Shared Knowledge and Competition for Attention in Information Markets[*]

Simone Galperti and Isabel Trevino[†]

UCSD

February 23, 2018

Abstract

Information consumers seek to learn about world events, but often also what others know about those events. Capturing consumers' attention is essential for information suppliers to thrive in the market. We find that competition for attention leads to homogeneity of information sources—in terms of accuracy and clarity—even when consumers would demand heterogeneous sources. Their equilibrium type exhibits higher clarity as consumers care more about predicting others' beliefs, while this desire does not affect accuracy. We also find that whenever attention becomes the "currency" whereby consumers pay for information, it causes novel market inefficiencies, whose form and size depend on the consumers' interest in others' beliefs.

Keywords: information supply, coordination, payoff interdependence, attention, accuracy, clarity, inefficiency.

JEL classification: C72, D62, D83, L10.

[*]We thank for comments and suggestions S. Nageeb Ali, Rahul Deb, John Duffy, Joseph Engelberg, Renato Gomes, Garrett Johnson, Stephen Morris, Alessandro Pavan, Joel Sobel, Philipp Strack, Xavier Vives, Rakesh Vohra, and Joel Watson and seminar participants at ESSET (Gerzensee), Yale, and Princeton. John Rehbeck provided excellent research assistance. All remaining errors are ours.

[†]Department of Economics, University of California, San Diego, 9500 Gilman Dr., La Jolla, CA, 92093. E-mail addresses: sgalperti@ucsd.edu; itrevino@ucsd.edu. First draft: July 2016

1 Introduction

People acquire information to learn about world events, but often also what others do (not) know about those events. Political protesters look for mutually known news that can catalyze a rally. Stock traders look for information others may not know, so as to anticipate price swings. Shared knowledge matters in many economic and social settings and mass media contribute to its creation (Chwe (2013)). According to Shiller (2015), "significant market events generally occur only if there is similar thinking among large groups of people, and the news media are essential vehicles for the spread of ideas." (p. 101) Recognizing that news sources not only help people learn some facts, but also others' (mutual) beliefs about those facts, is then crucial for understanding information markets.

Prominent research has investigated how this desire to learn some facts as well as others' beliefs about them affects the demand for information, treating its sources as exogenous. Little is known, however, about that desire's effects on the supply of information. To fill this gap, this paper endogenizes the information supply in a full-fledged competitive market. The paper also focuses on another aspect of information, which sets it apart from other commodities: Information consumes attention while being consumed. Since attention can be "harvested" and sold to third parties (usually advertisers), it becomes a key revenue source for information providers, who fiercely compete for it.[1]

We offer two sets of results. First, we show that competition for attention leads to a homogeneous supply of information, even when consumers would value accessing heterogeneous sources. We identify how the supplied type depends on the strength of the consumers' *fact-learning* motive relative to their *belief-learning* motive. Second, we discover an inefficiency specific to information markets, which arises despite perfect competition and the consumers' efficient use of information: Whenever attention happens to become the "currency" whereby consumers pay for information, the competitive outcome will be generically inefficient. The inefficiency can involve the volume as well as the type of supplied information, again depending on the strength of the two learning motives.

To model these motives, we follow the paradigm of games with payoff interdependencies that cause externalities in the use of information (Morris and Shin (2002); Angeletos and Pavan (2007); Hellwig and Veldkamp (2009)). Consider a society of identical agents who have to support some policy within a feasible set. They want to support the best policy, which depends on some unknown state of the world. They also like (or dislike)

[1] The importance of competition for attention when it comes to information sources has been stressed by many scholars across disciplines, including Simon (1971), Benkler (2006), Lanham (2006), Sunstein (2009), Davenport and Beck (2013), and Webster (2014).

conformism, namely, supporting what others support. Each agent then desires to learn the state as well as the others' (mutual) beliefs about it in order to coordinate or not with them. Let α measure this coordination motive relative to that of acting on the state: $\alpha > 0$ ($\alpha < 0$) captures a desire to coordinate (anti-coordinate); $\alpha = 0$ if the agents care only about the state. They can attend to various news sources that convey information about the state.[2] Each is characterized by the *accuracy* of its content and the *clarity* with which it conveys that content (as in Myatt and Wallace (2012)). For instance, a source may run a summary or in-depth report of the policies' pros and cons and broadcast it by radio or television, which differ in communication clarity. Clarity is complementary to attention for understanding content, as it raises the return per minute spent attending to a source. Realistically, distinct agents may interpret each source differently, introducing a risk of miscoordination.

In our model, each information source is produced by a distinct profit-maximizing supplier, broadly interpreted as traditional news outlets, social media, blogs, and news aggregators. Attention is their only revenue channel. We shut down the usual price channel in order to better understand the competition for attention, which seems under-studied yet important in the Internet era.[3] In the spirit of perfect competition, entry is free and suppliers act non-strategically.[4] Entrants choose their level of accuracy and clarity, each from a finite set, where higher levels cost more to produce. An accuracy-clarity pair is called a type. After the entrants commit to their types, the consumers observe them, allocate attention, update beliefs, and choose their actions.

Section 4 characterizes the supply of information and how it depends on the coordination motive α. Perhaps surprisingly, the equilibrium supply always involve only one type of sources. Importantly, this is not because the agents demand only one type—in fact, they would pay attention to multiple ones if produced—but because the competition for attention renders only one of them profitable.[5] This type and its dependence on α turn out to vary significantly in relation to accuracy and clarity.

To illustrate, suppose suppliers compete via accuracy—that is, they choose accuracy,

[2]Gentzkow (2007), Gentzkow and Shapiro (2011), Webster (2014), and Kennedy and Prat (2017) find that many people get their news from multiple sources.

[3]Setting a zero price for accessing information may be a deliberate choice for some news outlets. Webster (2014) discusses numerous examples and argues that "advertising now supports much of the world's media, both online and off." Gentzkow (2007) estimates that setting zero prices may be optimal for online outlets in the presence of a strong demand for online advertising.

[4]Perfect competition is of course an abstraction, which helps derive sharper results. However, news markets—especially in the broad sense used here—tend to be fairly competitive, at least in developed countries (see Footnote 15).

[5]Consistent with this result, Boczkowski (2010) argues that "the rise of homogenization in the news has led to a state of affairs that neither journalists nor consumers like but feel powerless to alter." (p. 6)

but all share some fixed clarity. Then, the type with the lowest *average* cost of accuracy wins the market, independently of α. Intuitively, having all the same clarity, each supplier offers the same return to attention and hence receives the same *positive* amount of attention (i.e., revenue) per unit of accuracy. This amount falls as more suppliers—possibly of different types—enter and compete for attention, so only the most efficient suppliers survive.

Suppose instead that suppliers compete via clarity—that is, they choose clarity, but all share some fixed accuracy. Then, the equilibrium type becomes less clear as α falls. Imagine clarity can be either high or low. High-clarity sources *always* capture some attention—as they offer a higher return to it—but possibly *less* attention than do low-clarity sources. This is where α matters: As α falls, the consumers shift attention from high- to low-clarity sources (Myatt and Wallace (2012); Pavan (2014)). This is because easily understandable content tends to be more public among its consumers (i.e., to induce more correlated interpretations). But public information is less worthy of costly attention for anti-conformist consumers who want to respond in opposite directions to the state and to others' beliefs. This gives low-clarity sources an advantage in terms of attention revenues as α falls. Importantly, although the size of this advantage depends on the number of high- and low-clarity entrants, its sign does not. Therefore, one type will always be more competitive and win the market. In short, we find that the endogeneity of supply magnifies the attention reallocation mechanism identified in the literature. This result can overturn the literature prediction that the clearest sources (and possibly others) always receive attention. Here, the demand is indeed biased towards those sources, yet the competition for attention can drive them out of business.

We consider scenarios where suppliers choose both accuracy and clarity. The equilibrium type continues to primarily depend on clarity, correlation, and the coordination motive, in ways similar to competition via clarity.[6]

We also examine the equilibrium entry and attention allocation. For competition via accuracy, each entrant captures the same constant attention, but entry shrinks in α, thereby reducing the total information consumed in equilibrium. For competition via clarity, the attention captured by each entrant increases in α, but entry is non-monotonic. This renders the effect of α on the consumed information ambiguous.

Our results imply that competition for attention need not promote the sources offering the *highest* return to attention. This challenges the common wisdom that in the

[6]Chwe (2013) observes that communication aimed at creating common knowledge and facilitate coordination often uses simple and repetitive language so as to ensure that the message gets through.

"attention economy" higher clarity is always better (Davenport and Beck (2013)). In fact, being too clear can *hurt* attention revenues and help less clear sources win the market. The paper highlights how this depends on the fact- and belief-learning motives of the consumers.

A second takeaway is to draw a distinction between quantity and diversity of information sources, where competition for attention pushes towards homogeneity. This echoes Webster's (2014) point that "the digital media marketplace [is] less diverse than its sheer numerical abundance might suggest" (p. 16) and "competition doesn't do much to improve the diversity of news products." (p. 58) This should not be confused with the variety of news *topics*, which can be easily explained by people's different interests. Our analysis can be viewed as focusing on one topic at a time.

Section 5 investigates the market inefficiencies caused by the very nature of information as the traded good. This is helped by our perfect-competition assumption. It is well known that, when it comes to information, consumers need not use it efficiently due to interdependencies in their final payoffs (Morris and Shin (2002), Angeletos and Pavan (2007), and Pavan (2014)). We explain how this distorts the supply of information. More importantly, suppose the consumers use information efficiently. Will the competitive equilibrium be efficient? The answer is no (under a general Pareto criterion to be defined below). In contrast to the laissez-faire outcome, the efficient supply may be higher or *lower*, require a different type of sources, or even multiple types. These inefficiencies arise precisely from the unique fact that for information—in contrast to other commodities—attention acts as the "currency" whereby consumers pay for it. This enables the existence of a market. But unlike the frictionless price mechanism, attention does not adjust freely to allow the trading sides to internalize costs and benefits of producing information. In short, when attention is costly and coordination is important, insufficiently correlated information is provided; when attention is cheap, too much information is provided. These findings matter for all real-life situations where we get news "for free" in exchange for our attention. They also speak to Webster's (2014) point that, "there is a broad consensus that the way people allocate their attention will go a long way toward determining digital media's social impact."

Related Literature. A rich literature studies information in strategic environments with coordination motives. Morris and Shin (2002) highlight the role of information as a coordination device and show that increasing the accuracy of public information can *decrease* welfare, as agents who strongly value coordination may underweight their private information when choosing actions. Angeletos and Pavan (2007) characterize the use and social value of information as a function of the coordination motives. In these papers the agents do not choose which information to acquire.

Subsequent research has endogenized information acquisition. Hellwig and Veldkamp (2009) show that information acquisition inherits the strategic motives from the underlying coordination game, which can lead to multiple equilibria. Colombo et al. (2014) study the effect of coordination motives on the efficiency in the acquisition and use of information, and the social value of free public information when agents can access costly private information. Myatt and Wallace (2012) study a framework where information sources differ in accuracy and clarity and attention is a continuous decision, which can remove the equilibrium multiplicity in Hellwig and Veldkamp (2009). Using this framework, Pavan (2014) studies the equilibrium and efficient allocation of attention. All these papers assume exogenous and heterogeneous sources. The present paper calls this assumption into question by finding that competition for attention pushes towards a homogeneous information supply. This literature has shown that decentralized information acquisition need not serve the social interest; this paper shows that its supply need not either.

Few papers endogenize information sources in coordination games; none analyze competitive information markets and their efficiency. Dewan and Myatt (2008, 2012) examine how party leaders adjust, at no cost, the clarity of their speeches to gain influence and guide activists concerned about party cohesion. In these papers, the sources' accuracy and quantity remain exogenous. Cornand and Heinemann (2008) show that increasing the accuracy of public information improves welfare, as long as one can restrict who can access it. Myatt and Wallace (2014) study how central banks design information to stabilize output, expectations, and volatility. In Chahrour (2014), a planner may withhold information if there is a risk that agents attend to different information sources.

A vast literature has studied the phenomenon of media bias and its political-economy consequences.[7] Media bias refers to deliberate and systematic distortion of information, so it is orthogonal and complementary to the focus of the present study. The idea of competition for attention and the distinction between accuracy and clarity are mostly absent from that literature, which asks whether competition—broadly defined—promotes media independence, timeliness, and unbiasedness. An exception is Chen and Suen (2017), who study a model of news markets where each outlet's owner chooses accuracy to attract attention and its editor chooses a reporting bias trading off helping readers make informed decisions and pursuing her agenda. Their readers do not exhibit coordination motives, thereby rendering the role of information sources as coordination devices irrelevant.

[7] See Gentzkow and Shapiro (2008) and Prat and Stromberg (2013) for literature reviews.

2 A Model of Information Markets

As we present the model, it helps to keep in mind our introduction story. Agents in a large society have to choose an action, broadly interpreted as supporting some policy with uncertain effects, forecasting some event, adopting a new technology, or merely picking a topic for conversation. They want to do the right thing, but also respond to (anti)conformist pressures—a desire to (not) follow popular trends or group norms. Coordination motives may also stem from network externalities—for instance, when adopting products that require a dense user base. Before acting, the agents acquire information from news sources, who compete for their attention to maximize profits.

2.1 Demand of Information

This part follows the broadly used setup of Myatt and Wallace (2012) and Pavan (2014).

Consumers, Information Sources, and Attention. There is a unit mass of ex-ante identical agents, the information consumers, indexed by n.[8] They have a common prior about the state of the world θ, given by $\theta \sim N(0, p^{-1})$. A higher precision p means that they initially know more about θ. They can acquire information about θ from news sources, indexed by m. If consumer n attends to source m, she privately observes a signal

$$s^{n,m} = \theta + z^m + x^{n,m},$$

where $z^m \sim N(0, \sigma^2_{z^m})$, $x^{n,m} \sim N(0, \sigma^2_{x^{n,m}})$, and θ, z^m, and $x^{n,m}$ are all mutually independent. The part $\theta + z^m$ represents the information contained in source m, which consists of the state plus some "sender noise." The latter determines source m's accuracy, defined by $a^m \equiv \sigma^{-2}_{z^m}$. The "receiver noise" $x^{n,m}$ reflects the idiosyncratic meaning each agent gives to a source's content. This noise depends *jointly* on how clearly the source conveys information and how attentively the consumer listens to it. Think of attention as the time agent n spends listening to source m, denoted by $e^{n,m} \geq 0$: The longer the time, the higher the chances of getting the content right. Given this, let c^m be source m's *clarity* and $\sigma^{-2}_{x^{n,m}} \equiv [c^m]^2 e^{n,m}$.[9] A higher c^m raises the return of attending to source m in terms of better understanding of its content (i.e., shrinking $\sigma^2_{x^{n,m}}$). Choosing $e^{n,m} = 0$ means ignoring source m, as $s^{n,m}$ is then just noise.

[8] Section 6 discusses the possibility of heterogeneous agents.

[9] This way of modeling communication is reminiscent of Dewatripont and Tirole (2005), if we reinterpret $[c^m]^2$ as the sender's effort and e^m as the receiver's effort in their model. We square c^m in the definition of $\sigma^2_{x^{n,m}}$ only to simplify some of the expressions below.

Actions and Payoffs. Agent n's payoff function is

$$u(k^n, K, \sigma, \theta) - C(\mathbf{e}^n),$$

where $k^n \in \mathbb{R}$ is her action, $K \equiv \int_0^1 k^n dn$ is the population average action, $\sigma^2 \equiv \int_0^1 [k^n - K]^2 dn$ is the action *dispersion*, and $\mathbf{e}^n$ is the vector of n's attention allocations $e^{n,m}$. The cost function satisfies $C(\mathbf{e}^n) = \tau \sum_m e^{n,m}$, where $\tau > 0$ is the opportunity cost of the time spent acquiring information or the shadow value of some time budget constraint. The substantive assumption, also studied by Myatt and Wallace (2012) and Pavan (2014), is that C depends only on the total attention paid, not how it is divided between sources. That is, in terms of opportunity cost attending one source is the same as attending another; their return may differ, but this is captured by clarity. Linearity is mostly a technical simplification to obtain closed-form attention allocations, which enable a cleaner analysis of the overall market.[10]

As is standard in the literature, (i) u is a second-order polynomial, (ii) σ^2 has only a non-strategic externality effect, so in terms of partial derivatives $u_{k\sigma} = u_{K\sigma} = u_{\theta\sigma} = 0$ and $u_\sigma(k, K, 0, \theta) = 0$ for all (k, K, θ), (iii) $u_{kk} < 0$, (iv) $\alpha = -u_{kK}/u_{kk} < 1$, and (v) $u_{k\theta} \neq 0$. By (i), the agent have linear best responses. By (ii), u is additively separable in σ^2, whose coefficient is denoted by $u_{\sigma\sigma}/2$. By (iv), the slope of best-response functions is less than 1, delivering uniqueness of equilibrium actions (Angeletos and Pavan (2007)). These assumptions allow for rich payoff externalities. A simple case is the loss function

$$-(1-\gamma)(k-\theta)^2 - \gamma(k-K)^2$$

for $\gamma \in (0, 1)$, often used in "beauty-contest" games à la Morris and Shin (2002).

Timing. After learning the accuracy and clarity of all sources, the agents simultaneously allocate their attention; they then observe their signals and update beliefs; finally, they simultaneously choose actions, and payoffs realize. Consistent with the literature, we consider Perfect Bayesian equilibria of this game, called "consumer equilibria" for short.

2.2 Supply of Information

We aim to model two key aspects: News sources compete with each other, and capturing attention is a key part of this competition. To do this in the simplest and starkest way, we assume that each supplier cares directly about the attention its source captures and

[10]Linearity (more generally convexity) of C rules out cost functions based on entropy as in the rational-inattention literature (Sims (2003), Sims et al. (2010)). As showed by Myatt and Wallace (2012), convexity of C ensures that the agents' behavior is described by a unique equilibrium, a useful property to study information supply. We leave the analysis with rational-inattention costs for future work.

cannot charge a price for accessing it.[11] While price competition has been extensively studied, competition for attention has not. Yet, many outlets supply news for free and rely on capturing attention to make money, usually by "selling eye balls" to advertisers.[12]

Production. Each supplier can produce one unit of the good "information," which takes the form of a signal with some accuracy and clarity. They can choose among finitely many signal types, each defined by some $a_t > 0$ and $c_t > 0$ where $t = 1, \ldots, T$ and $T \geq 2$. Think of accuracy as a reporting style (summary vs. in-depth reports) and clarity as a communication technology (TV vs. newspapers).[13] Producing a t-signal involves a fixed cost $d_t > 0$ but zero marginal costs of communicating it—a typical cost structure for information goods in the digital age (Hamilton (2004)). We will specify how d_t depends on a_t and c_t later. Suppliers can choose to not produce (i.e., set $a^m = c^m = 0$) at no cost. Note that they have no information about θ when choosing which signal to provide.

Profits. Supplier m's profit from a t-signal is

$$\int_0^1 e^{n,m} dn - d_t.$$

Suppliers have no preference over which agents attend to which source and how they use information. One interpretation is that they sell ad impressions to advertisers (not to be confused with $s^{n,m}$). As a first approximation and in the spirit of partial-equilibrium analysis, revenues are proportional to the amount of attention received with exogenous return normalized to 1. Intuitively, the more time agents spend attending a source, the more ads it can show them, and the more impressions it can charge to advertisers.[14]

Perfect Competition.[15] There is an arbitrarily large number of potential suppliers.

[11] More generally, suppliers may also compete in prices. This can introduce a trade-off between price and attention revenues (see, for example, Crampes et al. (2009)) and lead to multiplicity of equilibria as in Hellwig and Veldkamp (2009), due to the discreteness of the "price cost" of acquiring an information source. We leave the study of this potentially interesting issues for future work.

[12] These are key aspects of the "attention economy" as argued by Davenport and Beck (2013) and Webster (2014).

[13] We are agnostic on whether TV or newspapers have higher clarity. This ultimately seems an empirical question, whose answer may depend on the type of news considered (for example, financial or business news vs. general-interest news). Broadcast media have a multisensorial approach that enables potential higher-impact reporting than print media, but the static nature of print media allows for indefinite exposure, as opposed to the fleeting nature of broadcast media. Webster (2014) argues that the technology used to deliver media is a key dimension of audience-making strategies. The importance of technologies in news provision is also raised by Prat and Stromberg (2013).

[14] Webster (2014), Ch. 4, describes measures that media use to assess their audience which are consistent with this approximation. Davenport and Beck (2013) describe the "stickiness" of online news outlets—namely, their ability to grab and keep attention—as the time spent on a site and the number of visits and viewed pages per person.

[15] News markets tend to be fairly competitive, at least in developed countries. Webster (2014) argues that "perhaps the most astonishing thing about digital media is their numerical abundance." Kennedy

Entry and exit are free. When making its decisions, each supplier takes the existing suppliers as given and anticipates the agents' behavior as described by the consumer equilibrium. Suppliers produce a t-signal if and only if this is profitable. We will use this zero-profit condition for each type to solve the model. Let the number of t-suppliers be q_t. We say that $\mathbf{q}^* = (q_1^*, \ldots, q_T^*)$ is a *competitive equilibrium* if, given $\mathbf{q}^*$, no active supplier wants to exit and no new supplier wants to enter for every type of signals.

3 Attention Allocation: A Review

We first review the aspects of the consumer equilibria necessary for our analysis, leaving the details for Appendix B. Fix $\mathbf{q} = (q_1, \ldots, q_T)$. A consumer equilibrium consists of two things for each n: a vector $\mathbf{e}^n$ of attention allocations and a strategy $k^n(\mathbf{s}^n; \mathbf{e}^n)$ mapping $\mathbf{e}^n$ and signals $\mathbf{s}^n$ (one for each supplier) into actions. We follow the literature and focus on the unique equilibrium where $\mathbf{e}^n$ and k^n are the same for all n and k^n is linear in $\mathbf{s}^n$. We therefore drop the superscript n.

Our goal is to understand how the information supply depends on the consumers' desire to learn the state and others' beliefs about it. This hinges on their payoff motive of responding to θ relative to that of coordinating with others, which is measured by $\alpha = -\frac{u_{kK}}{u_{kk}}$. Indeed, as Angeletos and Pavan (2007) show, every equilibrium must satisfy

$$k(\mathbf{s}; \mathbf{e}) = \mathbb{E}[(1-\alpha)\kappa + \alpha K | \mathbf{s}, \mathbf{e}],$$

where $\kappa(\theta) \equiv \kappa_0 + \kappa_1 \theta$ is the unique equilibrium strategy under complete information, with $\kappa_0 \equiv -\frac{u_k(0,0,0,0)}{u_{kk}+u_{kK}}$ and $\kappa_1 \equiv -\frac{u_{k\theta}}{u_{kk}+u_{kK}}$. Appendix B provides a formula for $k(\cdot;\cdot)$. Different α's can correspond to different societies—some are more conformist than others—or news topics. Stock-market news may involve $\alpha < 0$, gossip news $\alpha > 0$.

We will assume that the demand is sufficiently strong in the following sense: For every $t = 1, \ldots, T$, if only t-signals were feasible, their supply would be positive, which of course also means that they receive positive attention. As we will see, this is implied by the next condition. Let

$$\phi = |\kappa_1| \sqrt{\frac{-u_{kk}}{2}}.$$

Assumption 1 (Strong demand). $\phi > p\sqrt{\tau}\left[\frac{c_t^{-1}}{1-\alpha} + \frac{d_t c_t}{a_t}\right]$ *for all* $t = 1, \ldots, T$.

This condition always holds for sufficiently small p and τ. Intuitively, the demand for

and Prat (2017) find concentration indexes of news markets that are fairly small relative to usual standards. See also Noam (2009). Finally, recall that we interpret information sources broadly as traditional new outlets, social media, blogs, etc..

information is strong if ex ante the agents know little about θ or their attention cost is small. Assumption 1 is akin to the standard assumption that the intercept of a demand curve is large enough to induce positive trade in equilibrium.

Consider now the attention allocation. As shown in Appendix B, the agents pay the same attention to each t-source, denoted by $e_t(\mathbf{q})$. Of course, $e_t(\mathbf{q}) = 0$ if $q_t = 0$. The next result describes $e_t(\mathbf{q})$ when $q_t > 0$.[16] Assume that $c_t \geq c_{t'}$ if $t > t'$ and define

$$T_{\mathbf{q}} = \{t : q_t > 0\} \quad \text{and} \quad T_e = \{t \in T_{\mathbf{q}} : e_t(\mathbf{q}) > 0\}.$$

Lemma 1 (Attention Functions)**.** *For every $t \in T_e$,*

$$e_t(\mathbf{q}) = \frac{a_t c_t^{-1}}{p + \sum_{t' \in T_e} a_{t'} q_{t'}} \left[\frac{\phi}{\sqrt{\tau}} - \frac{p c_t^{-1}}{1-\alpha} + \sum_{t' \in T_e \setminus \{t\}} \frac{a_{t'} q_{t'}}{1-\alpha} (c_{t'}^{-1} - c_t^{-1}) \right].$$

Let t_1 and t_2 be the first and second largest types in $T_{\mathbf{q}}$. When $c_{t_1} > c_{t_2}$, define

$$q_{t_1}(\alpha) = \left[\frac{\phi}{\sqrt{\tau}} (1-\alpha) c_{t_2} - p \right] \frac{c_{t_1} a_{t_1}^{-1}}{c_{t_1} - c_{t_2}}.$$

Corollary 1 (Attention Targets)**.** *If $c_t = c$ for all t, all active suppliers receive attention: $T_e = T_{\mathbf{q}}$. If $c_t > c_{t'}$ for all $t > t'$, then T_e contains all types above some $\hat{t} \in T_{\mathbf{q}}$, where $\hat{t} = t_1$ if and only if $q_{t_1} \geq q_{t_1}(\alpha)$.*

These properties review some key points in Myatt and Wallace (2012) and Pavan (2014). As expected, $e_t(\mathbf{q})$ decreases in the quantity of competitors of *any* type.[17] It also decreases if ex ante the agents know more about θ (i.e., p is higher), which weakens their incentive to acquire information. Higher clarity has ambiguous effects: By offering a higher return to attention, it can raise e_t, but by "consuming" less attention to convey the same content, it can lower e_t.[18]

The coordination motive α has two effects on attention. The first, given by $-\frac{p}{1-\alpha}$, is negative. As α rises, the agents' optimal actions depend less on their signals and more on their prior, since it captures shared knowledge (see Appendix B). This lowers the value of acquiring information in the first place. Thus, suppliers also compete with the agents' prior to attract attention. The second effect of raising α exists only if $c_t > c_{t'}$. It is negative for the lower-clarity sources and positive for the higher-clarity ones, which

[16] Its proof appears in Appendix A together with all the others.

[17] One can show that $\frac{\partial e_t(\mathbf{q})}{\partial q_{t'}} < 0$ if $t' \in T_e$.

[18] The property that agents pay attention to all sources of either type is clearly driven by their symmetry in the model. Although this is in part at odds with reality, there exists evidence of significant overlap in audiences across news sources (Gentzkow and Shapiro (2011), Webster (2014)).

reflects the latter's comparative advantage as coordination devices.[19] As the agents attend more to a source, they interpret more similarly its content, which then becomes more public among them, helping coordination. Clearer sources offer a higher return to attention and so attract more of it when α rises. That less clear sources have an advantage when the agents want to anti-coordinate may be counterintuitive, but has a simple logic. Each agent now wants to respond in opposite directions to shared information suggesting that θ is high *and* everybody else's action is high. As a result, such public information has little value. It is, however, less likely to arise from unclear and hence more private sources, which are then more worth attending for the agents.

Finally, the active suppliers of highest-clarity can crowd out attention so that nobody attends to lower-clarity suppliers (but not vice versa). Note that $q_{t_1}(\alpha)$ decreases in α: A larger α raises the value assigned to clarity and so the willingness to attend to the highest-clarity suppliers, which can now more easily exhaust the agent's attention.

4 Equilibrium Information Supply

We now turn to the analysis of competitive equilibria. We first derive the zero-profit conditions that characterize them, given the attention functions obtained before. Think of these functions as the analog of demand functions in standard markets. Recall that suppliers choose signals that can differ in clarity, accuracy, or both. We cover these three cases separately so as to better demonstrate the differences between accuracy and clarity.

4.1 Zero-profit Conditions

Consider the suppliers' entry and exit decision. Given that $\mathbf{q}$ suppliers already entered across types, a new t-supplier enters if and only if

$$e_t(q_t + 1, \mathbf{q}_{-t}) \geq d_t, \tag{1}$$

where $\mathbf{q}_{-t}$ is the vector of quantities excluding q_t. Since all t-suppliers are identical, if (1) holds for the last entrant, it holds for all t-suppliers *already* in the market, which implies

$$(q_t + 1)e_t(q_t + 1, \mathbf{q}_{-t}) \geq (q_t + 1)d_t.$$

Conversely, if this condition holds and only q_t suppliers of type t already entered, an extra t-supplier wants to enter. Letting $E_t(\mathbf{q}) = q_t e_t(\mathbf{q})$, we get the following observation.

[19]That clear sources are better coordination devices seems to be a general property that holds beyond the Gaussian framework considered here (see Chwe (2013)).

Lemma 2. $\mathbf{q}^*$ *is a competitive equilibrium if and only if, for all* $t = 1, \ldots, T$,

$$E_t(q_t^*, \mathbf{q}_{-t}^*) \geq q_t^* d_t \qquad \text{and} \qquad E_t(q_t^* + 1, \mathbf{q}_{-t}^*) < (q_t^* + 1) d_t.$$

These conditions characterize all equilibria (if any exists), but are inconvenient to use due to the discreteness of q_t. This is analogous to the issue that arises in textbook analysis of long-run competitive equilibria when profits jump from being strictly positive to being strictly negative if one more firm enters the market. To avoid this, it is typically assumed that firms are small so that equilibria can be characterized by a zero-profit condition. A similar approach can be adopted here by letting q_t be a non-negative real number for all t. Given this, the equilibrium conditions become

$$E_t(q_t^*, \mathbf{q}_{-t}^*) = q_t^* d_t, \quad t = 1, \ldots, T, \tag{2}$$
$$q_t > q_t^* \Rightarrow E_t(q_t, \mathbf{q}_{-t}^*) < q_t d_t, \quad t = 1, \ldots, T. \tag{3}$$

To understand the second, consider Figure 1. It represents the total production cost $q_t d_t$ and the attention function $E_t(q_t, \mathbf{q}_{-t})$ (when positive), which is strictly increasing and concave in q_t. The quantity $q_t = 0$ always satisfies (2), but can be part of an equilibrium only if no t-supplier can profitably enter given $\mathbf{q}_{-t}$. In Figure 1, this holds for d_t, but not for $\hat{d}_t$. In this case, condition (3) rules out $q_t = 0$ and $\mathbf{q}_{-t}$ as an equilibrium.

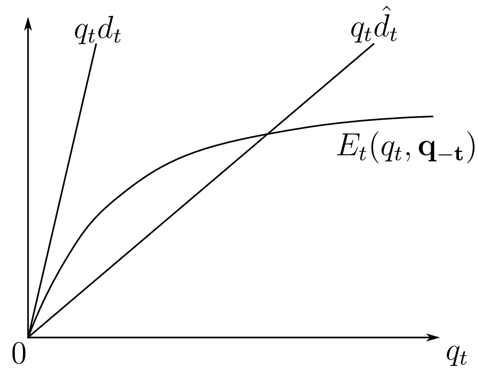


Figure 1: Attention-revenue and production-cost curves

The interpretation of continuous quantities is that each supplier is small relative to the whole market, which is consistent with the idea of perfect competition. In this model, however, letting the number of information sources grow arbitrarily involves some intricacies: Since each provides an independent signal about the state, they could eventually allow the agents to fully learn it. We avoid this issue and maintain the spirit of the model by appropriately rescaling the accuracies a_t as we let the number of sources grow. We present the details in Appendix C.

We can now see why under Assumption 1 there would be a positive supply of t-signals, if this were the only feasible type. Let $\overline{q}_t$ be the largest solution to $E_t(q_t, \mathbf{q}_{-t}) = q_t d_t$ for $\mathbf{q}_{-t} = \mathbf{0}$. One can show that

$$\overline{q}_t = \frac{c_t^{-1}}{d_t}\left[\frac{\phi}{\sqrt{\tau}} - \frac{pc_t^{-1}}{1-\alpha}\right] - \frac{p}{a_t},$$

which is strictly positive by our assumption.

4.2 Competition via Clarity

Consider the case where suppliers can use clarity as a lever to attract attention, but they all produce the same content. That is, $c_t > c_{t'}$ for $t > t'$ and $a_t = a$ for all t. For example, this content may be the narration of some event whose accuracy a stands for the currently available facts, and each supplier can communicate this either via a TV news channel (type t) or an online newspaper (type t'). Arguably, the same content cannot be conveyed in the same way using videos and sounds and using pictures and written words, and the attention required to process it might differ between media.[20] It is reasonable to assume that higher clarity costs more: $d_t = d(c_t)$ is a strictly increasing and weakly convex function of c_t. For simplicity, we leave the dependence of d on a implicit.

Despite Assumption 1, generically in equilibrium only one type of suppliers is active. To state this, define

$$\alpha_{t'}^{t} = 1 - a\frac{c_{t'}^{-1} - c_t^{-1}}{c_t d(c_t) - c_{t'} d(c_{t'})},$$

with the convention that $\alpha_0^1 = -\infty$ and $\alpha_T^{T+1} = 1$. The role of $\alpha_{t'}^{t}$ will become clear shortly, but it is worth anticipating that $\alpha_{t'}^{t}$ increases in c_t and $c_{t'}$, as we show below.

Proposition 1. *There exists a competitive equilibrium* $\mathbf{q}^*$*.*
Case 1: $\alpha \neq \alpha_{t'}^{t}$ for all t,t'. *The equilibrium is unique and satisfies* $q_{t^*}^* = \overline{q}_{t^*}$ *and* $q_t^* = 0$ *for all* $t \neq t^*$*, where* t^* *is the unique type for which* $\alpha_{t^*-1}^{t^*} < \alpha < \alpha_{t^*}^{t^*+1}$*.*
Case 2: $\alpha = \alpha_{t-1}^{t}$ for some t. *There is a continuum of equilibria which satisfy* $q_{t-1}^* \in [0, \overline{q}_{t-1}]$*,* $q_t^* = \frac{c_{t-1}d(c_{t-1})}{c_t d(c_t)}\left[\overline{q}_{t-1} - q_{t-1}^*\right]$*, and* $q_{t'}^* = 0$ *for all* $t' \neq t, t-1$*.*

To see the intuition, think of $\frac{d(c_t)}{ac_t^{-1}}$ as the cost per unit of content of a t-signal, taking into account its return to attention for consumers. Similarly, think of $\frac{e_t(\mathbf{q})}{ac_t^{-1}}$ as the revenue per unit of clarity-adjusted content (the analog of a "price") earned from a t-signal. When types t and $t' < t$ attract positive attention, one can show that

$$\frac{e_t(\mathbf{q})}{ac_t^{-1}} - \frac{e_{t'}(\mathbf{q})}{ac_{t'}^{-1}} = \frac{c_{t'}^{-1} - c_t^{-1}}{1-\alpha}. \tag{4}$$

[20] For a similar point, see Dewan and Myatt (2008) and Webster (2014).

Using the definition of $\alpha_{t'}^t$, we have that

$$\frac{c_{t'}^{-1} - c_t^{-1}}{1-\alpha} \gtrless \frac{d(c_t)}{ac_t^{-1}} - \frac{d(c_{t'})}{ac_{t'}^{-1}} \quad \text{if and only if} \quad \alpha \gtrless \alpha_{t'}^t. \tag{5}$$

That is, $\alpha > \alpha_{t'}^t$ (resp. $\alpha < \alpha_{t'}^t$) implies that the higher-clarity signal is more (resp. less) profitable than the lower-clarity one. This implies that when one type yields zero profits, the other must yield strictly negative profits and so no supplier wants to choose it. In particular, when $\alpha_{t-1}^t < \alpha < \alpha_t^{t+1}$, t-signals are more profitable than any higher and lower clarity signal and hence wins the entire market. Note that this result follows from properties of the attention functions that are derived (not assumed) from more primitive properties of how agents use information in a variety of social and economic contexts.

Although based on Section 3 one should expect that high α's favor clear suppliers and low α's favor opaque suppliers, it is perhaps surprising that, generically, only one type is active in equilibrium. Moreover, this is a consequence of the competition for attention, not the lack of demand for one type of sources. Except for large α's, one type of suppliers dominates the market not because the agents do not want to attend to the others at all, but because competition renders these types unprofitable. Indeed, the only type that can entirely crowd out the consumers' attention is T: if $q_T \geq q_T(\alpha)$, no other type can attract positive attention by Corollary 1. The zero-profit quantity $\overline{q}_T$ satisfies $\overline{q}_T \geq q_T(\alpha)$ if and only if

$$\alpha \geq \overline{\alpha} \equiv 1 - a\frac{c_{T-1}^{-1} - c_T^{-1}}{c_T d(c_T)},$$

where $\overline{\alpha} > \alpha_{T-1}^T$. Thus, if for instance $\alpha < \alpha_{T-1}^T$, given the equilibrium $\mathbf{q}^*$ at least T-signals would receive some attention if provided by some source. Yet, the comparative advantage of lower-clarity signals in serving the consumers' coordination motives renders entry unprofitable for T-sources.

Another interesting property is that sources of *low*-clarity (even the lowest one) can win the competition for attention. This may seem counterintuitive, as such sources offer a low return to attention and they can never steal attention entirely from higher-clarity sources. But the dependence of the result on the coordination motive being sufficiently weak helps explain the puzzle and demonstrates that recognizing the role of information sources as coordination devices is important to understand their supply. To see the intuition, suppose the agents' actions are strong strategic substitutes—so that $\alpha < \alpha_0^1$. We then know that the agents favor the lowest-clarity sources, as they better help them choose actions far from the population average. With endogenous sources competing for attention, this predilection for the lowest clarity depresses the profitability of higher-clarity sources, which can drive them out of business.

The thresholds $\alpha_{t'}^t$ have some noteworthy properties. They are independent of the agents' payoff function u (which determines κ_1 and u_{kk}), prior precision p, and attention cost τ. These aspects affect how much time the agents are willing to put *overall* into acquiring information, but not how they divide it between sources. Accordingly, the supplied quantity $\overline{q}_{t^*}$ is decreasing in p (higher prior knowledge weakens the incentive to acquire information), in $|\kappa_1|$ (a smaller $|\kappa_1|$ weakens the agents' desire to respond to θ and hence to acquire information), and in τ.[21] The threshold $\hat{\alpha}$ depends only on aspects of the supply side of information, as summarized in the next result.

Proposition 2. *For every t and t', $\alpha_{t'}^t$ decreases in the accuracy of information, a, and increases in the clarity of either type, c_t and $c_{t'}$.*

Of course, a affects the production cost d. The question here is what happens if sources can provide higher accuracy at the same cost. Intuitively, a higher common accuracy lowers $\alpha_{t'}^t$ because a richer content takes more time to be understood and hence is more prone to being interpreted differently, causing mis-coordination. Since this effect is mitigated by higher clarity, consumers assign a premium to it at even lower α's. The The threshold $\alpha_{t'}^t$ also falls when clarity worsens for the lower-clarity sources, because this boosts the extra profitability of the higher-clarity sources. These sources, however, do not benefit from an increase in their clarity: Being clearer can *decrease* the attention revenues and costs more at the margin by convexity of d; hence, it has to be offset by a stronger coordination motive for clearer sources to cover their production costs.

It is interesting to consider the consequences of the equilibrium outcome on the consumers' allocation of attention and the overall information they obtain.

Corollary 2. *In equilibrium, every active t-supplier receives attention $e_t(\mathbf{q}^*) = d(c_t)$. Therefore, as α increases, consumers allocate weakly more attention to each attended source of information.*

The first property follows from the zero-profit condition for active suppliers. The second follows from Proposition 1. As α rises, the equilibrium type of suppliers transitions from lower to higher clarity. Thus, as the coordination motive increases, the consumers' understand better each information source provided in equilibrium, because its clarity

[21] We conjecture that the equilibrium characterization in Proposition 1 (and 3 below) remains qualitatively unchanged if the attention cost function is $C(\mathbf{e}) = \overline{C}(\sum_m e^m)$, where the function $\overline{C}$ satisfies $\overline{C}'(0) > 0$ and $\overline{C}'' > 0$. Indeed, the revenue differential in (4) does not depend on attention costs. Also, the marginal attention cost enters symmetrically in all equilibrium zero-profit conditions, which implies—by the same logic of the proofs—that in equilibrium only one type of signals can win the whole market, generically.

increases and its received attention increases. This also implies that each source becomes more public, in the sense that its signals become more correlated across consumers. Following Myatt and Wallace (2012), the publicity of source m of type t is defined as the correlation between the signals of any pair (n, n') of its consumers, which satisfies

$$corr(s^{n,m}, s^{n',m}|\theta) = \frac{e_t(\mathbf{q}^*)c_t^2}{a + e_t(\mathbf{q}^*)c_t^2}.$$

Note that this conclusion is true only in equilibrium. Myatt and Wallace (2012) showed that, given a fixed supply of information, as α rises consumers shift attention from less to more clear sources. Therefore, while the latter become more public, the former become *less* public. So, it is not true that all attended sources become uniformly more public.

The total information consumed by the agents depends on attention as well as the quantity of sources. The latter varies non-monotonically in α. As α rises within each interval $(\alpha_{t-1}^t, \alpha_t^{t+1})$, the quantity $\overline{q}_t$ falls due to the effect of competition with the prior as a coordination device. The question is what happens when we transition from $\overline{q}_{t-1}$ to $\overline{q}_t$ at $\alpha = \alpha_{t-1}^t$. Straightforward algebra implies that $\overline{q}_{t-1} > \overline{q}_t$ at α_{t-1}^t if and only if

$$\frac{d(c_t) - d(c_{t-1})}{c_t - c_{t-1}} c_t^2 c_{t-1}^2 < \frac{a\phi}{p\sqrt{\tau}}. \tag{6}$$

Note that the right-hand side is strictly increasing in t due to the convexity of d.

Corollary 3. *Suppose we refine equilibria so that there exists a unique active type of sources for every $\alpha < 1$. The equilibrium quantity of information is strictly decreasing in α if and only if $\alpha < \alpha_{\overline{t}}^{\overline{t}+1}$, where $\overline{t}$ is the largest type for which (6) holds.*

Thus, if $\overline{t} = T$, the supply of information decreases monotonically as α rises. Otherwise, it jumps upward whenever there is a transition to a higher type, provided that α is sufficiently large. This is because for low clarity the effect of competition for attention with the prior as a coordination device always dominates as α rises. But for sufficiently high clarity, its higher return to attention wins as we transition to higher types. Since source-specific attention is monotonically increasing, the overall effect on total attention is ambiguous. This renders the effect of α on the quality of consumed information ambiguous. One can show that each consumer ultimately conditions his action on the average of the signals he attends, whose equilibrium total variance is $\frac{1}{p} + \frac{1}{a\overline{q}_{t^*}} + \frac{c_{t^*}^{-2}}{\overline{q}_{t^*} e_{t^*}(\mathbf{q}^*)}$ (see Appendix C). This variance need not vary monotonically in α.

4.3 Competition via Accuracy

Consider now the case where suppliers use accuracy as a lever to compete for attention, but they all communicate via the same technology. That is, $a_t > a_{t'}$ for $t > t'$ and $c_t = c$ for all t. As an illustrative example, consider a stylized scenario where there are only online newspapers and each can choose between running a summary report $(a_{t'})$ or an in-depth analysis $(a_t > a_{t'})$ of some event or topic of public interest. We assume that higher accuracy costs more: $d_t = d(a_t)$ is a strictly increasing function of a_t. Again, we leave the dependence of d on c implicit.

In this case, no type of suppliers can crowd out attention (Corollary 1): If $q_t > 0$,

$$e_t(\mathbf{q}) = \frac{a_t c^{-1}}{p + \sum_{t'=1}^{T} a_{t'} q_{t'}} \left[\frac{\phi}{\sqrt{\tau}} - \frac{p c^{-1}}{1-\alpha} \right] > 0, \tag{7}$$

where the inequality follows from Assumption 1. This already suggests that competition via accuracy is qualitatively different from competition via clarity.

Despite Assumption 1, the equilibrium supply of information is again dominated by one type of sources, under the generic property that the average cost of accuracy is minimized by only one type.[22]

Proposition 3. *Suppose* $\{t^*\} = \arg\min_t \frac{d(a_t)}{a_t}$*. There exists a competitive equilibrium* $\mathbf{q}^*$*, which is unique and satisfies* $q^*_{t^*} = \overline{q}_{t^*}$ *and* $q^*_t = 0$ *for all* $t \neq t^*$*.*

To gain intuition, again think of $\frac{d(a_t)}{a_t c^{-1}}$ as the per-unit cost of producing information of type t and $\frac{e_t(\mathbf{q})}{a_t c^{-1}}$ as the per-unit revenue (again, "the price") earned from supplying it. Because all types have the same clarity, they earn the same price:

$$\frac{e_t(\mathbf{q})}{a_t c^{-1}} = \frac{1}{p + \sum_{t=1}^{T} a_t q_t} \left[\frac{\phi}{\sqrt{\tau}} - \frac{p c^{-1}}{1-\alpha} \right], \quad t = 1, \dots, T.$$

Moreover, this price falls as more suppliers enter. Free entry then implies that only the suppliers with the cheapest production can profitably survive in the market.

A few remarks are in order. The result is again driven by the competition for attention. By (7), the agents would always pay attention to all active types, and one can find examples of $\mathbf{q}$ where multiple types could make positive profits. Yet, such situations cannot survive the competition for attention. Another interesting aspect is that the equilibrium type of suppliers does not depend on the agents' coordination motive: α only affects—negatively—the supplied quantity of information $(\overline{q}_{t^*})$ due to the aforementioned competition with the agents' prior. This is in stark contrast with the case of competition

[22] If multiple types have minimal average cost, these will share the market and the others will stay out in equilibrium. Details are available upon request.

via clarity. The effects of τ, p, and $|\kappa_1|$ on $\overline{q}_{t^*}$ follow the same logic as before. Finally, clarity has a non-monotonic effect on $\overline{q}_{t^*}$:

$$\frac{\partial \overline{q}_{t^*}}{\partial c} = \frac{c^{-2}}{d(a_{t^*})}\left[-\frac{\phi}{\sqrt{\tau}} + \frac{2pc^{-1}}{1-\alpha}\right] > 0 \quad \text{if and only if} \quad c < \frac{2p\sqrt{\tau}}{(1-\alpha)\phi}.$$

This reflects the non-monotonic effect of clarity on attention explained after Lemma 1: Excessive clarity can depress the attention revenues, thereby reducing market entry.

The consequences of the equilibrium outcome on the consumers' allocation of attention and the overall information they obtain are as follows.

Corollary 4. *In equilibrium, every active t^*-supplier receives attention $e_{t^*}(\mathbf{q}^*) = d(a_{t^*})$.*

Since the market-winning type is independent of α, so is the attention each source receives. However, since $\overline{q}_{t^*}$ is strictly decreasing in α, so is the total attention allocation (i.e., $E_{t^*}(\mathbf{q}^*) = \overline{q}_{t^*} d(a_{t^*})$). Overall, consumers get less information as α increases. One can show that each consumer ultimately conditions his action of the average of the signals he attends, whose equilibrium total variance is $\frac{1}{p} + \frac{1}{\overline{q}_{t^*} a_{t^*}} + \frac{c^{-2}}{\overline{q}_{t^*} e_{t^*}(\mathbf{q}^*)}$ (see Online Appendix C). This variance then increases as α rises.

4.4 Competition via Accuracy and Clarity

The case where the types of sources differ in both accuracy and clarity turns out to be qualitatively similar to the case where they differ only in clarity. Clarity remains the driver of the competitive outcome. For this reason, we only explain the differences from the previous analysis at an intuitive level for a two-type setting. The formal steps follow the same logic of the proof of Proposition 1. We have two possibilities to consider, depending on whether the choice between types involves a trade-off between accuracy and clarity. These possibilities relax the independence between accuracy and clarity that was implicit in the previous sections.

High-accuracy-low-clarity vs. low-accuracy-high-clarity. In this case, the feasible types are described by (a_h, c_l) and (a_l, c_h), where $a_h > a_l$ and $c_h > c_l$. Without risk of confusion, we label the first hl and the second lh. As an intuitive interpretation, suppose that each news source is conveying information on the desirability of some public policy. One option is to publish a short article stating that the policy is desirable in simple words which are easily understandable for everybody (type lh). Another option is to publish an in-depth report explaining the policy effects and all arguments in favor or against it, which may involve data, graphs, technical jargon, and subtle logics and hence are more likely to be interpreted in different ways by the agents (type hl). These types may correspond to the distinction in the news industry between "soft" and "hard" news.

To explain the outcomes of competition, we need the following notation:

$$\hat{\alpha} = 1 - \frac{c_l^{-1} - c_h^{-1}}{\frac{c_h}{a_l} d_{lh} - \frac{c_l}{a_h} d_{hl}} \quad \text{and} \quad \overline{\alpha} = 1 - \frac{c_l^{-1} - c_h^{-1}}{\frac{c_h}{a_l} d_{lh}} < 1.$$

Note that $\hat{\alpha}$ can exceed 1, in contrast to $\alpha_{t'}^{t} < 1$ in Section 4.2. This happens if the high-accuracy-low-clarity sources cost significantly more: $d_{hl} > \frac{c_h a_h}{c_l a_l} d_{lh}$. If $\hat{\alpha} < \overline{\alpha}$, the characterization of competitive equilibria is as in Proposition 1 for $T = 2$, except that $\hat{\alpha}$ replaces α_1^2. If instead $\overline{\alpha} < 1 < \hat{\alpha}$, for every $\alpha < 1$ there exists a unique equilibrium where low-accuracy-high-clarity suppliers dominate the market. Intuitively, when $d_{hl} > \frac{c_h a_h}{c_l a_l} d_{lh}$, a type-$hl$ source costs simply too much relative to the little attention it attracts because the agents have to process more content that is also more difficult to understand. Abstracting from details, consider the interpretation of type hl as hard news and type lh as soft news. There has been much discussion that soft news has been replacing hard news over the recent decades. This model suggests that such a transition may be driven not by increasing scarcity of consumers' attention—its cost τ does not affect $\hat{\alpha}$—but by rising social conformism or technological changes in news production that lower $\hat{\alpha}$.

High-accuracy-high-clarity vs. low-accuracy-low-clarity. In this case, the feasible type are described by (a_l, c_l) and (a_h, c_h), where again $a_h > a_l$ and $c_h > c_l$. A reasonable assumption here is that $d_{hh} > d_{ll}$. For illustration, consider a stylized situation where news providers can hire two types of journalists. Type hh is an expert, hard-working, perceptive journalist with excellent writing skills, who can produce accurate as well as clear articles; type ll is a novice, lazy, dull journalist with poor writing skills, who cannot achieve the same accuracy and clarity of type hh.

Turning to equilibrium analysis, we now need the following notation:

$$\hat{\alpha}' = 1 - \frac{c_l^{-1} - c_h^{-1}}{\frac{c_h}{a_h} d_{hh} - \frac{c_l}{a_l} d_{ll}} \quad \text{and} \quad \overline{\alpha}' = 1 - \frac{c_l^{-1} - c_h^{-1}}{\frac{c_h}{a_h} d_{hh}} < 1.$$

Again, it is possible that $\hat{\alpha}' > 1 > \overline{\alpha}'$. This happens if and only if accuracy differs significantly between types: $\frac{a_h}{a_l} > \frac{c_h}{c_l} \frac{d_{hh}}{d_{ll}}$. If $\hat{\alpha}' < \overline{\alpha}$, the characterization of competitive equilibria is again as in Proposition 1 with $T = 2$, except that $\hat{\alpha}'$ replaces α_1^2. Note that in this case, when $\alpha < \hat{\alpha}'$, in equilibrium suppliers provide the unambiguously dominated type of sources, which is worse in terms of both accuracy and clarity; and this occurs, even though the agents would be willing to pay some attention to the better sources hh. If instead $\overline{\alpha}' < 1 < \hat{\alpha}'$, we again have a unique equilibrium for every $\alpha < 1$ where type-hh suppliers dominate the market. Intuitively, when $\frac{a_h}{a_l} > \frac{c_h}{c_l} \frac{d_{hh}}{d_{ll}}$, a type-$ll$ source can simply attract too little attention to be profitable, due to its extremely low accuracy.

5 Inefficiencies in Information Markets

What inefficiencies can arise in information markets? Some can have familiar causes, such as monopolistic power. A reasonable starting point is to rule these out by considering competitive settings, so as to focus on inefficiencies specific to *information* markets. From Morris and Shin (2002), Angeletos and Pavan (2007), and Pavan (2014) (reviewed in the next section) we know that the agents' payoff interdependencies distort their use and acquisition of information. This can in turn distort the overall market outcome as we show in Section 5.3. But suppose we switch off these demand-driven distortions. Would the resulting competitive equilibrium be efficient (in some appropriate sense)? We consider this more novel question first. For the sake of brevity, we will focus on the more interesting case of competition via clarity and only discuss competition via accuracy (for the generic cases with a unique equilibrium).

5.1 Efficient Demand of Information

Following Angeletos and Pavan (2007) and Pavan (2014), we can identify the inefficiencies in the demand of information by studying the allocation of attention and use of information that maximize the agents' ex-ante utility for fixed information sources. This criterion takes as constraints that information is dispersed in the economy and cannot be transferred between agents.[23] Their failure to internalize payoff interdependencies leads to three kinds of inefficiencies:

• *complete-information externalities*: Even under complete information, each agent may incorrectly internalize how his action affects everybody else. The equilibrium action rule, $\kappa(\theta) = \kappa_0 + \kappa_1\theta$, can then differ from the first-best action rule, $\kappa^{**}(\theta) = \kappa_0^{**} + \kappa_1^{**}\theta$.[24]

• *socially optimal degree of coordination*: Under incomplete information, each agent may incorrectly internalize how aligning his action with its population average affects *dispersion* (i.e., $Var[k - K]$) and *non-fundamental volatility* (i.e., $Var[K - \kappa^{**}]$). The efficient action strategy k^{**} requires to use the coordination motive

$$\alpha^{**} = 1 - \frac{u_{kk} + 2u_{kK} + u_{KK}}{u_{kk} + u_{\sigma\sigma}},$$

[23] Angeletos and Pavan (2009) investigate policies that can achieve efficiency in the use of information.

[24] Angeletos and Pavan (2007) show that $\kappa_0^{**} = -\frac{u_k(0,0,0,0)+u_K(0,0,0,0)}{u_{kk}+2u_{kK}+u_{KK}}$ and $\kappa_1^{**} = -\frac{u_{k\theta}+u_{K\theta}}{u_{kk}+2u_{kK}+u_{KK}}$.

which can differ from α. A standard assumption is that $\alpha^{**} < 1$.[25]

• *social aversion to dispersion*: By assumption dispersion, σ^2, has a non-strategic effect on the agents' payoffs, so they do not take it into account when allocating their attention.

Importantly, these inefficiencies arise from the payoff structure of the underlying game among the agents. Hence, their existence and effects apply for every configuration of information supply. Removing all them delivers the efficient allocation of attention and use of information: These are given by similar conditions to those in Lemma 1 by replacing α with α^{**}, κ_1 with κ_1^{**}, and u_{kk} with $u_{kk} + u_{\sigma\sigma}$ (see (19) in Appendix B).

5.2 Inefficiency of Competitive Equilibria

Suppose the demand of information is efficient. Given the resulting market equilibrium, would a social planner prefer a different supply of information sources?

We first need to define social welfare. Since we do not have a price, we of course need a more general notion than the usual total surplus generated by a single market. We define welfare as a weighted sum of the consumers' surplus and the profit of each type of suppliers. The latter is $E_t(\mathbf{q}) - d_t q_t$. Pavan (2014) shows that, under the efficient use of information (i.e., k^{**}), given attention allocation $\mathbf{e}$ the consumers' surplus is (up to a constant)

$$\mathcal{V}(\mathbf{e}) = \mathcal{L}^*(\mathbf{e}) - C(\mathbf{e}),$$

where

$$\mathcal{L}^*(\mathbf{e}) = \frac{u_{kk} + 2u_{kK} + u_{KK}}{2} Var[K - \kappa^{**}|\mathbf{e}, k^{**}(\cdot;\mathbf{e})] + \frac{u_{kk} + u_{\sigma\sigma}}{2} Var[k - K|\mathbf{e}, k^{**}(\cdot;\mathbf{e})].$$

$\mathcal{L}^*$ measures the welfare loss due to volatility (first term) and dispersion (second term). It is worth noting that, although volatility is independent of the attention allocation of each agent individually,[26] they internalize it through $k^{**}(\cdot;\mathbf{e})$ when choosing $\mathbf{e}$. Recall that we also assume $u_{\sigma\sigma} = 0$. Let $\gamma = (\gamma_1, \ldots, \gamma_T)$, and $\gamma_c = 1 - \sum_t \gamma_t$, where $\gamma_t > 0$ for all t and $\gamma_c > 0$. We express welfare as

$$\mathcal{P}(\mathbf{q};\gamma) = \gamma_c \mathcal{V}(\mathbf{e}(\mathbf{q})) + \sum_{t=1}^{T} \gamma_t [E_t(\mathbf{q}) - d_t q_t]. \tag{8}$$

Note three things. The planner can neither transfer information between agents nor control how they allocate attention. $\mathcal{P}$ assigns the same weight to all agents and all suppliers of each type, a reasonable simplification since the members of each group are

[25] Another standard assumption is that $u_{kk} + 2u_{kK} + u_{KK} < 0$ and $u_{kk} + u_{\sigma\sigma} < 0$, which ensure that the first-best attention allocation is unique and bounded (see Pavan (2014)).

[26] This is because in this class of models the distribution of K is independent of the attention allocation of each agent for every linear $k(\cdot;\cdot)$.

identical. Even if $\gamma_t = \frac{1}{T+1}$ for all t, $\mathcal{P}$ is not equivalent to the total surplus for a single market, because here we do not have transferability via "money."

Welfare depends on the information supply in complex ways (see Appendix A.6 for details). Changing $\mathbf{q}$ affects the total amount of provided content and how agents allocate attention. This allocation in turn determines how they weigh their prior and signals when choosing an action, which affects both dispersion and volatility. Attention also affects dispersion by determining how differently the agents interpret each source. Finally, volatility depends on each source's common noise and hence on the total provided content.

Despite this complexity, we obtain an unambiguous result. Generically, the competitive equilibrium does *not* maximize welfare—even when the demand is efficient.

Proposition 4. *Suppose consumers acquire and use information efficiently (i.e., $\alpha = \alpha^{**}$, $\kappa = \kappa^{**}$, $u_{\sigma\sigma} = 0$), and $\alpha \neq \alpha^t_{t'}$ for all t, t'. Let $\mathbf{q}^{**}$ be the resulting competitive equilibrium. For every γ, generically*

$$\mathbf{q}^{**} \notin \arg\max_{\mathbf{q}} \mathcal{P}(\mathbf{q}; \gamma).$$

Thus, generically the competitive equilibrium fails Pareto efficiency, defined by treating all agents and all suppliers of each type as a single entity.

To gain intuition and add content to Proposition 4, we examine how the welfare effects of changing the information supply depend on the attention cost. This helps us highlight that attention is the source of inefficiency here. The explanation follows.

Proposition 5. *Fix γ and all parameters except the attention cost τ. Given α^{**}, let $\mathbf{q}^{**}$ be the equilibrium with active type t^{**}. For every type t of sources that would receive attention if supplied, there exists $\tau_t > 0$ such that $\frac{\partial \mathcal{P}(\mathbf{q}^{**})}{\partial q_t} \gtrless 0$ if and only if $\tau \gtrless \tau_t$. Also, $\tau_t > \tau_{t^{**}}$ if $t < t^{**}$. If $\tau = \tau_{t^{**}}$, then $\frac{\partial \mathcal{P}(\mathbf{q}^{**})}{\partial q_{t^{**}+1}} > 0$ provided α^{**} is sufficiently close to $\alpha^{t^{**}+1}_{t^{**}}$.*

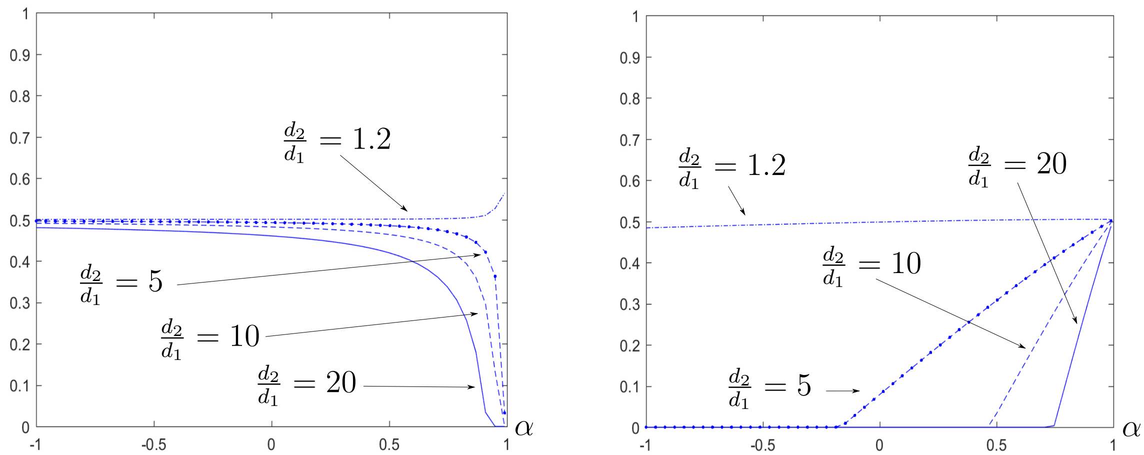
When the attention cost is high—perhaps due to long workdays—the agents pay little attention to information sources, giving suppliers weak incentives to enter. With few active sources, the marginal value of information is still high enough to justify its production cost for the planner (but not τ for the consumers). Thus, increasing the equilibrium supply improves welfare. By contrast, when attention is cheap—perhaps due to abundant free time or access to new technologies like the Internet—the agents pay lots of attention to information sources, incentivizing entry in large numbers. Eventually, the information value of the marginal entrant becomes very small and falls short of its production cost (but not τ!). In this case, reducing the equilibrium supply improves welfare. The possibility of *over*-supply of information indicates that the kind of inefficiency discovered here differs from the usual *under*-supply inefficiency in the IO literature, caused by consumers' not internalizing the full benefits of firms' entry.

Adding types of sources that are not supplied in equilibrium may also improve welfare—provided that they attract attention. Interestingly, if there is over-supply of the equilibrium type t^{**}, adding lower-clarity sources is never welfare improving, no matter what welfare weights γ we use. Only higher-clarity sources can improve welfare. This is because the consumers allocate too much attention due to its cheap cost; hence, welfare can improve only by adding sources with a higher return to attention. The proposition identifies an instance where adding sources of type $t^{**}+1$ strictly improves on $\mathbf{q}^{**}$.

It is worth emphasizing that the welfare effects of changing the supply of each signal type depend on the coordination motive. This is because α^{**} (but not τ) determines not only the equilibrium type t^{**}, but also which other types would receive attention if added to the supply and which attention cost would render this welfare improving. For instance, if α^{**} is very high so that $t^{**}=T$ and $\tau<\tau_T$, increasing the supply of any type decreases welfare.

These remarks are very important, as they reveal a novel and general cause of inefficiency for information markets: the fact that attention becomes the "currency" whereby consumers pay for information. To see why, note that this is a key difference from standard markets where consumers pay with "money." Also, in this model the suppliers of each type essentially generate a horizontal supply curve of level d_t (which is akin to a constant marginal cost of production for society). Therefore, if they faced a standard downward-sloping demand curve, the competitive equilibria would be efficient, because the price mechanism would convey to the trading sides the social marginal cost and benefit of the traded good. While prices flexibly adjust to serve this purpose, attention cannot because it is tied to its cost for the consumers. This is a distinctive aspect of information markets, where suppliers can monetize the attention that information consumes while being consumed. Granting suppliers more freedom of adjusting clarity might allow this dimension to play the role of a price. In reality, however, clarity need not be as adjustable as prices—due to granularity, technological constraints, or dependence on content—so that the inefficiencies found here may persist.

Proposition 5 shows the effects of changing the *equilibrium* supply of information. But what is the welfare-maximizing supply? Are equilibria inefficient only in terms of quantities or also in terms of the composition of source types? Answering these questions analytically is hard due to the intricate dependence of $\mathcal{P}(\mathbf{q};\gamma)$ on $\mathbf{q}$. Turning to numerical methods, we show that the efficient $\mathbf{q}$ can involve a positive quantity of multiple source types. Thus, while competition for attention pushes towards an homogeneous supply of information, welfare maximization calls for an heterogeneous supply. To see why, suppose there are two feasible types ($T = 2$). Consider a situation where there are only high-clarity sources, which cost less for the agents to acquire but more to produce. Since the marginal value of information is diminishing, at some point it equals the cost of the marginal high-clarity source (d_2). At this point, however, the marginal value of information can still exceed the cost of the cheaper low-clarity sources ($d_1 < d_2$). Hence, adding such sources increases welfare. On the other hand, a supply with only low-clarity sources cannot be efficient—at least for large α's—because high-clarity sources are superior coordination devices.



(a) Beauty-contest payoff (b) General payoff

Figure 2: Efficient supply

Figure 2 represents the efficient supply of information for different specifications of the agents' payoffs and the production costs. The curves describe the efficient share of high-clarity sources (i.e., $\frac{q_2}{q_2+q_1}$) as a function of $\alpha \in (-1, 1)$ for different values of d_2/d_1. Figure 2(a) is for the beauty-contest payoff specification of Myatt and Wallace (2012), which can be written as

$$u(k, K, \sigma, \theta) = -k^2 + 2\alpha kK - \alpha K^2 + 2(1-\alpha)k\theta - (1-\alpha)\theta^2$$

and by construction satisfies $\kappa = \kappa^{**}$, $\alpha = \alpha^{**}$, and $u_{\sigma\sigma} = 0$. Figure 2(b) is for a more general payoff function that satisfies

$$u(k, K, \sigma, \theta) = \frac{u_{kk}}{2}k^2 + u_{kK}kK + \frac{u_{KK}}{2}K^2 + u_{k\theta}k\theta,$$

where we impose that $u_{kK} = -u_{kk}\alpha$ and $u_{KK} = -u_{kK}$ to ensure that $\alpha^{**} = \alpha$. To ease the comparison with the beauty-contest case, we fix $u_{kk} = -2$. Also, we selected $u_{k\theta} = 0.3$.[27] Remarkably, the share of high-clarity sources is often interior and far from the equilibrium outcome. Also, the efficient supply can be qualitatively different depending on the nature of the agents' interaction. One important aspect is that in the beauty-contest case α controls *directly* the agents' desire to respond to the state as measured by $u_{k\theta} = 2(1-\alpha)$, which shrinks to zero as $\alpha \to 1$. By contrast, in the more general payoff $u_{k\theta}$ is independent of α. This explains the stark difference between the two panels in Figure 2.

5.3 Inefficient Use of Information and Supply of Information

Efficiency in the use and acquisition of information cannot be taken for granted. How do these demand-driven inefficiencies affect information supply? Recall that the efficient attention allocation is given by replacing α with α^{**}, κ_1 with κ_1^{**}, and u_{kk} with $u_{kk} + u_{\sigma\sigma}$ in the consumer equilibrium. Combining this with Proposition 1, we obtain the following.

Proposition 6. *Suppose $\alpha \neq \alpha_{t'}^{t}$ for all t, t'.*
• *Inefficiencies caused by complete-information externalities or social aversion to dispersion do not affect the equilibrium type of suppliers t^*. Considering its equilibrium quantity as a function of $(\kappa', u'_{\sigma\sigma}, \alpha')$, we have*

$$\overline{q}_{t*}(\kappa^{**}, 0, \alpha) > \overline{q}_{t^*}(\kappa, 0, \alpha) = q_{t^*}^{*} \quad \text{if and only if} \quad |\kappa_1^{**}| > |\kappa_1|,$$
$$\overline{q}_{t^*}(\kappa, u_{\sigma\sigma}, \alpha) > \overline{q}_{t^*}(\kappa, 0, \alpha) = q_{t^*}^{*} \quad \text{if and only if} \quad u_{\sigma\sigma} < 0.$$

• *Differences between private and socially optimal coordination motives can affect the equilibrium type of suppliers: If $\alpha < \alpha_{t^*}^{t^*+1} < \alpha^{**}$ (resp. $\alpha > \alpha_{t^*-1}^{t^*} > \alpha^{**}$), this inefficiency causes the equilibrium type t^* to be too opaque (resp. clear).*
• *If the equilibrium type is t^* under both α and α^{**}, we have*

$$\overline{q}_{t^*}(\kappa, 0, \alpha^{**}) < \overline{q}_{t^*}(\kappa, 0, \alpha) = q_{t^*}^{*} \quad \text{if and only if} \quad \alpha < \alpha^{**}.$$

Intuitively, any shortfall in the agents' willingness to acquire information lowers the attention revenues for suppliers and ultimately their equilibrium quantity. Such shortfalls can result from not internalizing the complete-information externalities, if this lowers the agents' desire to respond to the state (i.e., $|\kappa_1| < |\kappa_1^{**}|$). Similarly, not internalizing society's aversion to dispersion renders the agents less willing to pay attention to information sources, as doing so curbs the heterogeneity in beliefs that causes dispersion. An excessive coordination motive (i.e., $\alpha > \alpha^{**}$) leads the agents to overweight their

[27] We chose this value simply because it delivers a clear graph in Figure 2. Other simulations suggest that its qualitative properties are robust to changing $u_{k\theta}$ and allowing for $u_k \neq 0$, $u_K \neq 0$, or $u_{K\theta} \neq 0$.

prior when choosing their actions, which reduces the value of acquiring new information. When competition is via clarity, differences between the private and socially optimal coordination motive can also change the equilibrium type of suppliers, as that motive drives the agents' preference for clarity.

When competition is via accuracy, inefficiencies in the information demand affect only the supplied quantity of information, not its type. As Proposition 3 shows, α is irrelevant for which type of sources dominates the market. The distortions in the supplied quantities are qualitatively the same as in Proposition 6.

Finally, Proposition 6 shows how the supply of information changes if we remove inefficiencies in the use and acquisition of information. While doing so improves the agents' welfare *when information sources are fixed* (by definition), this need not be the case once their supply is endogenous.

6 Concluding Remarks

Information sources allow agents to learn about relevant events as well as about others' knowledge of those events, which is essential for coordination to occur in social and economic contexts. We examine how the dual role of information sources as learning and coordination devices affect how information markets work, in contexts where suppliers compete for consumers' attention. The paper offers two main takeaways. First, competition for attention pushes towards a homogenization of information sources, whose type is driven by the consumers' desires to coordinate in combination with the sources' clarities. Second, by becoming the "currency" whereby consumers pay for information, attention causes information markets to work inefficiently. This novel source of inefficiency is on top of those due to inefficient uses of information and suppliers' market power.

We close with three remarks. First, to focus on the consequences of competition for attention, this paper intentionally rules out price competition. Though in many settings information providers seem to optimally set zero prices for accessing their sources, this is not always the case. In future research, we plan to investigate how adding price competition affects information markets. On the one hand, it would be interesting to examine the trade-off that suppliers face between attention and price revenues and its solution under competition. On the other hand, it would be interesting to understand if and when the price mechanism can restore efficiency in competitive information markets. We conjecture that as long as attention provides a share of the revenues of information sources—which is often the case in reality—the inefficiencies we found in this paper should persist to some degree.

Second, a promising line for future research is to investigate policies that can address the market inefficiencies caused by the fact that the traded good is information. Angeletos and Pavan (2009) propose policies that can address inefficiencies originating in the demand side of the market. It would be interesting to consider policies directed at the supply side (e.g., entry regulations) that aim to address both the demand-side inefficiencies and the novel kind of inefficiencies found in this paper.

Finally, one could extend the model analyzed in this paper in several directions to offer a more realistic description of information markets. Real-world consumers of information are not identical. They differ in terms of conformism, topics of interest, and abilities or available time for acquiring information. Some of these aspects can be easily incorporated in the framework of this paper without changing its thrust. For instance, as suggested by Dewan and Myatt (2008, 2012), one can interpret the state θ not as the same ideal action for all agents in society, but as the average ideal action. Angeletos and Pavan (2009) allow for the possibility of multidimensional states with private and common components. In some cases, different topics and social groups give rise to different markets, where each can be studied using our model. Other extensions are more intricate. For instance, allowing for different coordination motives or attention costs within a single market have deep consequences already for the demand of information, which remain unexplored. Either way, allowing for heterogeneity of consumers is likely to result in heterogeneity of information sources. This should not be surprising. Yet, the point remains that, within market segments, competition for attention creates a force towards homogeneity and generic inefficiencies.

A Appendix: Proofs

A.1 Proof of Lemma 1

For $t \in T_e$, let $\delta_t = e_t c_t^2$, $b_t(\delta_t) = \frac{1-\alpha}{(1-\alpha)a_t^{-1}+\delta_t^{-1}}$, and $B = p + \sum_{t \in T_e} q_t b_t$, where we omit the dependence on δ_t for simplicity. Given this, equation (17) in Appendix B becomes

$$\delta_t = \frac{\phi}{\sqrt{\tau}} \frac{b_t}{B} c_t. \tag{9}$$

Substituting (9) in the expression for b_t yields

$$b_t = a_t - \frac{B a_t \sqrt{\tau}}{(1-\alpha)\phi c_t}. \tag{10}$$

It follows that

$$B - p = \sum_{t\in T_e} q_t b_t = \sum_{t\in T_e} a_t q_t - \frac{B\sqrt{\tau}}{(1-\alpha)\phi}\sum_{t\in T_e}\frac{a_t q_t}{c_t},$$

which yields

$$B = (p + \sum_{t\in T_e} a_t q_t)\left[1 + \frac{\sqrt{\tau}}{(1-\alpha)\phi}\sum_{t\in T_e}\frac{a_t q_t}{c_t}\right]^{-1}. \tag{11}$$

Now use (10) and (11) to replace in the expression of δ_t to obtain

$$\begin{aligned}
\delta_t &= \left[\left(1 + \frac{\sqrt{\tau}}{(1-\alpha)\phi}\sum_{t'\in T_e}\frac{a_{t'}q_{t'}}{c_{t'}}\right)\frac{a_t}{p + \sum_{t'\in T_e} a_t q_t} - \frac{a_t\sqrt{\tau}}{(1-\alpha)\phi c_t}\right]\frac{\phi}{\sqrt{\tau}}c_t \\
&= \left[1 + \frac{\sqrt{\tau}}{(1-\alpha)\phi}\sum_{t'\in T_e}\frac{a_{t'}q_{t'}}{c_{t'}} - \frac{p + \sum_{t'\in T_e} a_{t'}q_{t'}}{(1-\alpha)\phi c_t/\sqrt{\tau}}\right]\frac{a_t\phi c_t/\sqrt{\tau}}{p + \sum_{t'\in T_e} a_{t'}q_{t'}} \\
&= \left[1 + \frac{\sqrt{\tau}}{\phi}\sum_{t'\in T_e\setminus\{t\}}\frac{a_{t'}q_{t'}}{1-\alpha}(c_{t'}^{-1} - c_t^{-1}) - \frac{p\sqrt{\tau}}{(1-\alpha)\phi c_t}\right]\frac{a_t\phi c_t/\sqrt{\tau}}{p + \sum_{t'\in T_e} a_{t'}q_{t'}} \\
&= \left[\frac{\phi}{\sqrt{\tau}} + \sum_{t'\in T_e\setminus\{t\}}\frac{a_{t'}q_{t'}}{1-\alpha}(c_{t'}^{-1} - c_t^{-1}) - \frac{p}{(1-\alpha)c_t}\right]\frac{a_t c_t}{p + \sum_{t'\in T_e} a_{t'}q_{t'}}.
\end{aligned}$$

Finally, using the definition of δ_t and ϕ, we obtain

$$e_t(\mathbf{q}) = \frac{a_t c_t^{-1}}{p + \sum_{t'\in T_e} a_{t'}q_{t'}}\left[\frac{\phi}{\sqrt{\tau}} + \sum_{t'\in T_e\setminus\{t\}}\frac{a_{t'}q_{t'}}{1-\alpha}(c_{t'}^{-1} - c_t^{-1}) - \frac{pc_t^{-1}}{1-\alpha}\right]. \tag{12}$$

A.2 Proof of Corollary 1

The first part of the corollary is immediate from (12) and Assumption 1. Now, suppose $c_t > c_{t'}$ for all $t > t'$. Assumption 1 and part 5 of Lemma 4 imply that $e_{t_1}(\mathbf{q}) > 0$ whenever t_1 is the largest type in $T_{\mathbf{q}}$. Part 5 of Lemma 4 also implies that if $t \in T_e$, than all higher types belong to T_e. We now show when $T_e = \{t_1\}$.

Claim 1. Let t_2 be the second largest type in $T_{\mathbf{q}}$. For every $\alpha < 1$, we have $e_t(\mathbf{q}) = 0$ for all $t \in T_{\mathbf{q}} \setminus \{t_1\}$ if and only if $q_{t_1} \geq q_{t_1}(\alpha)$ where

$$q_{t_1}(\alpha) = \left[\frac{\phi}{\sqrt{\tau}}(1-\alpha)c_{t_2} - p\right]\frac{c_{t_1}a_{t_1}^{-1}}{c_{t_1} - c_{t_2}}.$$

Proof. (Only if) Suppose $e_t = 0$ for all $t \in T_{\mathbf{q}} \setminus \{t_1\}$. Then, by part 2 of Lemma 4, e_{t_1} must satisfy

$$e_{t_1} = \frac{a_{t_1}c_{t_1}^{-1}}{p + a_{t_1}q_{t_1}}\left[\frac{\phi}{\sqrt{\tau}} - \frac{pc_{t_1}^{-1}}{1-\alpha}\right]. \tag{13}$$

Also, by part 3 of Lemma 4, it must be that

$$
\begin{aligned}
\frac{\tau}{\phi^2 c_t^2} &\geq \left[\frac{p}{1-\alpha} + \frac{1}{(1-\alpha)a_{t_1}^{-1}q_{t_1}^{-1} + c_{t_1}^{-2}e_{t_1}^{-1}q_{t_1}^{-1}}\right]^{-2} = \left[\frac{p}{1-\alpha} + \left(\frac{1-\alpha}{a_{t_1}q_{t_1}} + \frac{1}{q_{t_1}e_{t_1}c_{t_1}^2}\right)^{-1}\right]^{-2} \\
&= \left[\frac{p}{1-\alpha} + a_{t_1}q_{t_1}\frac{\frac{\phi}{\sqrt{\tau}}c_{t_1} - \frac{p}{1-\alpha}}{(1-\alpha)\frac{\phi}{\sqrt{\tau}}c_{t_1} + a_{t_1}q_{t_1}}\right]^{-2},
\end{aligned}
$$

where we used the expression for e_{t_1} in (13). Let $q_t(\alpha)$ be defined as the unique value of q_t such that the previous inequality holds with equality:

$$
q_t(\alpha) = \left[\frac{\phi}{\sqrt{\tau}}(1-\alpha)c_t - p\right]\frac{c_{t_1}a_{t_1}^{-1}}{c_{t_1} - c_t} > 0, \tag{14}
$$

where the inequality follows from Assumption 1. It is then immediate that $e_t = 0$ for all $t < t_1$ implies $q_{t_1} \geq \max_{t<t_1} q_t(\alpha) = q_{t_1}(\alpha)$. Note that $q_{t_1}(\alpha)$ is strictly decreasing in α and reaches zero for some $\alpha < 1$.

(If) Let $q_{t_1} \geq q_{t_1}(\alpha)$. We reason by induction. Let $\underline{t}$ be the smallest $t \in T_{\mathbf{q}}$ and suppose $e_{\underline{t}} > 0$. Since $e_{t_1} > 0$, by expression (12) $e_{\underline{t}}$ must satisfy

$$
\begin{aligned}
e_{\underline{t}} &= \frac{a_{\underline{t}}c_{\underline{t}}^{-2}}{p + \sum_{t' \in T_e} a_{t'}q_{t'}}\left[\frac{\phi}{\sqrt{\tau}}c_{\underline{t}} + \frac{1}{1-\alpha}\sum_{t' \in T_e \setminus \{t\}} a_{t'}q_{t'}\left(\frac{c_{\underline{t}}}{c_{t'}} - 1\right) - \frac{p}{1-\alpha}\right] \\
&\leq \frac{a_{\underline{t}}c_{\underline{t}}^{-2}}{p + \sum_{t' \in T_e} a_{t'}q_{t'}}\left[\frac{\phi}{\sqrt{\tau}}c_{\underline{t}} + \frac{1}{1-\alpha}a_{t_1}q_{\underline{t}}(\alpha)\left(\frac{c_{\underline{t}}}{c_{t_1}} - 1\right) - \frac{p}{1-\alpha}\right] \\
&= \frac{a_{\underline{t}}c_{\underline{t}}^{-2}}{p + \sum_{t' \in T_e} a_{t'}q_{t'}}\left[\frac{\phi}{\sqrt{\tau}}c_{\underline{t}} - \frac{\phi}{\sqrt{\tau}}c_{\underline{t}}\right] = 0,
\end{aligned}
$$

where $q_{\underline{t}}(\alpha)$ is defined in (14). This contradicts $e_{\underline{t}} > 0$. Now suppose that $e_{t''} = 0$ for all $t'' < t$ in $T_{\mathbf{q}}$ and $e_t > 0$. Then, e_t must satisfy

$$
\begin{aligned}
e_t &= \frac{a_t c_t^{-2}}{p + \sum_{t' \in T_e} a_{t'}q_{t'}}\left[\frac{\phi}{\sqrt{\tau}}c_t + \frac{1}{1-\alpha}\sum_{t' \in T_e \setminus \{t\}} a_{t'}q_{t'}\left(\frac{c^1}{c_{t'}} - 1\right) - \frac{p}{1-\alpha}\right] \\
&\leq \frac{a_t c_t^{-2}}{p + \sum_{t' \in T_e} a_{t'}q_{t'}}\left[\frac{\phi}{\sqrt{\tau}}c_t + \frac{1}{1-\alpha}a_{t_1}q_t(\alpha)\left(\frac{c_t}{c_{t_1}} - 1\right) - \frac{p}{1-\alpha}\right] \\
&= \frac{a_t c_t^{-2}}{p + \sum_{t' \in T_e} a_{t'}q_{t'}}\left[\frac{\phi}{\sqrt{\tau}}c_t - \frac{\phi}{\sqrt{\tau}}c_t\right] = 0,
\end{aligned}
$$

where the inequality follows because T_e contains only types larger than t and $q_t(\alpha) < q_{t_1}(\alpha)$.

□

A.3 Proof of Proposition 1

Given $\mathbf{q}$, let $Q = \sum_{t=1}^{T} q_t$ and for all t define

$$\mu_t(\mathbf{q}) = \frac{1}{p + aQ}\left[\frac{\phi}{\sqrt{\tau}} - \frac{pc_t^{-1}}{1-\alpha} + \frac{a}{1-\alpha}\sum_{t' \neq t} q_{t'}(c_{t'}^{-1} - c_t^{-1})\right] - \frac{c_t d(c_t)}{a}.$$

Note that $\mu_t(q_t, \mathbf{q}_{-t})$ is strictly decreasing in q_t for all $\mathbf{q}_{-t}$. Also, $\mu_t(\mathbf{q}) > \mu_{t'}(\mathbf{q})$ if and only if

$$\frac{1}{p + aQ}\left\{\frac{a}{1-\alpha}\left[\sum_{\hat{t} \neq t} q^{\hat{t}}(c_{\hat{t}}^{-1} - c_t^{-1}) - \sum_{\hat{t} \neq t'} q^{\hat{t}}(c_{\hat{t}}^{-1} - c_{t'}^{-1})\right] + \frac{p}{1-\alpha}(c_{t'}^{-1} - c_t^{-1})\right\}$$

is strictly larger than $\frac{c_t d(c_t) - c_{t'} d(c_{t'})}{a}$. The quantity in square brackets equals

$$\begin{aligned}
&\sum_{\hat{t} \neq t,t'} q^{\hat{t}}(c_{\hat{t}}^{-1} - c_t^{-1}) + q_{t'}(c_{t'}^{-1} - c_t^{-1}) - \sum_{\hat{t} \neq t',t} q^{\hat{t}}(c_{\hat{t}}^{-1} - c_{t'}^{-1}) - q_t(c_t^{-1} - c_{t'}^{-1}) \\
&= \sum_{\hat{t} \neq t,t'} q^{\hat{t}}(c_{t'}^{-1} - c_t^{-1}) + q_{t'}(c_{t'}^{-1} - c_t^{-1}) + q_t(c_{t'}^{-1} - c_t^{-1}) = (c_{t'}^{-1} - c_t^{-1})Q.
\end{aligned}$$

Therefore, $\mu_t(\mathbf{q}) > \mu_{t'}(\mathbf{q})$ if and only if

$$\frac{c_{t'}^{-1} - c_t^{-1}}{1-\alpha} > \frac{c_t d(c_t) - c_{t'} d(c_{t'})}{a},$$

which holds if and only if $\alpha > \alpha_{t'}^{t}$.

Note that the zero-profit condition (2) is equivalent to $q_t^* \mu_t(q_t^*, \mathbf{q}_{-t}^*) = 0$ and the no-entry condition (3) is equivalent to $q_t \mu_t(q_t, \mathbf{q}_{-t}^*) < 0$ if $q_t > q_t^*$.

Case 1: $\alpha \neq \alpha_{t'}^{t}$ for all t and t'.

Claim 2. If $\mathbf{q}^*$ is an equilibrium, then $T_{\mathbf{q}^*} = \{t : q_t^* > 0\}$ is a singleton.

Proof. We know from Assumption 1 that $\mathbf{q} = \mathbf{0}$ cannot be an equilibrium. Suppose that $|T_{\mathbf{q}^*}| \geq 2$. Then, for every $t \in T_{\mathbf{q}^*}$ we must have $\mu_t(\mathbf{q}^*) = 0$. Take any $t, t' \in T_{\mathbf{q}^*}$ with $t > t'$. If $\alpha > \alpha_{t'}^{t}$, then $\mu_t(\mathbf{q}^*) > \mu_{t'}(\mathbf{q}^*) = 0$, which is a contradiction. If $\alpha < \alpha_{t'}^{t}$, then $\mu_t(\mathbf{q}^*) < \mu_{t'}(\mathbf{q}^*) = 0$, which is also a contradiction.

□

Claim 3. If $\mathbf{q}^*$ is an equilibrium and $\{t^*\} = T_{\mathbf{q}^*}$, then $q_{t^*}^* = \overline{q}_{t^*}$ and $\alpha_{t^*-1}^{t^*} < \alpha < \alpha_{t^*}^{t^*+1}$.

Proof. Since $\{t^*\} = T_{\mathbf{q}^*}$ means that $q_{t^*}^* > 0$, it must be that $q_{t^*}^* = \overline{q}_{t^*}$ by the definition of the latter as the unique positive solution to $E_{t^*}(q_{t^*}, \mathbf{0}) = q_{t^*} d(c_{t^*})$. This also implies that

$\mu_{t^*}(\overline{q}_{t^*}, \mathbf{0}) = 0$. Suppose $t^* < T$, so that suppliers of type t^*+1 would receive positive attention upon entering by Corollary 1. By condition (3), it must be that $q_{t^*+1}\mu_{t^*+1}(\overline{q}_{t^*}, q_{t^*+1}, \mathbf{0}) < 0$ for all $q_{t^*+1} > 0$, where $\mathbf{0}$ has $T-2$ elements. Therefore, $\mu_{t^*+1}(\overline{q}_{t^*}, q_{t^*+1}, \mathbf{0}) < 0$ for all $q_{t^*+1} > 0$. This holds if and only if $\mu_{t^*+1}(\overline{q}_{t^*}, 0, \mathbf{0}) \leq 0$ by the continuity and monotonicity of μ_{t^*+1} in q_{t^*+1}. By the previous calculations we know that $\mu_{t^*+1}(\overline{q}_{t^*}, \mathbf{0}) \leq 0 = \mu_{t^*}(\overline{q}_{t^*}, \mathbf{0})$ if and only if $\alpha \leq \alpha_{t^*}^{t^*+1}$. Since $\alpha \neq \alpha_{t^*}^{t^*+1}$, we established one inequality. If $t^* = 1$, we are done.

Now consider t^*-1 (if any). There are two possibilities. First, suppliers of type t^*-1 cannot attract attention, which requires that $\overline{q}_{t^*} \geq q_{t^*}(\alpha)$ by Corollary 1. This holds if and only if

$$\alpha \geq \overline{\alpha}^{t^*} \equiv 1 - a\frac{c_{t^*-1}^{-1} - c_{t^*}^{-1}}{c_{t^*}d(c_{t^*})}.$$

Second, suppliers of type t^*-1 would attract attention upon entering. In this case, by the same argument as before, it must be that $\mu_{t^*-1}(\overline{q}_{t^*}, 0, \mathbf{0}) \leq 0$. Substituting $\overline{q}_{t^*}$, we get that this condition is equivalent to $\alpha \geq \alpha_{t^*-1}^{t^*}$. Since $\overline{\alpha}^{t^*} > \alpha_{t^*-1}^{t^*}$ and $\alpha \neq \alpha_{t^*-1}^{t^*}$, we established the other inequality.

□

Claim 4. If $\alpha_{t^*-1}^{t^*} < \alpha < \alpha_{t^*}^{t^*+1}$, $\mathbf{q}^*$ with $\{t^*\} = T_{\mathbf{q}^*}$ and $q_{t^*}^* = \overline{q}_{t^*}$ is the unique equilibrium.

Proof. As shown in Proposition 2 (whose proof is next), $\alpha_{t'}^{t}$ is increasing in both c_t and $c_{t'}$. Therefore, $\alpha_{t^*-1}^{t^*} < \alpha < \alpha_{t^*}^{t^*+1}$ implies $\alpha_t^{t^*} < \alpha < \alpha_{t^*}^{t'}$ for all $t < t^* < t'$. Consequently, $\mu_t(\overline{q}_{t^*}, \mathbf{0}) < 0$ for all $t \neq t^*$. By the monotonicity property of μ_t in q_t, it follows that $\mu_t(\overline{q}_{t^*}, q_t, \mathbf{0}) < 0$ for all $q_t > 0$ and $t \neq t^*$. Thus, for every t, t-suppliers strictly prefer not to enter because, even if they could attract attention, they would make strictly negative profits. By definition, $\overline{q}_{t^*}$ is the unique solution to $q_{t^*}\mu_{t^*}(q_{t^*}, \mathbf{0}) = 0$; hence, $\mathbf{q}^* = (\overline{q}_{t^*}, \mathbf{0})$ is the unique equilibrium.

□

Case 2: If $\alpha = \alpha_{t-1}^{t}$ for some t.

Claim 5. If $\mathbf{q}^*$ is an equilibrium, $t' \notin T_{\mathbf{q}^*}$ for all $t' \neq t, t-1$.

Proof. If $\alpha = \alpha_{t-1}^{t}$ for some t (which cannot be $t = 1$), $\mu_t(\mathbf{q}) = \mu_{t-1}(\mathbf{q})$ for all $\mathbf{q}$. Also, since $\alpha_{t-1}^{t} < \alpha_{t-1}^{t'}$ for all $t' > t$ by Proposition 2, $\mu_{t'}(\mathbf{q}) < \mu_t(\mathbf{q})$ for all $\mathbf{q}$ and $t' > t$. Similarly, since $\alpha_{t-1}^{t} > \alpha_{t'}^{t}$ for all $t' < t-1$ by Proposition 2, $\mu_{t'}(\mathbf{q}) < \mu_{t-1}(\mathbf{q})$ for all $\mathbf{q}$ and $t' < t-1$. Therefore, if $t' < t-1$ belonged to $T_{\mathbf{q}^*}$, we would have to have $0 = \mu_{t'}(\mathbf{q}^*) < \mu_{t-1}(\mathbf{q}^*)$. If $q_{t-1}^* > 0$, this immediately contradicts the zero-profit condition for type $t-1$. If $q_{t-1}^* = 0$, continuity of μ_{t-1} in q_{t-1} implies that there exists $q_{t-1} > 0$ such that $\mu_{t-1}(q_{t-1}, \mathbf{q}_{-(t-1)}^*) > 0$, now contradicting the condition (3) for type $t-1$. A similar argument rules out that $t' > t$ belongs to $T_{\mathbf{q}^*}$.

□

Claim 6. For every $q_{t-1} \in [0, \overline{q}_{t-1}]$, there exists an equilibrium with $q^*_{t-1} = q_{t-1}$ and $q^*_t = \frac{c_{t-1}d(c_{t-1})}{c_t d(c_t)}[\overline{q}_{t-1} - q_{t-1}]$.

Proof. Consider $q_{t-1} = \overline{q}_{t-1}$. By definition of $\overline{q}_{t-1}$, $\mu_{t-1}(\overline{q}_{t-1}, \mathbf{0}) = 0$. By $\alpha = \alpha^t_{t-1}$, $\mu_t(\overline{q}_{t-1}, \mathbf{0}) = 0$. By monotonicity of μ_t in q_t, $\mu_t(\overline{q}_{t-1}, q_t, \mathbf{0}) < 0$ for all $q_t > 0$. Therefore, no t-suppliers want to enter. Consider $q_{t-1} = 0$. In this case,

$$\begin{aligned} q_t &= \frac{c_{t-1}d(c_{t-1})}{c_t d(c_t)} \left[\frac{1}{c_{t-1}d(c_{t-1})} \left(\frac{\phi}{\sqrt{\tau}} - \frac{p c_{t-1}^{-1}}{1-\alpha} \right) - \frac{p}{a} \right] \\ &= \frac{1}{c_t d(c_t)} \left(\frac{\phi}{\sqrt{\tau}} - \frac{p c_{t-1}^{-1}}{1-\alpha} + \frac{p c_t^{-1}}{1-\alpha} - \frac{p c_t^{-1}}{1-\alpha} \right) - \frac{c_{t-1}d(c_{t-1})}{c_t d(c_t)} \frac{p}{a} \\ &= \frac{1}{c_t d(c_t)} \left(\frac{\phi}{\sqrt{\tau}} - \frac{p c_t^{-1}}{1-\alpha} \right) - \frac{p}{a c_t d(c_t)} \left[\frac{c_{t-1}^{-1} - c_t^{-1}}{1-\alpha} - c_{t-1}d(c_{t-1}) \right] \\ &= \frac{1}{c_t d(c_t)} \left(\frac{\phi}{\sqrt{\tau}} - \frac{p c_t^{-1}}{1-\alpha} \right) - \frac{p}{a} = \overline{q}_t, \end{aligned}$$

where the forth equality uses $\alpha = \alpha^t_{t-1}$. By the same argument as before, given this q_t, no supplier of type $t-1$ wants to enter. Consider now $q_{t-1} \in (0, \overline{q}_t)$. By definition of $\overline{q}_{t-1}$, we know that $\mu_{t-1}(q_{t-1}, \mathbf{0}) > 0$. Therefore, to have an equilibrium, q_t must be positive and induce $\mu_{t-1}(q_t, q_{t-1}, \mathbf{0}) = \mu_t(q_t, q_{t-1}, \mathbf{0}) = 0$. This is equivalent to

$$c_t d(c_t) \left[\frac{p}{a} + q_t + q_{t-1} \right] = \frac{\phi}{\sqrt{\tau}} - \frac{p c_t^{-1}}{1-\alpha} + \frac{c_{t-1}^{-1} - c_t^{-1}}{1-\alpha} a q_{t-1},$$

or

$$c_t d(c_t) \left[\frac{p}{a} + q_t + q_{t-1} \right] = \frac{\phi}{\sqrt{\tau}} - \frac{p c_{t-1}^{-1}}{1-\alpha} + a \left[\frac{p}{a} + q_{t-1} \right] \frac{c_{t-1}^{-1} - c_t^{-1}}{1-\alpha}.$$

Using $\alpha = \alpha^t_{t-1}$, this becomes

$$c_t d(c_t) \left[\frac{p}{a} + q_t + q_{t-1} \right] = \frac{\phi}{\sqrt{\tau}} - \frac{p c_{t-1}^{-1}}{1-\alpha} + \left[\frac{p}{a} + q_{t-1} \right] [c_t d(c_t) - c_{t-1}d(c_{t-1})],$$

or

$$q_t = \frac{c_{t-1}d(c_{t-1})}{c_t d(c_t)} \left[\frac{1}{c_{t-1}d(c_{t-1})} \left(\frac{\phi}{\sqrt{\tau}} - \frac{p c_{t-1}^{-1}}{1-\alpha} \right) - \frac{p}{a} - q_{t-1} \right] = \frac{c_{t-1}d(c_{t-1})}{c_t d(c_t)} \left[\overline{q}_{t-1} - q_{t-1} \right].$$

Finally, given the suggested quantities of types t and $t-1$, no other type of suppliers want to enter the market. Since $\mu_t(q_t, q_{t-1}, 0) = \mu_{t-1}(q_t, q_{t-1}, 0) = 0$ and $\alpha = \alpha^t_{t-1}$, every type $t' \neq t, t-1$ will make negative profits after entering.

$\square$

A.4 Proof of Proposition 2

The first part of the proposition is immediate. For the second part, fix $t > t'$. Suppose $\hat{c}_t > c_t > c_{t'}$. Then, $\hat{\alpha}^t_{t'} > \alpha^t_{t'}$ if and only if

$$\frac{c_{t'}^{-1} - \hat{c}_t^{-1}}{\hat{c}_t d(\hat{c}_t) - c_{t'} d(c_{t'})} < \frac{c_{t'}^{-1} - c_t^{-1}}{c_t d(c_t) - c_{t'} d(c_{t'})},$$

or equivalently

$$c_t \left[\frac{d(c_t) - d(c_{t'})}{c_t - c_{t'}} c_t + d(c_{t'}) \right] < \hat{c}_t \left[\frac{d(\hat{c}_t) - d(c_{t'})}{\hat{c}_t - c_{t'}} \hat{c}_t + d(c_{t'}) \right].$$

By convexity of d, we have $\frac{d(c_t)-d(c_{t'})}{c_t - c_{t'}} \leq \frac{d(\hat{c}_t)-d(c_{t'})}{\hat{c}_t - c_{t'}}$. Since $\hat{c}_t > c_t$, the inequality holds. Now, suppose $c_t > \hat{c}_{t'} > c_{t'}$. Then, $\hat{\alpha}^t_{t'} > \alpha^t_{t'}$ if and only if

$$\frac{\hat{c}_{t'}^{-1} - c_t^{-1}}{c_t d(c_t) - \hat{c}_{t'} d(\hat{c}_{t'})} < \frac{c_{t'}^{-1} - c_t^{-1}}{c_t d(c_t) - c_{t'} d(c_{t'})},$$

or equivalently

$$c_{t'} \left[c_{t'} \frac{d(c_t) - d(c_{t'})}{c_t - c_{t'}} + d(c_t) \right] < \hat{c}_{t'} \left[\hat{c}_{t'} \frac{d(c_t) - d(\hat{c}_{t'})}{c_t - \hat{c}_{t'}} + d(c_t) \right].$$

Using convexity of d, the same argument as before establishes that this inequality holds.

A.5 Proof of Proposition 3

We can write the equilibrium equations (2)–(3) as $q_t^* \mu_t(q_t^*, \mathbf{q}^*_{-t}) = 0$ and $q_t \mu_t(q_t, \mathbf{q}^*_{-t}) < 0$ if $q_t > q_t^*$, where now

$$\mu_t(\mathbf{q}) = \frac{1}{p + \sum_{t=1}^T a_t q_t} \left[\frac{\phi}{\sqrt{\tau}} - \frac{pc^{-1}}{1-\alpha} \right] - c \frac{d(a_t)}{a_t}.$$

Note that, for every $\mathbf{q}$, $\mu_t(\mathbf{q}) > \mu_{t'}(\mathbf{q})$ if and only if $\frac{d(a_t)}{a_t} < \frac{d(a_{t'})}{a_{t'}}$. Also, for all t, $\mu_t(q_{t'}, \mathbf{q}_{-t'})$ is strictly decreasing in $q_{t'}$ for every t'.

By Assumption 1, we know that there does not exist an equilibrium with $\mathbf{q}^* = \mathbf{0}$. By assumption $\arg\min_t \frac{d(a_t)}{a_t} = \{t^*\}$. If $\mathbf{q}^*$ is an equilibrium, we must have $\{t^*\} = T_{\mathbf{q}^*}$. If not, $q_t^* > 0$ for some $t \neq t^*$ and $q_t \mu_t(\mathbf{q}^*) = 0$, which implies that $\mu_t(\mathbf{q}^*) = 0$. But then, by continuity, there exists $q_{t^*} > 0$ such that $\mu_{t^*}(q_{t^*}, \mathbf{q}^*_{-t^*}) > 0$ and hence $q_{t^*} \mu_{t^*}(q_{t^*}, \mathbf{q}^*_{-t^*}) > 0$, which contradicts condition (3). This shows that $t^* \in T_{\mathbf{q}^*}$. Given this, we must have $\mu_{t^*}(\mathbf{q}^*) = 0$ by condition (2). Since then $\mu_t(\mathbf{q}^*) < 0$ for every other t, it follows that $t \notin T_{\mathbf{q}^*}$ for all $t \neq t^*$.

It follows immediately that $q^*_{t^*} = \overline{q}_{t^*}$ and $q^*_t = 0$ for all $t \neq t^*$ is an equilibrium. By the previous points, this is also the unique equilibrium.

A.6 Proof of Propositions 4 and 5

We start by unpacking the components of the planner's objective function. Recall that the symmetric consumer equilibria considered in this paper ultimately depend on the total attention allocated to each type of sources, that is, $\mathbf{E} = (E_1, \ldots, E_T)$.

Lemma 3. *Given* $\mathbf{q}$ *and* $\mathbf{E}$*, let* $W^{**}_t(\mathbf{E}) = q_t w^{**}_t\left(\frac{E_1}{q_1}, \ldots, \frac{E_T}{q_T}\right)$ *where* w^{**}_t *is the weight assigned to each* t*-signal by the efficient strategy* $k^{**}(\cdot;\cdot)$.[28] *Then, we have*

$$Var[k - K|\mathbf{E}, k^{**}(\cdot;\mathbf{E})] = [\kappa^{**}_1]^2 \sum_{t=1}^{T} \frac{[W^{**}_t(\mathbf{E})]^2}{c^2_t E_t}$$

$$Var[K - \kappa^{**}|\mathbf{E}, k^{**}(\cdot;\mathbf{E})] = [\kappa^{**}_1]^2 \left\{ \sum_{t=1}^{T} \frac{[W^{**}_t(\mathbf{E})]^2}{a q_t} + \frac{[W^{**}_p(\mathbf{E})]^2}{p} \right\},$$

where $W^{**}_p(\mathbf{E}) = 1 - \sum_{t=1}^{T} W^{**}_t(\mathbf{E})$.

Proof. Suppose first that, for all t, q_t is a fixed non-negative integer. When each consumer n follows the efficient strategy $k^{**}(\mathbf{s}^n;\mathbf{E})$, we have

$$k^{**}(\mathbf{s}^n;\mathbf{E}) = \kappa^{**}_0 + \kappa^{**}_1 \sum_{t=1}^{T} \sum_{m \in q_t} w^{**}_t\left(\frac{E_1}{q_1}, \ldots, \frac{E_T}{q_T}\right) s^{n,m} = \kappa^{**}_0 + \kappa^{**}_1 \sum_{t=1}^{T} W^{**}_t(\mathbf{E}) S^n_t,$$

where $W^{**}_t(\mathbf{E}) = q_t w^{**}_t\left(\frac{E_1}{q_1}, \ldots, \frac{E_T}{q_T}\right)$ and (if $q^i > 0$)

$$S^n_t = \frac{1}{q_t} \sum_{m \in q_t} s^{n,m} = \frac{1}{q_t} \sum_{m \in q_t} (\theta + y^m + x^{n,m}) = \theta + \frac{1}{q_t} \sum_{m \in q_t} (y^m + x^{n,m}).$$

Given this, the average action satisfies

$$K^{**}((\mathbf{s}^n)_{n \in [0,1]};\mathbf{E}) = \int_0^1 k^{**}(\mathbf{s}^n;\mathbf{E}) dn = \kappa^{**}_0 + \kappa^{**}_1 \sum_{t=1}^{T} W^{**}_t(\mathbf{E}) \int_0^1 S^n_t dn,$$

where (again, if $q_t > 0$)

$$\int_0^1 S^n_t dn = \theta + \frac{1}{q_t} \sum_{m \in q_t} \left(y^m + \int_0^1 x^{n,m} dn\right) = \theta + \frac{1}{q_t} \sum_{m \in q_t} y^m,$$

[28] In this lemma, we adopt the convention that $\frac{0}{0} = 0$ to simplify its statement.

since with i.i.d. error terms and a continuum of agents $\int_0^1 x^{n,m} dn$ equals the mean of each $x^{n,m}$, which is zero. Thus, letting $X_t^n = \frac{1}{q_t}\sum_{m \in q_t} x^{n,m}$, we have

$$[k^{**} - K^{**}]((\mathbf{s}^n)_{n\in[0,1]}; \mathbf{E}) = \kappa_1^{**}\left[\sum_{t=1}^{T} W_t^{**}(\mathbf{E})\left(S_t^n - \int_0^1 S_t^n dn\right)\right] = \kappa_1^{**}\sum_{t=1}^{T} W_t^{**}(\mathbf{E}) X_t^n.$$

Since for each t the variable X_t^n is normally distributed with mean zero and variance $[c_t^2 E_t]^{-1}$ and X_t^n is independent of $X_{t'}^n$ for $t \neq t'$, we obtain

$$Var[k - K|\mathbf{E}, k^{**}(\cdot;\mathbf{E})] = [\kappa_1^{**}]^2 \sum_{t=1}^{T} \frac{[W_t^{**}(\mathbf{E})]^2}{c_t^2 E_t}.$$

Now letting $Y_t = \frac{1}{q_t}\sum_{m\in q_t} y^m$, we have

$$\begin{aligned} [K^{**} - \kappa^{**}]((\mathbf{s}^n)_{n\in[0,1]}; \mathbf{E}) &= \kappa_1^{**}\left[\sum_{t=1}^{T} W_t^{**}(\mathbf{E})\left(\int_0^1 S_t^n dn\right) - \theta\right] \\ &= \kappa_1^{**}\left[\sum_{t=1}^{T} W_t^{**}(\mathbf{E})\left(\theta + Y_t\right) - \theta\right] \\ &= \kappa_1^{**}\left[\sum_{t=1}^{T} W_t^{**}(\mathbf{E}) Y_t - W_p^{**}(\mathbf{E})\theta\right], \end{aligned}$$

where $W_p^{**}(\mathbf{E}) = 1 - \sum_{t=1}^{T} W_t^{**}(\mathbf{E})$. Since $Y_t \sim N(0, a^{-1}q_t^{-1})$ for each t, $\theta \sim N(0, p^{-1})$, and θ and the Y_t's are mutually independent, we obtain

$$Var[K - \kappa^{**}|\mathbf{E}, k^{**}(\cdot;\mathbf{E})] = [\kappa_1^{**}]^2 \left\{\sum_{t=1}^{T} \frac{[W_t^{**}(\mathbf{E})]^2}{a q_t} + \frac{[W_p^{**}(\mathbf{E})]^2}{p}\right\}.$$

Thus, we proved the result for every vector $\mathbf{q}$ of non-negative integers. As argued in Appendix C, it is possible to allow q_t to be any non-negative real number for all t, thereby simplifying the analysis while maintaining the interpretation of the above expressions.

□

We now proceed with the proof of the proposition. To simplify notation, hereafter let $\chi = u_{kk} + 2u_{kK} + u_{KK}$ and $\gamma_c = 1 - \sum_{t=1}^{T}\gamma_t$. Note that $\chi < 0$ because $\alpha^{**} = 1 - \frac{\chi}{u_{kk}} < 1$ and $u_{kk} < 0$ by assumption. Using Lemma 3 and letting $d_t = d(c_t)$, we can write the planner's objective function as

$$\mathcal{P}(\mathbf{q}) = \sum_{t=1}^{T} \gamma_t (E_t - d_t q_t) + \gamma_c \frac{\chi[\kappa_1^{**}]^2}{2}\left\{\sum_{t=1}^{T} \frac{[W_t^{**}(\mathbf{E})]^2}{a q_t} + \frac{[W_p^{**}(\mathbf{E})]^2}{p}\right\} \qquad (15)$$

$$+\gamma_c\left\{\frac{u_{kk}}{2}[\kappa_1^{**}]^2\sum_{t=1}^{T}\frac{[W_t^{**}(\mathbf{E})]^2}{E_tc_t^2}-\tau\sum_{t=1}^{T}E_t\right\},$$

where we will leave the dependence of each E_t on $\mathbf{q}$ implicit. Note that for every $\mathbf{q}$ the derivatives of the last line in (15) are always equal to zero by a standard envelope argument, because consumers always choose $\mathbf{E}$ optimally. Moreover, when consumers pay attention to t-sources, by Lemma 4 (adapted to the case of $\alpha=\alpha^{**}$, $\kappa_1=\kappa_1^{**}$, and $u_{\sigma\sigma}=0$) we have that

$$W_t^{**}(\mathbf{E})=\frac{c_t\sqrt{\tau}}{\phi}E_t.$$

Combining these observations, we can reduce the object of study to

$$\hat{\mathcal{P}}(\mathbf{q}) \;=\; \gamma^c\frac{\chi[\kappa_1^{**}]^2}{2}\left\{\sum_{t=1}^{T}\frac{\tau c_t^2E_t^2}{\phi^2aq_t}+\frac{1}{p}\left[1-\frac{\sqrt{\tau}}{\phi}\sum_{t=1}^{T}c_tE_t\right]^2\right\}+\sum_{t=1}^{T}\gamma_t(E_t-d_tq_t).$$

Using this, we have

$$\begin{aligned}\frac{\partial\hat{\mathcal{P}}}{\partial q_t} \;=\; & \gamma^c\chi[\kappa_1^{**}]^2\left\{\sum_{t'\neq t}\left[\frac{\tau c_{t'}^2E_{t'}}{\phi^2aq_{t'}}\right]\frac{\partial E_{t'}}{\partial q_t}-\frac{\sqrt{\tau}}{p\phi}\left[1-\frac{\sqrt{\tau}}{\phi}\sum_{t''=1}^{T}c_{t''}E_{t''}\right]\left[\sum_{t''=1}^{T}c_{t''}\frac{\partial E_{t''}}{\partial q_t}\right]\right\} \quad (16)\\ & +\gamma^c\chi[\kappa_1^{**}]^2\left\{\frac{\tau c_t^2E_t}{\phi^2aq_t}\left[\frac{\partial E_t}{\partial q_t}-\frac{E_t}{2q_t}\right]\right\}+\sum_{t'\neq t}\gamma_{t'}\frac{\partial E_{t'}}{\partial q_t}+\gamma_t\left[\frac{\partial E_t}{\partial q_t}-d_t\right].\end{aligned}$$

To simplify this, we use the formula of E_t to calculate $\frac{\partial E_t}{\partial q_t}$ and $\frac{\partial E_{t'}}{\partial q_t}$ for $t'\neq t$:

$$\begin{aligned}\frac{\partial E_t}{\partial q_t} &= \frac{\partial}{\partial q_t}\left\{\frac{q_tc_t^{-1}}{\frac{p}{a}+\sum_{t'\in T_e}q_{t'}}\left[\frac{\phi}{\sqrt{\tau}}+\sum_{t''\in T_e\setminus\{t\}}\frac{aq_{t''}(c_{t''}^{-1}-c_t^{-1})}{1-\alpha^{**}}-\frac{pc_t^{-1}}{1-\alpha^{**}}\right]\right\}\\ &= \frac{\frac{p}{a}+\sum_{t'\in T_e\setminus\{t\}}q_{t'}}{\left(\frac{p}{a}+\sum_{t'\in T_e}q_{t'}\right)^2c_t}\left[\frac{\phi}{\sqrt{\tau}}+\sum_{t''\in T_e\setminus\{t\}}\frac{aq_{t''}(c_{t''}^{-1}-c_t^{-1})}{1-\alpha^{**}}-\frac{pc_t^{-1}}{1-\alpha^{**}}\right],\end{aligned}$$

$$\begin{aligned}\frac{\partial E_{t'}}{\partial q_t} &= \frac{\partial}{\partial q_t}\left\{\frac{q_{t'}c_{t'}^{-1}}{\frac{p}{a}+\sum_{t''\in T_e}q_{t''}}\left[\frac{\phi}{\sqrt{\tau}}+\sum_{t''\in T_e\setminus\{t'\}}\frac{aq_{t''}(c_{t''}^{-1}-c_{t'}^{-1})}{1-\alpha^{**}}-\frac{pc_{t'}^{-1}}{1-\alpha^{**}}\right]\right\}\\ &= \frac{-q_{t'}c_{t'}^{-1}}{\left(\frac{p}{a}+\sum_{t''\in T_e}q_{t''}\right)^2}\left[\frac{\phi}{\sqrt{\tau}}+\sum_{t''\in T_e\setminus\{t'\}}\frac{aq_{t''}(c_{t''}^{-1}-c_{t'}^{-1})}{1-\alpha^{**}}-\frac{pc_{t'}^{-1}}{1-\alpha^{**}}\right]\\ &\quad+\frac{q_{t'}c_{t'}^{-1}}{\frac{p}{a}+\sum_{t''\in T_e}q_{t''}}\left[\frac{a(c_t^{-1}-c_{t'}^{-1})}{1-\alpha^{**}}\right]\end{aligned}$$

$$= \frac{-q_{t'}c_{t'}^{-1}}{\left(\frac{p}{a}+\sum_{t''\in T_e} q_{t''}\right)^2}\left[\frac{\phi}{\sqrt{\tau}}-\frac{pc_t^{-1}}{1-\alpha^{**}}+\sum_{t''\in T_e}\frac{aq_{t''}(c_{t''}^{-1}-c_t^{-1})}{1-\alpha^{**}}\right].$$

Given α^{**}, let t^{**} be the type of sources supplied in the ensuing equilibrium $\mathbf{q}^{**}$. We will first evaluate (16) with respect to $q_{t^{**}}$ at the equilibrium $(0,\overline{q}_{t^{**}})$ for $\overline{q}_{t^{**}}=\frac{1}{d_{t^{**}}c_{t^{**}}}\left[\frac{\phi}{\sqrt{\tau}}-\frac{pc_{t^{**}}^{-1}}{1-\alpha^{**}}\right]-\frac{p}{a}>0$. We have,

$$\frac{\partial E_{t^{**}}(0,\overline{q}_{t^{**}})}{\partial q_{t^{**}}} = \frac{\frac{p}{a}c_{t^{**}}^{-1}}{\left(\frac{p}{a}+\overline{q}_{t^{**}}\right)^2}\left[\frac{\phi}{\sqrt{\tau}}-\frac{pc_{t^{**}}^{-1}}{1-\alpha^{**}}\right]=\frac{\frac{p}{a}c_{t^{**}}d_{t^{**}}^2}{\frac{\phi}{\sqrt{\tau}}-\frac{pc_{t^{**}}^{-1}}{1-\alpha^{**}}},$$

and clearly $\frac{\partial E_t(0,\overline{q}_{t^{**}})}{\partial q_{t^{**}}}=0$ for $t\neq t^{**}$. Recall that $E_{t^{**}}(0,\overline{q}_{t^{**}})=\overline{q}_{t^{**}}d_{t^{**}}$, $E_t(0,\overline{q}_{t^{**}})=0$ for $t\neq t^{**}$. Combining terms, we obtain

$$\begin{aligned}
\frac{\partial\hat{\mathcal{P}}(0,\overline{q}_{t^{**}})}{\partial q_{t^{**}}} &= -\gamma^c\chi[\kappa_1^{**}]^2\frac{\sqrt{\tau}}{p\phi}\left[1-\frac{\sqrt{\tau}}{\phi}c_{t^{**}}\overline{q}_{t^{**}}d_{t^{**}}\right]\left[\frac{\frac{p}{a}c_{t^{**}}^2d_{t^{**}}^2}{\frac{\phi}{\sqrt{\tau}}-\frac{pc_{t^{**}}^{-1}}{1-\alpha^{**}}}\right] \\
&\quad +\gamma^c\chi[\kappa_1^{**}]^2\frac{\tau c_{t^{**}}^2d_{t^{**}}}{\phi^2 a}\left[\frac{\frac{p}{a}c_{t^{**}}d_{t^{**}}^2}{\frac{\phi}{\sqrt{\tau}}-\frac{pc_{t^{**}}^{-1}}{1-\alpha^{**}}}-\frac{d_{t^{**}}}{2}\right]+\gamma_{t^{**}}\left[\frac{\frac{p}{a}c_{t^{**}}d_{t^{**}}^2}{\frac{\phi}{\sqrt{\tau}}-\frac{pc_{t^{**}}^{-1}}{1-\alpha^{**}}}-d_{t^{**}}\right] \\
&= -\gamma^c\chi[\kappa_1^{**}]^2\frac{\tau}{\phi^2}\left[\frac{c_{t^{**}}^{-1}}{1-\alpha^{**}}+\frac{c_{t^{**}}d_{t^{**}}}{a}\right]\left[\frac{\frac{p}{a}c_{t^{**}}^2d_{t^{**}}^2}{\frac{\phi}{\sqrt{\tau}}-\frac{pc_{t^{**}}^{-1}}{1-\alpha^{**}}}\right] \\
&\quad +\gamma^c\chi[\kappa_1^{**}]^2\frac{\tau c_{t^{**}}^2d_{t^{**}}}{\phi^2 a}\left[\frac{\frac{p}{a}c_{t^{**}}d_{t^{**}}^2}{\frac{\phi}{\sqrt{\tau}}-\frac{pc_{t^{**}}^{-1}}{1-\alpha^{**}}}-\frac{d_{t^{**}}}{2}\right]+\gamma_{t^{**}}\left[\frac{\frac{p}{a}c_{t^{**}}d_{t^{**}}^2}{\frac{\phi}{\sqrt{\tau}}-\frac{pc_{t^{**}}^{-1}}{1-\alpha^{**}}}-d_{t^{**}}\right] \\
&= -\gamma^c\chi[\kappa_1^{**}]^2\frac{\tau}{\phi^2}\left\{\frac{1}{1-\alpha^{**}}\left[\frac{\frac{p}{a}c_{t^{**}}d_{t^{**}}^2}{\frac{\phi}{\sqrt{\tau}}-\frac{pc_{t^{**}}^{-1}}{1-\alpha^{**}}}\right]+\frac{c_{t^{**}}^2d_{t^{**}}^2}{2a}\right\} \\
&\quad +\gamma_{t^{**}}d_{t^{**}}\left[\frac{\frac{p}{a}c_{t^{**}}d_{t^{**}}}{\frac{\phi}{\sqrt{\tau}}-\frac{pc_{t^{**}}^{-1}}{1-\alpha^{**}}}-1\right].
\end{aligned}$$

Note that

$$\lim_{\tau\to 0}\frac{\partial\hat{\mathcal{P}}(0,\overline{q}_{t^{**}})}{\partial q_{t^{**}}}=-\gamma_{t^{**}}d_{t^{**}}<0.$$

On the other hand, since $\chi<0$, $\frac{\partial\hat{\mathcal{P}}(0,\overline{q}_{t^{**}})}{\partial q_{t^{**}}}$ is strictly increasing in τ and is unbounded from above. Therefore, there exists τ is sufficiently large (and p sufficiently small so that $\overline{q}_{t^{**}}$ remains positive) that renders $\frac{\partial\hat{\mathcal{P}}(0,\overline{q}_{t^{**}})}{\partial q_{t^{**}}}$ strictly positive. Thus, $\frac{\partial\hat{\mathcal{P}}(0,\overline{q}_{t^{**}})}{\partial q_{t^{**}}}>0$ if and only if $\tau>\tau_{t^{**}}(\gamma)$ for some unique $\tau_{t^{**}}(\gamma)>0$.

We now evaluate (16) with respect to q_t at the equilibrium $(0,\overline{q}_{t^{**}})$ for types $t\neq t^{**}$ that, given α^{**} and $q_{t^{**}}$, would receive attention if available to consumers (recall Corollary 1). First calculate

$$\begin{aligned}
\frac{\partial E_t(0,\bar{q}_{t^{**}})}{\partial q_t} &= \frac{c_t^{-1}}{\frac{p}{a}+\bar{q}_{t^{**}}}\left[\frac{\phi}{\sqrt{\tau}}+\frac{a\bar{q}_{t^{**}}(c_{t^{**}}^{-1}-c_t^{-1})}{1-\alpha^{**}}-\frac{pc_t^{-1}}{1-\alpha^{**}}\right] \\
&= c_t^{-1}\left[d_{t^{**}}c_{t^{**}}+\frac{a(c_{t^{**}}^{-1}-c_t^{-1})}{1-\alpha^{**}}\right]
\end{aligned}$$

$$\begin{aligned}
\frac{\partial E_{t^{**}}(0,\bar{q}_{t^{**}})}{\partial q_t} &= \frac{-\bar{q}_{t^{**}}c_{t^{**}}^{-1}}{\left(\frac{p}{a}+\bar{q}_{t^{**}}\right)^2}\left[\frac{\phi}{\sqrt{\tau}}-\frac{pc_t^{-1}}{1-\alpha^{**}}+\frac{a\bar{q}_{t^{**}}(c_{t^{**}}^{-1}-c_t^{-1})}{1-\alpha^{**}}\right] \\
&= \frac{-\bar{q}_{t^{**}}c_{t^{**}}^{-1}}{\frac{p}{a}+\bar{q}_{t^{**}}}\left[d_{t^{**}}c_{t^{**}}+\frac{a(c_{t^{**}}^{-1}-c_t^{-1})}{1-\alpha^{**}}\right]
\end{aligned}$$

and of course $\frac{\partial E_{t'}(0,\bar{q}_{t^{**}})}{\partial q_t}=0$ for $t'\neq t,t^{**}$. Also, recall that

$$\begin{aligned}
\frac{E_t(0,\bar{q}_{t^{**}})}{q_t} &= \frac{c_t^{-1}}{\frac{p}{a}+\sum_{t''\in T_e}q_{t''}}\left[\frac{\phi}{\sqrt{\tau}}+\sum_{t'\in T_e\setminus\{t\}}\frac{aq_{t'}(c_{t'}^{-1}-c_t^{-1})}{1-\alpha^{**}}-\frac{pc_t^{-1}}{1-\alpha^{**}}\right]\Bigg|_{(0,\bar{q}_{t^{**}})} \\
&= c_t^{-1}\left[d_{t^{**}}c_{t^{**}}+\frac{a(c_{t^{**}}^{-1}-c_t^{-1})}{1-\alpha^{**}}\right].
\end{aligned}$$

Letting $\xi_t = d_{t^{**}}c_{t^{**}}+\frac{a(c_{t^{**}}^{-1}-c_t^{-1})}{1-\alpha^{**}}$ and combining terms, we obtain

$$\begin{aligned}
\frac{\partial\hat{\mathcal{P}}(0,\bar{q}_{t^{**}})}{\partial q_t} &= \gamma^c\chi[\kappa_1^{**}]^2\left\{\left[\frac{\tau c_{t^{**}}^2E_{t^{**}}}{\phi^2 aq_{t^{**}}}\right]\frac{\partial E_{t^{**}}}{\partial q_t}-\frac{\sqrt{\tau}}{p\phi}\left[1-\frac{\sqrt{\tau}}{\phi}c_{t^{**}}E_{t^{**}}\right]\left[c_{t^{**}}\frac{\partial E_{t^{**}}}{\partial q_t}+c_t\frac{\partial E_t}{\partial q_t}\right]\right\} \\
&\quad +\gamma^c\chi[\kappa_1^{**}]^2\frac{\tau c_t^2E_t}{\phi^2 aq_t}\left[\frac{\partial E_t}{\partial q_t}-\frac{E_t}{2q_t}\right]+\gamma_{t^{**}}\frac{\partial E_{t^{**}}}{\partial q_t}+\gamma_t\left[\frac{\partial E_t}{\partial q_t}-d_t\right] \\
&= \gamma^c\chi[\kappa_1^{**}]^2\left\{\left[\frac{\tau c_{t^{**}}^2 d_{t^{**}}}{\phi^2 a}\right]\frac{-\bar{q}_{t^{**}}c_{t^{**}}^{-1}}{\frac{p}{a}+\bar{q}_{t^{**}}}-\frac{\sqrt{\tau}}{p\phi}\left[1-\frac{\sqrt{\tau}}{\phi}c_{t^{**}}d_{t^{**}}\bar{q}_{t^{**}}\right]\left[\frac{-\bar{q}_{t^{**}}}{\frac{p}{a}+\bar{q}_{t^{**}}}+1\right]\right\}\xi_t \\
&\quad +\gamma^c\chi[\kappa_1^{**}]^2\frac{\tau\xi_t^2}{2\phi^2 a}+\gamma_{t^{**}}\frac{-\bar{q}_{t^{**}}c_{t^{**}}^{-1}}{\frac{p}{a}+\bar{q}_{t^{**}}}\xi_t+\gamma_t\left[\frac{\xi_t}{c_t}-d_t\right] \\
&= \gamma^c\chi[\kappa_1^{**}]^2\frac{\tau}{\phi^2}\left\{-\frac{c_{t^{**}}^{-1}}{1-\alpha^{**}}\left[\frac{-\bar{q}_{t^{**}}}{\frac{p}{a}+\bar{q}_{t^{**}}}\right]-\left[\frac{c_{t^{**}}^{-1}}{1-\alpha^{**}}+\frac{c_{t^{**}}d_{t^{**}}}{a}\right]+\frac{\xi_t}{2a}\right\}\xi_t \\
&\quad +\left[\frac{\gamma_{t^{**}}}{c_{t^{**}}}\frac{-\bar{q}_{t^{**}}}{\frac{p}{a}+\bar{q}_{t^{**}}}+\frac{\gamma_t}{c_t}\right]\xi_t-\gamma_t d_t \\
&= -\gamma^c\chi[\kappa_1^{**}]^2\frac{\tau}{a\phi^2}\left\{\left[\frac{\frac{pc_{t^{**}}^{-1}}{1-\alpha^{**}}}{\frac{p}{a}+\bar{q}_{t^{**}}}\right]+\frac{\xi_t}{2}\right\}\xi_t+\frac{\gamma_{t^{**}}\xi_t}{c_{t^{**}}}\left[\frac{\frac{p}{a}}{\frac{p}{a}+\bar{q}_{t^{**}}}-1\right]+\gamma_t\left[\frac{\xi_t}{c_t}-d_t\right] \\
&= -\gamma^c\chi[\kappa_1^{**}]^2\frac{\tau}{a\phi^2}\left\{d_{t^{**}}c_{t^{**}}\left[\frac{1}{2}+\frac{\frac{pc_{t^{**}}^{-1}}{1-\alpha^{**}}}{\frac{\phi}{\sqrt{\tau}}-\frac{pc_{t^{**}}^{-1}}{1-\alpha^{**}}}\right]+\frac{a(c_{t^{**}}^{-1}-c_t^{-1})}{2(1-\alpha^{**})}\right\}\xi_t
\end{aligned}$$

$$+\frac{\gamma_{t^{**}}\xi_t}{c_{t^{**}}}\left[\frac{\frac{p}{a}d_{t^{**}}c_{t^{**}}}{\frac{\phi}{\sqrt{\tau}}-\frac{pc_{t^{**}}^{-1}}{1-\alpha^{**}}}-1\right]+\frac{\gamma_t}{c_t}(\xi_t-d_tc_t).$$

We need to consider whether $t > t^{**}$ or $t < t^{**}$. If $t > t^{**}$, it means that $\alpha^{**} < \alpha_t^{t^{**}}$ and hence $0 < \xi_t < c_t d_t$. If $t < t^{**}$, it means that $\alpha^{**} > \alpha_t^{t^{**}}$ and hence again $\xi_t < c_t d_t$, but also that $\alpha^{**} < \overline{\alpha}^{t^{**}}$ as defined in the proof of Proposition 1.[29] The latter condition implies that $\xi_t > 0$. Therefore, either way, $\frac{\partial\hat{\mathcal{P}}(0,\overline{q}_{t^{**}})}{\partial q_t}$ is strictly increasing in τ and unbounded from above, and $\lim_{\tau\to 0}\frac{\partial\hat{\mathcal{P}}(0,\overline{q}_{t^{**}})}{\partial q_t} = -\frac{\gamma_{t^{**}}}{c_{t^{**}}}\xi_t + \frac{\gamma_t}{c_t}(\xi_t - d_t c_t) < 0$. Therefore, there exists a unique $\tau_t > 0$ such that $\frac{\partial\hat{\mathcal{P}}(0,\overline{q}_{t^{**}})}{\partial q_t} > 0$ if and only if $\tau > \tau_t$.

We now prove that $\tau_t > \tau_{t^{**}}$ if $t < t^{**}$. This follows from showing that, if $\frac{\partial\hat{\mathcal{P}}(0,\overline{q}_{t^{**}})}{\partial q_{t^{**}}} = 0$ at $\tau_{t^{**}}$, then $\frac{\partial\hat{\mathcal{P}}(0,\overline{q}_{t^{**}})}{\partial q_t} < 0$ at $\tau_{t^{**}}$ for $t < t^{**}$, and by using the monotonicity property of $\frac{\partial\hat{\mathcal{P}}(0,\overline{q}_{t^{**}})}{\partial q_t}$ in τ. If $\frac{\partial\hat{\mathcal{P}}(0,\overline{q}_{t^{**}})}{\partial q_{t^{**}}} = 0$, then

$$\gamma^c\chi[\kappa_1^{**}]^2\frac{\tau}{a\phi^2} = \frac{\gamma_{t^{**}}\left[\frac{\frac{p}{a}c_{t^{**}}d_{t^{**}}}{\frac{\phi}{\sqrt{\tau}}-\frac{pc_{t^{**}}^{-1}}{1-\alpha^{**}}}-1\right]}{\frac{1}{1-\alpha^{**}}\left[\frac{pc_{t^{**}}d_{t^{**}}}{\frac{\phi}{\sqrt{\tau}}-\frac{pc_{t^{**}}^{-1}}{1-\alpha^{**}}}\right]+\frac{c_{t^{**}}^2d_{t^{**}}}{2}}.$$

Therefore, $\frac{\partial\hat{\mathcal{P}}(0,\overline{q}_{t^{**}})}{\partial q_t} < 0$ if and only if

$$\frac{\gamma_{t^{**}}\left[\frac{\frac{p}{a}c_{t^{**}}d_{t^{**}}}{\frac{\phi}{\sqrt{\tau}}-\frac{pc_{t^{**}}^{-1}}{1-\alpha^{**}}}-1\right]\left\{d_{t^{**}}c_{t^{**}}\left[\frac{\frac{pc_{t^{**}}^{-1}}{1-\alpha^{**}}}{\frac{\phi}{\sqrt{\tau}}-\frac{pc_{t^{**}}^{-1}}{1-\alpha^{**}}}\right]+\frac{\xi_t}{2}\right\}}{\frac{1}{1-\alpha^{**}}\left[\frac{pc_{t^{**}}d_{t^{**}}}{\frac{\phi}{\sqrt{\tau}}-\frac{pc_{t^{**}}^{-1}}{1-\alpha^{**}}}\right]+\frac{c_{t^{**}}^2d_{t^{**}}}{2}} > \frac{\gamma_{t^{**}}}{c_{t^{**}}}\left[\frac{\frac{p}{a}d_{t^{**}}c_{t^{**}}}{\frac{\phi}{\sqrt{\tau}}-\frac{pc_{t^{**}}^{-1}}{1-\alpha^{**}}}-1\right]+\frac{\gamma_t}{c_t\xi_t}(\xi_t-d_tc_t),$$

or equivalently

$$\frac{\gamma_{t^{**}}\left[\frac{-c_{t^{**}}d_{t^{**}}\overline{q}_{t^{**}}}{\frac{\phi}{\sqrt{\tau}}-\frac{pc_{t^{**}}^{-1}}{1-\alpha^{**}}}\right]\left[\frac{\frac{pc_{t^{**}}^{-1}}{1-\alpha^{**}}}{\frac{\phi}{\sqrt{\tau}}-\frac{pc_{t^{**}}^{-1}}{1-\alpha^{**}}}+\frac{1}{2}+\frac{a(c_{t^{**}}^{-1}-c_t^{-1})}{2c_{t^{**}}d_{t^{**}}(1-\alpha^{**})}\right]}{c_{t^{**}}\left[\frac{\frac{pc_{t^{**}}^{-1}}{1-\alpha^{**}}}{\frac{\phi}{\sqrt{\tau}}-\frac{pc_{t^{**}}^{-1}}{1-\alpha^{**}}}+\frac{1}{2}\right]}+\gamma_{t^{**}}\left[\frac{d_{t^{**}}\overline{q}_{t^{**}}}{\frac{\phi}{\sqrt{\tau}}-\frac{pc_{t^{**}}^{-1}}{1-\alpha^{**}}}\right] > \frac{\gamma_t}{c_t\xi_t}(\xi_t-d_tc_t),$$

[29]If $\alpha^{**} \geq \overline{\alpha}^{t^{**}}$, it means that $\overline{q}_{t^{**}} \geq q_{t^{**}}(\alpha)$ and hence no $t < t^{**}$ can attract attention by Corollary 1.

or equivalently

$$-\overline{q}_{t^{**}}\frac{\gamma_{t^{**}}a(c_{t^{**}}^{-1}-c_t^{-1})}{c_{t^{**}}(1-\alpha^{**})}>\frac{\gamma_t(\xi_t-d_tc_t)}{c_t\xi_t}\left[\frac{\phi}{\sqrt{\tau}}+\frac{pc_{t^{**}}^{-1}}{1-\alpha^{**}}\right].$$

The last inequality holds because the left-hand side is positive and the right-hand side is negative given $\xi_t < d_tc_t$.

Finally, suppose that $\tau = \tau_{t^{**}}$ so that $\frac{\partial\hat{\mathcal{P}}(0,\overline{q}_{t^{**}})}{\partial q_{t^{**}}} = 0$. Then, by the previous calculations, $\frac{\partial\hat{\mathcal{P}}(0,\overline{q}_{t^{**}})}{\partial q_t} > 0$ if and only if

$$\overline{q}_{t^{**}}\frac{\gamma_{t^{**}}a(c_t^{-1}-c_{t^{**}}^{-1})}{c_{t^{**}}(1-\alpha^{**})}<\frac{\gamma_t(\xi_t-d_tc_t)}{c_t\xi_t}\left[\frac{\phi}{\sqrt{\tau}}+\frac{pc_{t^{**}}^{-1}}{1-\alpha^{**}}\right].$$

For $t = t^{**}+1$, the left-hand side is strictly negative. The right-hand side is negative and approaches zero as $\xi_{t^{**}+1} \to c_{t^{**}+1}d_{t^{**}+1}$. Given the definition of $\xi_{t^{**}+1}$, this happens when $\alpha^{**} \to \alpha_{t^{**}}^{t^{**}+1}$.

B Appendix: Consumer Equilibrium

In order to describe the equilibrium $k(\cdot;\cdot)$, using Myatt and Wallace's (2012) language and symmetry, define the total *precision* of source m of type t as

$$\pi_t^m(e^m) = [Var(z^m + x^{n,m})]^{-1} = \frac{a_te^m[c_t]^2}{a_t + e^m[c_t]^2}$$

and its *publicity* as

$$\rho_t^m(e^m) = corr(z^m + x^{n,m}, z^m + x^{n',m}) = \frac{e^m[c_t]^2}{a_t + e^m[c_t]^2}.$$

We drop the subscript m when π_t^m and ρ_t^m are the same across all t-sources. Abusing notation, let q_t denote the set of active t-suppliers.

Lemma 4 (Consumer behavior)**.** *Given $T_{\mathbf{q}} = \{t : q_t > 0\}$, there exists a unique symmetric equilibrium $\mathbf{e}$ and $k(\cdot;\cdot)$, which satisfies the following properties:*
1. $e^m = e_t$ for all $m \in q_t$ and all $t \in T_{\mathbf{q}}$;
2. if $e_t > 0$, then

$$-\frac{u_{kk}}{2}\left[\frac{\kappa_1 w_t(\mathbf{e})}{c_te_t}\right]^2 = \tau, \tag{17}$$

where

$$w_t(\mathbf{e}) = \frac{(1-\alpha)\pi_t(e_t)}{1-\alpha\rho_t(e_t)}\left[p + \sum_{t'=1}^{T} q_{t'}\frac{(1-\alpha)\pi_{t'}(e_{t'})}{1-\alpha\rho_{t'}(e_{t'})}\right]^{-1};$$

3. if $e_t = 0$*, then*

$$-\frac{u_{kk}}{2}[c_t\kappa_1]^2\left[\frac{p}{1-\alpha}+\sum_{t'\neq t} q_{t'}\frac{(1-\alpha)\pi_{t'}(e_{t'})}{1-\alpha\rho_{t'}(e_{t'})}\right]^{-2}\leq\tau; \tag{18}$$

4. for all $n\in[0,1]$*,*

$$k^n = k(\mathbf{s}^n;\mathbf{e}) = \kappa_0+\kappa_1\sum_{t\in T_{\mathbf{q}}} w_t(\mathbf{e})\sum_{m\in q_t} s^{n,m}; \tag{19}$$

5. there exists $r>0$ *such that, if* $e_t>0$ *then* $c_t>r\sqrt{\tau}$*, and if* $e_t=0$ *then* $c_t\leq r\sqrt{\tau}$*.*

Lemma 4 follows from Proposition 1 and Corollary 1 in Pavan (2014) and the property that all t-suppliers offer identical sources of information. We omit its proof.

To gain intuition, note that given $k(\cdot;\cdot)$ in (19) each consumer's expected payoff can be written as (see Pavan (2014))

$$\mathbb{E}[u(K,K,\sigma_k,\theta)|\mathbf{e},k(\cdot;\mathbf{e})]+\frac{u_{kk}}{2}Var[k-K|\mathbf{e},k(\cdot;\mathbf{e})]-C(\mathbf{e}) \tag{20}$$

where the variation of individual actions around their mean—called *dispersion*—satisfies

$$Var[k-K|\mathbf{e},k(\cdot;\mathbf{e})]=\kappa_1^2\sum_{t=1}^{T}\sum_{m\in q_t}\frac{[w_t^m(\mathbf{e})]^2}{e^m[c_t]^2},$$

with

$$w_t^m(\mathbf{e})=\frac{(1-\alpha)\pi_t^m(e^m)}{1-\alpha\rho_t^m(e^m)}\left[p+\sum_{t'=1}^{T}\sum_{m'\in q_{t'}}\frac{(1-\alpha)\pi_{t'}^{m'}(e^{m'})}{1-\alpha\rho_{t'}^{m'}(e^{m'})}\right]^{-1}.$$

Importantly, when all consumers follow strategy $k(\cdot;\cdot)$, the distribution of the average action K is independent of their attention allocation—hence, consumers treat the first term in (20) as a constant. Keeping k fixed by the envelope theorem, we get that the right-hand side of (17) measures the private benefit of marginally increasing the attention to source m. To obtain part 1 of the lemma, note that by Corollary 1 in Pavan (2014) if one t-supplier receives positive attention, then all t-suppliers must receive positive attention. Since condition (17) must then hold for all of them, using $w_t^m(\mathbf{e})$, we get that

$$\frac{(1-\alpha)\pi_t^m(e^m)}{e^m(1-\alpha\rho_t^m(e^m))}=\frac{(1-\alpha)\pi_t^{m'}(e^{m'})}{e^{m'}(1-\alpha\rho_t^{m'}(e^{m'}))}$$

for every two t-suppliers m and m'. This condition leads to $e^m=e^{m'}$.

C Appendix: Continuum of Suppliers

From the characterization of the unique equilibrium in the continuation game between consumers (Lemma 4), it is easy to see that what ultimately matters for each consumer is how much attention she allocates to the group of t-suppliers *as a whole*—this amount is then divided evenly among all its members. Of course, this simplification follows from the property that in the model, all t-suppliers are identical sources of information. Indeed, we can write condition (17) as

$$\tau = \frac{-u_{kk}}{2}\kappa_1^2 \left[\frac{q_t w_t(\mathbf{e})}{c_t q_t e_t}\right]^2,$$

where

$$q_t w_t(\mathbf{e}) = \frac{1-\alpha}{(1-\alpha)[a_t q_t]^{-1} + ([c_t]^2 q_t e_t]^{-1}} \left[p + \sum_{t' \in T_e} \frac{1-\alpha}{(1-\alpha)[a_{t'} q_{t'}]^{-1} + ([c_{t'}]^2 q_{t'} e_{t'})^{-1}}\right]^{-1};$$

similarly, condition (18) becomes

$$\tau \geq \frac{-u_{kk}}{2}[c_t \kappa_1]^2 \left[\frac{p}{1-\alpha} + \sum_{t' \in T_e} \frac{1-\alpha}{(1-\alpha)[a_{t'} q_{t'}]^{-1} + ([c_{t'}]^2 q_{t'} e_{t'})^{-1}}\right]^{-2},$$

and the optimal action policy becomes

$$k(\mathbf{s}^n; \mathbf{e}) = \kappa_0 + \kappa_1 \sum_{t \in T_e} q_t w_t(\mathbf{e}) S_t^n,$$

where we let $S_t^n = \frac{1}{q_t} \sum_{m \in q_t} s^{n,m}$. Letting $E_t = q_t e_t$ for all t, we see that all these conditions depend only on the group quantities E_t. So, for every $\mathbf{q}$, the consumers' behavior is characterize by

$$\tau = \frac{-u_{kk}}{2}\kappa_1^2 \left[\frac{W_t(\mathbf{E})}{c_t E_t}\right]^2,$$

where

$$W_t(\mathbf{E}) = \frac{1-\alpha}{(1-\alpha)[a_t q_t]^{-1} + ([c_t]^2 E_t)^{-1}} \left[p + \sum_{t' \in T_e} \frac{1-\alpha}{(1-\alpha)[a_{t'} q_{t'}]^{-1} + ([c_{t'}]^2 E^{t'})^{-1}}\right]^{-1};$$

similarly, condition (18) becomes

$$\tau \geq \frac{-u_{kk}}{2}[c_{t'} \kappa_1]^2 \left[\frac{p}{1-\alpha} + \sum_{t' \in T_e} \frac{1-\alpha}{(1-\alpha)[a_{t'} q_{t'}]^{-1} + ([c_{t'}]^2 E^{t'})^{-1}}\right]^{-2},$$

and the optimal action policy becomes

$$k(\mathbf{s}^n; \mathbf{E}) = \kappa_0 + \kappa_1 \sum_{t \in T_e} W_t(\mathbf{E}) S_t^n.$$

Also, note that the random variable S_t^n is normally distributed with mean zero and variance $\frac{1}{p} + \frac{1}{q_t a_t} + \frac{1}{[c_t]^2 E_t}$. Thus, we can interpret the consumers as basing their actions on the sufficient statistic S_t, which summarizes all the information they will get from the group of t-suppliers; given this, each consumer can be viewed as choosing how much attention to allocate to the source of S_t.

We now want to argue that we can let q_t be any positive real number and continue to use the above equations to characterize the consumers' behavior, while preserving the economic meaning of the model. To this end, first note that if q_t is a positive rational number (that is, $q_t \in \mathbb{Q}_+$), then we can write $a_t q_t$ as $\frac{a_t}{\zeta^t}\chi^t$ where $\chi^t, \zeta^t \in \mathbb{Z}_+$. Moreover, if we take $\zeta \in \mathbb{Z}_+$ sufficiently large, we can approximate all q_t in $\mathbb{Q}_+$ with $\frac{\chi^t}{\zeta}$ for some $\chi^t \in \mathbb{Z}_+$ for all t.[30] In this case, $\frac{a_t}{\zeta}\chi^t$ can be interpreted as a situation where there are χ^t suppliers (which is a positive integer) of type t in the market and the informational content of each potential supplier is $\frac{a_t}{\zeta}$. Note that $\frac{a_t}{\zeta}$ decreases to zero as ζ increases, which can be interpreted as saying that the informational content of each supplier becomes arbitrarily small when there is a large number of potential suppliers. This is consistent with a notion of perfect competition in information markets defined as the property that each supplier is "small" in terms of the amount of information it can provide. Since $\mathbb{Q}_+$ is dense in $\mathbb{R}_+$, by letting ζ become arbitrarily large and χ^t adjust correspondingly across t's, in this way we can approximate every $\mathbf{q} \in \mathbb{R}_+^T$ and hence every level of $a_t q_t$ across t's. In the limit, the interpretation of a_t is that it measures the rate at which t-suppliers entering the market contribute to the total amount of information that they provide to the consumers.

References

Angeletos, G.-M. and A. Pavan (2007). Efficient use of information and social value of information. *Econometrica 75*(4), 1103–1142.

Angeletos, G.-M. and A. Pavan (2009). Policy with dispersed information. *Journal of the European Economic Association 7*(1), 11–60.

Benkler, Y. (2006). *The wealth of networks: How social production transforms markets and freedom.* Yale University Press.

[30] This extra step takes care of the case in which the types of signals have the same accuracy, but different clarities.

Boczkowski, P. J. (2010). *News at work: Imitation in an age of information abundance.* University of Chicago Press.

Chahrour, R. (2014). Public communication and information acquisition. *American Economic Journal: Macroeconomics 6*(3), 73–101.

Chen, H. and W. Suen (2017). Competition for attention in the news media market. *mimeo, University of Hong Kong.*

Chwe, M. S.-Y. (2013). *Rational ritual: Culture, coordination, and common knowledge.* Princeton University Press.

Colombo, L., G. Femminis, and A. Pavan (2014). Information acquisition and welfare. *Review of Economic Studies.*

Cornand, C. and F. Heinemann (2008). Optimal degree of public information dissemination. *Economic Journal 118*(528), 718–742.

Crampes, C., C. Haritchabalet, and B. Jullien (2009). Advertising, competition and entry in media industries. *Journal of Industrial Economics 57*(1), 7–31.

Davenport, T. H. and J. C. Beck (2013). *The attention economy: Understanding the new currency of business.* Harvard Business Press.

Dewan, T. and D. P. Myatt (2008). The qualities of leadership: Direction, communication, and obfuscation. *American Political Science Review 102*(03), 351–368.

Dewan, T. and D. P. Myatt (2012). On the rhetorical strategies of leaders: Speaking clearly, standing back, and stepping down. *Journal of Theoretical Politics 24*(4), 431–460.

Dewatripont, M. and J. Tirole (2005). Modes of communication. *Journal of Political Economy 113*(6), 1217–1238.

Gentzkow, M. (2007). Valuing new goods in a model with complementarity: Online newspapers. *The American Economic Review 97*(3), 713–744.

Gentzkow, M. and J. M. Shapiro (2008). Competition and truth in the market for news. *Journal of Economic Perspectives 22*(2), 133–154.

Gentzkow, M. and J. M. Shapiro (2011). Ideological segregation online and offline. *The Quarterly Journal of Economics 126*(4), 1799–1839.

Hamilton, J. (2004). *All the news that's fit to sell: How the market transforms information into news*. Princeton University Press.

Hellwig, C. and L. Veldkamp (2009). Knowing what others know: Coordination motives in information acquisition. *Review of Economic Studies 76*(1), 223–251.

Kennedy, P. and A. Prat (2017). Where do people get their news? *Mimeo, Columbia University.*

Lanham, R. A. (2006). *The economics of attention: Style and substance in the age of information*. University of Chicago Press.

Morris, S. and H. S. Shin (2002). Social value of public information. *American Economic Review 92*(5), 1521–1534.

Myatt, D. P. and C. Wallace (2012). Endogenous information acquisition in coordination games. *Review of Economic Studies 79*(1), 340–374.

Myatt, D. P. and C. Wallace (2014). Central bank communication design in a Lucas-Phelps economy. *Journal of Monetary Economics 63*, 64–79.

Noam, E. M. (2009). *Media Ownership and Concentration in America*. Oxford University Press.

Pavan, A. (2014). Attention, coordination and bounded recall. *Mimeo, Northwestern University.*

Prat, A. and D. Stromberg (2013). The political economy of mass media. In *Advances in Economics and Econometrics: Volume 2, Applied Economics: Tenth World Congress*, Volume 50, pp. 135. Cambridge University Press.

Shiller, R. J. (2015). *Irrational exuberance*. Princeton university press.

Simon, H. (1971). Computers, communications and the public interest. *Computers, communications, and the public interest. Johns Hopkins Press, Baltimore*, 40–41.

Sims, C. A. (2003). Implications of rational inattention. *Journal of monetary Economics 50*(3), 665–690.

Sims, C. A. et al. (2010). Rational inattention and monetary economics. *Handbook of Monetary Economics 3*, 155–181.

Sunstein, C. R. (2009). *Republic. com 2.0.* Princeton University Press.

Webster, J. G. (2014). *The marketplace of attention: How audiences take shape in a digital age*. Mit Press.