Abstract

We develop a model of asset pricing in which buyers are either unable or unwilling to buy an asset at a price substantially above its price in recent transactions. This constraint could result from legal restrictions on appraisals, behavioral preferences, or agency problems. The model features price-predictability, differential pricing for identical assets, “buyer’s” and “seller’s” markets, and associations between price appreciation, volume, and liquidity. We apply the model to the market for residential real estate, in which a bank’s willingness to lend for a home purchase is limited by the appraisal, which is, in turn, generated by recent transaction prices of similar properties. We provide stylized facts consistent with all seven predictions of the model. We also provide evidence of price-predictability in several other settings in which the model is likely to apply – the markets for taxi medallions, airplanes, and commercial and industrial real estate.

Keywords: Momentum, Asset Pricing Anomalies, Real Estate, Appraisals

JEL: G11, R30
1 Introduction

Asset pricing models typically assume that investors are willing to pay up the expected value of an asset: the quantities that they demand may be limited by their wealth or their allowed leverage, but the prices that they are willing to pay are not directly constrained.\(^1\) This means that, if the value of an asset jumps, its price will jump as well. In this manuscript, we analyze constrained prices, in which buyers are unable or unwilling to pay substantially more than prices in recent transactions.

We argue that this assumption holds in a wide variety of settings and generates unusual asset-pricing implications. Most importantly, markets subject to constrained prices feature price-predictability, even at very long time horizons. We provide simple empirics consistent with the model: the prices of airplanes, taxi medallions, shopping malls, single-family homes, and many other assets are all highly predictable at the annual level. The model provides a bevy of additional implications for price levels, volatility, returns, and buy- and sell-side liquidity, all of which are consistent with evidence from the $25 trillion United States (US) residential real estate market.

Price constraints may be more common than one would think, and we begin by offering three examples. First are legal constraints which bind offers. For example, consider the market for single-family homes, a market in which most buyers need loans and a bank’s willingness to lend is limited by the appraised value of a property. As appraisals are generated by recent transactions of similar properties, they constitute a direct limit on the prices that most buyers can offer sellers.

Second are agency constraints, in which the agent making a bid for an asset has more

\(^1\)For example, Miller (1977) discusses the effect of short-sale constraints on asset prices, and He and Krishnamurthy (2013) analyze the effects of leverage constraints. A substantial literature arose in the early 1990s surrounding the failure of the Consumption Capital Asset Pricing Model (CCAPM) to account for equity returns: He and Modest (1995) analyze the effects of short-sale, leverage, and solvency constraints, and argue that these can reconcile the CCAPM with stock price return data. Lucas (1994) disagrees. Garleanu and Pedersen (2011) analyze asset pricing when investors face margin requirements.

Other forms of constraints on investors have been studied as well. For example, Eun and Janakiramanan (1986) analyze a setting in which foreign ownership is constrained. We do not provide a thorough literature review on these models here.
information than the principal for whom she works and is exposed to asymmetric risks from over- and under-bidding. For example, consider an associate working at a venture capital firm who is considering a deal for a new round of equity for a portfolio firm. Even if she believes the fair price for shares in the firm to be substantially higher than in the last round of financing, she may be unwilling to offer that price: if the firm ultimately fails, then she could be accused of overpaying, with the much lower share price in the preceding round as evidence. If the firm ultimately succeeds, then she could receive a pat on the back. Herding can be an optimal response to asymmetric risks in agency settings.  

Third are behavioral constraints, in which most buyers are subject to the bias of not wanting to feel “ripped off.” For example, a taxi driver may be considering a bid on a taxi medallion, which gives him the right to drive a taxi in New York. Even if all current offer prices of medallions are $800,000 and above, and he believes this to be a fair price, if recent transactions all featured prices below $600,000, he may not be willing to offer much above $600,000.  

### 1.1 Theoretical Overview

Our model assumes a unit measure of identical assets, and a measure $N > 1$ of potential owners (agents) who have heterogeneous valuations for those assets.  

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2Another example concerns general managers considering hiring an athlete. Suppose that several elite midfielders have recently been signed to contracts worth $10 million per year, but a general manager believes her target midfielder to be worth $15 million per year. If she makes a $15 million offer and the player succeeds, then the manager will receive a pat on the back. If the player fails, then the manager may well be fired. Asymmetric payoffs from abnormal hires limit the willingness of general managers to pay more for talent than apparently comparable players are paid. Indeed, this argument can be expanded to any market in which the agent in charge of the purchase has an incentive to herd.

3There are informational models in which the buyer will, rationally, be unwilling to pay more than prices in recent transactions, but this constraint can arise in behavioral models as well.

4Inefficient use of “comparable” transaction prices in setting prices has recently gained traction outside of real estate circles. For example, the market for syndicated lending appears subject to excessive reliance on comparable prices. Murfin and Pratt (2016) show this for debt at separate firms. Dougal et al. (2015) find an overuse of a firm’s past borrowing rates in setting rates for a new loan.

5Heterogeneous valuation is present in all three of the preceding examples. Differential job opportunities, family sizes, and incomes yield differential valuations for a given house. The abilities of venture capitalists match differently with the needs of different potential portfolio firms. Differential abilities of taxi drivers yield differential valuations of medallions.
price of the asset equals the valuation placed upon it by the marginal agent. If cash flows, risk, discount rates, cash flow growth rates, etc., change, then the value placed on the asset by the marginal buyer changes and its price changes. In a standard model, the price would adjust immediately to its new valuation. In our model, the speed of adjustment is limited by potential buyers’ willingness or ability to pay more than recent transaction prices.

To be more concrete, suppose that prices are in long-run equilibrium, so that if cash flows are $c$ this year, are expected to grow at $g$ per year, and the annual discount rate is $r_0$, then the price of the asset is $p_0 = \frac{c}{r_0 - g}$. Now suppose, for example, that interest rates fall substantially, to $r_1$. In a well-functioning market, the price would rise immediately to $p_1 = \frac{c}{r_1 - g} > p_0$. If, however, buyers are unable or unwilling to pay substantially more than $p_0$, then prices are bound below $p_1$.

What will rational sellers do? At $p_0$, with the new interest rate $r_1$, the asset yields a return above $r_1$, and many sellers who would have sold at $r_0$ will be unwilling to sell. Because agents have heterogeneous valuations for the asset, some sellers will be willing to sell even though the return is high, thus generating some (diminished) volume of transactions. These transactions can take place at prices marginally above $p_0$, raising the baseline from which potential buyers derive comparisons. As prices rise above $p_0$, the average selling price rises and the maximum price which buyers are willing or able to pay rises as well. This process continues until prices reach $p_1 = \frac{c}{r_1 - g}$.

What does this model imply? The buyers’ constraints on offers and the fraction of sellers who are motivated to sell determine the “speed limit” of price increases in a market. This speed limit can vary from market to market, generating heterogeneity in the rate of appreciation. Because speed limits are likely to be similar from month-to-month and year-to-year within a market, price changes are highly predictable. Because different markets have different speed limits, prices and appreciation rates can vary substantially across markets even when cash flows, discount rates, and cash flow growth rates are identical. In rapidly appreciating markets, queues of buyers will form, so that liquidity on the buyer’s side is
severely diminished and a “seller’s market” emerges. Given the empirical prevalence of buyers’ and sellers’ markets in real estate, it is easy to forget just how odd those concepts are in a standard asset pricing framework. There is no such thing as a seller’s market in stocks and bonds.

Once the discount rate has fallen to \( r_1 \), prices begin to rise and earnings-to-price ratios fall. Both over time within a market, and at a given time across markets, *markets with higher earnings-to-price ratios will tend to earn higher total returns*. There is no risk in the model, so this is a serious pricing anomaly.\(^6\)

Finally, heterogeneous valuations generate differential ownership of the asset, but it is changes in agents’ valuations that generate trade. As the rate of change in agent valuations increases, trade increases. Faster trade allows faster appreciation, so *markets with more flux in valuations across agents will experience faster appreciation*. While this comparative static seems abstract, it is testable in practice – for example, real estate markets in which many individuals enter and leave the market in a given year (e.g., Washington, D.C.) will experience faster appreciation after a decrease in interest rates than markets with less churn (e.g., Dayton, OH).

### 1.2 Empirical Support

#### 1.2.1 Price Predictability

Our theory applies in any market in which buyers typically need loans with high loan-to-value ratios, and in which the amounts that banks will lend are limited by recent transaction prices. Perhaps the clearest prediction of the theory is that these markets will be characterized by momentum in both the short- and long-run.\(^7\) Figure 1a plots scatters of contemporaneous

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\(^6\)Note that, once the discount rate falls to \( r_1 \), the fundamental attributes of the asset never change, but its price and return does. In a sense, this is *excess volatility*, in that the fundamental value is not changing, but its price is. Excess volatility is a well-documented fact in the literatures surrounding a variety of assets, including real estate. We do not focus on the concept of volatility in this paper, because the model is deterministic, but the implication of the model for the data is clear.

\(^7\)Momentum has been well-documented in a wide variety of assets, including stocks, bonds, foreign exchange, precious metals, and real estate (Moskowitz et al. (2012), Cutler et al. (1991)). While there are
and lagged quarterly or monthly returns for six assets (details of the underlying data can be found in Section 4). There is clear price-predictability for aircraft, taxi medallions, US single family homes, and US commercial real estate, all of which feature assets which are typically purchased with collateralized debt. For comparison, Figure 1a also provides similar plots for large US stocks and for seats on the New York Stock Exchange. Neither asset is typically purchased with collateralized debt, and neither is predictably at the monthly level. Figure 1b repeats the exercise using annual returns, and the results are largely similar.

1.2.2 US Single Family Homes

The theory provides at least six additional testable implications, beyond price-predictability. In order to evaluate these implications empirically, we focus on the market for US residential real estate, a market which is economically important and, not coincidentally, affords better data than markets for airplanes and taxi medallions! Single-family real estate is clearly subject to the assumption that most buyers are unable to pay substantially more than prices in recent transactions, as bank loans are limited by appraisals.8

As anyone who has been through the process of purchasing a house in the United States knows, appraisals based on recent sales of comparable homes are important for a mortgage to be approved. Indeed, this is codified into law: real estate loans of $250,000 or more models of momentum that apply to assets generally (e.g., Hong and Stein (1999)), a substantial theoretical literature has arisen to explain momentum in real estate specifically. Piazzesi and Schneider (2009), Head et al. (2014), Anenberg (2014), Wheaton (1990), Guren (2016), and Albrect et al. (2016) offer search models which exhibit momentum. Spiegel (2001), Titman et al. (2014), Glaeser et al. (2014), and Glaeser and Nathanson (2015) offer alternative explanations. Most similar to our model, Quan and Quigley (1991) note that appraisals are backward-looking and that they are biased downward in periods of price appreciation, but they do not formally model the price process resulting from an appraisal constraint. Glaeser et al. (2014), Cutler et al. (1991), and Guren (2016) also document long-run reversals in returns on housing. Our baseline model cannot account for this fact, though Hong and Stein (1999) and Glaeser and Nathanson (2015) can. A natural extension would allow for the supply of the asset to respond to prices – e.g., the supply of housing would increase if prices were high enough to justify new construction. We suspect that this would allow for (partial) reversal of prices, but do not formally model it.

8Consistent with this, Guren (2016) shows empirically that the likelihood of selling a house falls substantially when it is priced only a few percentage points above the prices of comparable houses. While he does not establish the reason for this fact, it is consistent with the assumption that most buyers are close to the maximum loan-to-value limits for a mortgage, and that the “value” in that limit comes from comparisons to comparable homes.
require an appraisal, and the United States Congress authorizes The Appraisal Foundation to write the Uniform Standards of Professional Appraisal Practice. According to the Appraisal Foundation, “Often the primary approach to develop an opinion of value for a residential property, the sales comparison approach utilizes recent sales of comparable properties.” Unless a buyer has enough cash to make up the difference, her ability to pay a given price for a house is legally limited by prices of recent transactions. This makes residential real estate an excellent laboratory to test our model’s implications.

The model’s first implication is that appraisals lag market prices. This lag, in addition to the importance of appraisals for determining maximum loan amounts and the importance of maximum loan amounts for maximum bids, puts a brake on price increases, generating the theory’s predictions. The Federal Housing Finance Agency (FHFA) is a government agency responsible for overseeing aspects of the secondary mortgage market in the US. As part of their research focus, they have maintained two house price indices which we reproduce in this paper. One index is, like the S&P Case-Shiller indices, built using only transaction data. The other uses a combination of transaction prices and appraisals. Figure 2 plots the values of these two indices from January, 2000, until January, 2016. Especially after 2006, the index which includes appraisals clearly lags the purchase-only index.

The model’s second implication is that, as prices are rising faster, the gap between appraisals and market prices will be larger. This should be apparent already in Figure 3, but to make it more so, we document this explicitly in Figure 4. Figure 4 shows a scatter comparing the rate of monthly appreciation in the purchase-only price index to the gap between the levels of the purchase-only and the purchase-plus-appraisal indices. It is clear

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9In spite of the results of this paper, requiring appraisals may still be consistent with efficient contracting. Banks, government sponsored entities (Fannie Mae and Freddie Mac), mortgage insurers, and mortgage-backed security buyers need some measure of the value of a property against which the loan amount can be compared. As originators have a clear incentive to inflate that value, some objective constraint on valuations must be chosen.

10This implication follows immediately from our assumptions and should not be revelatory.

11While the benefit of including appraisals may be non-obvious, Kleiner and Hizmo (2016) find that, within a market, houses trade more or less often depending on the appreciation of the sub-market, leading to bias in the Case-Shiller indices. If appraisals are timely and accurate, they can remove this effect.
that when prices are rising or falling faster, the indices diverge by more.

The model’s third implication is that prices are highly predictable, and our analysis of this implication in the market for single family homes is already discussed above.

The model’s fourth implication is that total returns will be higher for lower-priced assets. The reason is that, during the period of price adjustment, prices are rising but cash flows are not, implying a negative association between total returns and prices. Figure 5 shows total returns on rental real estate versus rent-price ratios for 200 major metro areas analyzed by Zillow.\textsuperscript{12} The higher the rent-price ratio, the higher the total return.\textsuperscript{13} A simple regression of annual return on rent-price ratio generates a coefficient of approximately 2/3, implying that, for every extra 3% of rents earned, price increases in the following year are lower by only 1%.

The model’s fifth implication is that prices vary substantially for apparently similar assets. Figure 5 confirms this: prices range from as low as 5 times rents to as high as 25 times rents. In the stock market, variation like this in Price-to-Earnings ratios is associated with inverse variation in expected growth rates in earnings: prices are high relative to earnings because earnings are expected to grow. As we see in Figure 5, while it is true that more expensive cities experience faster price appreciation, they actually experience \textit{lower} total returns.

The model’s sixth implication is that assets for which the natural owner changes frequently experience faster appreciation. This is because more change in natural ownership causes more trade, which allows for more recent comparable prices. As prices are generally increasing, more recent prices mean higher comparable prices, which allows for a higher offer price. Figure 6 plots the relationship between turnover – the fraction of homes in a neighborhood that changed hands in a given year – and price appreciation. As the model suggests, in the domain in which prices are increasing, higher turnover is associated with faster appreciation. There is no reason in standard theory why these variables should be

\textsuperscript{12}Price-to-rent ratios and price appreciation data are for the years 2010-2013.
\textsuperscript{13}The relationship between rent-price ratios and total returns has previously been documented by Case and Shiller (1990). Our data are from a more recent time period, confirming the persistence of this relationship.
associated.

The model’s seventh implication is that liquidity on the buyer’s side will be diminished when prices are rising. Figure 7 shows the average number of days on market for a property, compared to the market’s annual price appreciation. The existence of “sellers’” and “buyers’” markets is clear: when prices are increasing quickly, houses sell quickly.\footnote{The existence of hot and cold markets has been well documented, and several theories have attempted to explain them, both at the seasonal level (e.g., Ngai and Tenreyro (2014)), and at lower frequencies (e.g., Ngai and Tenreyro (2014), Burnside et al. (2011)).}

We do not model the situation in which the value of the asset falls, either because of an increase in the discount rate or a decrease in the utility that it provides to owners. This is because a limit on what buyers are willing or able to pay will not bind when prices are falling. We believe, however, that an analogous model to ours could provide similar implications for declining markets. For example, a seller who bought a home with a high loan-to-value mortgage may be financially unable to sell for less than she owes on the loan. If comparable home prices have fallen below that level, her lender may allow her to sell and absorb the loss itself, making prices in recent comparable sales an important limit on the price that a seller will accept.\footnote{There is empirical support for the claim that sellers are unwilling or unable to sell for much less than their purchase price (e.g., Genesove and Mayer (1997) and Genesove and Mayer (2001)), and this fact has already generated at least one theory, Stein (1995). There is less support that recent transaction prices are the determinant of that lower bound on a sales price but, to our knowledge, this possibility has not been studied.} Similar arguments follow for behavioral and agency constraints.

Such a model would deliver price predictability, variation in prices of apparently identical assets (excess volatility, in the time-series), higher total returns for lower priced assets, faster depreciation in markets in which the natural owner fluctuates more, and buyers’ markets. Our previously-discussed stylized facts hold in the domain of price decreases as well as price increases, assuming that sellers are limited in a similar way to buyers.

The model has implications for monetary policy. Central banks respond to the onset of recessions by cutting interest rates. The goal is to spur investment, and much of that response must come from investment in real estate. Low interest rates will spur investment more if prices of real estate rise more quickly in response to low rates. Our model establishes...
a limit to price increases driven by the requirement/structure of appraisals. The effectiveness of monetary policy is therefore limited by appraisal constraints. The ability to relax these constraints could be a useful additional tool for policy-makers looking to respond to recessions.

The model also has welfare implications. During the period in which prices are rising, assets will be mis-allocated. Some owners who should sell retain the asset in order to capture the expected price appreciation. Others who should buy will be unable to find a seller. If the price were able to adjust immediately, then the asset will always be held by the person who most values it.

2 Preliminaries

The model is substantially different from standard models of asset pricing. We therefore build it in stages, so that the reader can become familiar with each addition prior to moving to more sophisticated versions of the model.

There is a measure one mass of an asset (e.g., houses, shares in a firm, taxi medallions, airplanes, etc.). An agent owns one unit of the asset, in which case she receives flow utility $V - x$, or she doesn’t, in which case her flow utility is 0.

There is a mass of agents of measure $N > 1$, whose types $x$ are distributed uniformly over $[0, N]$. That is, there are $N$ times as many agents as assets. Time begins at $t = -T$, and continues to time $\infty$.

Initially, agents with types $x \in [0, 1]$ are endowed with assets and the remainder do not own the asset. Agents of all types are allowed to trade at no cost, and the price of the asset, $p$, adjusts so that the supply of the asset equals its demand. Recalling that the flow utility for an agent who owns the asset is $V - x$, agents with types in $x \in [0, 1]$ receive the highest utility from owning the asset, so it is efficient for them to own the asset in equilibrium.

We begin by working out simple models, in each case adding components that make the
model more interesting and dynamic. Generally speaking, we do not formally prove or derive the results in the simpler models, as they are there to help readers build intuition. No result should be complicated enough to warrant a formal derivation.

2.1 The Static Model

Let the discount rate be $r$. Then if the assets are allocated efficiently, the flow utility of the marginal owner, whose type is $x = 1$, equals the flow disutility of owning the asset, $(r \cdot p) dt$. That is, $(V - 1) dt = (r \cdot p) dt$, so

$$p(t) = \frac{V - 1}{r}$$

for all $t$. Any price other than $p = \frac{V - 1}{r}$ will not equate supply and demand of the asset. At lower prices, there will be excess demand, and at higher prices, there will be excess supply. The price must adjust so that the marginal owner, of type $x = 1$, is indifferent between owning and not owning.

Note that the price is constant, so equating the flow utility of the marginal agent with the flow expense is legitimate. If prices were increasing or decreasing, this method would not be valid.

2.2 A Simple Dynamic Model

In the static model, no trading takes place. The agents with the highest utility from owning the asset are endowed with the asset, and there is never a reason for anybody to trade. In this section, we induce trading by randomly shocking each agent’s type. In any time interval of length $dt$, the measure of agents whose types are shocked is $\lambda dt$, drawn uniformly over $[0, N]$. That is, in any time $dt$, and for any set $\varepsilon \subseteq [0, N]$, with total mass $\tau$, the mass of shocked agents equals $\lambda \frac{\tau}{N} dt$. We assume that, once shocked, an agent’s new type is drawn uniformly from $[0, N]$. Because (i) there is initially a uniform distribution of types over $[0, N]$, (ii) shocks occur for all types equally often, and (iii) new types are drawn from the
uniform $[0, N]$ once a shock occurs, the distribution of types will remain uniform at all times.

We now show that, in this simple dynamic model, the price remains $p(t) = \frac{V-1}{r}$. For any agent whose type after being shocked is $x_{post}$, her flow utility from owning the asset, net of the price, is

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(V - x_{post}) dt - (r \cdot p) dt = \left( V - x_{post} - r \cdot \frac{V - 1}{r} \right) dt = (1 - x_{post}) dt,
$$

so long as the price is $p(t) = \frac{V-1}{r}$. This flow utility is weakly greater than zero if $x_{post} \in [0, 1]$, and strictly less than zero if $x_{post} \in (1, N]$. Therefore, agents will wish to own the asset if $x_{post} \in [0, 1]$, regardless of their pre-shock types. This should be intuitive. Suppose the asset is a house, and that there is a unit mass of houses which constitute a town. A shock to a type might be a good new job opportunity. If the opportunity is in the town, then the agent will want to move to the town, so long as housing prices are not too high. If the opportunity is elsewhere, then she will want to live elsewhere. These facts are true regardless of where her previous job was located. \footnote{Innate geographic or family preferences for an area could be allowed for in the model. This would just require an association between agents’ pre- and post-shock types. This adds complexity to the model without providing interesting implications, so we abstract away from it.}

There are four possibilities for the pair of pre- and post-shock types, as shown in Table 4. An agent can have a type $x \in [0, 1]$ or $x \in (1, N]$. If the former, then she optimally owns the asset. If the latter, she does not. Therefore, if her type is shocked from $[0, 1]$ to $[0, 1]$, then she owns the asset pre- and post-shock. If it is shocked from $[0, 1]$ to $(1, N]$, then she sells the asset. If it is shocked from $(1, N]$ to $[0, 1]$, then she buys the asset. If it is shocked from $(1, N]$ to $(1, N]$, she never owns the asset.

As before, the marginal type owning the asset is $x = 1$, so we must simply confirm that, at the price of $p(t) = \frac{V-1}{r}$, the number of agents wanting to buy the asset equals the number wanting to sell.

An agent whose type before being shocked is $x_{pre} \in [0, 1]$ is subsequently shocked into $x_{post} \in (1, N]$ with probability $\frac{N-1}{N}$ (the remaining probability, $1/N$, is associated with being...
shocked into $x_{post} \in [0, 1]$). In any instant $dt$, the number of agents shocked from $x_{pre} \in [0, 1]$ to $x_{post} \in (1, N]$ is therefore $\frac{1}{N} \cdot \frac{N-1}{N} \cdot \lambda dt$.

Meanwhile, agents who are initially of types $x_{pre} \in (1, N]$ and are shocked so that $x_{post} \in [0, 1]$ will want to buy the asset. The total number of agents shocked from $x_{pre} \in (1, N]$ to $x_{post} \in [0, 1]$ equals $\frac{N-1}{N} \cdot \frac{1}{N} \cdot \lambda dt$.

Therefore, the number of buyers equals the number of sellers at $p(t) = \frac{V-1}{r}$, so this is the equilibrium price in this dynamic model. The trade flow in any time increment is $\phi(t)dt = \left(\frac{1}{N} \cdot \frac{N-1}{N}\right) \lambda dt$.

We note again that the price is constant, so expectations of future price changes do not affect the decision to buy and sell the asset. It is sufficient for an agent to compare the flow utility she receives from the asset to the flow cost of owning it.

### 2.3 The Dynamic Model with a Shock to Interest Rates

In order to evaluate more realistic price dynamics, we need a shock to equilibrium prices. The simplest shocks are to $V$ or $r$. We will evaluate shocks to $r$, though the intuition applies just as easily to shocks to $V$.

Recall that time begins at $t = -T$. We have assumed thus far that agents believe that nothing will ever change: $r$ and $V$ and the distribution of types are expected to be constant forever, as in the last two sections. Or, we have at least assumed that agents believe that any changes are equally likely to increase or decrease prices, so they are willing to compare flow utilities with flow costs of ownership, without thinking about the future direction of prices. So far in our set-up, this belief has been correct, as prices are $p(t) = \frac{V-1}{r}$ at all times. Suppose now that there is a surprise shock at time $t = 0$ from $r_0$ to $r_1 < r_0$, and suppose that agents believe that this is a one-off shock which will not happen again (as it will not in this model).\(^{17}\)

\(^{17}\)Alternately, we can suppose that agents believe that future shocks are as likely to increase as decrease prices, and therefore see the expected price appreciation from owning the asset as zero. This allows them to ignore potential appreciation when deciding whether to own the asset.
In this case, it is straightforward to see that prices jump from \( p(t) = \frac{V - 1}{r_0} \) for \( t < 0 \) to \( p(t) = \frac{V - 1}{r_1} \) for \( t > 0 \). The flow utility for the marginal owner, type \( x = 1 \), remains \((V - 1) dt\), but the flow cost of ownership at the old price falls from \( r_0 \cdot p(t) = r_0 \cdot \frac{V - 1}{r_0} = V - 1 \) to \( r_1 \cdot p(t) = r_1 \cdot \frac{V - 1}{r_0} = \frac{r_1}{r_0} (V - 1) < V - 1 \). Unless the price rises, there will be more agents wanting to own the asset than there are shares of the asset, forcing the price higher. Equilibrium can only be restored if the price rises to \( p(t) = \frac{V - 1}{r_1} \). This adjustment is shown as the lightly dashed line in Figure 8: The price jumps immediately from \( p_L = \frac{V - 1}{r_0} \) to \( p_H = \frac{V - 1}{r_1} \).

3 The Model with Price Constraints

3.1 A Model with Naïve Agents

Our conclusion in the last section that prices respond instantly to interest rate shocks is exactly how models of financial assets typically work. If rates suddenly drop, then expected returns for all assets drop accordingly, and prices rise instantly to their new levels. We see relatively instantaneous adjustment in liquid financial markets, which is why prices become unpredictable shortly after any shock. The purpose of this model is to show why some asset prices are different – why they do not respond to shocks in the way that the simple model thus far suggests that they should.

In this section, we begin with a model in which agents are “naïve”, in that they assume that future returns are always, in expectation, zero. Therefore, when they decide whether to trade, they only compare the flow utility from owning the asset to the flow disutility from paying for the asset. If future returns are expected to be zero, they need not consider expected changes in either of those flows. As we will see, this assumption is equivalent to assuming that buyers and sellers trade immediately if they are shocked.

The advantage of beginning with this non-fully-rational model is that it is considerably simpler to solve than the full model, yet still provides a substantial share of the empirical
implications that the full model yields. The intuition behind these implications can therefore be provided without the complexity of a fully rational model. That said, in Section 3.2, we provide a fully rational version of the model, confirm the implications from the naïve model, and also provide several additional predictions which line up well with the stylized facts.

To basis of this manuscript is the following assumption:

**Assumption 1.** Agents are unable or unwilling to pay substantially more than recent transaction prices. Specifically, an offered price must be weakly less than $L > 1$ times the average price of the most recent mass of $C > 0$ transactions.

As before, time begins at $t = -T$ and trading takes place under the assumption that any shocks to prices are as likely up as down, so can be ignored in price setting. At time $t = 0$ there is a shock to interest rates from $r_0$ to $r_1 < r_0$. We study how prices respond to this shock, in the presence of a pricing constraint.

It will be useful henceforth to define prices that would obtain in the absence of Assumption 1. We define the *fair price* accordingly:

**Definition 1.** The fair price of the asset is $p = \frac{V - 1}{r}$ for any given $r$.

We will see that Assumption 1 can keep the price below the fair price.

As discussed above, before moving to the model with fully sophisticated agents, we have one more stop to make. In this section, we assume that buyers and sellers believe that the price of the asset behaves the way prices of most assets behave: its return is not predictable at any given time. This assumption is equivalent to assuming that buyers and sellers do not choose when to trade. Instead, if an owner’s type is shocked to $x \in (1, N]$, she sells her asset immediately. If a non-owner is shocked to $x \in [0, 1]$, she immediately buys the asset. We assume that the price is either the fair price of $p = \frac{V - 1}{r_1}$, or is the highest price that buyers can offer for the asset, whichever is lower. We note that this is not a rational model, and is not indicative of the flexibility that we allow for our agents in the fully rational model. It is simply a useful example showing price dynamics with an appraisal constraint, which should
make understanding the full model easier. It is also, for the behaviorally inclined reader, an exposition of what aspects of the model are robust to allowing (a certain type of) irrational agents.

Define the trading intensity to be $\phi(t)$. If agents trade when they are shocked, then the volume of assets traded in instant $dt$ is

$$\phi_0 = \lambda \cdot \frac{1}{N} \cdot \frac{N - 1}{N} \cdot dt.$$  

This is because the number of shocked owners in time $dt$ is $\frac{1}{N} \lambda dt$, and $\frac{N-1}{N}$ of them are shocked to types $x_{post} \in (1, N]$. Under the assumption that prices will not further adjust, these types will prefer not to own the asset at whatever the market clearing price is. Similarly, the number of shocked non-owners is $\frac{N-1}{N} \lambda dt$, a share $\frac{1}{N}$ of whom prefer to own the asset. The numbers of buyers and sellers are equal, so the market clears at volume $\phi_0 = \frac{N-1}{N^2} \lambda dt$ when agents trade when shocked.

Suppose that the interest rate at $t = 0$ drops from $r_0$ to $r_1 < r_0$, as before. Then the fair price of the asset increases immediately from $p_L = \frac{V-1}{r_0}$ to $p_H = \frac{V-1}{r_1}$. With no pricing constraint, as established in the last section, the actual price jumps immediately, as shown in the lightly dashed line in Figure 8. Under Assumption 1, if $p_L << p_H$, and if agents trade as soon as their types/valuations are shocked, then the price will gradually adjust from $p_L$ to $p_H$, as shown in the heavily dashed line in Figure 8.

How do we determine the equation for the price trajectory? Assumption 1 states that a buyer’s ability or willingness to pay is limited by a mass $C$ of recent sale prices (“comps”). Over the period $[t - C\phi_0^{-1}, t]$, a mass $C$ of trades occur.\(^{18}\)

---

\(^{18}\)This formula works because $\phi_0$ is constant. In the fully rational model, the length of time over which $C$ trades occur varies as the trading intensity varies. As is standard, this makes the full model a fixed point problem, which is more difficult to solve.
The average price over that interval is

\[
\bar{p}(t) = \frac{1}{C} \int_{t-C\phi_0^{-1}}^{t} p(s)\phi_0 ds.
\]

By Assumption 1, the maximum purchase price is \(L\) times the average price. It will be helpful to have a definition for the maximum price that buyers will offer. Because real estate is the leading example in the paper, we will use the term \textit{appraised price}.

\begin{definition}
The appraised price is \(L\) times the average price: \(p_A(t) \equiv L\bar{p}(t)\). The actual transaction price is then \(p(t) \equiv \min\{p_A(t), p_H\}\).
\end{definition}

In this section, this becomes

\[
p_A(t) = \frac{L}{C} \int_{t-C\phi_0^{-1}}^{t} p(s)\phi_0 ds,
\]

\[
p(t) \equiv \min\{p_A(t), p_H\}.
\]

There are three periods of interest in this model. For \(t < 0\), prior to the shock to interest rates, \(p(t) = p_L\). We call this the pre-shock period. At some time \(\bar{t}\), the appraised price will equal \(p_H\), after which it is not a binding constraint on prices. We call times \(t > \bar{t}\) the post-adjustment period, and \(\bar{t}\) the time-to-adjustment. Finally, for \(0 \leq t < \bar{t}\), prices are constrained by the appraised price and increase predictably. We call this time interval the period-of-adjustment, and it is the interval of interest in the model.

Note that, if \(p_H \leq Lp_L\), there is no period of adjustment. The permanent increase in the price is sufficiently small that the appraised price never constrains buyers’ offers. If, however, \(p_H > Lp_L\), there is a period in which the price is constrained.

To find the price path during the adjustment period, we first note that, once the shock occurs at time 0, the price immediately jumps to \(Lp_L\). From there, and until the adjustment time (at which \(p = p_H\)), the price path is given by \(p_A(t)\). We can find this path by
differentiating the integral equation $p_A(t) = \frac{L}{C} \int_{t - C\phi_0^{-1}}^{t} p(s)\phi_0 ds$ with respect to $t$. We have

$$p'(t) = \frac{L}{C} \cdot \left( p(t) - p(t - C\phi_0^{-1}) \frac{d}{dt} (t - C\phi_0^{-1}) \right) \cdot \phi_0.$$  \hspace{1cm} (5)

The above simplifies to$^{19}$

$$p'(t) = \frac{L}{C} \cdot (p(t) - p_L) \cdot \phi_0.$$  \hspace{1cm} (6)

Putting this together, the price is $p_L$ for $t < 0$, jumps to $Lp_L$ at $t = 0$, follows the solution of Equation 6,

$$p(t) = p_L + (L - 1)p_L \exp \left\{ \frac{L}{C} \cdot \phi_0 \cdot t \right\},$$  \hspace{1cm} (7)

until the adjustment time $\bar{t}$, where

$$\bar{t} = \frac{C}{L\phi_0} \cdot \log \left( \frac{p_H - p_L}{Lp_L - p_L} \right).$$  \hspace{1cm} (8)

At $\bar{t}$, the price reaches $p_H$ and remains there.

Comparative statics can be obtained easily and, even though the model is not yet fully rational, these comparative statics establish several implications/insights that we will get out of the model.

1. The speed of adjustment, $p'(t)$, is increasing with the trading intensity, $\phi_0$. More trading means more recent comps. As prices are increasing over time, more recent comps imply higher average selling prices off of which an appraisal is based, and therefore higher current appraisals. This weakens the appraisal constraint, allowing faster appreciation.

2. The speed of adjustment, $p'(t)$, is increasing with the flexibility in appraisals, $L$.

3. The speed of adjustment, $p'(t)$, is decreasing with the required comps $C$. Naturally, the

$^{19}$As we formalize in the fully sophisticated model, we assume that parameters are such that $t - C\phi_0^{-1} < 0$ for all $t \in (0, \bar{t})$, so that $p(t - C\phi_0^{-1}) = p_L$. 

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more comps are required, the farther back in time the appraiser must look. As prices
are increasing over time, looking farther back for comps reduces the average price in
an appraisal, and therefore reduces the speed of appreciation.

4. The time to adjustment, $\bar{t}$, is decreasing with the size of the adjustment $p_H/p_L$. When
shocks are larger, the amount of adjustment must be larger, and therefore the time to
adjustment must be larger. The speed of adjustment $p'(t)$ does not depend upon $p_H,$
so any gap between $p_L$ and $p_H$ must be covered by the time to adjustment, $\bar{t}$. This has
important empirical implications: prices that are more “out of equilibrium” will not
necessarily experience faster price appreciation. Instead, they will experience longer
sustained price appreciation.

5. The time to adjustment, $\bar{t}$, is decreasing with the flexibility in appraisals, $L$. As the
appraisal process is more flexible, appreciation is faster (see (2) above) and therefore
full adjustment comes more quickly.

We also note some simple facts which are not comparative statics:

6. Price changes are predictable. Prices do not increase for $t < 0$ or $t > \bar{t}$. In the
adjustment period, prices are given by $p(t) = p_L + (L - 1)p_L \exp \left\{ \frac{1}{\kappa} \cdot \phi_0 \cdot t \right\}$. This is
a smooth process: if one knows prices increased 10% in the last $\Delta t$, then one knows
that they will increase something in the neighborhood of 10% in the next $\Delta t$, so long
as that does not take the time past $\bar{t}$.

7. Prices and appreciation rates can vary substantially among assets that appear to offer
identical cash flows and risks. This is true within a single asset over time: the relevant
asset pricing variables $V$ and $r$ are constant during the adjustment period, yet prices
are rising. This means that, at different points in the adjustment period, the same asset
will have different prices and different total returns. This is also true across assets.
The model only has a single asset, but presumably the economy would have more than
one. For two assets with identical values of $V$ and $r$, but different parameters $L, C,$
or $\lambda$, appreciation rates will differ. Apparently equivalent assets will feature different
pricing and returns.

8. Related to (7) above, less expensive assets will have higher total returns.\textsuperscript{20} During the adjustment period, assets with identical values of \(V\) and \(r\) will generate identical flow-utilities to their owners, but will have different prices. The lower the price, the higher return.

In the full model, the trading intensity is not constant. It will depend on the price. More importantly, it will depend on expectations about price \textit{appreciation}. We will establish in the full model that the price will adjust even more slowly than in the naïve model, as shown in the solid line in Figure 8. Rationality does not solve this problem – it exacerbates it. We will see that rationality creates additional problems as well, most notably buyers’ and sellers’ markets.

### 3.2 The Model with Fully Rational Agents

In this section, we develop the model with fully rational agents, in that both buyers and sellers form expectations of future price increases when they decide whether to trade. While buyers are still limited by the appraisal constraint in what they can pay, both buyers and sellers are aware during the adjustment period that prices will rise in the future, making buying more attractive and selling less attractive. As we will see, because prices are constrained, there will be excess demand for the asset, and a queue of buyers will form. This also will prompt “investor demand”, in which agents buy the asset purely to earn a profit during the adjustment period and sell when adjustment is complete, at time \(\bar{t}\). Further, some owners who would prefer to sell the asset, if prices were not constrained, will instead retain it.

These facts generate new dynamics that bring the model more into line with both what economic intuition would suggest should occur, and what does actually occur in the real estate market.

\textsuperscript{20} Specifically, if total return is defined as \(R(t) = \frac{V - 1 + p'(t)}{p(t)}\), then total return is decreasing in \(t\) during the adjustment period, and therefore decreasing in \(p\).
3.2.1 Model Assumptions

We make several technical assumptions that allow the model to be solved analytically.

**Assumption 2.** *The game ends at time T. At time T, assets are liquidated for their fair prices.*

We are interested in considering a price response to rate or utility shocks that occur over several years. We are not interested in evaluating how prices move until the end of time. Therefore, we place an end to the game: time lasts from time $-T$ until time $T$, where $T = N/\lambda$. At time $T$, assets are liquidated for their fair prices. This assumption is technical and—as it should be clear once the model is solved—not particularly restrictive.

**Assumption 3.** *Agents are shocked at most once.*

Assumption 3 will be helpful because prices will not be static in this model. An agent’s beliefs about her future type will affect her buying and selling decisions. For our purposes, it is simplest to assume that she is shocked only once. Once shocked, she does not need to worry about future shocks when making her buying and selling decisions. We note that this assumption could be loosened to “agents are shocked at most once during the adjustment period”, and the results would follow. We do not like this alternative assumption, because the adjustment period is endogenous, but it should be clear that we are only ruling out multiple shocks to an investor’s valuation during periods of adjustment, which may be short.

**Assumption 4.** *During the adjustment period, agents can trade at most once.*

We make Assumption 4 for tractability. We note that this assumption still allows agents to make multiple trades, so long as only one is during the adjustment periods (for example, buyers are allowed to buy the asset for its appreciation and then sell as soon as appreciation

---

21$\lambda$ is not the hazard rate. The hazard rate associated with $\lambda$ is given by

$$\tilde{\lambda}(t) = \frac{\lambda}{1 - \lambda t}. \quad (9)$$

Note that $\tilde{\lambda}(t) \to \infty$ as $t \to \lambda^{-1}$. 

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is complete). In unreported numerical work (available upon request), we allow an unlimited number of trades for any agent at any time, and find that all results appear to hold under that assumption as well. We are simply unable to find a closed-form solution for the price path, making it less interesting as the core model.

**Assumption 5.** Agents do not trade before they are shocked.

Assumption 5 is helpful for the same reason as Assumption 3: buying and selling decisions that depend on the possibility of future shocks make agents’ objectives complicated enough that the model becomes intractable. If they act as though their types will not change, the model can be solved.

**Assumption 6.** If there are more buyers than sellers at any given time, and prices are unable to adjust to equate the two, then buyers are assigned the asset according to a “first-in-first-out” (FIFO) rationing rule.

We will see that Assumption 1 will prevent prices from fully adjusting instantaneously. Prior to full adjustment, there will be a supply-demand imbalance. In order to derive an analytical solution for the price trajectory, we assume a FIFO assignment rule. FIFO captures the idea that in rapidly appreciating markets, it takes longer for buyers to find a willing seller. We note that nearly any reasonable rule would yield the same price path as we find – we simply need to choose one, and FIFO is simple. As we will see, the price path is determined by the flow of sellers, not the flow of buyers, and the rule governing how buyers exit a queue does not affect seller behavior.

**Assumption 7.** Agents do not queue to buy or sell unless they will be able to trade before the time of adjustment, \( \bar{t} \), and do not leave the queue before trading.

This assumption rules out trivial variations on the equilibrium we find in which agents who will be unable to trade before prices fully adjust jump into and out of a queue arbitrarily. Clearly, this behavior does not affect their welfare, as they will not trade before adjustment,
so they are indifferent to it. This behavior is therefore part of any equilibrium with the same price path as the one we find, but this form of alternate equilibrium is trivially different and we rule it out. Agents only queue if they will buy before $\bar{t}$ and do not leave the queue until they purchase. Note that at any time after adjustment is complete, buyers and sellers can trade as much as they like.

### 3.2.2 Parameter Assumptions

We make three assumptions on parameters. First, define $\Delta p \equiv p_H - p_L$ as the change in the fair price caused by the shock. All three assumptions ensure that we do not run out of agents to shock, and Assumption 8 ensures that the price constraint binds for at least some time after the shock at $t = 0$.

**Assumption 8.** The price is appropriately flexible:

$$\sqrt{\frac{p_H}{p_L}} < L < \frac{p_H}{p_L}. \quad (10)$$

If $L \geq p_H/p_L$, then prices are not constrained and the model trivially reverts to the standard model of immediate adjustment. Because we are interested in situations in which the price does not adjust immediately, we assume that $L < p_H/p_L$. As we will show in Lemma 1, the assumption that $L > \sqrt{p_H/p_L}$ implies that the price trajectory satisfies an ordinary differential equation, instead of a more cumbersome delay differential equation. As solving the delay differential equation is not feasible, we make this assumption for tractability.

**Assumption 9.** The population is sufficiently large:

$$N > \frac{\lambda}{r_1 \log(\frac{\Delta p}{L_{PL} - p_L})}. \quad (11)$$

Assumption 3 states that agents are shocked at most once. Assumption 9 restricts parameters so that there are agents to shock up until time $T$. Put differently, it guarantees that $\lambda T < N$. 

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**Assumption 10.** The mass of required comps is sufficiently small:

\[ C < \lambda \frac{L N - 1}{N^2} \left( (2 - V) + \frac{r_1}{r_0} (V - 1) \right). \]  

This assumption goes hand-in-hand with Assumption 8 in that, while we assume that buyers are constrained in what the can or will pay, the constraint cannot be too severe. If \( C \) is very large or \( L \) is close to unity, then this constraint binds so severely that the price adjusts too slowly to model. For example, if \( C \) is very large, then the average price over those comps will stay close to \( p_L \) even as a long time passes, because most comps will come from the pre-shock period \( t < 0 \). Prices will not move much above \( Lp_L \) before we run out of agents to shock. The same argument follows for \( L \) close to unity.

### 3.2.3 Setup

Now that the model’s assumptions on parameters and behavior are set, it is worth a brief description of what we will now show. Once the shock occurs at time \( t = 0 \), all agents will rationally anticipate the coming price increase. This makes fewer shocked owners willing to sell. We assume in Assumption 9 that the population is sufficiently large. This means that, even knowing that prices will appreciate, some owners will be shocked to types where the utility of owning, \( V - x \), is low enough that they will still sell. This generates a low-but-positive flow of sellers. That flow is less than \( \frac{N - 1}{N^2} \lambda dt \), which is the flow in the naïve case.

Non-owners are also shocked. Of this group, those in \( x_{post} \in [0, 1] \) will want to buy the asset and hold it until the game ends at time \( T \). The flow of this type of buyer is \( \frac{N - 1}{N^2} \lambda dt \). Those with types \( x_{post} > 1 \) may also want to purchase the asset in order to capture the coming price appreciation, and they will sell at the adjustment time \( \bar{t} \), after which prices settle at \( p_H \). We call these types “investors”, because they are buying specifically for expected appreciation, not because they are the most natural owners.\(^{22}\) The flow of buyers is therefore

\(^{22}\)In real estate parlance, these agents would be called “flippers”.
greater than $\frac{N-1}{N} \lambda dt$.

As should be clear, this means that there will be an imbalance between buyers and sellers during the adjustment period. Immediately, at time $t = 0$, a queue of buyers forms, and it continues to grow up until the point where the number of buyers in the queue equals the aggregate future flow of sellers up to time $\bar{t}$. The queue ceases to form then, because getting in the queue at that time is equivalent to getting waiting until $\bar{t}$ to trade.

This process is depicted in Figures 9 and 8.

During this time, the ownership of the asset will not be efficient. Some owners who are shocked and now have types $x_{\text{post}} > 1$ continue to hold the asset for its appreciation. Some non-owners who are shocked and have new types $x_{\text{post}} > 1$ experience the shock early enough to buy the asset during the adjustment period, and become investors. Meanwhile, some non-owners who are shocked into $x_{\text{post}} \in [0,1]$ experience the shock too late in the adjustment period to buy the asset before adjustment is complete, and are stuck waiting out the adjustment.

At time $\bar{t}$, prices reach $p_H$ and stop appreciating. Adjustment is complete. At this time, all owners with $x_{\text{post}} > 1$ sell the asset and all non-owners with types $x_{\text{post}} \in [0,1]$ buy the asset. The asset allocation to agents is efficient and, after this time, trade flows are identical to the naïve model – buyers and sellers trade as soon as they are shocked.

Let $N(t)$ denote the mass of non-owners who have yet to be shocked, $N(t)$ the mass of non-owners who have been shocked, but have yet to queue to buy, and $B(t)$ the mass of non-owners who have queued to buy. Some non-owners find it optimal to queue immediately after being shocked. They enter the queue at a rate of $\varphi_B(t)$. Others find it optimal to queue some time after being shocked. They enter the queue at a rate of $\varphi_{B'}(t)$. Define the total flow of non-owners to trade to be

$$\varphi_B(t) \equiv \varphi_B(t) + \varphi_{B'}(t).$$ (13)
Similarly, let $O(t)$ denote the mass of owners who have yet to be shocked, $O(t)$ the mass of owners who have been shocked, but have yet to queue to sell, and $S(t)$ the mass of owners who have queued to sell. Some owners find it optimal to queue immediately after being shocked. They enter the queue at a rate of $\varphi_S(t)$. Others find it optimal to queue some time after being shocked. They enter the queue at a rate of $\varphi_S(t)$. Define the total flow of non-owners to trade to be

$$\varphi_s(t) \equiv \varphi_S(t) + \varphi_s(t). \quad (14)$$

These stocks and flows are illustrated in Figures 9 and 8.

Having defined notation to track agents throughout the market, we now turn our attention to the trade flow, $\phi(t)$. When buyers queue, $S = 0$ and $B > 0$, the trade flow is the seller flow: $\phi(t) = \varphi_S(t)$. Conversely, when sellers queue, $S > 0$ and $B = 0$, the trade flow is the buyer flow: $\phi(t) = \varphi_B$. When $S(t) > 0$ and $B(t) > 0$, the trade flow is undefined. Suppose that $B(t) > S(t) > 0$. Then the mass $S(t)$ of sellers can match with a mass $S(t)$ of buyers, leaving a mass $B(t) - S(t)$ unmatched. A positive mass of the asset changes hands instantaneously, so the trade flow is “infinite.” Formally, define

$$\mathcal{T} \equiv \{ t \in [0, T] \mid \varphi_S(t) > 0 \text{ and } S(t) = 0, \text{ or } \varphi_B(t) > 0 \text{ and } B(t) = 0 \} \quad (15)$$

and let the trade flow, $\phi : \mathcal{T} \to (0, \infty)$, be given by

$$\phi(t) \equiv \begin{cases} 
\varphi_S(t) & \text{if } S(t) = 0 \\
\varphi_B(t) & \text{if } B(t) = 0 
\end{cases} \quad (16)$$

For $t < 0$, define $\phi(t) \equiv \phi_0$, the trade flow in the naïve equilibrium. As it will be useful in the exposition that follows, define the cumulative trade flow after the shock at time $t = 0$ to be

$$\Phi(t) = \int_0^t \phi(s)ds. \quad (17)$$
Note that since $\phi > 0$, $\Phi$ is invertible.

Let $w_S$ and $w_B$ denote sellers' and buyers' queueing times respectively. If a seller enters the selling queue at time $t$, then she sells at time $t + w_S(t)$; if a buyer enters the buying queue at time $t$, then he buys at time $t + w_B(t)$. Agents exit their queues at a rate equal to the trading intensity $\phi$. Under Assumption 6, $w_S$ and $w_B$ can be defined implicitly as

$$\int_t^{t+w_S(t)} \phi(s)ds = S(t);$$  \hspace{1cm} (18)  
$$\int_t^{t+w_B(t)} \phi(s)ds = B(t).$$  \hspace{1cm} (19)

When, for example, a buyer enters the buyers' queue, there is a mass $B(t)$ of buyers ahead of her. $w_B(t)$ is exactly the amount of time it will take for all of the buyers ahead of her to exit the queue by buying from sellers.\footnote{This queueing process is assumed by the FIFO rule. Other queueing processes would yield identical price paths, but would induce different queueing behavior on the part of buyers. For example, if buyers were assumed to be randomly picked from the queue, then these integrals would be much more complicated. Still, the outcome is easy to intuit: “investor” buyers would continue to queue up to time $\tilde{t}$ because, until that time, there is a possibility of buying for less than $p_H$ and selling for $p_H$ at $\tilde{t}$. The flow of trade is determined by sellers, however, so this additional queueing would not affect sales volume and, therefore, prices.}

**Definition 3 (Admissible Price Paths).** A price path $p$ is said to be admissible if there is $\tilde{t} \in [0, T]$ such that $p(0) = L_p$, $p(t) = p_H$ for all $t \geq \tilde{t}$, and $p$ is twice continuously differentiable in $t$. Let $P$ denote the set of all admissible price paths.

Definition 3 says that a price path must start at $L_p$ at time 0, end at $p_H$ at time $\tilde{t}$, and remain at $p_H$ for all times thereafter. Moreover, it must be twice continuously differentiable between times 0 and $\tilde{t}$.

For $j \in \{S, B\}$, define $\tilde{t}_j \in (0, \tilde{t}]$ to be the time after which

$$w_j(t) = \tilde{t} - t \text{ for all } t \in [\tilde{t}, \tilde{t}].$$  \hspace{1cm} (20)

Such a $\tilde{t}$ must always exist, as $\tilde{t}$ may equal $\tilde{t}$. It may not equal 0 as $w_J(0) = 0$. We will refer to $\tilde{t}$ as the *trade horizon*, as it is the last date on which a non-owner can enter the buyers'
queue and buy the asset for less than $p_H$. Suppose that $B(\tilde{t}) > S(\tilde{t}) = 0$. At time $\tilde{t}$, a buyer who enters the buyers’ queue will have to wait until time $\tilde{t}$ to buy. She is indifferent between entering the queue at time $t$ (when there is wait of $\tilde{t} - t$), and entering the queue at time $\tilde{t}$ (when there is no wait). The existence and uniqueness of $\tilde{t}$ is necessary for the solution of the non-owners’ problem.

**Definition 4 (Admissible Wait Times).** Fix $j \in \{S, B\}$ and $p \in P$. A wait time $w_j$ is said to be admissible if there is $\tilde{t} \in [0, \bar{t}]$ such that $w_j(0) = 0$, $w_B(t) = \tilde{t} - t$ for all $t \in [\tilde{t}, \tilde{t}]$, $w_j(t) = 0$ for all $t \in [\tilde{t}, T]$, and $w_j$ is twice continuously differentiable over $(0, \tilde{t})$. Let $W_j$ denote the set of all admissible wait times.

Consider (for concreteness) the buyers’ wait time. Definition 4 says that a wait time (whether it be for sellers or buyers) must start at 0 at time 0. Once it reaches $\tilde{t}_B(t)$, it decreases at a rate of $-1$ until time $\bar{t}$, at which time the market clears. Moreover, it must be twice continuously differentiable between times 0 and $\tilde{t}_j$ (note that $w_B$ has kinks at times $\tilde{t}_j$ and $\tilde{t}$).

Given a price path $p \in P$, a sellers’ wait time $w_S \in W_S$, and a buyers’ wait time $w_B \in W_B$, we define the owner’s and non-owner’s problems to be

$$ V_O(t, x) \equiv \max_{t_s \in [t, T]} \int_t^{t_s + w_S(t_s)} (v - x)e^{-rs}ds + p(t_s + w_S(t_s))e^{-r(t_s + w_S(t_s))} + V_N(t_s + w_S(t_s), x) $$

$$ V_N(t, x) \equiv \max_{t_b \in [t, T]} -p(t_b + w_B(t_b))e^{-r(t_b + w_B(t_b))} + V_O(t_b + w_B(t_b), x). $$

The shocked non-owner chooses a time $t_b$ at which to enter the buying queue. At time $t_b + w_B(t_b)$, he pays the sale price $p(t_b + w_B(t_b))$ and receives the flow utility $v - x$ from $t_b + w_B(t_b)$ until she sells the asset. The shocked owner chooses a time $t_s$ at which to enter the selling queue. The queue may be empty, in which case she proceeds directly to trade.

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24 These problems, as stated, do not allow for owners not to sell, or non-owners not to buy. However, the problems are equivalent to problems in which these options are present. A purchase (sale) at $T$, or even $\tilde{t}$, can be immediately offset by a sale (purchase) at the same date.
She receives a flow utility of \( v - x \) from the asset until \( t_s + w_S(t_s) \). At time \( t_s + w_S(t_s) \), she receives the sale price \( p(t_s + w_S(t_s)) \).

By Assumption 4, agents can trade at most once during the adjustment period. We implement this assumption in the following way. Define

\[
\tau_s(t_s) \equiv \max\{t_s + w_S(t_s), \bar{t}\} \quad (23)
\]
\[
\tau_b(t_b) \equiv \max\{t_b + w_B(t_b), \bar{t}\}. \quad (24)
\]

The owner’s and non-owner’s problems can then be written as

\[
V_O(t, x) \equiv \max_{t_s \in [t, \bar{t}]} \int_t^{t_s+w_S(t_s)} (v - x)e^{-rs}ds + p(t_s + w_S(t_s))e^{-r(t_s+w_S(t_s))} + V_N(\tau_b(t_b), x) \quad (25)
\]
\[
V_N(t, x) \equiv \max_{t_b \in [t, \bar{t}]} -p(t_b + w_B(t_b))e^{-r(t_b+w_B(t_b))} + \int_{t_b+w_B(t_b)}^{\tau_b(t_b)} (v - x)e^{-rs}ds + V_O(\tau_s(t_s), x). \quad (26)
\]

An owner who chooses to sell during the period of adjustment becomes a non-owner, but cannot re-purchase until time \( \bar{t} \). Similarly, a non-owner who chooses to buy during the period of adjustment becomes an owner, but cannot re-sell until time \( \bar{t} \). Between the time that he buys, and \( \bar{t} \), he receives the flow utility \( V - x \) from the asset.

**Definition 5 (Equilibrium).** An equilibrium is a price path \( p \in P \), a sellers’ wait time \( w_S \in W_S \), and a buyers’ wait time \( w_B \in W_B \), such that \( p \) satisfies

\[
p(t) = \frac{L}{C} \int_{\tau(t)}^{t} p(s)\phi(s)ds, \quad (27)
\]

where \( \phi \) is the trade flow implied by the solutions to the owner’s and non-owner’s problems under \( p \), \( w_S \), and \( w_B \), and \( \tau(t) = \Phi^{-1}(\Phi(t) - C) \).

For ease of notation, define \( \mathcal{E} = P \times W_S \times W_B \), so that an equilibrium can be described as a triple \( (p, w_S, w_B) \in \mathcal{E} \).
The following Lemma gives us an ordinary differential equation for the price dynamics.

**Lemma 1.** If \( \tau(t) < 0 \) for all \( t \in [0, T) \), then the price evolves according to

\[
p'(t) = \frac{L}{C} \cdot (p(t) - p_L) \cdot \phi(t, p)
\]

subject to \( p(0) = Lp_L \).

We refer to the differential equation in Lemma 1 as the *pricing equation*. Because \( \phi \) depends on \( p \), the dynamics for \( p \) are non-linear. Prices appreciate faster when they have greater flexibility (\( L \) is large) and require fewer comps (\( C \) is small), when the difference between the current price and the old price is large (\( p(t) - p_L \) is large), and when the trade flow is high (\( \phi(t, p) \) is large). As a corollary, we compute the cumulative trade flow:

**Corollary 1.** The cumulative trade flow is given by

\[
\int_0^t \phi(s) ds = \frac{C}{L} \log \left( \frac{p(t) - p_L}{Lp_L - p_L} \right).
\]

Since the trade flow is strictly positive, we immediately obtain the following corollary.

**Corollary 2.** \( p \) is strictly increasing.

### 3.2.4 Equilibrium and Comparative Statics

The space of solutions to the differential equation in Lemma 1 is large, so it will be difficult to rule out the possibility of multiple equilibria. We will instead show that, within a certain natural subset of \( \mathcal{E} \), there exists a unique equilibrium. There may be equilibria outside this subset, but we cannot confirm this. We will note in Section 4 which of our results probably generalize to any solution, and which may not.

Specifically, we will look for equilibria with two properties. The more important, and more restrictive, is that we require that the price path be sufficiently convex: \( r''(t) > r_1 p'(t) \).
The price path in the naïve equilibrium of Section 3.1 satisfies this equation, which makes it a natural place to look for an equilibrium in the fully rational model.

As it turns out, the condition \( r''(t) > r_1 p'(t) \) ensures threshold strategies in which agents either buy/sell immediately when they are shocked, or wait until adjustment is complete to buy/sell. As the price is going up faster and faster, if it is optimal to own over the period of length \( dt \) at time \( t \), then it makes sense to own over the period of length \( dt \) at time \( t' > t \).

This generates a threshold strategy.

If prices were less convex, or even concave, then some types would buy/sell immediately upon being shocked, some would wait until a time which is a function of their new types and the equilibrium price path, and some would wait until full adjustment. This is clearly a much more difficult equilibrium to analyze, and it remains the province of future work.

The other property that we require is that the wait time for buyers and sellers is greater than \(-1\) for any time periods in which a queue is forming: \( w'_j(t) > -1 \) for all \( t \in (0, \bar{t}_j), j \in \{B, S\} \). This is sufficient (but far from necessary) for the buyers’ and sellers’ problems to be quasi-concave and quasi-convex, respectively. In words, we assume that arriving in a queue at a later time does not enable agents to trade sooner: the length of the queue, measured in units of time, cannot shrink by more than one second every second. Given FIFO, this requirement is satisfied so long as there is a positive flow arriving to buy and sell at all times up to the point where a queue is so long that, upon entering it, an agent will not exit before adjustment is complete. We believe that this requirement of an equilibrium is highly likely to hold in any equilibrium, but do not prove this.

Agents will take different actions depending on their types. Our first step is to partition the type-space \([0, N]\) into different actions. Fix some price trajectory \( p \in P \). There is some \( \bar{t} \in (0, T) \) such that \( p(t) = p_H \) for all \( t \geq \bar{t} \). Define \( z(t, \bar{t}) \equiv \exp(-r_1(\bar{t} - t)) \) and

\[
\bar{x}(t, p) \equiv 1 + r_1 \left( \frac{p_H - p(t)}{1 - z(t, \bar{t})} \right).
\] (30)
\( x(t, p) \) is the type of the owner who is indifferent between selling at time \( t \) and time \( \bar{t} \):

\[
\int_t^{\bar{t}} \left( V - \tilde{x}(t) \right) e^{-rt} ds + p_H e^{-rt} = p(t) e^{-rt}.
\] (31)

In the rest of the paper, we will use the shorthand “\( \bar{x}(t) \)” for “\( \bar{x}(t, p) \).” The dependence of \( \bar{x} \) on \( p \) should be understood.

Next, define

\[
\tilde{y}(t, p) \equiv V + p'(t) - r_1 p(t).
\] (32)

\( \tilde{y} \) denotes the non-owner who’s indifferent between queueing to buy now and queueing to buy later. He is the non-owner for whom the marginal benefit of owning equals the marginal cost of foregone price appreciation:

\[
V - \tilde{y}(t) = -p(t)(\rho(t) - r_1).
\] (33)

where \( \rho(t) := p'(t)/p(t) \). As with \( \bar{x} \), we will use the shorthand “\( \tilde{y}(t) \)” for “\( \tilde{y}(t, p) \).” We have the following facts about \( \bar{x} \) and \( \tilde{y} \):

**Lemma 2.** If \( p \in P \), then for all \( t \in [0, \bar{t}) \), \( \bar{x}'(t) > 0 \), \( \tilde{y}'(t) > 0 \), and \( \bar{x}(t) > \tilde{y}(t) > 1 \).

These facts will be useful in establishing the existence of an equilibrium. In the meantime, the two facts about \( \bar{x} \), together with Assumptions 8 – 10, ensure that \( \lim_{t \uparrow \bar{t}} p'(t) < N - 1 \) and guarantee that \( \bar{x}(t) \in (0, N) \) for all \( t \in (0, \bar{t}) \).

We now characterize a subset of \( \mathcal{E} \), which will be the focus of our discussion. Let

\[
\mathcal{E}_0 \equiv \{(p, w_S, w_B) \in \mathcal{E} \mid p'' > r_1 p' \text{ for all } t \in (0, \bar{t}); w_j(t) > -1 \text{ for all } t \in (0, \tilde{t}_j), j \in \{B, S\}\}.
\] (34)

The following lemma shows that elements of \( \mathcal{E}_0 \) admit a unique functional form for the trade flow.
Lemma 3. If \((p, w_S, w_B) \in \mathcal{E}_0\), then the trade flow induced by \((p, w_S, w_B)\) is

\[
\phi(t, p) = \lambda \cdot \frac{1}{N} \cdot \frac{N - \bar{x}(t, p)}{N}.
\]  

(35)

Moreover, sellers do not wait to trade: \(w_S = 0\).

Since \(\phi(t) = \Phi'(t)\), we can write the trade flow in Lemma 3 as

\[
d\Phi = \lambda dt \cdot \frac{1}{N} \cdot N - \bar{x}(t, p).
\]

(36)

During a time increment \(dt\), a mass \(\lambda dt\) of agents are shocked, of which a fraction \(1/N\) are owners, of which a fraction \((N - \bar{x}(t))/N\) are shocked to \([\bar{x}(t), N]\). Recall that \(\bar{x}(t)\) is the type of the agent who is indifferent between selling at times \(t\) and \(\bar{t}\). Therefore, all agents for whom \(x > \bar{x}(t)\) will sell at time \(t\), while agents for whom \(x \leq \bar{x}(t)\) will sell at time \(\bar{t}\).

Buyers’ and sellers’ strategies are illustrated in Figure 10.

To ease the exposition, define the price \(u(t) \equiv p(t) - p_L\), and the parameters

\[
u_0 \equiv Lp_L - p_L, \quad \alpha \equiv \frac{\lambda L}{CN^2}, \quad \beta \equiv \frac{N - 1}{r_1}.
\]

(37)

Given an equilibrium, Lemma 3 says that the trade flow is determined by the rate at which owners are shocked to \([\bar{x}(t), N]\). Let \(p^* : [0, T] \times (0, T) \to \mathbb{R}\) be given by

\[
p^*(t; \bar{t}) \equiv p_L + \frac{z(t, \bar{t})^\alpha (\beta - \Delta p)(1 - z(t, \bar{t}))^\alpha}{\int_{\bar{t}}^1 s^\alpha (\beta - \Delta p - 1)(1 - s)^\alpha \Delta p - 1 ds},
\]

(38)

for \(t \in [0, \bar{t}]\), and \(p(t, \bar{t}) = p_H\) for \(t \in [\bar{t}, T]\). \(\bar{t}\) is the unique solution of \(p^*(0; \bar{t}) = Lp_L\) (the existence and uniqueness of \(\bar{t}\) is shown in the proof of Proposition 1).

The following proposition states that there exists a unique equilibrium in \(\mathcal{E}_0\).

**Proposition 1 (Existence and Uniqueness).** There exists a unique equilibrium in \(\mathcal{E}_0\), and \(p^*\) is its price path.
Figure 8 shows a numerical solution of the fully rational model. The dashed line shows the solution under naïve trade flow, while the solid line shows the solution under sophisticated trade flow. The naïve trade flow trajectory converges to the new price faster than the fully rational trade flow trajectory.

Similar results hold for the rate of appreciation, price predictability, differential pricing of assets with the same cash flows and discount rates, and differential total returns based on asset prices, as in the naïve model.

We can derive comparative statics for $p^*$, and find that all of the comparative statics from the naïve model continue to hold in the fully rational model.

**Corollary 3.** The time to adjustment, $\bar{t}$, is

1. decreasing with the shock intensity, $\lambda$;
2. decreasing with appraisers’ flexibility, $L$;
3. increasing with the required comps, $C$;
4. increasing with the size of the adjustment, $\Delta p$.

We can also do comparative statics on the average price appreciation.

**Corollary 4.** Define the average rate of appreciation to be $\mu(\bar{t}) \equiv \frac{1}{\bar{t}} \int_0^{\bar{t}} p'(s) ds$. It is

1. increasing with the shock intensity, $\lambda$;
2. increasing with appraisers’ flexibility, $L$;
3. decreasing with the required comps, $C$;
4. decreasing with the size of the adjustment, $\Delta p$.

**Proof.** Observe that

$$\mu(\bar{t}) = \frac{1}{\bar{t}} \int_0^{\bar{t}} p'(s) ds = \frac{p(\bar{t}) - p(0)}{\bar{t}} = \frac{p_H - p_L}{\bar{t}}, \quad (39)$$

so that $\mu'(\bar{t}) < 0$. The comparative statics follow immediately from Corollary 3 and the chain rule.

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Corollary 5. Returns are predictable, in the sense that for a given \( t \in (0, \bar{t}) \), and \( s \in (0, \bar{t} - t) \),

\[
\frac{p'(t + s)}{p(t + s)} = \left[ \frac{p'(t)}{p(t)} \right] + \left[ \frac{p''(t)p(t) - p'^2}{p(t)^2} \right] \cdot s + O(s^2) \tag{40}
\]

Proof. Direct application of Taylor’s Theorem.

The linearization in Corollary 5 should be seen as the as model’s equivalent of the AR(1) regressions that we run using real estate data.

Corollary 6. Consider two markets, \( I \) and \( J \), for which \( p^I_L = p^J_L \), \( p^I_H = p^J_H \), and \( r^I = r^J \), but either \( \lambda^I \neq \lambda^J \), \( L^I \neq L^J \), \( C^I \neq C^J \), or \( N^I \neq N^J \). Then for all \( t \in (0, \max\{\bar{t}^I, \bar{t}^J\}) \), \( p^I(t) \neq p^J(t) \).

4 Discussion and Empirical Results

4.1 Additional Implications that Result Only in the Fully Rational Model

We now discuss the results from the fully rational model. Given that results are so similar in the naïve and fully rational models, one could ask why we bother with the complexity of the fully rational model. There are four major reasons.

The first reason is that it is useful to show that rationality does not solve the problems that arise in the naïve model. Indeed, it makes them worse, as it lengthens the time to adjustment:

Corollary 7. The time to adjustment in the fully rational trading model, \( \bar{t}_s \), is greater than the time to adjustment in the naïve trading model, \( \bar{t}_n \).

If prices will appreciate from \( L_{pL} \) to \( p_H \), then owners who have received a sufficiently negative utility shock (i.e., owners for whom \( x_{post} > 1 \)) are willing to receive negative flow utility in exchange for a higher selling price in the future. In the naïve model, this does not happen.
Agents who have been shocked sell immediately. Since the trade flow is strictly lower in the fully rational model, it takes longer for prices to adjust. A numerical example is shown in Figure 11, panel (a), which displays the price paths for the cases of instant adjustment, the naïve model, and the rational model.

The remaining reasons to develop the fully rational model relate to new implications of the model which do not arise in the naïve model.

The first new implication from the rational model is that we observe sellers’ markets, in which sellers are able to sell the asset whenever they wish, but buyers must queue and wait to purchase. In real estate, it is common for both participants and brokers to talk about sellers’ and buyers’ markets, terms and concepts which are foreign to standard models of financial assets. Indeed, Figure 7 shows that the length of time that houses sit on the market really does differ substantially depending on the rate of price appreciation. While our model alone cannot produce this fact, we will argue in Section 4.2 that an equivalent model, in which interest rates rise rather than fall, would produce queues of sellers rather than queues of buyers. Combining the two would yield the relationship that sale times are shorter when markets are appreciating and longer when they are depreciating. These facts are consistent neither with standard asset pricing theories, nor with the naïve model in this paper. Only the fully rational model can produce it. We see this in Figure 11, panel (c), which displays the time between when a buyer or seller queue and when they can transact. For sellers, the wait time is always zero. For buyers, it is zero before and after adjustment, but during adjustment, it can be considerably longer.

The second new implication of the rational model is that it generates investors, who buy the asset not because they are the most natural holders, but because they foresee price appreciation and move in early in the process to take advantage of it. It would be unnatural in a paper concerned with the pricing of financial assets if such investors were not present, as they are not in the naïve model. We see this in Figure 11, panel (b), which plots the flows of buyers and sellers. The flat sections pre- and post-adjustment represent normal trade...
flow. Seller flow drops during adjustment, as some sellers delay selling to profit from the anticipated price increase. Buyer flow spikes at time 0, and the difference between the level of that curve and the flat section just before represents the flow of investors. At time $\tilde{t}_b$, the queue of buyers is so long that new buyers stop queueing and the buyer flow drops until adjustment is complete. The resulting growth and shrinkage of the queue is shown in Figure 11, panel (d).

The third new implication of the rational model is that the friction preventing buyers from offering the fair price for the asset yields welfare losses. There are owners with less flow utility from owning the asset who do not sell to potential buyers with a greater flow utility. This does not happen in the naïve model, but it may happen in practice.

4.2 Extension to Positive Shocks to Rates or Negative Shocks to Flow Utility

As noted above, our model only discusses the effect of limits to what buyers are willing or able to pay for an asset, and therefore only has novel predictions for markets in which the fair price of an asset rises. We believe, however, that the model can apply equally well to settings in which there are limits to what sellers are willing to accept and the fair price falls. As with limits on buyers, limits on sellers could be legal/financial, behavioral, or agency-based. For example, continuing with the leading example of this paper of the market for real estate, suppose that an owner pays $100,000 for a house with a $10,000 down payment. After a year, she owes $88,000 on the house, but the fair price has fallen to $75,000. She may understand that $75,000 is the new fair price but, if she is financially constrained, she may not have the $13,000 to send to the bank in the case of a short sale.

Banks sometimes accept that sellers must sell the house and repay less than the outstanding principal value of the mortgage.\textsuperscript{25} It is likely that, as house prices fall, the amount that a bank would accept on a sale would depend critically on recent comparable transactions. This

\textsuperscript{25}If the alternative is default, then a short sale may be more profitable than costly foreclosure proceedings.
example, therefore, would suggest an inability (on the part of the owner) or unwillingness (on the part of the lender) to allow a sale for substantially less than the $88,000 owed on the house, and a relationship between that level and the level of comparable transactions.

There is also evidence (Genesove and Mayer, 1997) that sellers are loss-averse, and are reluctant to sell for prices below their purchase prices, even if they have a low loan-to-value ratio and do not need cash. This finding is especially observable when the purchase was recent. This suggests a (potentially behavioral) justification for an assumption on sellers’ willingness-to-sell at prices below those of recent transactions.

If we believe that a similar argument to that which we apply to buyers would apply to sellers as well, then the implications of the model should hold when the fair price falls. A lower fair price would cause an increase in the number of shocked owners wanting to sell, and a decrease in the number of shocked owners wanting to buy. The result would be a seller queue; that is, increased inventory on the market. A prolonged, predictable decline in real estate prices would ensue. Some owners would have to sell, because their disutilities of owning would be prohibitively high (perhaps because of unemployment), and these sales would lower comparable prices for others, thus lowering the prices at which they would sell. Eventually, prices would reach a new, lower, level.

For the application to real estate (and taxi medallions and airplanes) in the next section, we consider that the model probably speaks to price decreases as well, so that the existence of buyers’ markets, inventory gluts, persistent price decreases, and differential total returns for similar assets, all apply in sinking markets.

4.3 Application to Residential Real Estate

We have already discussed how many of our assumptions apply in the market for residential real estate, and how empirical facts about that market appear consistent with the model’s predictions. In this section we formally list the model’s predictions and compare those predictions to stylized facts regarding the real estate market. We do not attempt a serious
analysis proving that our model is the correct model of that market, and that other models are incorrect. This is a theory paper.

The following five predictions arise in the naïve model, the fully-rational model, and ought to be true when prices fall as well, so long as sellers face similar constraints to their willingness-to-accept as buyers face in their willingness-to-pay in our model.

1. Appraisals lag prices, and the lag is larger the faster prices are changing. In order to evaluate whether this is true in practice, we use data provided by the Federal Housing Finance Agency (FHFA), a government agency responsible for overseeing aspects of the secondary mortgage market in the US. They maintain two house price indices which we reproduce in this paper. One index is, like the S&P Case-Shiller indices, built using only transaction data. The other uses a combination of transaction prices and appraisals.

Figure 2 plots the values of these two indices from January, 2000, until January, 2016. Especially after 2006, the index which includes appraisals clearly lags the purchase-only index. This is consistent with the idea that appraisals are backward-looking, as we assume in this paper, and that the resulting gap between appraisals and market prices can be large. In Figure 3, we focus on the period of rapidly increasing prices during recovery from the Great Recession – the second quarter of 2011 to the second quarter of 2016. For the 100 largest cities in the US, we plot the cumulative increases over that five year period in the purchase-only and purchase-plus-appraisal indices. Cities are ordered, from left to right, by the lowest cumulative return to highest, using the averages of the two indices. It should be clear that the facts that appraisals lag price increases, and that the gap is larger where returns have been larger, are true for essentially every city – there is not much variation from city to city.

It is also clear from the figure that the gap between appraisals and market prices is larger when prices are rising or falling faster, but to make it more so, we document this explicitly in Figure 4. Figure 4 shows a scatter comparing the rate of monthly
appreciation in the purchase-only price index to the gap between the levels of the purchase-only and the purchase-plus-appraisal indices. It is clear that when prices are rising or falling faster, the indices diverge by more.

2. Prices are predictable. Price predictability in real estate was identified in Case and Shiller (1989) and has been highly studied theoretically and empirically since then. For reference, Figures 1a (Figure 1b) present scatter-plots of contemporaneous and lagged monthly (annual) changes in the value of the S&P Case-Shiller 20-City Composite index from January 1991 through September 2015. It is clear that, even in the 26 years since the publication of Case and Shiller (1989), price predictability remains prevalent in real estate.

The figures are striking enough, but we also present the data in Table 3, as a list of coefficient estimates from AR(1) regressions of monthly and annual returns on lagged returns, for each component city of the S&P Case-Shiller 20-City Composite. Returns are highly auto-correlated, with AR(1) coefficients as high as 90% (75%) in quarterly (annual) samples.

This predictability, as discussed, has been widely studied, though there is debate over whether current models are adequate for explaining the sheer magnitude of predictability. Rather than wade into that debate, we focus on the other asset pricing anomalies that appear in real estate markets and are less heavily studied theoretically.

3. Prices and appreciation rates can vary substantially, both over time, for a given asset whose fundamentals do not change, and across apparently identical assets at the same time. Figure 2 presents two scatter-plots comparing the rent-price ratio to price appreciation (panel A) and to total returns (panel B) for rental real estate in 200 metro areas in the years 2010-2013, as calculated by Zillow. The first thing to notice is that there is substantial variation in the price of rental housing, as prices regularly range from five to 25 times annual rents. While this type of variation is present in the stock market as well, rental properties are much more similar than stocks in terms of likely
growth in earnings. Indeed, panel A shows that the variation in price appreciation between properties whose prices – relative to rents – differ by a factor of five, have only a 6% difference in annual price appreciation. As shown in panel B, this appreciation is not nearly enough to account for the lower rents that accrue to expensive properties. This figure suggests substantial variation in the pricing of similar properties.

4. Lower-priced assets tend to earn higher total returns. This is because, after the shock to rates or valuations, the fundamentals of an asset do not change, but the price does. Whenever the price is lower, therefore, the total return is higher. Panel B of Figure 2 plots total return for rental properties versus their rent-price ratios. It is immediately clear that less expensive properties earn substantially higher total returns. In a standard asset-pricing framework, this would be viewed either as an anomaly, or accounted for by an unexplained “risk factor”. It seems unlikely that inexpensive properties face some unexplained risk that is sufficient to deliver returns more than 10 percentage points above those of expensive properties.

5. Markets with a higher natural rate of change in the identity of the optimal owner experience faster appreciation or depreciation. This is because trade generates recent comparable prices which are less stale, allowing prices to move more quickly toward their new fair values. Figure 4 presents a scatter-plot of price appreciation versus turnover for 200 metro areas over the years 2010-2014, where turnover is the fraction of homes that sold in the prior year. Also presented are lines-of-best-fit for the regions in which appreciation is positive and in which it is negative. When prices are increasing, higher turnover is associated with faster appreciation, as the model suggests that it should be. When prices are decreasing, higher turnover is associated with faster depreciation, as the model suggests that it should be.

The following two predictions arise only in the fully rational model.

6. Buyers’ and sellers’ markets emerge when prices are rising or falling, respectively. In a buyer’s market, prices are rising, queues of buyers form, and the time between when a
typical buyer starts searching for a house and when she purchases one is high. Sellers are able to sell relatively quickly. In a seller’s market, prices are falling, queues of sellers form, and the time-on-market for a house is high. It is difficult to find good data for average time-to-buy a house. While we have many anecdotal examples of quickly appreciating markets in which bidding wars are common (or in which buyers approach owners whose houses aren’t even on the market, in the hopes of making an offer that the owners cannot refuse), we do not have actual data on buyer search time or the length of a buyer queue.\textsuperscript{26}

We can, however, observe average days-on-market for sellers, and the length of the seller queue. Figure 6 is a scatter-plot comparing the average days on market for houses which sell in a given year with the price appreciation that year. Periods of increasing prices should feature no seller queues and short periods between a house being listed and sold.\textsuperscript{27} Periods of decreasing prices should experience seller queues, yielding higher days-on-market for sold homes. Figure 6 shows this rather clearly, and there appears to be a floor of average days-on-market in the neighborhood of 90 days, reached when appreciation hits 10% per year.

Another way to measure the seller queue is inventories, typically measured as the ratio of current houses for sale divided by monthly sales (to account for seasonal variation in inventories and longer-term growth in the size of the market). In Figure 12, we plot the inventory-to-sales ratio from January 1987 to January 2016, as well as the Case-Shiller 10-City Composite index. There is always some inventory of unsold homes on the market, simply because some search is required in real estate, there is time between a house going under contract and the sale closing, and because some people choose

\textsuperscript{26}http://www.forbes.com/sites/erincarlyle/2016/04/13/bidding-wars-for-homes-have-become-the-new-normal-in-many-u-s-cities/#23ea53fc688a

\textsuperscript{27}In the model, when prices are appreciating, there is no seller queue and sales take place as soon as houses are listed. In practice, even if a house goes under contract on the day it is listed, it still will take 30-60 days to close, yielding a practical minimum of average days-on-market of somewhere in the neighborhood of at least 45 days, depending on local market rules. As the real estate market also features search and as some sellers might initially overprice their houses, average days-on-market will typically be higher than 60 days, even in a seller’s market.
unreasonable list prices. This minimum inventory appears to be in the neighborhood of four months worth of homes. As Figure 12 shows, inventories oscillate around six months of supply while prices are stable. Beginning in 1997, prices begin a steady rise, and inventories fall to four months supply, staying remarkably stable until September 2005. This is a seller’s market, in which prices are predictably increasing and the seller’s queue is as close to empty as appears empirically possible.

By the summer of 2006, nine months later, prices flatten and begin to fall, falling in earnest until May 2009. The seller’s queue reaches a peak four months prior, and reaches a new normal of eight months supply, two months later. Prices begin rising in earnest again in February 2012, precisely the same month in which inventories reach their new normal. It should be immediately clear that (i) periods of rising prices are those in which inventories are low and stable, (ii) periods of falling prices are those in which inventories grow, and (iii) the end of a period of declining prices is associated with a substantial reduction/elimination of excess inventories. All three of these predictions are consistent with the model, in which (i) sellers’ markets are associated with rising prices and no seller’s queue, (ii) buyers’ markets feature a growing queue of sellers, and (iii) queues evaporate when prices reach their new long-run levels.

7. In a buyer’s market, some purchasers are investors. These people are not the most natural owners for a property, but enter the market to enjoy the expected price appreciation. Investors exit the market when price adjustment is complete. There is no good database of investor-owners – simply looking at non-owner-occupied property is not sufficient, as it is not efficient for everyone to own her residence. However, we offer two pieces of evidence that, during periods of rapid price appreciation, many owners only intend to own the property during the period of expected appreciation. First, anecdotal evidence suggests that the rate at which homeowners choose to rent out their homes after they move has increased substantially since the housing crisis.
of 2007-2010.\footnote{http://money.cnn.com/2014/06/17/real_estate/homeowner-landlords/} There has also been a substantial increase in the ownership of single family rentals by Wall Street investment funds, like BlackStone.\footnote{30} The six largest buyers have purchased $28 billion on properties since the crisis. This welfare loss suggests room for policy interventions.

Second, many individual (non-institutional) investors appear to get into the business of buying houses simply for appreciation – not rental income or the opportunity to live in them – when prices are appreciating. This practice is known as “flipping” a house, which also may include renovations during the ownership phase. Figure 13 shows the Google Trends search frequency of the term “house flipping” among US searches from January 2004 to September 2015. The term is rarely searched until July 2005, the date of the series premier of the A&E show *Flip This House*, a reality show featuring various teams of real estate investors who flip houses. Though it does not show in the Google Trends data, house flipping was popular prior to the premier of the show, though the use of the term “flipping” to describe the practice coincided with the premier. Once the term house flipping enters the lexicon, the correlation of the online search volume for “house flipping” with the level of the Case-Shiller 10-City Composite index is 67\%.\footnote{30} We believe that this provides evidence that, during periods of price appreciation, individual investors buy property simply to enjoy the appreciation, not to act as landlords, owner-occupiers, or other long-term investors. This is consistent with our model’s predictions.

### 4.4 Application to Alternative Assets

Lest readers wonder whether there is something unique about real estate and question the need for a general model of constrained asset prices, we analyze the prices of assets other

\footnote{Not coincidentally, Trump University, a for-profit school ostensibly devoted to teach the art of flipping and managing real estate, was also founded in 2005, and aimed to profit from the already-popular practice. Also not coincidentally, Trump University folded in 2010, when interest in flipping waned.}
than single family real estate. We show that the basic prediction of price-predictability holds for seven classes of airplane, four classes of commercial real estate, apartment buildings, and taxi medallions. These assets share little in common beyond the fact that they are all often purchased with debt, collateralized by the asset itself. We conclude by showing that price-predictability is not inherent to all assets. Seats on the New York Stock Exchange, which share a number of important properties with taxi medallions, do not exhibit price-predictability.

Consider the market for taxi medallions. If you want to drive a taxi in New York, you must either own a medallion or rent one from somebody else. There is an active market for both individual and corporate (rental) medallions, and medallion transactions are recorded by the New York City Taxi & Limousine Commission. We were able to acquire the data using a Freedom of Information Law request, and have monthly transaction volume and average prices from 1947 to 2016.

An individual or company purchasing a medallion could finance the purchase with a loan, cash, or some combination of the two. If the medallion purchase is financed with a loan, then it may be constrained by the appraisal process just as in residential real estate. If it is financed with cash, then the buyer might be subject to a behavioral constraint of not wanting to feel like a chump.

As we see in Figures 1a and 1b, both at the monthly and annual levels, medallion prices are predictable. They are less predictable than single-family homes, suggesting that the constraint may be weaker for medallions than for houses, but they are clearly predictable.

We repeat the analysis using indices for airplane prices. Aircraft price indices were provided by VREF, an aggregator of airplane transaction data and order books. VREF creates price indices by calculating average transaction prices for all equivalent planes within a class-vintage at the quarterly level. For example, the “Light Single” class is comprised of transaction data for the Tiger AA5B, the Beechcraft C23 Sundowner, the Cessna 172P, the Cardinal, the Piper Warrior, and the Archer, all from the late 1960s. The index is simply
average transaction prices of these airplanes, adjusted for condition (e.g., the age of key components), over time. The vintage is not changing over time, so prices tend to decline as the aircraft age. That said, prices of the smallest and least expensive planes have been considerably more stable than, say, the prices of cars.\footnote{Note that these data are for much smaller and older airplanes than those studied by Benmelech and Bergman (2009, 2011a,b). Our data cover planes which are often owned by individuals or firms which lease them to individuals.}

As we see in Figures 1a and 1b, both at the monthly and annual levels, airplane prices are predictable. Table 1 splits the sample into seven classes of plane, which range from very inexpensive (the Light Single class described above, whose average selling price in the fourth quarter of 2016 is approximately $45 thousand) to the more expensive (the Large Jet class had an average selling price in the fourth quarter of 2016 of over $2.5 million). Regardless of class, there is substantial predictability.

We also make use of price indices for commercial, industrial, and multi-family residential property provided by Moody’s/Real Capital Analytics for the years 2000 to 2015.\footnote{A description of the process by which these indices are created can be found at the Real Capital Analytics website: https://www.rcanalytics.com/our-data/data-process/ (accessed February 9, 2017).} As we see in Figures 1a and 1b, both at the monthly and annual levels, commercial (including industrial and multi-family residential) real estate prices are predictable. Table 2 cuts the sample 10 ways, splitting by major versus non-major markets, or by property type: office (central business district or suburban), industrial, retail, core commercial, and apartment. Regardless of the cut, there is substantial predictability at the monthly level. At the annual level, there is substantial predictability in all categories except for central business district office properties.

But perhaps predictability is present for all but the most liquid financial assets for some reason other than our model. Are there assets similar to medallions, for example, which do not exhibit predictability? If so, why don’t they?

An investment house wanting to trade on the floor of the New York Stock Exchange (NYSE) is required to own a seat. As seats are in limited supply, a firm wanting to trade must
buy one from another firm willing to sell. These seats are similar to taxi medallions, in that both a seat and a medallion give the bearer the right to engage in some profitable business—trading stock or driving a taxi. Those profits are heterogeneous (some traders and drivers are better than others), generating heterogeneous valuations for the assets and, therefore, trade. The main differences, for the purpose of our model, are that investment houses probably do not need to borrow to afford a seat, and they are probably not behaviorally biased in valuing seats, as they regularly trade financial assets. We would therefore expect that seats on the NYSE would not exhibit the same pricing patterns as taxi medallions. Indeed, they do not.

The predictability of returns on NYSE seats has been thoroughly studied in Schwert (1977), who finds no predictability.\(^{33}\) We also analyze monthly closing prices from 1880 to 1925, a period prior to the period studied by Schwert (1977) for which we have closing price data (and not just monthly high-low data).\(^{34}\) As we see in Figures 1a and 1b, at both the monthly and annual levels, we find no price predictability. Indeed, in untabulated results, we find a precisely estimated zero coefficient in an AR(1) regression of monthly returns on lagged monthly returns. While a seat on the NYSE is, in many important ways, similar to a taxi medallion, their investor bases are not. As the model predicts, it is not the asset which drives predictability, but rather constraints on the investor base.

5 Conclusion

We have developed a simple model of asset prices under the assumption that agents are unable or unwilling to pay substantially more than recent transactions. We have argued that legal, agency, or behavioral constraints are all possible sources of this inability or unwillingness. Our model generates price predictability, differential pricing for identical assets, and higher returns on lower priced assets. Markets which experience higher turnover in the optimal owner of the asset experience more rapid conversion to the long-run price. Perhaps

\(^{33}\)We would like to thank David Gross for the idea of looking at the pricing of NYSE seats.

\(^{34}\)These data were courteously provided by Asaf Bernstein.
surprisingly, when agents are rational and anticipate these pricing anomalies, each problem becomes more severe. When prices are rising, a seller’s market emerges: sellers are able to sell their holdings rapidly, but buyers need a long time to locate a seller.

We formally model the case in which buyers are constrained and prices are rising, but the argument likely holds for the case in which sellers are constrained and prices are falling. In this case, a buyer’s market emerges, in which case buyers are able to quickly find an asset to buy, but sellers list assets for sale for a longer time before finding a buyer.

We show that the model’s core empirical prediction of price-predictability is present in several disparate markets: residential real estate, commercial and industrial real estate, taxi medallions, and airplanes. This prediction is not borne out in stocks or seats on the NYSE, markets which are not subject to the core constraint defining the model.

The model’s additional predictions, of which there are many, are also consistent with stylized facts from the market for US residential real estate. We begin by replicating the Case and Shiller (1989) predictability results, and show that real estate is highly predictable, both when it is rising and when it is falling, at both the monthly and annual levels. We then confirm each of the above predictions of the model: different rental markets can have very different price-rent ratios, and lower-priced markets earn markedly higher total returns. Appreciating markets feature low days-on-market for sold homes, suggesting a seller’s market, whereas depreciating markets feature high days-on-market, suggesting seller queues. Appreciating markets feature low and stable inventories of unsold homes, while depreciating markets feature high and rising levels of unsold homes. These last four facts are all consistent with the predictions of the model.

Higher turnover markets feature faster adjustment to the long-run price, whether that is higher or lower than the current price. Therefore, higher turnover markets should experience more rapid depreciation and appreciation. Low turnover markets should experience neither. We show empirically that this appears to be the case for real estate.

Finally, in appreciating markets, many buyers will step in simply to profit from the
anticipated price appreciation, not because they are the most natural owners of the house. Using data on online searches for the term “house flipping”, we find suggestive evidence that this practice is indeed more common in rising markets, though it correlates more with historical increases than future price increases. This may provide evidence that people form beliefs about future increases based upon recent experience, but we leave further theorizing for future work.

Our theory has implications for monetary policy. Central banks respond to the onset of recessions by cutting interest rates. The goal is to spur investment, and much of that response must come from investment in residential real estate. Low interest rates will spur investment more if prices of real estate rise more quickly in response to low rates. Our model establishes a limit to price increases driven by the requirement/structure of appraisals. The effectiveness of monetary policy is therefore limited by appraisal constraints. The ability to relax these constraints could be a useful additional tool for policy-makers looking to respond to recessions.

The model also has welfare implications. During the period in which prices are rising, assets will be mis-allocated. Some owners who should sell retain the asset in order to capture the expected price appreciation. Others who should buy will be unable to find a seller. If the price were able to adjust immediately, then the asset would always be held by the person who most values it. Especially if we believe that owner-occupancy serves some good (outside the model), then reducing the strength of appraisal constraints can improve welfare.

References


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6 Appendix A: Auxiliary Results

This section is self-contained. Let $a, b > 0$, and $m : [0, 1) \rightarrow [0, b)$ be given by

$$m(z) = \frac{z^a(1-z)^b}{\int_z^1 s^{a-1}(1-s)^{b-1}ds}.$$  \hfill (41)

It will be useful to continue $m$, $m'$, and $m''$ to $z = 1$. To that end, define

$$m(1) \equiv m_1^0 \equiv b,$$  \hfill (42)

$$m'(1) \equiv m_1^1 \equiv \frac{(a+b)b}{1+b}, \text{ and}$$  \hfill (43)

$$m''(1) \equiv m_1^2 \equiv \frac{2b(a-1)(a+b)}{(2+b)(1+b)^2}.$$  \hfill (44)

By construction, $m$, $m'$, and $m''$ are continuous at $z = 1$, as we will presently show.

**Lemma 4.** $\lim_{z \uparrow 1} m(z) = m_1^0$.

**Proof.** By L'Hôpital’s rule,

$$\lim_{z \uparrow 1} m(z) = \lim_{z \uparrow 1} \frac{az^{a-1}(1-z)^b - bz^a(1-z)^{b-1}}{-az^{a-1}(1-z)^{b-1}} = \lim_{z \uparrow 1} (a(1-z) - bz) = b$$  \hfill (45)

as desired. \hfill $\blacksquare$

**Lemma 5.** $\lim_{z \uparrow 1} m'(z) = m_1^1$.
Proof. If \( \lim_{z \uparrow 1} m'(z) < \infty \), then

\[
\lim_{z \uparrow 1} m'(z) = \lim_{z \uparrow 1} \left( \left( \frac{a}{z} - \frac{b}{1-z} \right) m(z) + \left( \frac{1}{z} + \frac{1}{1-z} \right) m(z)^2 \right) \tag{46}
\]

\[
= \lim_{z \uparrow 1} \frac{a(1-z) - bz m(z) + m(z)^2}{z(1-z)} \tag{47}
\]

\[
= \lim_{z \uparrow 1} \frac{-(a+b)m(z) + (a(1-z) - bz)m'(z) + 2m(z)m'(z)}{1-2z} \tag{48}
\]

\[
= (a + b)b + b \lim_{z \uparrow 1} m'(z) - 2b \lim_{z \uparrow 1} m'(z) \tag{49}
\]

\[
= (a + b)b - b \lim_{z \uparrow 1} m'(z). \tag{50}
\]

We conclude that

\[
\lim_{x \uparrow 1} m'(x) = \frac{b(a + b)}{1 + b} \tag{51}
\]

by the uniqueness of the limit. \( \blacksquare \)

Before showing that \( \lim_{z \uparrow 1} m''(z) = m_1^2 \), we need to set some additional notation. Define

\[
A(z) \equiv 2, \tag{52}
\]

\[
B(z) \equiv (3a-1)(1-z) - (3b-1)z, \tag{53}
\]

\[
C(z) \equiv a(a-1)(1-z)^2 - 2abz(1-z) + b(b-1)z^2, \tag{54}
\]

and

\[
Q(z, m) \equiv A(z)m^2 + B(z)m + C(z). \tag{55}
\]

Note that \( Q \) is a quadratic in \( m \).

**Lemma 6.** \( \lim_{z \uparrow 1} m''(z) = m_1^2 \).

Proof. A simple (albeit tedious) calculation shows that

\[
z^2(1-z)^2m''(z) = m(z)Q(z, m(z)). \tag{56}
\]
Another simple calculation shows that
\[
\lim_{z \uparrow 1} \frac{d}{dz} Q(z, m(z)) = 0, \tag{57}
\]
while a less simple calculation shows that
\[
\lim_{z \uparrow 1} \frac{d^2}{dz^2} Q(z, m(z)) = -\frac{2(a - 1)(b - 1)(a + b)}{(1 + b)^2} + (1 + b) \lim_{z \uparrow 1} m''(z). \tag{58}
\]

Note that \(Q(1, b) = 0\). If \(\lim_{z \uparrow} m''(z) < \infty\), then
\[
\lim_{z \uparrow 1} m''(z) = \lim_{z \uparrow 1} \frac{m(z)Q(z, m(z))}{z^2(1 - z)^2}
= \lim_{z \uparrow 1} \frac{m'(z)Q(z, m(z)) + m(z) \frac{d}{dz} Q(z, m(z))}{2z(1 - z)^2 - 2z^2(1 - z)}
= \lim_{z \uparrow 1} \frac{m''(z)Q(z, m(z)) + 2m'(z) \frac{d}{dz} Q(z, m(z)) + m(z) \frac{d^2}{dz^2} Q(z, m(z))}{2(1 - z)^2 - 4z(1 - z) - 4z(1 - z) + 2z^2}
= \frac{b}{2} \lim_{z \uparrow 1} \frac{d^2}{dz^2} Q(z, m(z))
= -\frac{b(a - 1)(b - 1)(a + b)}{(1 + b)^2} + \frac{b(1 + b)}{2} \lim_{z \uparrow 1} m''(z). \tag{63}
\]

We conclude that
\[
\lim_{z \uparrow 1} m''(1) = \frac{2b(a - 1)(a + b)}{(2 + b)(1 + b)^2} \tag{64}
\]
by the uniqueness of the limit. ■

In the next lemma, we show that \(m\) is the unique solution of a certain boundary value problem. Let \(X, Y : (0, 1) \to \mathbb{R}\) be given by
\[
X(z) = \frac{a}{z} - \frac{b}{1 - z}, \tag{65}
Y(z) = \frac{1}{z} + \frac{1}{1 - z}. \tag{66}
\]

Lemma 7. \(m\) is strictly increasing.
Proof. Put \( f(z) = z^a(1 - z)^b \) and \( g(z) = \int_z^1 s^{a-1}(1 - s)^{b-1} ds \), so that \( m = f/g \). Note that

\[
 f'(z)g(z) = (az^{a-1}(1 - z)^b - bz^a(1 - z)^b) \int_z^1 s^{a-1}(1 - s)^{b-1} ds
\]

\[
 = z^{a-1}(1 - z)^b \int_z^1 (as^{a-1}(1 - z)(1 - s)^{b-1} - bzs^{a-1}(1 - s)^{b-1}) ds
\]

\[
 > z^{a-1}(1 - z)^b \int_z^1 (as^{a-1}(1 - s)^b - bs^a(1 - s)^b) ds
\]

\[
 = -z^{2a-1}(1 - z)^{2b-1}
\]

\[
 = f(z)g'(z).
\]

(67)

Therefore, \( m'(z) = (f'(z)g(z) - f(z)g'(z))/g(z)^2 > 0 \) as desired.

Proposition 2. \( m \) is the unique, strictly increasing element of \( C^1[0, 1] \) satisfying \( m(0) = 0 \), \( m(1) = b \), and

\[
m'z = X(z)m(z) + Y(z)m(z)^2
\]

(d.e.)

for all \( z \in (0, 1) \).

Proof. Suppose there is \( \ell \in C^1[0, 1] \), \( \ell \neq m \), satisfying the three properties of \( m \) enumerated above. Since \( (\ell - m)(0) = 0 \), and \( (\ell - m)(1) = 0 \), there is \( z_0 \in (0, 1) \) such that \( (\ell - m)'(z_0) = 0 \) (by the mean value theorem):

\[
0 = (\ell - m)'(z_0) = X(z_0)(\ell - m)(z_0) + Y(z_0)(\ell^2 - m^2)(z_0) = (X(z_0) + Y(z_0)(\ell + m)(z_0))(\ell - m)(z_0).
\]

(72)

There are two cases.

1. If \( (\ell - m)(z_0) = 0 \), then we argue that \( \ell = m \) in a neighborhood of \( z_0 \). This argument is standard, and goes as follows. If \( u \) is a solution of \( u' = Xu + Y u^2 \) subject to \( u(z_0) = u_0 \), then \( v \equiv 1/u \) is a solution of \( v' + Xv + Y = 0 \) subject to \( v(z_0) = 1/u_0 \). \( X \) and \( Y \) are continuously differentiable in a neighborhood of \( z_0 \), and hence they are locally Lipschitz. By the Picard-Lindelöf Theorem, there is but one such \( u \) in a neighborhood \( U \) of \( z_0 \). Therefore, there is but one such \( u \) in \( U \). Therefore, \( \ell = m \) in \( U \). Since \( \ell \) and
m are analytic on (0, 1), and ℓ = m on U, we have that ℓ = m everywhere, which is a contradiction.

2. If \( X(z_0) + Y(z_0)(\ell + m)(z_0) = 0 \), then we argue that \( m'(z_0) = \ell'(z_0) < 0 \). Since \( m \) and \( \ell \) are strictly increasing, and \( m(0) = \ell(0) = 0 \), we must have \( m(z_0) > 0 \) and \( \ell(z_0) > 0 \). Moreover, \( Y(z_0) \). We conclude that

\[
0 = (X(z_0) + Y(z_0)(\ell + m)(z_0))m(z_0) = m'(z_0) + Y(z_0)\ell(z_0)m(z_0) > m'(z_0). \tag{73}
\]

But then \( m \) is strictly decreasing in a neighborhood of \( z_0 \), which is a contradiction.

We conclude that \( m \) is the unique. \( \blacksquare \)

**Lemma 8.** If \( a > 1 \), then \( m \) is strictly convex.\(^{35}\)

**Proof.** Recall that

\[
z^2(1 - z)^2m''(z) = m(z)Q(z, m(z)). \tag{74}
\]

Let \( D \) be the discriminant of \( Q \): \( D(z) \equiv B(z)^2 - 4A(z)C(z) \). Let \( u \) be the upper root of \( Q \):

\[
u(z) \equiv \frac{-B(z) + \sqrt{D(z)}}{2A(z)}. \tag{75}
\]

Since \( a > 0 \), \( Q \) is a smiling parabola in \( m \). The idea of the proof is to show that \( m > u \) on \((0, 1)\), and hence \( m'' > 0 \) on \((0, 1)\). Three facts will prove useful.

1. \( u(1) = m(1) \). By direct calculation,

\[
u(1) = \frac{1}{4} \left( -B(1) + \sqrt{D(1)} \right) = b = m(1). \tag{76}
\]

2. \( u'(1) = m'(1) \). Also by direct calculation,

\[
u'(1) = \frac{1}{4} \left( -B'(1) + \frac{D'(1)}{\sqrt{D(1)}} \right) = \frac{(a + b)b}{1 + b} = m'(1). \tag{77}
\]

\(^{35}\)The authors would like to thank Robert Van Wesep for proving this fact.
3. $u$ is strictly concave if $b < 1$ and strictly convex if $b > 1$. To show this, define

$$\tilde{A} \equiv (a + b - 2)^2,$$

$$\tilde{B} \equiv -2(a^2 + ab - a - 3b + 2), \quad \text{and}$$

$$\tilde{C} \equiv (1 + a)^2.$$  

(78) \hspace{1cm} (79) \hspace{1cm} (80)

Another simple calculation shows that

$$D(z) = \tilde{A}z^2 + \tilde{B}z + \tilde{C}.$$  

(81)

Let $\tilde{D}$ be the discriminant of $D$: $\tilde{D} \equiv \tilde{B}^2 - 4\tilde{A}\tilde{C}$. It transpires that

$$u''(z) = -\frac{\tilde{D}}{16\sqrt{D(z)^3}} = \frac{2(a + b)(a - 1)(b - 1)}{\sqrt{D(z)^3}}.$$  

(82)

The fact follows by observation.

Next, we show that $u$ is real-valued on $[0, 1]$. If $b \geq 1$, then $\tilde{D} \leq 0$, and hence $D(z) \geq 0$ for all $z \in [0, 1]$. If $b < 1$, then

$$\tilde{B} + 2\tilde{A} = 2(b^2 + ab - 3a - b + 2) > 2(ab - 3a + 2) > 2(a - 3a + 2) > 0,$$  

(83)

which implies that $-\tilde{B}/2\tilde{A}$, the point at which $D$ attains its minimum, is strictly greater than one. Together with the fact that $D(1) = \tilde{A} + \tilde{B} + \tilde{C} = (1 + b)^2 > 0$, we can conclude that $D(z) > 0$ for all $z \in [0, 1]$. Next, we show that $m$ and $u$ do not intersect on $(0, 1)$. To that end, define

$$Z \equiv \{z \in (0, 1) \mid m(z) = u(z)\}.$$  

(84)

$Z$ is the set of roots of $m - u$ on $(0, 1)$ (which we claim to be empty). For each $r > 0$ and
\[ z \in \mathbb{R}, \text{ define the } r\text{-ball to be} \]

\[ B_r(z) = \{ z' \in \mathbb{R} \mid |z - z'| < r \}. \tag{85} \]

There are two cases.

1. \( b > 1 \): Suppose that \( Z \neq \emptyset \). Put \( \bar{z} \equiv \inf Z \). Choose a sequence \((z_n)\) in \( Z \) converging to \( \bar{z} \). Now \((m - u)(z_n) = 0\) for each \( n \in \mathbb{N}, \bar{z} \in [0, 1]\), and \( m - u \) is continuous on \([0, 1]\), so \( m(\bar{z}) = u(\bar{z}) \), and hence \( m''(\bar{z}) = 0 \). Moreover, \( m(0) = 0 > (1 - a)/2 = u(0) \), so \( \bar{z} \neq 1 \). \( m - u \) is analytic on \((0, 1)\), so \( \bar{z} \) is isolated in \( Z \). Formally, there is \( r > 0 \) such that \( m \neq u \) on \( B_r(\bar{z}) \setminus \{ \bar{z} \} \). Choose the largest such \( r \):

\[ r^* \equiv \sup \{ r' > 0 \mid m \neq u \text{ on } b_{r'}(\bar{z}) \setminus \{ \bar{z} \} \}. \tag{86} \]

Note that \( r^* \leq 1 - z_0 \), since \( m(1) = u(1) \). Put \( z = \bar{z} + r \). Since \( m - u \) has no zeros less than \( \bar{z} \), it must vanish at \( z \). For ease of notation, put \( R = (\bar{z}, z) \). There are two cases: either \( m > u \) on \( R \), or \( m < u \) on \( R \). Suppose that \( m > u \) on \( R \). Observe that \( \bar{z} \) is a local minimizer of \( m - u \): \((m - u)(\bar{z}) = 0\), and \( m - u > 0 \) on \( B_r(\bar{z}) \setminus \{ \bar{z} \} \). Since \( \bar{z} \) is a local minimizer of \( m - u \), it must satisfy the second order necessity condition \((m - u)''(\bar{z}) \geq 0\).

But then \( m''(\bar{z}) \geq u''(\bar{z}) > 0 \), which is a contradiction, since \( m''(\bar{z}) = 0 \). Now suppose that \( m < u \) on \( R \). Since \( m(\bar{z}) = u(\bar{z}), m(z) = u(z), \) and \( m - u \) is continuous, \( m - u \) has a local minimizer, say \( x \in R \), at which the second order necessity condition must be satisfied: \((m - u)''(x) \geq 0\). Again, \( m''(x) \geq u''(x) > 0 \), which is a contradiction, since \( m'' < 0 \) on \( R \). \( Z = \emptyset \).

2. \( b \leq 1 \): Suppose that \( Z \neq \emptyset \). Choose \( z_0 \in Z \). Since \( m(z_0) = u(z_0) \) and \( m(1) = u(1) \), there is \( x_0 \in (z_0, 1) \) such that \( m'(x_0) = u'(x_0) \) (by the mean value theorem), and since \( m'(1) = u'(1) \), there is \( y_0 \in (x_0, 1) \) such that \( m''(y_0) = u''(y_0) \) (again, by the mean value theorem). There are two cases.

(a) \( b = 0 \): \( u \) is affine, so \( m''(y_0) = u''(y_0) = 0 \). Put \( z_1 = y_0 \).
(b) $b < 1$: $u$ is strictly concave, so $m''(y_0) = u''(y_0) < 0$, and $m''(1) > 0$, so there is $z_1 \in (y_0, 1)$ such that $m''(z_1) = 0$ (by the intermediate value theorem).

In either case, $m''(z_1) = 0$, which implies that $m(z_1) = u(z_1)$. Continuing in this fashion, we obtain a sequence $(z_n)$ such that $m''(z_n) = 0$ for each $n \in \mathbb{N}$. Now $(z_n)$ is bounded, so it has a convergent subsequence $(z_{n_k})$ with limit $\bar{z} \in [z_0, 1]$. Since $m''$ is continuous, $m''(\bar{z}) = 0$. If $\bar{z} = 1$, then $m''(1) = 0$, which is a contradiction (see Lemma 6). Observe that $m''$ is analytic on $(0, 1)$. If $\bar{z} \in [z_0, 1)$, then it is isolated in $Z$. Formally, there is $r > 0$ such that $m''(\bar{z}) \neq 0$ on $B_r(\bar{z}) \setminus \{\bar{z}\}$. But this is a contradiction, since $z_{n_k} \to \bar{z}$: there is $N \in \mathbb{N}$ such that for all $k \geq N$, $z_{n_k} \in B_r(\bar{z})$ and $m''(z_{n_k}) = 0$. 

$Z = \emptyset$.

Now $m(0) = 0 > (1 - a)/2 = u(1)$, and $m$ is continuous, so there is a neighborhood of $z = 0$ on which $m > u$. But $m$ and $u$ never cross ($Z$ is empty), so $m > u$ on $(0, 1)$. We conclude that $m'' > 0$ on $(0, 1)$.

7 Appendix B: Proofs

Proof of Lemma 1. First, differentiate the comparables equation:

$$C = \int_{\tau(t)}^t \phi(s) ds \Rightarrow \phi(t) - \phi(\tau(t)) \tau'(t) = 0. \quad (87)$$

Next, differentiate the appraiser’s equation:

$$p(t) = \frac{L}{C} \int_{\tau(t)}^t p(s) \phi(s) ds. \Rightarrow p'(t) = \frac{L}{C} \cdot (p(t) \phi(t) - p(\tau(t)) \phi(\tau(t)) \tau'(t)) \quad (88)$$

$$= \frac{L}{C} \cdot (p(t) - p(\tau(t))) \phi(t) \quad (89)$$

$$= \frac{L}{C} \cdot (p(t) - p_L) \cdot \phi(t) \quad (90)$$

where the last line follows because $\tau(t) < 0$ and hence $p(\tau(t)) = p_L$. ■
Proof of Corollary 1. Observe that
\[
\frac{L}{C} \int_0^t \phi(s) ds = \int_0^t \frac{u'(s)}{u(s)} \cdot ds = \log(u(s)) \bigg|_0^t = \log \left( \frac{u(t)}{u_0} \right). \tag{91}
\]

Proof of Corollary 2. By Corollary 1, we have that for \( t > 0 \)
\[
0 < \frac{L}{C} \int_0^t \phi(s) ds = \log \left( \frac{u(t)}{u_0} \right) \Rightarrow u(t) > u_0 \Rightarrow u'(t) = \frac{L}{C} \cdot u(t) \cdot \phi(t, p) > 0, \tag{92}
\]
where we’ve used the fact that \( \phi > 0 \) by definition. ■

Proof. We prove each fact in order.

Fact 1: \( \bar{x}(t) \geq 1 \). \( p(t) \leq p_H \) by definition. Therefore,
\[
\bar{x}(t) = 1 + r \left( \frac{p_H - p(t)}{1 - z(t, t)} \right) > 1 \tag{93}
\]
as desired.

Fact 2: \( \bar{x}(t) \) is increasing. Fix \( t \in (0, \bar{t}) \), and let \( t' \in (t, \bar{t}) \). Define
\[
\eta(s, \bar{x}(t)) \equiv V - \bar{x}(t) + p'(s) - rp(s). \tag{94}
\]
Since \( p'' > rp \) on \((0, \bar{t})\), \( \eta(\bullet, \bar{x}(t)) \) is strictly increasing (in its first argument). By the definition of \( \bar{x} \), we have that
\[
0 = \int_t^{\bar{t}} \eta(s, \bar{x}(t)) e^{-rs} ds; \tag{95}
\]
\[
0 = \int_t^{\bar{t}} \eta(s, \bar{x}(t')) e^{-rs} ds > \int_t^{\bar{t}} \eta(s, \bar{x}(t')) e^{-rs} ds, \tag{96}
\]
where the second line follows because \( \eta(\bullet, \bar{x}(t')) \) is strictly increasing in its first argu-
ment and it changes sign between \( t' \) and \( \bar{t} \). Observe that

\[
0 < \int_{t}^{\bar{t}} (\eta(s, \bar{x}(t)) - \eta(s, \bar{x}(t')))e^{-rs}ds = \int_{t}^{\bar{t}} (\bar{x}(t') - \bar{x}(t))e^{-rs}ds. \tag{97}
\]

Taking \( t' \to t \), we conclude that \( \bar{x}'(t) \geq 0 \).

**Fact 3:** \( \bar{y}(t) > 1 \). \( \bar{y}(t) = V + p'(t) - rp(t) = V - 1 + p'(t) - rp(t) = r(p_H - p(t)) + 1 + p'(t) > 1 \).

**Fact 4:** \( \bar{x}(t) > \bar{y}(t) \). Since \( \bar{y} \) is increasing,

\[
0 = \int_{t}^{\bar{t}} (V - \bar{x}(t) + p'(s) - rp(s))e^{-rs}ds = \int_{t}^{\bar{t}} (\bar{y}(s) - \bar{x}(t))e^{-rs}ds > \int_{t}^{\bar{t}} (\bar{y}(t) - \bar{x}(t))e^{-rs}ds, \tag{98}
\]

which implies that \( \bar{x}(t) > \bar{y}(t) \).

**Proof of Lemma 3.** The goal of the proof is to show that the flow of buyers to trade always exceeds the flow of sellers to trade, and hence the trade flow is the seller flow.

**Objectives.** Fix \( t \in [0, T] \) and \( x \in [0, N] \). Recall that the shocked owner chooses a time \( t_s \) to enter the sellers’ queue, while the shocked non-owner chooses a time \( t_b \) to enter the buyers’ queue:

\[
V_O(t, x) = \max_{t_s \in [t, T]} \int_{t}^{t_s + w_S(t_s)} (v - x)e^{-rs}ds + p(t_s + w_S(t_s))e^{-r(t_s + w_S(t_s))} + V_N(\tau_s(t_s), x) \tag{99}
\]

\[
V_N(t, x) = \max_{t_b \in [t, T]} -p(t_b + w_B(t_b))e^{-r(t_b + w_B(t_b))} + \int_{t_b + w_B(t_b)}^{\tau_b(t_b)} (v - x)e^{-rs}ds + V_O(\tau_b(t_b), x) \tag{100}
\]
where
\[ \tau_s(t_s) \equiv \max\{t_s + w_S(t_s), \bar{t}\} \quad (101) \]
\[ \tau_b(t_b) \equiv \max\{t_b + w_B(t_b), \bar{t}\}. \quad (102) \]

For tractability, we’ve assumed that agents can only trade once before time \( \bar{t} \). They may trade as often as they like thereafter. Let \( f_O(\bullet; t, x), f_N(\bullet; t, x) : [0, T] \to \mathbb{R} \) be given by
\[
 f_O(t_s; t, x) = \int_t^{t_s+w_S(t_s)} (v - x)e^{-rs}ds + p(t_s + w_S(t_s))e^{-r(t_s+w_S(t_s))} (103)
\]
\[
 f_N(t_b; t, x) = -p(t_b + w_B(t_b))e^{-r(t_b+w_B(t_b))} + \int_{t_b+w_B(t_b)}^{\tau_b(t_b)} (v - x)e^{-rs}ds \quad (104)
\]
so that
\[
 V_O(t, x) = \max_{t_s\in[t,T]} f_O(t_s; t, x) + V_N(\tau_s(t_s), x) \quad (105)
\]
\[
 V_N(t, x) = \max_{t_b\in[t,T]} f_N(t_b; t, x) + V_O(\tau_b(t_b), x). \quad (106)
\]

It will be useful to split the owner’s and non-owner’s problems into the pre-adjustment period, \([0, \bar{t}]\), and the post-adjustment period, \([\bar{t}, T]\).

**Post-Adjustment.** Suppose that \( t \in [\bar{t}, T] \). Then for all \( s \in [t, T] \), \( p(s) = p_H, w_B(s) = 0 \), and \( w_S(s) = 0 \). The owner’s and non-owner’s problems are
\[
 V_O(t, x) = \max_{t_s\in[t,T]} f_O(t_s; t, x) + V_N(t_s, x) \quad (107)
\]
\[
 V_N(t, x) = \max_{t_b\in[t,T]} f_N(t_b; t, x) + V_O(t_b, x). \quad (108)
\]
The owner’s value is

\[
V_O(t, x) = \begin{cases} 
\int_t^\infty (v - x)e^{-rs}ds & \text{if } x < 1 \\
pHe^{-rt} & \text{otherwise}
\end{cases}
\] (109)

She can either hold the asset forever, in which case she receives \(v - x\) in perpetuity, or sell the asset, in which case she receives \(p_H\). The non-owner’s value is

\[
V_N(t, x) = V_O(t, x) - pHe^{-rt}.
\] (110)

It is the owner’s value, less the price \(p_H\) of becoming an owner.

**Pre-Adjustment.** Suppose that \(t \in [0, \bar{t}]\), and let \(t_s \in (0, \bar{t}_s)\) and \(t_b \in (0, \bar{t}_b)\), so that \(w'_S(t_s) > 0\) and \(w'_B(t_b) > 0\). In what follows, primes denote differentiation with respect to the first argument:

\[
f'_O(t_s; t, x) = (v - x + p'(t_s + w_S(t_s)) - rp(t_s + w_S(t_s)))(1 + w'_S(t_s))e^{-r(t_s + w_S(t_s))}
\] (111)

\[
f'_N(t_s; t, x) = -(v - x + p'(t_b + w_B(t_b)) - rp(t_b + w_B(t_b)))(1 + w'_B(t_b))e^{-r(t_b + w_B(t_b))}.
\] (112)

If \(f'_O(t_s; t, x) = 0\), then

\[
f''_O(t_s; t, x) = (p''(t_s + w_S(t_s)) - rp'(t_s + w_S(t_s)))(1 + w'_S(t_s))e^{-r(t_s + w_S(t_s))}
\]

\[
- \left[ \frac{w''_S(t_s)}{1 + w'_S(t_s)} - r(1 + w'_S(t_s)) \right] f'_O(t_s; t, x)
\]

\[
= (p''(t_s + w_S(t_s)) - rp'(t_s + w_S(t_s)))(1 + w'_S(t_s))^2 > 0.
\] (113)
Hence, \( f_O(\bullet; t, x) \) is strictly quasiconvex on \([0, \tilde{t}_s] \). If \( f'_N(t_b; t, x) = 0 \), then

\[
\begin{align*}
  f''_N(t_b; t, x) &= -(p''(t_b + w_B(t_b)) - rp'(t_b + w_B(t_b))(1 + w'_B(t_b))^2 e^{-r(t_b + w_B(t_b))}) \\
  &\quad - \left[ \frac{w''_B(t_b)}{1 + w'_B(t_b)} - r(1 + w'_B(t_b)) \right] f'_N(t_b; t, x) \\
  &= -(p''(t_b + w_B(t_b)) - rp'(t_b + w_B(t_b))(1 + w'_B(t_b))^2) < 0.
\end{align*}
\]

Hence, \( f'_N(\bullet; t, x) \) is strictly quasiconcave on \([0, \tilde{t}_b] \).

**The Owner’s Problem.** The owner’s problem is to

\[
\max_{t_s \in [t, \tilde{t}]} f_O(t_s; t, x) + V_N(\bar{t}, x). \tag{119}
\]

Since \( f_O \) is strictly quasiconvex, on \([0, \tilde{t}_s] \), and constant on \([\tilde{t}_s, \bar{t}], t^*_s(t, x) \in \{t, \bar{t}\} \). By definition, \( \bar{x}(t + w_S(t)) \) is the type who is indifferent between queueing to sell at time \( t \) and time \( \bar{t} \): \( f_O(\bar{t}; t, \bar{x}(t)) = f_O(t; t, x) \). Therefore,

\[
t^*_s(t, x) = \begin{cases} 
  t & \text{if } x > \bar{x}(t + w_S(t)) \\
  \bar{t} & \text{otherwise}
\end{cases} \tag{120}
\]

The seller flow is

\[
\varphi_S(t) = \lambda \cdot \frac{1}{N} \cdot \frac{N - \bar{x}(t + w_S(t))}{N}. \tag{121}
\]

for \( t \in [0, \tilde{t}_s] \), and \( \varphi_S(t) = \phi_0 \) for \( t \in [\tilde{t}_s, \bar{t}] \). During a time increment \( dt \), a mass \( \lambda dt \) of agents are shocked, of which a fraction \( 1/N \) are owners, of which a fraction \((N - \bar{x}(t + w_S(t)))/N\) are shocked to \([\bar{x}(t + w_S(t)), N]\). For \( t \in (\bar{t}, \tilde{\bar{t}}] \), \( \varphi_B(t) = \phi_0 \).
The Non-Owner’s Problem. The non-owner’s problem is to

\[
\max_{t_b \in [t, \bar{t}]} \ f_N(t_b; t, x) + V_O(\bar{t}, x). \tag{122}
\]

Observe that \( f'_N(t_b; t, x) > 0 \) if and only if \( x > \bar{y}(t_b + w_B(t_b)) \). For \( t \in [0, \bar{t}] \), there are three categories of non-owners.

1. Non-owners for whom \( f'_N(t) < 0 \) (i.e., non-owners for whom \( x < \bar{y}(t + w_B(t)) \)) find it optimal to queue immediately because their objectives are quasiconcave. Therefore,

\[
\phi_B(t) = \lambda \cdot \frac{N - 1}{N} \cdot \frac{\bar{y}(t + w_B(t))}{N}. \tag{123}
\]

During a time increment \( dt \), a mass \( \lambda dt \) of agents are shocked, of which a fraction \( (N - 1)/N \) are non-owners, of which a fraction \( \bar{y}(t + w_B(t))/N \) are shocked to \([0, \bar{y}(t + w_B(t))].\)

2. Non-owners for whom \( f'_N(t) < 0 \) (i.e., non-owners for whom \( x > \bar{y}(t + w_B(t)) \)) find it optimal to wait to queue:

\[
o(t) = \lambda \cdot \frac{N - 1}{N} \cdot \frac{N - \bar{y}(t + w_B(t))}{N}. \tag{124}
\]

During a time increment \( dt \), a mass \( \lambda dt \) of agents are shocked, of which a fraction \( (N - 1)/N \) are non-owners, of which a fraction \( (N - \bar{y}(t + w_B(t)))/N \) are shocked to \([\bar{y}(t + w_B(t)), N].\)

3. Some non-owners who found it optimal to wait to queue now find it optimal to queue. Consider the mass \( N(t) \) of non-owners who have been shocked but have yet to queue to buy. Since \( \bar{y} \) is increasing, these non-owners are distributed uniformly on \([\bar{y}(t), N].\)

Over a time increment \( dt \), a mass \( d\bar{y}(t + w_B(t))(1 + w'_B(t)) \) now find it optimal to queue to buy:

\[
\phi_B(t) \cdot dt = \frac{d\bar{y}(t + w_B(t))(1 + w'_B(t))}{N - \bar{y}(t + w_B(t))} \cdot N(t). \tag{125}
\]
Therefore, \( N(t) \) solves the initial value problem

\[
N'(t) = \lambda \cdot \frac{N - 1}{N} \cdot \frac{N - \bar{y}(t + w_B(t))}{N} - \frac{\bar{y}'(t + w_B(t))(1 + w'_B(t))}{N - \bar{y}(1 + w'_B(t))} \cdot N(t)
\]  

(126)

subject to \( N(0) = 0 \), which can be readily solved using the method of integrating factors:

\[
N(t) = \lambda \cdot \frac{N - 1}{N} \cdot \frac{N - \bar{y}(t + w_B(t))}{N} \cdot t
\]  

(127)

and hence,

\[
\varphi_B(t) = \lambda \cdot \frac{N - 1}{N} \cdot \frac{t\bar{y}'(t + w_B(t))(1 + w'_B(t))}{N}
\]  

(128)

Therefore, the total flow of non-owners to trade is

\[
\varphi_B(t) = \varphi_B(t) + \varphi_B(t) = \lambda \cdot \frac{N - 1}{N} \cdot \frac{\bar{y}(t + w_B(t)) + t\bar{y}'(t + w_B(t))(1 + w'_B(t))}{N}
\]  

(129)

Fortuitously, \( \varphi_B \) integrates without much effort:

\[
\int_0^t \varphi_B(s)ds = \lambda \cdot \frac{N - 1}{N} \cdot \int_0^t \frac{\bar{y}(s + w_B(s)) + s\bar{y}'(s + w_B(s))(1 + w'_B(s))}{N} \]

(130)

\[
= \lambda \cdot \frac{N - 1}{N} \cdot \left[ \frac{s\bar{y}(s + w_B(s))}{N} \right]_0^t
\]  

(131)

\[
= \lambda \cdot \frac{N - 1}{N} \cdot \frac{t\bar{y}(t + w_B(t))}{N}
\]  

(132)

Equivalently,

\[
\frac{L}{C} \int_0^t \varphi_B(s)ds = \alpha \beta r \bar{y}(t + w_B(t)).
\]  

(133)

For \( t \in (\bar{t}_b, \bar{t}) \), \( \varphi_B(t) = \phi_0 \).

**Buyers and Sellers.** Note that

\[
\varphi_B(0) = \lambda \cdot \frac{N - 1}{N} \cdot \frac{\bar{y}(0)}{N} \geq \lambda \cdot \frac{1}{N} \cdot \frac{N - \bar{y}(0)}{N} > \lambda \cdot \frac{1}{N} \cdot \frac{N - \bar{x}(0)}{N} = \varphi_B(0).
\]  

(134)
where the first inequality follows since $\bar{y}(0) \geq 1$, and the second since $\bar{x}(0) > \bar{y}(0)$. Put $\tilde{t} = \min\{\tilde{t}_s, \tilde{t}_b\}$. Now $\bar{y}$ is strictly increasing, $w_B(t) \geq -1$, and hence $\varphi_B$ is strictly increasing. On the other hand, $\bar{x}$ is increasing, and hence $\varphi_S$ is decreasing. Since $\varphi_B(0) > \varphi_S(0)$, we have $\varphi_B > \varphi_S$ on $[0, \tilde{t}]$ (and hence $\phi = \varphi_S$, $S = 0$, and $B > 0$ on $[0, \tilde{t}]$). But then $\tilde{t} = t_b$. For all $t \in [\tilde{t}_b, \tilde{t})$, $w_B(t) = \tilde{t} - t$, and hence

$$B(t) = \int_t^{\tilde{t} + w_B(t)} \phi(s) ds = \int_t^{\tilde{t}} \phi(s) ds > 0 \quad (135)$$

by the definitions of $\tilde{t}_b$ and $w_B$. Hence, $\phi = \varphi_S$, and $S = 0$ on $[\tilde{t}_b, \tilde{t})$. We conclude that $\phi = \varphi_S$ on $[0, \tilde{t})$.

**Proof of Proposition 1.** We consider the price path, sellers’ wait time, and the buyers’ wait time in order.

**Price Path (Existence I).** We first verify the following facts about $p^*$:

**Fact 1:** Since $v$ is strictly increasing (Lemma 7), $v(0) = 0, v(1) = \alpha \Delta p$, and $\alpha u_0 < \alpha \Delta p$, there is a unique $z_0 \in (0, 1)$ such that $v(z_0) = \alpha u_0$. Put $\bar{t} = -r^{-1} \log(z_0)$, so that $z_0 = z(0, \bar{t})$. Hence,

$$p(0) = p_L + \alpha^{-1} v(z(0, \bar{t})) = p_L + \alpha^{-1} v(z_0) = p_L + u_0 = L p_L \quad (136)$$

**Fact 2:** By Corollary 1,

$$\frac{L}{C} \int_0^{\bar{t}} \phi(s) ds = \log\left(\frac{p_H - p_L}{L p_L - p_L}\right) < \frac{p_H - p_L}{L p_L - p_L} - 1 = \frac{p_H/p_L - 1}{L - 1} - 1 = \frac{L^2 - 1}{L - 1} - 1 = L \quad (137)$$

which implies that $\int_0^{\bar{t}} \phi(s) ds < C$.

**Fact 3:** By Lemma 8, $v$ is convex, and hence $v(z) < v(1)z = \alpha \Delta p z$ for all $z \in (0, 1)$. Therefore,

$$\bar{t} = -\frac{1}{r} \cdot \log(z(0, \bar{t})) < -\frac{1}{r} \cdot \log\left(\frac{v(z(0, \bar{t}))}{\alpha \Delta p}\right) = -\frac{1}{r} \cdot \log\left(\frac{\alpha u_0}{\alpha \Delta p}\right) = \frac{1}{r} \cdot \log\left(\frac{\Delta p}{u_0}\right) < T \quad (138)$$
where the last line follows by Assumption 10.

Fact 4: 
\[ p^*(\bar{t}, \bar{t}) = p_L + \alpha^{-1}v(z(\bar{t}, \bar{t})) = p_L + \alpha^{-1}v(1) = p_L + \Delta p = p_H. \]

Fact 5: By Lemma 6,
\[ p'(\bar{t}) = \alpha^{-1}r(\bar{t}, \bar{t})v'(z(\bar{t}, \bar{t})) = \alpha^{-1}r \cdot \frac{\alpha \Delta p(\beta - \Delta p) + \alpha \Delta p}{\alpha(1 + \alpha \Delta p)} = \frac{\alpha \Delta p}{1 + \alpha \Delta p} \cdot (N-1) < N-1. \]  
(139)

Fact 6: Note that 
\[ p'(t) = \alpha^{-1}r(t, \bar{t})v'(z(t, \bar{t})). \]  
By Lemma 8 and Assumption 9,
\[ p''(t) = \alpha^{-1}r^2(t, \bar{t})v'(z(t, \bar{t})) + \alpha^{-1}r^2(t, \bar{t})^2 v''(z(t, \bar{t})) > \alpha^{-1}r^2(t, \bar{t})v'(z(t, \bar{t})) = rp'(t). \]  
(140)

We conclude that \( p^* \) is admissible, and satisfies the assumptions of Lemma 3. Two facts follow immediately from Lemma 3. First, the trade flow is
\[ \phi(t) = \lambda \cdot \frac{1}{N} \cdot \frac{N - \bar{x}(t)}{N}, \]  
(141)
and second, \( w^*_S = 0. \)

**Price Path (Existence II and Uniqueness).** We claim that there is only one price path that satisfies the pricing equation under \( \phi \), namely, \( p^* \). By Lemma 1, and the definitions of \( \alpha \), \( \beta \), and \( \bar{x} \), we obtain
\[ p'(t) = \frac{L}{C} \cdot (p(t) - p_L) \cdot \phi(t) = \alpha r \cdot (p(t) - p_L) \cdot \left( \beta - \left( \frac{p_H - p(t)}{1 - z(t, \bar{t})} \right) \right). \]  
(142)

Using the transformation \( u = p - p_L \), and the definition of \( \Delta p \), we further obtain
\[ u'(t) = \alpha r \cdot u(t) \cdot \left( \beta - \left( \frac{\Delta p - u(t)}{1 - z(t, \bar{t})} \right) \right). \]  
(143)
Define $z^{-1}(z, \bar{t}) \equiv \bar{t} + r^{-1} \log(z)$, and $v(z) \equiv \alpha u(z^{-1}(z, \bar{t}))$, and observe that

$$v'(z) = \frac{\alpha u'(z^{-1})}{rz}$$

$$= \frac{\alpha u(z^{-1})}{z} \cdot \left( \alpha \beta - \frac{\alpha \Delta p - \alpha u(z^{-1})}{1 - z} \right)$$

$$= \left( \frac{\alpha \beta}{z} - \frac{\alpha \Delta p}{z(1 - z)} \right) \cdot \alpha u(z^{-1}) + \left( \frac{1}{z(1 - z)} \right) \cdot \alpha (\alpha u(z^{-1}))^2$$

$$= X(z) \cdot (\alpha u(z^{-1})) + Y(z) \cdot (\alpha u(z^{-1}))^2$$

$$= X(z) \cdot v(z) + Y(z) \cdot v(z)^2$$

By Corollary 2 and Proposition 2, the unique solution is

$$v(z(t, \bar{t})) = \frac{z(t, \bar{t})^{\alpha(\beta - \Delta p)}(1 - z(t, \bar{t}))^{\alpha \Delta p}}{\int_{z(t, \bar{t})}^{1} s^{\alpha(\beta - \Delta p) - 1} (1 - s)^{\alpha \Delta p - 1} ds}$$

where $v(z(\bar{t}, \bar{t})) = v(1) = \alpha \Delta p$, and hence

$$p(t; \bar{t}) \equiv p_L + \frac{z(t, \bar{t})^{\alpha(\beta - \Delta p)}(1 - z(t, \bar{t}))^{\alpha \Delta p}}{\int_{z(t, \bar{t})}^{1} s^{\alpha(\beta - \Delta p) - 1} (1 - s)^{\alpha \Delta p - 1} ds}$$

as desired.

Proof of Corollary 7. According to proposition 1, the sophisticated equilibrium has stopping time

$$\bar{t}_s = -r^{-1} \log(v^{-1}(\alpha u_0)).$$

The na"ive equilibrium has stopping time

$$\bar{t}_n = \frac{1}{\alpha \beta r} \log(u_0^{-1} \Delta p).$$

69
Since $v$ is strictly convex, $\alpha u_0 = v(z_0) < \alpha \Delta p z_0 = \alpha \Delta p v^{-1}(\alpha u_0)$, and hence

$$t_n - t_s = -r^{-1} \log(v^{-1}(\alpha u_0)) - \alpha^{-1} \beta^{-1} r^{-1} \log(u_0^{-1} \Delta p)$$

(154)

$$= -r^{-1} \log(v^{-1}(\alpha u_0)) + \alpha^{-1} \beta^{-1} r^{-1} \log((\alpha u_0)(\alpha \Delta p)^{-1})$$

(155)

$$> -r^{-1} \log(v^{-1}(\alpha u_0)) - \beta^{-1} (\beta - \Delta p) r^{-1} \log(m^{-1}(\alpha u_0))$$

(156)

$$= -\alpha^{-1} \beta^{-1} r^{-1} (\alpha \beta + \alpha (\beta - \Delta p)) \log(v^{-1}(\alpha u_0)) > 0$$

(157)

as desired. □

**Proof of Corollary 3.** Recall that $\bar{t}$ is the unique solution to

$$p(0, \bar{t}) = p_L + \alpha^{-1} m(z(0, \bar{t})).$$

(158)

Equivalently,

$$\alpha u_0 = \frac{z_0^\alpha (\beta - \Delta p)(1 - z_0)^\alpha \Delta p}{\int_{z_0}^1 s^{\alpha (\beta - \Delta p) - 1} (1 - s)^{\alpha \Delta p - 1} ds}$$

(159)

where $z_0 = z(0, \bar{t})$. We call this equation the **stopping equation**. For ease of notation, put $a \equiv \alpha (\beta - \Delta p)$ and $b \equiv \alpha (\beta - \Delta p)$, so that

$$\alpha u_0 = \frac{z_0^a (1 - z_0)^b}{\int_{z_0}^1 s^{a - 1} (1 - s)^{b - 1} ds}. \quad (160)$$

Taking logs, we obtain

$$\log(\alpha u_0) + \log \left( \int_{z_0}^1 s^{a - 1} (1 - s)^{b - 1} ds \right) = a \log(z_0) + b \log(1 - z_0).$$

(161)

In what follows, we’ll show that $z_0$ is decreasing with $\Delta p$ and increasing with $\alpha$.

$z_0$ is decreasing with $\Delta p$. In what follows, primes denote differentiation with respect to
Differentiating the stopping equation with respect to $\Delta p$, we obtain

$$
\left( -\alpha \log(z_0) + \frac{z_0'}{z_0} + \alpha \log(1 - z_0) - b \frac{z_0'}{1 - z_0} \right) \left( \int_{z_0}^1 s^{a-1}(1 - s)^{b-1} ds \right) = -z_0'z_0^{-a-1}(1 - z_0)^{b-1} + \alpha \int_{z_0}^1 (\log(1 - s) - \log(s)) s^{a-1}(1 - s)^{b-1} ds
$$

(162)

Equivalently,

$$
\left( \frac{z_0'}{z_0} - b \frac{z_0'}{1 - z_0} \right) \frac{z_0^a(1 - z_0)^b}{\alpha u_0} - \left( \alpha \log(z_0) - \alpha \log(1 - z_0) \right) \left( \int_{z_0}^1 s^{a-1}(1 - s)^{b-1} ds \right) = -z_0'z_0^{-a-1}(1 - z_0)^{b-1} + \alpha \int_{z_0}^1 (\log(1 - s) - \log(s)) s^{a-1}(1 - s)^{b-1} ds.
$$

(163)

Collecting terms, we obtain

$$
(a(1 - z_0) - bz_0 + \alpha u_0) z_0' = \alpha \int_{z_0}^1 \log \left( \frac{z_0}{s} \cdot \frac{1 - s}{1 - z_0} \right) s^{a-1}(1 - s)^{b-1} ds.
$$

(164)

(165)

Now $a(1 - z_0) - bz_0 + \alpha u_0 = z_0(1 - z_0)m'(z_0)/m(z_0) > 0$, and $z_0(1 - s) < s(1 - z_0)$. Therefore, $z_0' > 0$ as desired.

$z_0$ is increasing with $\alpha$. In what follows, primes denote differentiation with respect to $\alpha$.

Differentiating the stopping equation with respect to $\alpha$, we obtain

$$
\left( (\beta - \Delta p) \log(z_0) + \frac{z_0'}{z_0} + \Delta p \log(1 - z_0) - b \frac{z_0'}{1 - z_0} - \frac{1}{\alpha} \right) \int_{z_0}^1 s^{a-1}(1 - s)^{b-1} ds = -z_0'z_0^{-a-1}(1 - z_0)^{b-1} + \int_{z_0}^1 ((\beta - \Delta p) \log(s) + \Delta p \log(1 - s)) s^{a-1}(1 - s)^{b-1} ds.
$$

(166)
Equivalently,

\[
\left( \frac{z_0' - b}{z_0} - \frac{z_0'}{1 - z_0} \right) \frac{z_0^a (1 - z_0)^b}{\alpha u_0} + \left( a \log(z_0) + b \log(1 - z_0) - 1 \right) \frac{1}{\alpha} \int_{z_0}^{1} s^{a-1} (1 - s)^{b-1} ds \quad (169)
\]

\[
= -z_0' z_0^{-a} (1 - z_0)^{b-1} + \frac{1}{\alpha} \int_{z_0}^{1} (a \log(s) + b \log(1 - s)) s^{a-1} (1 - s)^{b-1} ds. \quad (170)
\]

Collecting terms, we obtain

\[
(a(1 - z_0) - bz_0 + \alpha u_0) z_0' = \frac{1}{\alpha} \int_{z_0}^{1} \left( a \log \left( \frac{s}{z_0} \right) + b \log \left( \frac{1 - s}{1 - z_0} \right) + 1 \right) s^{a-1} (1 - s)^{b-1} ds
\]

\[
> \frac{1}{\alpha} \int_{z_0}^{1} \left( b \log \left( \frac{1 - s}{1 - z_0} \right) + 1 \right) s^{a-1} (1 - s)^{b-1} ds \quad (171)
\]

\[
> \frac{1}{\alpha} z_0^{a-1} (1 - z_0)^{b-1} \int_{z_0}^{1} \left( b \log \left( \frac{1 - s}{1 - z_0} \right) + 1 \right) \left( \frac{1 - s}{1 - z_0} \right)^{b-1} ds
\]

\[
= 0. \quad (172)
\]

Again, \(a(1 - z_0) - bz_0 + \alpha u_0 = z_0(1 - z_0)m'(z_0)/m(z_0) > 0\). Therefore, \(z_0' > 0\) as desired.

\(z_0\) is increasing with \(u_0\). In what follows, primes denote differentiation with respect to \(u_0\). \(m\) doesn’t depend explicitly on \(u_0\). Therefore,

\[
\alpha u_0 = m(z_0) \Rightarrow \alpha = m'(z_0) z_0' \Rightarrow z_0' > 0. \quad (173)
\]

**Results.** We’ve established that \(z_0\) is decreasing with \(\Delta p\) and increasing with \(\alpha\). We can
now conclude that

\[
\frac{\partial \tilde{t}}{\partial \lambda} = \frac{d \tilde{t}}{dz_0} \cdot \frac{\partial z_0}{\partial \alpha} \cdot \frac{\partial \alpha}{\partial \lambda} = -\frac{r}{z_0} \cdot \frac{\partial z_0}{\partial \alpha} \cdot \frac{L}{CN^2} \leq 0; \quad (\lambda)
\]

\[
\frac{\partial \tilde{t}}{\partial L} = \frac{d \tilde{t}}{dz_0} \cdot \left( \frac{\partial z_0}{\partial \alpha} \cdot \frac{\partial \alpha}{\partial L} + \frac{\partial z_0}{\partial u_0} \cdot \frac{\partial u_0}{\partial L} \right) = -\frac{r}{z_0} \cdot \left( \frac{\partial z_0}{\partial \alpha} \cdot \frac{\lambda}{CN^2} + \frac{\partial z_0}{\partial u_0} \cdot p_L \right) \leq 0; \quad (L)
\]

\[
\frac{\partial \tilde{t}}{\partial C} = \frac{d \tilde{t}}{dz_0} \cdot \frac{\partial z_0}{\partial \alpha} \cdot \frac{\partial \alpha}{\partial C} = -\frac{r}{z_0} \cdot \frac{\partial z_0}{\partial \alpha} \cdot \frac{\lambda L}{C^2 N^2} \geq 0; \quad (C)
\]

\[
\frac{\partial \tilde{t}}{\partial \Delta p} = \frac{d \tilde{t}}{dz_0} \cdot \frac{\partial z_0}{\partial \Delta p} = -\frac{r}{z_0} \cdot \frac{\partial z_0}{\partial \Delta p} \geq 0; \quad (\Delta p)
\]

as desired.
8 Tables and Figures

<table>
<thead>
<tr>
<th>Aircraft</th>
<th>Quarterly Returns</th>
<th>Yearly Returns</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Autocorr.</td>
<td>t-Stat</td>
</tr>
<tr>
<td>Large Jet</td>
<td>0.68</td>
<td>8.43</td>
</tr>
<tr>
<td>Light Jet</td>
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<td>7.72</td>
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<td>Pressurized Twin</td>
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<td>Light Single</td>
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</tr>
<tr>
<td>Turbo Prop</td>
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<td>8.03</td>
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</table>

Table 1: Aircraft. The above table shows results of AR(1) regressions of quarterly and annual returns for seven types of aircraft. Data are from 1994 Q4 to 2016 Q4. Data are provided by VREF. Returns are clearly predictable at both the quarterly and annual levels.

<table>
<thead>
<tr>
<th>Property</th>
<th>Monthly Returns</th>
<th>Yearly Returns</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>Autocorr.</td>
<td>t-Stat</td>
</tr>
<tr>
<td>Non-Major Markets</td>
<td>0.93</td>
<td>34.47</td>
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<tr>
<td>Major Markets</td>
<td>0.88</td>
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<tr>
<td>Office</td>
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<td>Office — Suburban</td>
<td>0.90</td>
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<td>Office — CBD</td>
<td>0.83</td>
<td>19.31</td>
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<tr>
<td>Industrial</td>
<td>0.82</td>
<td>19.33</td>
</tr>
<tr>
<td>Retail</td>
<td>0.84</td>
<td>21.08</td>
</tr>
<tr>
<td>Core Commercial</td>
<td>0.91</td>
<td>29.22</td>
</tr>
<tr>
<td>Apartment</td>
<td>0.92</td>
<td>31.39</td>
</tr>
<tr>
<td>National All-Property</td>
<td>0.93</td>
<td>33.11</td>
</tr>
</tbody>
</table>

Table 2: Multi-Family Residential and Commercial Real Estate. The above table shows results of AR(1) regressions of monthly and annual returns for several types of multi-family residential and commercial real estate between December 2000 and December 2015. Data are from Moody’s/Real Capital Analytics. Returns are clearly predictable at both the monthly and annual levels.
<table>
<thead>
<tr>
<th>Property</th>
<th>Monthly Returns</th>
<th>Yearly Returns</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Autocorr.</td>
<td>t-Stat</td>
</tr>
<tr>
<td>Atlanta</td>
<td>0.82</td>
<td>25.01</td>
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<td>Boston</td>
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<td>Cleveland</td>
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<td>Chicago</td>
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<td>17.95</td>
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<td>Detroit</td>
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<td>Las Vegas</td>
<td>0.85</td>
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<td>Los Angeles</td>
<td>0.94</td>
<td>51.85</td>
</tr>
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<td>Miami</td>
<td>0.87</td>
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<td>Minneapolis</td>
<td>0.68</td>
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<td>New York</td>
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<td>Phoenix</td>
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<td>San Diego</td>
<td>0.85</td>
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<tr>
<td>Seattle</td>
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<tr>
<td>San Francisco</td>
<td>0.88</td>
<td>34.96</td>
</tr>
<tr>
<td>Tampa</td>
<td>0.83</td>
<td>27.14</td>
</tr>
<tr>
<td>Washington DC</td>
<td>0.88</td>
<td>34.66</td>
</tr>
</tbody>
</table>

Table 3: **Single-Family Residential Real Estate.** The above table shows results of AR(1) regressions of monthly and annual returns for the 20 constituents of the S&P Core Logic Case-Shiller 20-City Composite index between January 1987 and June 2016 (depending on availability). Data are from S&P Core Logic. Returns are clearly predictable at both the monthly and annual levels.

<table>
<thead>
<tr>
<th>$x_{pre}$ \ $x_{post}$</th>
<th>[0, 1]</th>
<th>(1, N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>[0, 1]</td>
<td>Maintain ownership</td>
<td>Sell</td>
</tr>
<tr>
<td>(1, N)</td>
<td>Buy</td>
<td>Maintain non-ownership</td>
</tr>
</tbody>
</table>

Table 4: $x_{pre}$ is an agent’s type prior to her type being shocked. $x_{post}$ is her type after being shocked. In the simple versions of the model, prior to the full development in Section 3.2, agents of type $x \in [0, 1]$ own the asset and those of type $x \in (1, N)$ do not. Therefore, shocks only affect ownership if an agent’s type is shocked from one region to the other.
Figure 1: Predictability.
Figure 1: Predictability.
Panels 1a and 1b: **Predictability.** Panels 1a and 1b plot future versus current returns for aircraft (top-left), taxi medallions (top-right), single family homes (middle-left), multi-family residential and commercial real estate (middle-right), the S&P 500 (bottom-left), and seats on the NYSE (bottom-right). Panel 1a plots month-over-month returns, with the exception of the aircraft data, which are only available quarter-over-quarter. Panel 1b plots year-over-year returns. Data on aircraft cover the period 1994 Q4 to 2016 Q4, and are provided by VREF. Data on taxi medallions cover the period March 1992 to December 2008, and are provided by the New York City Taxi and Limousine Commission. Data on single family residential real estate cover the period January 2000 to June 2016, and are obtained from S&P Core-Logic. Data on multi-family and commercial real estate cover the period December 2000 to December 2015, and are obtained from Moody’s/Real Capital Analytics. Data on the S&P 500 cover the period January 1987 to December 2015, and are obtained from CRSP. Data on NYSE seats cover the period 1880 to 1925, and were collected from the New York Times by Asaf Bernstein.
Figure 2: **Appraisal Time Series.** This figure shows the time-series of the purchase-plus-appraisal and purchase only indices between 2000 Q1 and 2016 Q3. Data are from the FHFA.

Figure 3: **The Rise.** This figure shows the distribution of cumulative returns over the period 2011 Q2 to 2016 Q2 for the 100 largest US metropolitan areas for the purchase-plus-appraisal index and the purchase-only indices. Data are from the FHFA.
Figure 4: **Purchase-Only and Purchase-Plus-Appraisal Indices.** This scatter plot compares the rate of quarterly appreciation in the purchase-only price index to the gap between the levels of the purchase-only and the purchase-plus-appraisal indices between 2000 Q1 and 2016 Q3. Data are from the FHFA. It is clear that when prices are rising or falling faster, the indices diverge by more.
Figure 5: **Prices and Returns.** Panel 5a plots the total return (price appreciation plus rental income) against the rent-to-price ratio for the 200 largest US metropolitan areas between 2010 and 2013. Panel 5b plots price appreciation against the rent-to-price ratio. Data are from Zillow. While price appreciation is higher for more expensive metro areas, the difference is only 1/3 of what is necessary to equate total returns. Therefore, lower-priced metro areas earn substantially higher total returns.
Figure 6: **Turnover.** This figure plots the average turnover (percent of homes that have changed hands in the last year) against yearly price appreciation for the 200 largest US metropolitan areas between 2010 and 2014. Data are from Zillow. When prices are rising (falling), they rise (fall) faster when turnover is higher.

Figure 7: **Days on Market.** This figure plots the average days on market against yearly price appreciation for the 200 largest US metropolitan areas between 2010 and 2013. Data are from Zillow. Days on market appears to have a floor, which corresponds with the fact that there is a minimum time between when a house is listed for sale and when a sale can close, during which the house is labeled “on the market” even though it is under contract. Days on market is lower when prices are appreciating faster, consistent with the existence of buyers’ and sellers’ markets.
Figure 8: **Numerical Solution.** This figure shows a numerical solution of the instant adjustment, naïve, and sophisticated models for $p_H = 400$, $p_L = 4$, $r = .05$, $\lambda = 1$, $L = \sqrt{p_H/p_L}$, $N = \lambda \log \left( (p_H - p_L)(Lp_L - p_L) \right) / r$, and $C = \lambda L((N - 1)/r - (p_H - p_L))/(2N^2)$. 
Figure 9: Queueing. $N(t)$ is the mass of non-owners who have yet to be shocked, $\mathcal{N}(t)$ is the mass of non-owners who have been shocked, but have yet to queue to buy, and $\mathcal{B}(t)$ is the mass of non-owners who have queued to buy. Some non-owners find it optimal to queue immediately after being shocked. They enter the queue at a rate of $\mathcal{B}(t)$. Others find it optimal to queue some time after being shocked. They enter the waiting state at a rate of $n(t)$. They enter the queue at a rate of $\mathcal{B}(t)$. $O(t)$ is the mass of owners who have yet to be shocked, $\mathcal{O}(t)$ the mass of owners who have been shocked, but have yet to queue to sell, and $\mathcal{S}(t)$ is the mass of non-owners who have queued to sell. In the case of an upward adjustment, it will transpire that $\mathcal{S}(t) = 0$. Some owners find it optimal to queue immediately after being shocked. They enter the queue at a rate of $\mathcal{S}(t)$. Others find it optimal to queue some time after being shocked. They enter the waiting state at a rate of $o(t)$. They enter the queue at a rate of $\mathcal{S}(t)$. non-owners become owners at a rate of $\phi$; owners become non-owners at the same rate.
Figure 10: **Buyers’ and Sellers’ Strategies.** The above diagram shows strategies for buyers and sellers. Owners whose new types lie in $[0, 1]$ hold on to the asset, while owners whose new types lie in $(\bar{x}(t), N]$ sell immediately. Owners whose new types lie in $(1, \bar{x}(t)]$ wait until the price adjusts to sell. Non-owners whose new types lie in $[0, y(t)]$ buy immediately, while non-owners whose new types lie in $(y(t), N]$ buy later (or never).
Figure 11: **Numerical Solution.** This figure shows a numerical solution of the sophisticated model for $p_H = 200$, $p_L = 150$, $r = .05$, $\lambda = 1$, $L = \sqrt{p_H/p_L}$, $N = \lambda \log ((p_H - p_L)(Lp_L - p_L))/r$, and $C = \lambda L((N - 1)/r - (p_H - p_L))/(2N^2)$. Panel (a) shows the price paths for the instant adjustment, naïve, and sophisticated models. Panel (b) shows the flow of buyers and sellers to trade. Panel (c) shows the buyers’ and sellers’ wait times. Panel (d) shows the sizes of the buyers’ and sellers’ queues. Panels (b) through (d) all show paths only for the fully rational model.
Figure 12: **Buyers’ and Sellers’ Markets.** This figure plots the time series for the monthly supply of housing inventory, and the time series of house prices, as measured by the Case-Shiller 20-City Composite, between January 1987 and July 2015. Data are from US Census Bureau and Core Logic, respectively. Inventory appears to have a floor, which corresponds with the fact that there is a minimum time between when a house is listed for sale and when a sale can close, during which the house is labeled “inventory” even though it is under contract. Inventory rises when prices are falling, consistent with growth in a seller’s queue during down markets. Inventory is stable at its minimum level with prices are rising, consistent with the lack of a seller’s queue in rising markets.
Figure 13: **House Flipping.** This figure plots the time series of the frequency of US searches for “house flip” on Google and the time series of house prices, as measured by the Case-Shiller 20-City Composite, between January 2004 and September 2015. Data are from Google Trends and Core Logic, respectively. Flipping is more searched when prices are higher. One interpretation is that flipping is more searched when appreciation was higher in the preceding years.