mu_{-1}^T ((1 - \tau_H) (x + c) + (1 - \tau_L) (x - c)) \\
= \frac{1}{3} t (R_H' - R_L') + \frac{1}{3} \mu_{-1}^T (R_H - R_L - 2 (1 - \tau_H) x) \\
\]

When the system is neutral, \(t_n = \frac{1}{2}\), and \(R_H' - R_L' = R_H - R_L\), thus \(\kappa_n^{T,F} = \frac{1}{3} (1 - t) (R_H' - R_L') + \frac{1}{3} (1 - \mu_1^T) (R_H - R_L + 2 (1 - \tau_L) x)\).

When the system is conservative, \(t_c > \frac{1}{2} > 1 - t_c, \mu_1^{T-1} > 1 - \mu_1^T\), and \((R_H' - R_L')_c > R_H - R_L\).

Thus \(\kappa_c^{T,NF} = \min \{\pi_{sell}^{T,F}, \pi_{buy}^{T,F}\}\). When \(\xi\) is not too big, \(\pi_{sell}^{T,F} < \pi_{buy}^{T,F}\) and \(\kappa_c^{T,NF} = \frac{1}{3} t (R_H' - R_L') + \frac{1}{3} \mu_{-1}^T (R_H - R_L - 2 (1 - \tau_H) x)\); when \(\xi\) is sufficiently large, \(\pi_{sell}^{T,F} > \pi_{buy}^{T,F}\) and \(\kappa_c^{T,NF} = \frac{1}{3} (1 - t) (R_H' - R_L') + \frac{1}{3} (1 - \mu_1^T) (R_H - R_L + 2 (1 - \tau_L) x)\).

When the system is aggressive, \(t_a < \frac{1}{2} < 1 - t_a, \mu_1^{T-1} < 1 - \mu_1^T\) and \((R_H' - R_L')_a < R_H - R_L\). Thus \(\kappa_a^{T,NF} = \pi_{sell}^{T,F} = \frac{1}{3} t (R_H' - R_L') + \frac{1}{3} \mu_{-1}^T (R_H - R_L - 2 (1 - \tau_H) x)\).

3. **Buy-no-sell equilibrium**

BNS equilibrium sustains if \(\pi_{buy}^{BNS,F} > \kappa > \pi_{sell}^{BNS,F}\).

\[
\pi_{buy}^{BNS,F} = \frac{2}{3} (1 - t) (R_H' - R_L') \\
\pi_{sell}^{BNS,F} = \frac{1}{3} t (R_H' - R_L') + \frac{1}{3} \mu_{-1}^T (R_H - R_L - 2 (1 - \tau_H) x) \\
\]

When the system is neutral, \(t_n = \frac{1}{2}\), and \(R_H' - R_L' = R_H - R_L\), thus BNS sustains if \(\kappa_n^{T,F} < \kappa < \kappa_n^{NT,F}\).

When the system is conservative, \(t_c > \frac{1}{2} > 1 - t_c, \mu_1^{T-1} > 1 - \mu_1^T\), and \((R_H' - R_L')_c > R_H - R_L\). Thus \(\frac{1}{3} (t_c + \mu_{T,c}^{-1}) (R_H' - R_L')_c < \kappa < \frac{2}{3} (1 - t_c) (R_H' - R_L')_c\). When \(\xi\) is not too big, \(\pi_{buy}^{T,F} > \pi_{sell}^{T,F}\), and

\[
\kappa_c^{T,F} = \kappa_c^{BNS,NF} \\
\kappa_c^{BNS,F} = \frac{2}{3} (1 - t_c) (R_H' - R_L')_c < \frac{2}{3} t_c (R_H' - R_L')_c = \kappa_c^{NT,F} \\
\]
When the system is aggressive, \( t_a < \frac{1}{2} < 1 - t_a, \mu_T^{-1} < 1 - \mu_1^T \) and \( (R'_H - R'_L)_a < R_H - R_L \). Thus BNS sustains for \( \kappa \) in the range \( \kappa_{T,F}^{BNS,F} = \kappa_{BNS,F}^{BNS,F} < \kappa < \kappa_{T,F}^{BNS,F} = \kappa_{T,F}^{BNS,F} \).

4. Sell-no-buy equilibrium

\[
\pi_{buy}^{SNB,F} = \frac{1}{3} (1 - t) (R'_H - R'_L) + \frac{1}{3} (1 - \mu_1^T) (R'_H - R'_L) + \frac{1}{3} (1 - \mu_1^T) ((1 - \tau_a) (x - c) + (1 - \tau_a) (x + c))
\]

\[
= \frac{1}{3} (1 - t) (R'_H - R'_L) + \frac{1}{3} (1 - \mu_1^T) (R_H - R_L + 2(1 - \tau_L) x)
\]

\[
\pi_{sell}^{SNB,F} = \frac{2}{3} t (R'_H - R'_L)
\]

When the system is neutral, \( t_n = \frac{1}{2} \), and \( R'_H - R'_L = R_H - R_L \), thus SNB sustains if \( \kappa_{n}^{SNB,F} < \kappa < \kappa_{n}^{NT,F} \):

\[
\kappa_{n}^{SNB,F} = \frac{1}{3} (1 - t_n) (R_H - R_L) + \frac{1}{3} (1 - \mu_{1,n}^T) (R_H - R_L + 2(1 - \tau_{L,n}) x) > \kappa_{n}^{T,F}
\]

When the system is conservative, \( t_c > \frac{1}{2} > 1 - t_c, \mu_T^{-1} > 1 - \mu_1^T \), and \( (R'_H - R'_L)_c > R_H - R_L \).

\[
\kappa_{c}^{T,F} < \kappa_{n}^{SNB,F} < \kappa < \kappa_{c}^{SNB,F} = \kappa_{c}^{NT,F}
\]

When the system is aggressive, \( t_a < t_n = \frac{1}{2} < 1 - t_a, \mu_T^{-1} < 1 - \mu_1^T \) and \( (R'_H - R'_L)_a < R_H - R_L \). Thus \( \frac{1}{3} (2 - t_a - \mu_{1,a}^T) (R'_H - R'_L)_a < \kappa < \frac{2}{3} t_a (R'_H - R'_L)_a \)

\[
\tilde{\kappa}_{a}^{SNB,F} = \frac{2}{3} t_a (R'_H - R'_L)_a < \frac{2}{3} (1 - t_a) (R'_H - R'_L)_a = \kappa_{a}^{NT,F}
\]

\[
\kappa_{a}^{SNB,F} = \frac{1}{3} (1 - t) (R'_H - R'_L) + \frac{1}{3} (1 - \mu_1^T) (R_H - R_L + 2(1 - \tau_L) x)
\]

\[
> \frac{1}{3} t (R'_H - R'_L) + \frac{1}{3} \mu_1^T (R_H - R_L - 2(1 - \tau_H) x)
\]

\[
\pi_{sell}^{T,F} = \kappa_{a}^{T,F}
\]

Proof of Proposition 3

This proposition claims that when feedback effect exists, to maximize investment efficiency the optimal reporting policy can not be aggressive or neutral. We prove by contradiction. Suppose the optimal policy
has \( \xi^* \leq 0 \). Since \( \kappa \in [0, \bar{\kappa}] \), investment efficiency \( Q \) is determined by

\[
E (v (\theta, d)) = \frac{1}{2} (R_H + R_L) + f (x - c) + (1 - f) \int_0^{\kappa_T} \left( \frac{2}{3} \lambda (x - c) - \frac{2}{3} (1 - \lambda) c \right) g (\kappa) d\kappa
\]

\[
+ (1 - f) \int_\kappa^{\bar{\kappa}} \left( \frac{1}{2} \left( \frac{1}{3} + \frac{1}{3} \right) \lambda (x - c) - \frac{1}{3} (1 - \lambda) c \right) g (\kappa) d\kappa
\]

\[
= \frac{1}{2} (R_H + R_L) + f (x - c) + (1 - f) G (\kappa_T) \frac{1}{3} (\lambda x - c)
\]

Thus \( E (v (\theta, d)) \) increases with \( \kappa_T \), the upper bound of the trading equilibrium.

1. If \( \xi^* = 0 \). We can show that another policy with the same \( \beta \) but a \( \xi' \) slightly positive will be better for investment efficiency. When \( \xi' \) is not too large, \( \pi^{T,F}_{\text{buy},n} > \pi^{T,F}_{\text{buy},c} > \pi^{T,F}_{\text{sell},c} > \pi^{T,F}_{\text{sell},n} \), and thus the upper bound for trading equilibrium \( \kappa_{T,F}^T > \kappa_{n,F}^T \), and the trading region expands.

2. If \( \xi^* = \xi_a < 0 \). We can show that another policy with the same \( \beta \) but \( \xi_a = -\xi_a \) can achieve higher investment efficiency. While the feedback effect always makes buying on good news more profitable than selling on bad news, the price effect depends on the asymmetry in the reporting system: it makes buying on good news more profitable under an aggressive system, and selling on bad news more profitable under a conservative system. Thus overall the upper bound of trading region is determined as follows: \( \kappa_a^T = \min \{ \pi^{T,F}_{\text{sell},a}, \pi^{T,F}_{\text{buy},a} \} = \pi^{T,F}_{\text{sell},a} \), and \( \kappa_{c}^T = \min \{ \pi^{T,F}_{\text{sell},c}, \pi^{T,F}_{\text{buy},c} \} \).

If \( \pi^{T,F}_{\text{sell},c} < \pi^{T,F}_{\text{buy},c} \), then the trading region is larger under conservative reporting:

\[
\pi^{T,F}_{\text{sell},a} < \pi^{T,F}_{\text{sell},n} < \pi^{T,F}_{\text{sell},c}
\]

\[
\kappa_a^T < \kappa_{c}^T
\]

If \( \pi^{T,F}_{\text{sell},c} > \pi^{T,F}_{\text{buy},c} \), then the upper bound is determined by comparing \( \pi^{T,F}_{\text{sell},a} \) and \( \pi^{T,F}_{\text{buy},c} \):

\[
\pi^{T,F}_{\text{buy},c} = \frac{1}{3} (1 - t_c) (R'_H - R'_L)_c + \frac{1}{3} (1 - \mu^T) (R_H - R_L + 2 (1 - \tau_L) x)
\]

\[
\pi^{T,F}_{\text{sell},a} = \frac{1}{3} t_a (R'_H - R'_L)_a + \frac{1}{3} \mu^T (R_H - R_L - 2 (1 - \tau_H) x)
\]

Note that \( t_c = 1 - t_a \), and \( (1 - \mu^T) = \mu^T_{-1,c} \), and \( (R'_H - R'_L)_c > (R'_H - R'_L)_a \). Thus while the price effect is the same in the two cases, both the corrective action based on the firm’s internal information as well as the information revealed from stock market makes the trading on bad news under a conservative regime more profitable. As a result, \( \pi^{T,F}_{\text{sell},a} = \kappa_a^T < \kappa_{c}^T = \pi^{T,F}_{\text{buy},c} \). Thus, an aggressive regime or a neutral regime is never optimal for investment efficiency.
Proof of Proposition 4

When $\kappa < \kappa_n^T$, under neutral system trading equilibrium prevails, and investment efficiency is maximized. Thus to maximize stock price in the primary market is the same as minimizing the speculator’s trading profit.

\[
E(\pi) = (1 - f\beta) \{ t_\text{buy} (1 - t) \pi_\text{sell} \}
\]

\[
= \left\{ t \left( \frac{1}{3} (1 - t) (R_H' - R_L') + \frac{1}{3} (1 - \mu_1^T) (R_H - R_L + 2 (1 - \tau_L) x) \right) + (1 - t) \left( \frac{1}{3} t (R_H' - R_L') + \frac{1}{3} \mu_{-1}^T (R_H - R_L - 2 (1 - \tau_H) x) \right) \right\}
\]

\[
= \frac{2}{3} t (1 - t) (R_H' - R_L') + \frac{1}{3} (t (1 - \mu_1^T) + (1 - t) \mu_{-1}^T) (R_H - R_L)
\]

\[
+ 2x \left\{ \frac{1}{3} (t (1 - \mu_1^T) (1 - \tau_L) - (1 - t) \mu_{-1}^T (1 - \tau_H)) \right\}
\]

We show that for a given conservative system with $\xi_c > 0$, an aggressive system with $\xi_a = -\xi_c$ can give a lower $E(\pi)$. This is because $(R_H' - R_L')$ is higher under a conservative system, and $t (1 - t)$, $(t (1 - \mu_1^T) + (1 - t) \mu_{-1}^T)$ are the same, while the item in the bracket is positive when the system is conservative and negative while it is aggressive. Thus holding under the trading equilibrium, conservatism increases the speculator’s trading profit and is not optimal.

\[
\frac{(t (1 - \mu_1^T) (1 - \tau_L) - (1 - t) \mu_{-1}^T (1 - \tau_H))}{t (1 - \lambda) (1 - t) 1 - f} = \frac{1 - f_1 (1 - \lambda) (1 - t) 1 - f}{1 - t + (1 - \lambda) t 1 - f_1 \beta_L} - \frac{1 - f_2 (1 - \lambda) (1 - t) 1 - f}{1 - t + (1 - \lambda) t 1 - f_2 \beta_H}
\]

\[
\propto \frac{t (1 - \lambda) (1 - t) 1 - f}{1 - t + (1 - \lambda) t 1 - f_1 \beta_L} - \frac{t (1 - \lambda) (1 - t) 1 - f}{1 - t + (1 - \lambda) t 1 - f_2 \beta_H}
\]

\[
\approx \frac{t (1 - \lambda) (1 - t) 1 - f}{1 - t + (1 - \lambda) t}
\]

Proof of Proposition 5

\[
E_c(\pi) = (1 - f\beta) \{ t_\text{buy} (1 - t) \pi_\text{sell} \}
\]

\[
= \left\{ t \left( \frac{1}{3} (1 - t) (R_H' - R_L') + \frac{1}{3} (1 - \mu_1^T) (R_H - R_L + 2 (1 - \tau_L) x) \right) + (1 - t) \left( \frac{1}{3} t (R_H' - R_L') + \frac{1}{3} \mu_{-1}^T (R_H - R_L - 2 (1 - \tau_H) x) \right) \right\}
\]
Under a neutral regime the equilibrium is BNS:

\[
E_n(\pi) = (1 - f\beta) \frac{2}{3} \left(1 - \frac{1}{2}\right) (R_H - R_L) = \frac{1}{6} (1 - f\beta) (R_H - R_L)
\]

\[
E_c(\pi) = (1 - f\beta) \left\{ t \left(\frac{1}{3} (1 - t) (R'_H - R'_L) + \frac{1}{3} (1 - \mu \tau_1) (R_H - R_L + 2 (1 - \tau_L) x) \right) + (1 - t) \left(\frac{1}{3} t (R'_H - R'_L) + \frac{1}{3} \mu \tau_1 (R_H - R_L - 2 (1 - \tau_H) x) \right) \right\}
\]

When \( \xi \) is sufficiently small, the speculator’s profit under the trading regime is as follows. There is also a discontinuity in the speculator’s expected profit. Thus,

\[
\lim_{\xi \to 0} E_c(\pi) = \lim_{\xi \to 0} \frac{1}{3} (1 - f\beta) t (1 - t) + t (1 - \mu \tau_1) (R_H - R_L)
\]

\[
= \frac{1}{3} (1 - f\beta) t (1 - t) \left( 2 + \frac{(1 - \lambda)}{1 - \lambda (1 - t)} + \frac{(1 - \lambda)}{1 - \lambda \tau_1} \right) (R_H - R_L)
\]

\[
= \frac{1}{3} (1 - f\beta) \left( \frac{1}{2} + \frac{1 - \lambda}{2 - \lambda} \right) (R_H - R_L)
\]

Thus if conservatism benefits ex ante firm value if \( E_c(v(\theta, d)) - E_n(v(\theta, d)) \geq E_c(\pi) - E_n(\pi) \), i.e.,

\[
\frac{1}{3} (1 - f) (\lambda x - c) \geq \frac{1}{3} (1 - f\beta) \frac{1 - \lambda}{2 - \lambda} (R_H - R_L)
\]

Thus, conservatism’s benefit on ex-ante firm value is higher if: (1) \( f \) is smaller; (2) \( \lambda \) is higher; (3) \( x \) is higher; (4) \( c \) is lower; (5) \( R_H - R_L \) is smaller.

**Proof of Corollary 2**

Following the proof of Proposition 3, we can now write \( E(v_j(\theta, d)) \) under different disclosure bias \( j \in \{c, n, a\} \) as

\[
E(v_j(\theta, d)) = \frac{1}{2} (R_H + R_L) + f(x - c) + (1 - f) \int_{\kappa_j}^{\kappa_j^T} \left( \frac{2}{3} \lambda (x - c) - \frac{2}{3} (1 - \lambda) c \right) g(\kappa) d\kappa
\]

\[
+ (1 - f) \int_{\kappa_j}^{\kappa_j^T} \left( \frac{1}{3} (x - c) - \frac{1}{3} \lambda (x - c) - \frac{1}{3} (1 - \lambda) c \right) g(\kappa) d\kappa
\]

\[
= \frac{1}{2} (R_H + R_L) + f(x - c) + (1 - f) \left[ G(\kappa_j^T) + G(\kappa) \right] \frac{1}{3} (\lambda x - c)
\]

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Since $\kappa^T_c > \kappa^T_n > \kappa^T_a$, $E(v_c(\theta, d)) > E(v_n(\theta, d)) > E(v_a(\theta, d))$, which is the results we get from Proposition 3, conservative reporting, by providing the largest trading regime, results in the strongest feedback effect and thus highest firm value.

The second component, the expected speculator’s profit, $E(v_j(\theta, d))$ under different disclosure bias $j \in \{c, n, a\}$, will thus be\(^{11}\)

\[
E(\pi_c(\theta, d)) = \int_0^{\kappa^T_c} (1 - f\beta) \{t\pi_{buy} + (1 - t)\pi_{sell}\} d\kappa + \int_{\kappa^T_c}^{\kappa^T_n} (1 - f\beta) \frac{2}{3} (1 - t) (R'_H - R'_L) d\kappa
\]

\[
= \{t \left( \frac{1}{3} (1 - t) (R'_H - R'_L) + \frac{1}{3} (1 - \mu_1^T) (R_H - R_L + 2(1 - \tau_L)x) \right) + (1 - t) \left( \frac{1}{3} t (R'_H - R'_L) + \frac{1}{3} \mu_{T-1}^T (R_H - R_L - 2(1 - \tau_L)x) \right) \} (1 - f\beta) G(\kappa^T_c)
\]

\[
+ \frac{1}{3} (1 - t) (R'_H - R'_L) (1 - f\beta) [1 - G(\kappa^T_c)]
\]

\[
E(\pi_n(\theta, d)) = \int_0^{\kappa^T_n} (1 - f\beta) \{t\pi_{buy} + (1 - t)\pi_{sell}\} d\kappa + \int_{\kappa^T_n}^{\kappa^T_a} (1 - f\beta) \frac{2}{3} (1 - t) (R'_H - R'_L) d\kappa
\]

\[
= \left( \frac{1}{6} (R_H - R_L) + \frac{1}{3} (1 - \lambda) (R_H - R_L) \right) (1 - f\beta) G(\kappa^T_n) + \frac{1}{6} (R_H - R_L) (1 - f\beta) [1 - G(\kappa^T_a)]
\]

\[
E(\pi_a(\theta, d)) = \int_0^{\kappa^T_a} (1 - f\beta) \{t\pi_{buy} + (1 - t)\pi_{sell}\} d\kappa + \int_{\kappa^T_a}^{\kappa^T} (1 - f\beta) \frac{2}{3} (1 - t) (R'_H - R'_L) d\kappa
\]

\[
= \{t \left( \frac{1}{3} (1 - t) (R'_H - R'_L) + \frac{1}{3} (1 - \mu_1^T) (R_H - R_L + 2(1 - \tau_L)x) \right) + (1 - t) \left( \frac{1}{3} t (R'_H - R'_L) + \frac{1}{3} \mu_{T-1}^T (R_H - R_L - 2(1 - \tau_L)x) \right) \} (1 - f\beta) G(\kappa^T_a)
\]

\[
+ \frac{1}{3} (1 - t) (R'_H - R'_L) (1 - f\beta) [1 - G(\kappa^T_a)]
\]

Note that $E(\pi_c(\theta, d)) > E(\pi_n(\theta, d)) > E(\pi_a(\theta, d))$ as conservative reporting has the largest trading equilibrium region and thus the largest amount of trading profit. Note that a sufficient condition for $E(v_c(\theta, d) - \pi_c(\theta, d)) > E(v_n(\theta, d) - \pi_n(\theta, d))$ and for $E(v_c(\theta, d) - \pi_c(\theta, d)) > E(v_a(\theta, d) - \pi_a(\theta, d))$ to hold is that $E(v(\theta, d) - \pi(\theta, d))$, which is a function of $\xi$, is monotonically increasing in $\xi$. Similar to proof of proposition 5, we consider the case where $\xi \rightarrow 0$, i.e. $\xi$ is sufficiently small. If we can show that

\(^{11}\text{As will be discussed below, we are only interested in the case when } \xi \text{ is very small. In this case, as shown by proposition 2, BNS will be the only partial trading equilibrium for all disclosure regimes.}\)
\frac{\partial (E(v(\theta, d) - \pi(\theta, d)))}{\partial \xi} |_{\xi=0^+} > 0 \quad \text{and} \quad \frac{\partial (E(v(\theta, d) - \pi(\theta, d)))}{\partial \xi} |_{\xi=0^-} > 0 \quad \text{then we know that conservative accounting is optimal at least when} \ \xi \ \text{is sufficiently close to zero by continuity.}

From the proof of proposition 2, \( \kappa_c^T = \kappa_a^T = \pi_{\text{sell}} \) when \( \xi \) is sufficiently small as BNS will be the unique partial-trading equilibrium. Thus

\[
\frac{\partial (E(v_\varepsilon(\theta, d) \mid v_\varepsilon(\theta, d)))}{\partial \xi} |_{\xi=0^+} = (1 - f) g(\pi_{\text{sell}}|_{\xi=0^+}) \frac{\partial \pi_{\text{sell}}}{\partial \xi} |_{\xi=0^+} = \frac{1}{6} (R_H - R_L) + \frac{11 - \lambda}{32 - \lambda} (R_H - R_L) - \frac{2}{3} \frac{1 - f}{1 - f\beta} x \times \frac{f\{2(1 - f)(2 - \lambda)^2 + (1 - f\beta)(8 - (8 - \lambda)\lambda)(R_H - R_L) - 2(1 - f)[2 - \lambda(14 - 3\lambda)]x\}}{6(1 - f\beta)^2(2 - \lambda)^2}
\]

This results in

\[
\frac{\partial (E(v(\theta, d) - \pi(\theta, d)))}{\partial \xi} |_{\xi=0^+} = (1 - f) g(\pi_{\text{sell}}|_{\xi=0^+}) \frac{\partial \pi_{\text{sell}}}{\partial \xi} |_{\xi=0^+} - (1 - f\beta) \frac{11 - \lambda}{32 - \lambda} (R_H - R_L) g(\pi_{\text{sell}}|_{\xi=0^+}) \frac{\partial \pi_{\text{sell}}}{\partial \xi} |_{\xi=0^+} - (1 - f\beta) G(\pi_{\text{sell}}|_{\xi=0^+}) \frac{f\{2(1 - f)(2 - \lambda)^2 + (1 - f\beta)(8 - (8 - \lambda)\lambda)(R_H - R_L) - 2(1 - f)[2 - \lambda(14 - 3\lambda)]x\}}{6(1 - f\beta)^2(2 - \lambda)^2} + \frac{1}{3} (1 - f\beta)(1 - G(\pi_{\text{sell}}|_{\xi=0^+}) \frac{f[(1 - f\beta)(R_H - R_L) + 2(1 - f)(x - c)]}{2(1 - f\beta)^2}
\]

We can similarly have

\[
\frac{\partial (E(v_\alpha(\theta, d))}{\partial \xi} |_{\xi=0^-} = (1 - f) g(\pi_{\text{sell}}|_{\xi=0^-}) \frac{\partial \pi_{\text{sell}}}{\partial \xi} |_{\xi=0^-} = \frac{1}{6} (R_H - R_L) + \frac{11 - \lambda}{32 - \lambda} (R_H - R_L) - \frac{2}{3} \frac{1 - f}{1 - f\beta} x \times \frac{f\{2(1 - f)(2 - \lambda)^2 + (1 - f\beta)(8 - (8 - \lambda)\lambda)(R_H - R_L) - 2(1 - f)[2 - \lambda(14 - 3\lambda)]x\}}{6(1 - f\beta)^2(2 - \lambda)^2}
\]

This results in

\[
\frac{\partial (E(v(\theta, d) - \pi(\theta, d))}{\partial \xi} |_{\xi=0^-}
\]

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\[
(1 - f)g(\pi_{sell}|\xi=0^-)\frac{\partial \pi_{sell}}{\partial \xi}|_{\xi=0^-} - (1 - f\beta)\left\{\frac{1}{3} \frac{1 - \lambda}{2 - \lambda} \left(R_H - R_L\right)\right\}g(\pi_{sell}|\xi=0^-)\frac{\partial \pi_{sell}}{\partial \xi}|_{\xi=0^-}
\]

\[
- (1 - f\beta) G(\pi_{sell}|\xi=0^-) f\left\{\frac{-2c(1 - f)(2 - \lambda)^2 - 16(1 - f)x(1 - f\beta) - \lambda^2 [(1 - f\beta)(R_H - R_L) + (1 - f)2x] + 2\lambda(1 - f\beta)(R_H - R_L)}{6(1 - f\beta)^2(2 - \lambda)^2}\right\}
\]

\[
+ \frac{1}{3} (1 - f\beta)[1 - G(\pi_{sell}|\xi=0^-)] f\left\{\frac{(1 - f\beta)(R_H - R_L) + 2(1 - f)(x - c)}{2(1 - f\beta)^2}\right\} = \frac{\partial(E(v(\theta,d) - \pi(\theta,d))|_{\xi=0^+}}{\partial \xi}
\]

Thus, conservative reporting is optimal if

\[
\frac{\partial(E(v(\theta,d) - \pi(\theta,d))|_{\xi=0^-}}{\partial \xi} > 0
\]

\[
= \{(1 - f) - (1 - f\beta)\frac{1}{3} \frac{1 - \lambda}{2 - \lambda} \left(R_H - R_L\right)\}g(\pi_{sell}|\xi=0^-)\times f\left\{\frac{-2c(1 - f)(2 - \lambda)^2 + (1 - f\beta)[8 - (8 - \lambda)\lambda](R_H - R_L) - 2(1 - f)2\lambda(1 - f\beta)(R_H - R_L)}{6(1 - f\beta)^2(2 - \lambda)^2}\right\}
\]

\[
- (1 - f\beta) G(\pi_{sell}|\xi=0^-) f\left\{\frac{-2c(1 - f)(2 - \lambda)^2 - 16(1 - f)x(1 - f\beta) - \lambda^2 [(1 - f\beta)(R_H - R_L) + (1 - f)2x] + 2\lambda(1 - f\beta)(R_H - R_L)}{6(1 - f\beta)^2(2 - \lambda)^2}\right\}
\]

\[
+ \frac{1}{3} (1 - f\beta)[1 - G(\pi_{sell}|\xi=0^-)] f\left\{\frac{(1 - f\beta)(R_H - R_L) + 2(1 - f)(x - c)}{2(1 - f\beta)^2}\right\} > 0
\]

where

\[
\pi_{sell}|_{\xi=0^+} = \frac{1}{6} \left(R_H - R_L\right) + \frac{1}{3} \frac{1 - \lambda}{2 - \lambda} \left(R_H - R_L\right) - \frac{2}{3} \frac{1 - f}{1 - f\beta} (x - c)
\]

and

\[
\frac{\partial \pi_{sell}}{\partial \xi}|_{\xi=0^+} = f\left\{\frac{-2c(1 - f)(2 - \lambda)^2 + (1 - f\beta)[8 - (8 - \lambda)\lambda](R_H - R_L) - 2(1 - f)[12 - \lambda(14 - 3\lambda)]x}{2(1 - f\beta)^2(2 - \lambda)^2}\right\}
\]

If we assume that \(\lambda \to 1\), i.e. when there is maximum feedback effect, we can simplify the conditions as
\[(1 - f)g(\frac{1}{6}(R_H - R_L) - \frac{2}{3} \frac{1 - f}{1 - f\beta}x) + 1 - f\beta > 2(1 - f\beta)G(\frac{1}{6}(R_H - R_L) - \frac{2}{3} \frac{1 - f}{1 - f\beta}(x - c))\]

If we further assume that \(G\) is uniform, then the condition reduces to

\[
\frac{1 - f}{1 - f\beta}[1 + \frac{4}{3}(x - c) + \kappa] > \frac{1}{3}(R_H - R_L)
\]

which is more likely to hold when \(x\) is larger, \(c\) is smaller and \(f\) is lower, i.e. qualitatively the same condition as that in Proposition 5. which requires learning benefit to be larger.

**Proof of Proposition 6**

**Proof.** Note that due to the feedback effect, when the reporting is neutral, \(\pi_{\text{buy}}^T > \pi_{\text{sell}}^T\) regardless of \(\beta\). Since \(\tau_o = \frac{f(1 - \beta_o)}{1 - f\beta_o}\) and decreases with \(\beta_o\), thus the feedback effect increases with \(\beta_o\). This makes sense as \(\beta\) increases, the more likely that the firm does not know and learns from stock market, which makes feedback effect more significant. With neutral reporting,

\[
\pi_{\text{buy}}^T = \frac{1}{3} \left( \frac{1}{2} + \frac{1 - \lambda}{2 - \lambda} \right) (R_H - R_L) + \frac{2}{3} \frac{1 - f}{1 - f\beta} \frac{1 - \lambda}{2 - \lambda} x
\]

\[
\pi_{\text{sell}}^T = \frac{1}{3} \left( \frac{1}{2} + \frac{1 - \lambda}{2 - \lambda} \right) (R_H - R_L) - \frac{2}{3} \frac{1 - f}{1 - f\beta} \frac{1 - \lambda}{2 - \lambda} x
\]

Easy to see that \(\pi_{\text{buy}}^{T,F} > \pi_{\text{sell}}^{T,F}\), and the gap between \(\pi_{\text{buy}}^{T,F}\) and \(\pi_{\text{sell}}^{T,F}\) is due to feedback effect and is larger when \(\beta\) is larger, and even if \(\beta = 0\) the gap still exists. That is, withholding disclosure can alleviate, but never eliminate the feedback effect.

\[
\pi_{\text{buy}}^{T,F} = \frac{1}{3} (1 - t) (R'_H - R'_L) + \frac{1}{3} (1 - \mu^T) (R_H - R_L + 2(1 - \tau_L) x)
\]

\[
\pi_{\text{sell}}^{T,F} = \frac{1}{3} t (R'_H - R'_L) + \frac{1}{3} \mu^T (R_H - R_L - 2(1 - \tau_H) x)
\]
In this case, the firm’s problem is to choose \( \beta \) and \( \xi \) such that:

\[
\max_{\beta, \xi} E^{F,T}(v(\theta, d)) - E^{F,T}(\pi)
\]

s.t.

- \( d \in \arg \max \max_{\{-1,0,1\}} E(v(\theta, d) | \delta, X) \)
- \( s \in \arg \max \max_{\{-1,0,1\}} s E(v - p(X) | \eta) \)
- \( p(X) = E(v(\theta, d(\delta, X)) | X) \)
- \( \pi^{T,F}_{buy} \geq \kappa \)
- \( \pi^{T,F}_{sell} \geq \kappa \)

To prove that the optimal system must has \( \xi > 0 \), we proceed in three steps.

Step 1. At most one of the two constraints about trading profits (\( \pi^{T,F}_{buy} \geq \kappa \) and \( \pi^{T,F}_{sell} \geq \kappa \)) is binding. Otherwise, the manager can increase \( \beta \) by an infinitesimal amount if \( \beta < \bar{\beta} \), or decrease \( \xi \) by an infinitesimal amount if \( \beta = \bar{\beta} \), which raise net firm value, without violating the constraints.

Step 2. The constraint on selling must be binding. To see this, suppose it is not binding under the optimal solution \((\beta^*, \xi^*)\) sustains trading equilibrium and has \( \pi^{T,F}_{buy} = \kappa \) and \( \pi^{T,F}_{sell} > \kappa \). Fix \( \beta^* \). Since \( \pi^{T,F}_{buy} < \pi^{T,F}_{sell} \), it must be the case that \( \xi > 0 \). Then the firm can slightly lower \( \xi \) by an infinitesimal amount, and as a result, \( \pi^{T,F}_{buy} > \kappa \) and \( \pi^{T,F}_{sell} > \kappa \), the constraints are satisfied, and the expected loss to informed traders is lower, and thus the firm is better off. So the constraint on selling is binding.

Step 3. Now we have

\[
\kappa = \pi^{T,F}_{sell} = \frac{1}{3} t (R_H' - R_L') + \frac{1}{3} \bar{x} \left( R_H - R_L - \frac{1-f}{1-f(\beta + \xi)} \bar{x} \right) \tag{3}
\]
\[
\pi^{T,F}_{buy} = \frac{1}{3} (1-t) (R_H' - R_L') + \frac{1}{3} \bar{x} \left( R_H - R_L + \frac{1-f}{1-f(\beta + \xi)} \bar{x} \right) \tag{4}
\]

Note \( \mu_T^{-1} = \frac{(1-\lambda)}{\pi(1-\lambda)(1-t)} \) and \( \mu_T^1 = \frac{t}{\pi(1-\lambda)(1-t)} \), and we can write

\[
(R_H' - R_L') = \left( \frac{3\kappa}{t} - (1-\lambda) \frac{1-f}{1-f\beta} (R_H - R_L) + \frac{(1-\lambda)}{1-\lambda t} \frac{1-f}{1-f\beta} 2\bar{x} \right)
\]
We can rewrite the firm’s expected loss to informed trader as:

\[
E_{\beta,t}(\pi) = \frac{1}{2} \lambda (1 - f \beta_H) \pi_{buy} + \frac{1}{2} \lambda (1 - f \beta_L) \pi_{sell}
\]

\[
= \frac{1}{2} \lambda (1 - f \beta_H) \left( \frac{1}{3} (1 - t) (R_H - R_L) + \frac{1}{3} \frac{(1 - \lambda)(1 - t)}{1 - \lambda(1 - t)} \left( R_H - R_L + 2 \frac{1 - f}{1 - f \beta_L} x \right) \right)
\]

\[
+ \frac{1}{2} \lambda (1 - f \beta_L) \left( \frac{1}{3} t \left( R_H' - R_L' \right) + \frac{1}{3} t \frac{(1 - \lambda)}{1 - \lambda t} \left( R_H' - R_L' - 2 \frac{1 - f}{1 - f \beta_L} x \right) \right)
\]

\[
= \frac{1}{3} (1 - f \beta_H) \lambda t (1 - t) \left( R_H - R_L + \frac{(1 - \lambda)}{1 - \lambda(1 - t)} \left( R_H - R_L + 2 \frac{1 - f}{1 - f \beta_L} x \right) \right)
\]

\[
+ (R_H - R_L) \frac{(1 - \lambda)}{1 - \lambda t} \left( R_H - R_L - \frac{1 - f}{1 - f \beta_L} x \right)
\]

\[
= \frac{1}{3} (1 - f \beta_H) \lambda t (1 - t) \left( 2 \left( R_H - R_L \right) + \frac{(1 - t)}{1 - \lambda(1 - t)} \left( R_H - R_L + \frac{1 - f}{1 - f \beta_L} x \right) \right)
\]

\[
+ \frac{(1 - \lambda)}{1 - \lambda t} \left( R_H - R_L + \frac{1 - f}{1 - f \beta_L} x \right)
\]

\[
= \frac{1}{3} (1 - f \beta_H) \lambda t (1 - t) \left( \frac{3 \lambda t}{1 - \lambda t} \left( R_H - R_L \right) + \frac{1 - \lambda}{1 - \lambda t} \frac{1 - f}{1 - f \beta_L} x \right)
\]

\[
+ \frac{(1 - \lambda)}{1 - \lambda t} \left( R_H - R_L + \frac{1 - f}{1 - f \beta_L} x \right)
\]

\[
= \frac{(1 - f \beta_H) \lambda (1 - t) \left( 2 \frac{1 - \lambda}{1 - \lambda t} \lambda t \right) + \frac{1 - \lambda}{1 - \lambda t} \left( 1 - f \right) \left( \frac{1 - \lambda}{1 - \lambda t} \left( 1 - f \right) \right) x}
\]

\[
+ \frac{1}{3} \lambda \left( 1 - \lambda \right) (1 - f) \left( \frac{1 - \lambda}{1 - \lambda t} \lambda t \right)
\]

\[
= \frac{1}{3} \lambda \left( 1 - \lambda \right) (1 - f) \left( \frac{1 - \lambda}{1 - \lambda t} \lambda t \right)
\]

\[
= \frac{1}{3} \lambda \left( 1 - \lambda \right) (1 - f) \left( \frac{1 - \lambda}{1 - \lambda t} \lambda t \right)
\]

\[
= \frac{1}{3} \lambda \left( 1 - \lambda \right) (1 - f) \left( \frac{1 - \lambda}{1 - \lambda t} \lambda t \right)
\]

\[
= \frac{1}{3} \lambda \left( 1 - \lambda \right) (1 - f) \left( \frac{1 - \lambda}{1 - \lambda t} \lambda t \right)
\]

\[
= \frac{1}{3} \lambda \left( 1 - \lambda \right) (1 - f) \left( \frac{1 - \lambda}{1 - \lambda t} \lambda t \right)
\]

\[
= \frac{1}{3} \lambda \left( 1 - \lambda \right) (1 - f) \left( \frac{1 - \lambda}{1 - \lambda t} \lambda t \right)
\]

\[
= \frac{1}{3} \lambda \left( 1 - \lambda \right) (1 - f) \left( \frac{1 - \lambda}{1 - \lambda t} \lambda t \right)
\]

Compare two cases: \( \xi_0 = 0 \) and \( \xi_c > 0 \). Then from the binding constraint 3 we have \( \beta_0 < \beta_c \); intuitively, if the firm does not use asymmetric reporting, it has to withhold more information to make trading profitable and thus to elicit feedback. Thus, easy to see \( H_i(\xi_c) < H_i(\xi_0) \), and \( H_3(\xi_c) < 0 = H_1(\xi_0) \) and take FOC:

\[
H_2 = \text{Constant} \times \left( \frac{1 - t}{1 - \lambda t} + \frac{t}{1 - \lambda (1 - t)} \right)
\]

\[
= C \times \frac{(1 - t)(1 - \lambda (1 - t)) + t (1 - \lambda t)}{(1 - \lambda t)(1 - \lambda (1 - t))}
\]

\[
= C \times \frac{1 - \lambda (1 - t)^2 - \lambda^2}{(1 - \lambda t)(1 - \lambda (1 - t))}
\]

\[
\frac{\partial H_2}{\partial t} = \frac{\partial H_2}{\partial t} \frac{\partial t}{\partial \xi} = \frac{\partial H_2}{\partial t}
\]

\[
= C \times \frac{-2 \lambda (2t - 1)(1 - \lambda t)(1 - \lambda (1 - t)) - \lambda^2 (1 - 2t) \left( 1 - \lambda (1 - t)^2 - \lambda^2 t^2 \right)}{(1 - \lambda t)^2 (1 - \lambda (1 - t))^2}
\]

\[
= C \times \frac{(2t - 1) \lambda}{(1 - \lambda t)^2 (1 - \lambda (1 - t))^2} \left( -2 \lambda (1 - \lambda t)(1 - \lambda (1 - t)) + \lambda \left( 1 - \lambda (1 - t)^2 - \lambda^2 t^2 \right) \right)
\]

\[
= C \times \frac{(2t - 1) \lambda}{(1 - \lambda t)^2 (1 - \lambda (1 - t))^2} \left( -2 + 2 \lambda + \lambda - 2 \lambda^2 t (1 - t) - \lambda^2 (1 - t)^2 - \lambda^2 t^2 \right)
\]

\[
= C \times \frac{(2t - 1) \lambda}{(1 - \lambda t)^2 (1 - \lambda (1 - t))^2} \left( -2 + \lambda \right) (1 - \lambda)
\]

\[
< 0
\]
So $H_2(\xi_c) < H_2(\xi_0)$. So the loss to trading profit to informed traders is lowest under a conservative regime, followed by a neutral system, and the highest under a liberal system. So in this case a conservative system is optimal.

References


