Abstract

We consider vertical contracts where the retail market may involve search frictions. Minimum advertised price restrictions (MAP) act as a restraint on customers’ information and so can increase search frictions in the retail sector. Such restraints, thereby, soften retail competition—an impact also generated by resale price maintenance (RPM). However, by accommodating (consumer or retailer) heterogeneity, MAP can allow for higher manufacturer profits than RPM. We show that they can do so through facilitating price discrimination among consumers; encouraging service provision; and facilitating manufacturer collusion. Thus, welfare effects may be positive or negative compared to RPM or to the absence of such restrictions.
1 Introduction

When a retailer enters into a distribution agreement with a manufacturer, it is well understood that the agreement may impose ‘vertical’ restraints on the retailer that restrict price or non-price aspects of retailer activity.\(^1\) Price restraints include resale price maintenance (RPM), in which the manufacturer imposes floors and/or ceilings in retail pricing. Commonly considered non-price restraints typically include exclusivity provisions or the imposition of sales territories. This paper considers a form of a third type of vertical restraint, information restraints: specifically, minimum advertised price (MAP) policies.\(^2\) MAP policies impose a floor on the price at which retailers can advertise a product, but, crucially, not the price at which it can be sold to a consumer. That is, even if a MAP policy imposes a floor of $10 a unit on advertised prices, nothing restricts the retailer from selling it to a consumer for $7.

MAP policies are a ubiquitous example of a broader set of restraints that restrict the ability of retailers to communicate information to consumers. The reach and impact of these restrictions may be significant. For instance, European Commission (2017), reports on a survey of more than a thousand retailers engaged in ecommerce across Europe, and suggests that a large fraction face non-price restrictions, includ-

\(^1\)For a survey of common practices and the related economic issues, see Rey and Vergé (2008).

\(^2\)MAP policies appear the most common form of vertical information restraint. As an illustration of the scope of their use, the first 40 policies found in a search for “minimum advertised price pdf” using the Google search engine on April 13, 2016 covered the following product categories (companies): Adjustable office and industrial chairs (Neutral Posture); Antennas (Winegard Company); Aquarium filtration systems (Lifegard Aquatics); Baby and toddler pillows (Smugg-wugg); Batteries, flashlights, solar energy equipment (NOCO); Cameras, security cameras (Sony); Chainstays, gripping bar tape (Lizard Skins); Chairs (Allseating); Clutches, flywheels, accessories (Midway Industries); Data storage electronics (Seagate); Designer faucets, showers, and bath accessories (Sonoma Forge); Digital display mounting (Ergotron); Enzymes (Enzyme Science); Exhaust systems and mufflers (Flowmaster); Games (Cool Mini or Not); Health bars, drinks, and powders (Powercrunch); High-pressure hydraulic tools (Enerpac); Holster clips (Utiliclip); Home, mobile and automobile electronics (JVC Professional Video Products); Instruments for measuring and calibrating (Meriam Process Technologies); LCD and plasma tvs (Microtek); Mobile cameras (GoPro); Mobile telephone cases (Urban Armor Gear); Outerwear, corporate wear and imprinted apparel (Charles River Apparel); Pistols (Glock); Rock-crawling and off-road cars (GMADE); Security cameras and surveillance equipment (Samsung Techwin America); Scented pencils, stickers and paper clips (Scentoco); Shallow water anchors (JL Marine Systems); Speakers (Eminence); Stove ranges, broilers, ovens, kitchen equipment (Montague Company); Temperature-controlled food display cases (Structural Concepts); Thermal imaging infrared cameras (FLR Commercial Systems); Toy car track tape (InRoad Toys and PlayTape); Triathlon clothing and gear (Pearl Izumi); Ventilation fans (Panasonic Home & Environment Co.); Wall art, picture frames, jewelry, mugs, pillows (Glory Haus); Water bottles (Blender Bottle); Wood flooring (Kahrs International); Work boots (Thorogood Shoes). All these policies are available at www.johnasker.com/MAP.zip.
ing limitations to sell through market places (18%), websites other than their own (11%), and the use of price comparison sites (9%). These kinds of restrictions can be understood as making it more difficult for consumers to easily find products. In this paper we focus on MAP restrictions as a lens through which to begin to explore the economics of this class of restrictions.

At first sight, such restrictions seem odd. Advertising and information play an important role in ensuring that customers know of the existence of a firm or product. Price advertising can also reassure customers who are unsure about prices and so might be put off from shopping. Thus there is clearly a role for advertising prices and for a manufacturer to encourage retailers to do so whether to compete against other brands or to mitigate consumer uncertainty. However, this intuition focuses on the role of advertising in inter-brand competition. In this paper, we switch the focus to competition between a manufacturer’s retailers (intra-brand competition) so as to highlight the forces that may make restrictions on price advertising attractive for a manufacturer.

Superficially, MAP and RPM policies appear similar. If an advertised price is constrained to be equal to an actual price, then a MAP policy appears to operate as de facto RPM. Indeed, as we discuss below, such a perspective appears to underlie the thinking of antitrust agencies and commentators on policy. However, this view of MAP, misses the crucial distinction between MAP and RPM, which is that the MAP price does not restrict the retailer from offering an actual (transaction) price at some level below the advertised price. We show that this distinction, and the flexibility that the retailer retains therein, results in MAP having an economically distinct effect on markets.

Two themes emerge. First, for MAP policies to have any market impact, consumers need to have an informational (search) friction that advertising helps alleviate. Hence, in every setting we consider search frictions are a central feature. If consumers can costlessly observe all product offerings, MAP is clearly irrelevant to market outcomes.

Second, and of central economic interest, MAP policies soften competition by obscuring prices. Thus, consumers allocate themselves to competitors somewhat randomly or at least relying less on prices than they would if they were perfectly informed. By contrast, RPM softens competition by equalizing prices (and by doing so reducing the flexibility of retailers to respond to idiosyncratic conditions). When consumers, or retailers, are heterogenous, MAP is helpful in retaining the flexibility

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3There are models where these are equivalent. For example, consider the Diamond (1970) model with homogeneous consumers; customers would anticipate that all firms would charge this advertised ceiling of prices and so it acts as a minimum price restraint.
to profitably accommodate heterogeneity. Thus, MAP will have an impact on markets when search and heterogeneity among similarly situated economic actors are important features of the environment. By contrast, in these environments, RPM tends to have more limited efficacy.

We illustrate these themes through a number of simple models. It is useful to present several models so as to engage with the vast literature on RPM and the variety of pro- and anti-competitive effects of RPM therein. (These were already sufficient for a book-length treatment more than half a century ago. See Yamey, 1954). Rather than comparing MAP to RPM in every model of RPM that has been introduced in the literature, we present three examples—one where RPM can play no role; and two that show how MAP modifies canonical examples of the classic pro- and anti-competitive roles of RPM: namely, encouraging service provision and facilitating collusion. Specifically, these are:

- The price discrimination model: in which it is shown that MAP policies allow manufacturers to imperfectly separate high and low search cost consumers, and better extract surplus from high value consumers with high search costs. That is, the MAP policy obscures actual prices, allowing search patterns to be leveraged as a screening device.\(^4\)

- The service model: in which it is shown that by obscuring prices, MAP policies soften competition and protect retailer profits, while allowing retailers to optimize subject to their heterogeneous marginal costs of retailing. This can increase the returns to retailers from providing service that expands the market (such as informative advertising). By softening competition, while retaining retailer flexibility, MAP can dominate RPM as a means for manufacturers to profitably incentivize retailers.\(^5\)

- The collusion model: in which MAP raises cartel profits and stability by allowing manufacturers to more easily monitor each other’s behavior. Notably

\(^4\)Throughout we assume that retailers cannot price discriminate. Instead, discrimination occurs through the consumers’ choice of retailer. Chen (1999) considers the effect of RPM when retailers can engage in price discrimination. Although we do not consider it in this paper, MAP might facilitate retailer price discrimination and, thereby, boost industry (and manufacturer) profits in addition to retailers’ incentives to provide service.

\(^5\)Telser (1960) is typically cited for the idea that RPM promotes service (though it is also discussed in Yamey (1954), for example). More recent formalizations and developments include Matthewson and Winter (1984); Klein and Murphy (1988); and Deneckere, Marvel and Peck (1997). We follow the literature in considering pro-consumer service, although note that, as in the model of exclusion in Asker and Bar-Isaac (2014), service need not be helpful to the consumer.
this is done without sacrificing the ability of cartel members to tailor actual transaction prices to local market conditions.\textsuperscript{6} That is, whereas RPM may be viewed as having features of a price-fixing scheme, MAP may be viewed as analogous to a market division scheme.

In antitrust policy, investigations and discussions of RPM, and by extension MAP, tend to be focused on the last two models; that is, the tension between the service provision and the facilitation of collusion.

In understanding the way MAP restrictions work, and in particular how they differ from the price restraint embodied in an RPM restriction, it is useful to examine a typical MAP provision. As an example, consider the January 1, 2016, MAP policy of Samsung Techwin America (a manufacturer of security cameras and surveillance equipment).\textsuperscript{7} The MAP price is specified as a percentage of the manufacturer’s suggested retail price (in this instance, 40% of MSRP). The policy explicitly applies to all advertisements in all media, including online. The MAP restriction only applies to advertised prices and not to the price at which products are actually sold. In physical stores, this means that the posted price in the store is not affected by the MAP restriction. As regards online pricing, the policy states:

Pricing listed on an internet site is considered an “advertised price” and must adhere to the MAP policy. Once the pricing is associated with an actual purchase (an internet order), the price becomes the selling price and is not bound by this MAP policy. Statements such as “we will match any price”, and “call for price” are acceptable.

In particular, such policies allow retailers to advise customers that they will be able to see the price when the item is in a (digital) shopping basket.

Lastly, Samsung reserves the right to punish non-compliance with termination.\textsuperscript{8}

This paper contributes to the well established literature, in policy and academic circles, on the antitrust implications of vertical contracts.\textsuperscript{9} Where MAP provisions have been considered in this literature, the point of departure (as in this paper) has

\textsuperscript{6}In formalizing this argument, we adapt the framework of Jullien and Rey (2007), developed to consider RPM as facilitating upstream collusion, to accommodate MAP.

\textsuperscript{7}Available at www.johnasker.com/MAP.zip

\textsuperscript{8}Interestingly, the policy also describes its purpose, which in this instance is to “to help ensure the legacy of STA as a top producer of high performance, high quality, professional security products and to protect the reputation of its name and products ...[and]... to ensure dealers, retailers and distributors have the incentive to invest resources into services for STA’s customers.”.

\textsuperscript{9}See Rey and Vergé (2008) for a survey.
been RPM restrictions. As with many vertical restraints, the approach to MAP and RPM taken in US and European law differs, with the E.U. being the less permissive. In 2007 in the Leegin case federal U.S. law reversed earlier precedent, shifting RPM from a complicated version of a per se offense to a rule of reason regime.\textsuperscript{10} Even prior to the Leegin case, US law had ruled in favor of MAP provisions, acknowledging their pro-competitive potential in preserving service incentives.\textsuperscript{11} Post-Leegin cases in the U.S. have also failed to gain traction.

By contrast, in Europe, MAP provisions have tended to be viewed with more suspicion. In particular, MAP has been found to be a de facto form of RPM.\textsuperscript{12} In turn, RPM has been found to impose a sufficient degree of harm to competition that there is no need to examine its effects in determining liability (that is, making it an restriction of competition by object, somewhat similar to a per se offense in U.S. law).\textsuperscript{13}

Hence, jurisdictions differ in their approach to regulating MAP provisions. This paper contributes to understanding this divergence by providing the precise frameworks needed to make economic arguments supporting each approach. By doing so it helps clarify the trade-offs implicit in arriving at any policy position vis-a-vis MAP.

This paper is also closely related to the nascent literature (notably Lubensky (2014), Janssen and Shelegia (2015), and Janssen and Reshidi (2017)) that examines the effects of vertical contracts when the final goods market is characterized by search frictions. In particular, in such markets it has long been understood that the law of


\textsuperscript{12}See Commission Decision 16 July 2003 PO/Yamaha (COMP/37.975), OFT Decision Agreements between Lladro Comercial SA and UK retailers fixing the price for porcelain and stoneware figures, CP/0809-01, 31 March 2003 and, recently, the Decision of the UK Competition and Markets Authority Online resale price maintenance in the commercial refrigeration sector Case CE/9856/14. All these cases apply E.U. law. Hughes (2017) provides a broad discussion and criticism of the approaches of the UK Competition and Markets Authority and the EU Commission towards MAP and RPM.

\textsuperscript{13}See, for instance, the overview of relevant E.U. law in the Decision of the UK Competition and Markets Authority Online resale price maintenance in the commercial refrigeration sector Case CE/9856/14.
one price need not hold, and price dispersion may arise. Naturally, such frictions have implications for the contracts between manufacturer and retailer and for a manufacturer’s profitability. A minimum advertised price restriction (MAP) which limits the price that retailers may advertise (with no restriction on the price they may charge) can make it more difficult for consumers to find the retailers charging the lowest prices and lead to retail price dispersion (if not advertised price dispersion). As is illustrated in this paper, a manufacturer (and even consumers) may benefit.

The paper is also related to the large and growing literature on obfuscation in search markets. (See, for example, Ellison and Wolitzky (2012), Wilson (2010), Piccione and Spiegler (2012), Chioveanu and Zhou (2013), Gamp (2016) and Petrikaite (2016)). Much of the economic force of a MAP restriction is through making firms identical from the point of view of consumers. In this way, MAP plays a role in obfuscating the actual as opposed to advertised price (indeed this is its only role in our analysis). This paper is distinct from the rest of the obfuscation literature (to our knowledge) by explicitly considering the opportunities obfuscation gives to an upstream manufacturer, as opposed to a retailer.

Only four papers, that we know of, consider MAP policies in the economic literature. Kali (1998) and Cetinkaya (2009) explore theoretical models that treat MAP as an RPM provision with an additional advertising subsidy. A very different approach is adopted in this paper. Charness and Chen (2002) conduct an experimental study of the determinants of MAP compliance. Israeli, Anderson and Coughlan (forthcoming) empirically examine detected violations of MAP, using data from a firm engaged, on behalf of manufacturers, in monitoring MAP compliance of online retailers. They also provide an informative discussion of MAP’s prevalence. Importantly for our study, Israeli et al find that 14-22% of authorized dealers violate MAP, as compared to 46-54% of unauthorized dealers. This suggests that MAP policies are genuine restraints on retailer behavior.

The rest of this paper is organized as follows. Section 2 explores the price discrimination model and introduces some modeling structure that is used in the other models. Section 3 investigates how MAP can enhance customer service. Section 4 shows how a MAP program can help coordinate a manufacturer cartel. Lastly,

\footnote{This idea goes back to Stigler (1961) at least. Baye, Morgan and Scholten (2006) provide a useful overview.}

\footnote{More broadly, recent literature has explored retailers’ policies which influence search and pricing behavior in search markets. Such practices include stochastic discounts off the list price in Gill and Thanassouli (forthcoming), lowest price guarantees in Janssen and Parakhonyak (2013), and exploding offers in Armstrong and Zhou (2016).}

\footnote{The data used in Israeli et al comes from a firm offering MAP compliance monitoring of internet based sellers.}
Section 5 offers some final remarks.

2 MAP as price discrimination

We begin by laying out a simple model. Its essential feature is the presence of search frictions which imply that advertised prices might not reveal the actual prices to consumers. We consider variations in the sections below to illustrate different effects of MAP. In this section, we assume that consumers are heterogeneous both in search costs and valuations. An advertising restriction allows the industry to segment consumers and, thereby, raise profits. Welfare consequences, as is the case with most price discrimination, are ambiguous.

2.1 Model

Consumers have unit demand for a good, which they can buy from one of two retailers. Retailers source the good from a single manufacturer.

A fraction $\lambda$ of consumers have a low valuation of the good, and derive utility $l - p$ when buying the good at price $p$; and the remainder have a high valuation, $h$ (we abuse notation slightly and also use $l$ or $h$ to denote the consumer’s type).

Minimum advertised prices only have bite if there are limits on the ability of consumers to freely compare different retailers’ prices. We model these limits in a simple way. Specifically, some fraction of consumers, $\sigma$, can search costlessly and observe actual transaction prices. The remainder are assumed to be high-search cost, such that they visit the store offering the lowest advertised price (equivalently, highest advertised consumer surplus). If observing the same advertised price at both retailers, they will choose one at random. That is, they have sufficiently high search costs that they will only visit one store and, from the point of view of these consumers, advertised prices are the only point of differentiation between retailers.

We write $\sigma$ to denote the fraction of consumers in the whole population who search both retailers, with the the remainder visiting only a single retailer (high search cost). We write $\lambda_S$ to denote the fraction of searchers who are low valuation consumers and $\lambda_N$ to denote the fraction of non-searchers who are low valuation consumers. It is

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17Having high and low search cost consumers is an extreme but convenient, and often-used, way to examine search frictions and consumers who are heterogeneous in their search behavior. See, for example, Varian (1980). Alternatively, an ex-ante costly decision to engage in search in the style of Burdett and Judd (1983) would allow the fraction of searchers to be endogenously determined (at the cost of additional notation for these search costs, and a somewhat more involved analysis).
immediate that MAP can play no role if $\sigma = 1$. Therefore, we assume that $\sigma < 1$.\textsuperscript{18}

We also model consumers as non-strategic. That is, they act mechanically in choosing the lowest (advertised) price. This rules out perverse behavior that may arise from off-equilibrium beliefs regarding the signal content of advertised prices being unrestricted in a perfect Bayesian equilibrium. Essentially, we assume that consumers believe that a firm offering a lower advertised price offers more surplus from a transaction.\textsuperscript{19}

The two retailers are denoted $R_1$ and $R_2$. We suppose, in this section, that they incur no costs beyond payments to the manufacturer. Later, in Section 3, we allow the retailers to have positive, heterogeneous, marginal costs of retailing, but we assume throughout that the manufacturer offers the retailers identical terms.\textsuperscript{20} These retailers are indistinguishable to consumers other than through their advertising.

Retailers simultaneously set their prices, $p_1$ and $p_2$. We suppose truthful advertising in the following sense: the advertising price can be no lower than the retail transaction price $p_j$, i.e. $p^a_j \geq p_j$.\textsuperscript{21} If the manufacturer imposes a minimum advertised price restriction, $p^\text{MAP}$, then $p^a_j \geq \max\{p_j, p^\text{MAP}\}$. Combined with the model of consumer behavior described above, this means that no firm has an advantage in advertising a price higher than $\max\{p_j, p^\text{MAP}\}$.\textsuperscript{22} As a result, the discussion that follows explores the reduced form of this game and sets $p^a_j = \max\{p_j, p^\text{MAP}\}$.

Retailers buy the product from the manufacturer ($M$) according to a two-part tariff with linear wholesale price $w$ and fixed fee $T$.\textsuperscript{23} In the absence of vertical restrictions, retailers take the wholesale price $w$ as given and set the retail transaction price and advertised price. If manufacturers impose RPM, retailers are bound to charge at least the RPM price, $p^\text{RPM}$ (and it is assumed, without loss, that there is no advertising restriction).\textsuperscript{24} If manufacturers impose MAP, retailers are bound to advertise at the MAP price, $p^\text{MAP}$, or higher as described above. The manufacturer’s marginal cost is equal to zero.

\textsuperscript{18}Indeed, to highlight the role of these non-searchers, in the service and collusions models we set it equal to 0. That is, in these models, consumers only visit one store.

\textsuperscript{19}The analysis in Appendix B shows how, even if consumers are strategic, this behavior can be supported in equilibrium.

\textsuperscript{20}This may be due to legal restrictions, or be induced by the timing assumptions below.

\textsuperscript{21}There is a literature on false advertising. See Corts (2013,14), Rhodes and Wilson (2016) and Piccolo, Tedeschi, and Ursino (2015) for recent contributions.

\textsuperscript{22}The reasoning mirrors that of the standard homogenous good Bertrand model.

\textsuperscript{23}Since we assume that the manufacturer cannot distinguish between retailers, there are no subscripts on these variables to denote different retailers.

\textsuperscript{24}That is, we consider minimum RPM.
2.1.1 Timing

Timing is as follows.

1. The manufacturer sets the contract terms \((T, w)\) and any restraints (RPM, MAP etc. and the associated prices \(p^{RPM}\) or \(p^{MAP}\)).

2. Retailers accept or reject the offered contract.

3. Retailers each set their price, \(p_j\). The advertised price is \(p_j\) or, if a MAP restriction is in place, \(\max\{p_j, p^{MAP}\}\), where \(p^{MAP}\) is the minimum advertised price.

4. Given the advertised or actual prices that they observe, consumers sort into exploring purchases from \(R_1\) or \(R_2\). In particular, they choose to visit the retailer where they expect lower price, as long as the expected surplus is non-negative. If indifferent, consumers are equally likely to visit either retailer.

5. Purchases are made and profits realized.

In this model, since there is no relevant private information, the relevant notion of equilibrium is subgame perfect Nash equilibrium.

2.2 Analysis

We show that MAP can facilitate price discrimination. That is, in the face of consumer preference heterogeneity, MAP can be used to sort consumers among retailers in ways that extract greater surplus than might otherwise be the case. This is because MAP by preventing all consumers from searching amongst retailers costlessly can allow for price dispersion. As noted by Salop (1977), an integrated monopolist would value such price dispersion as it both segments the market and charges higher prices to less efficient searchers. An appropriate MAP contract enables the manufacturer both to allow for such price dispersion, and to extract the surplus that this creates in the industry as a whole.

With a two part-tariff, it is immediate that even without the use of vertical restraints, the manufacturer can induce a retail price of \(h\) or a retailer price of \(l\) and enjoy the full surplus of doing so.\(^{25}\) If industry profit is maximized by selling to all consumers at \(p = l\), by setting \(w = l\), the manufacturer can induce retailers who

\(^{25}\)As is well understood, the use of a two-part rather than linear tariff allows the manufacturer to overcome the familiar double-marginalization problem highlighted in Spengler (1950).
Bertrand compete to set prices at \( p = l \). This extracts all the retailer surplus, and the manufacturer sets \( T = 0 \). Conversely, if industry profit is maximized by selling to only high value consumers, all surplus can be extracted by the manufacturer (for example, by setting \( p = h \) and \( T = 0 \)). Thus, the manufacturer’s profit is equal to \( \max \{ l, (1 - \lambda)h \} \).

Note that RPM does not provide an avenue through which to raise profits above this level. Setting \( p^{\text{RPM}} > w \) provides a binding price floor on the retailers’ prices, since they are in homogenous bertrand competition. However this does not aid in surplus extraction.

However, the addition of MAP to this environment may allow for further surplus extraction by the manufacturer. It does so by inducing the different retailers to charge different prices (it is clear that optimally, in this simple environment one will charge a price \( l \) and the other a price \( h \)). When one retailer prices high and the other low, consumers with a high valuation and high search costs will purchase from the high price store if they are randomly allocated to that store. Consumer with zero search costs, or the luckier high valuation consumers will purchase from the low price store.

We begin by characterizing the optimal MAP-based discriminatory scheme for the case where \( \lambda_s = 1 \) (no high value consumers search).\(^{26}\) We then illustrate, through a simple example, that such a MAP policy can bring the manufacturer strictly greater profits than can be achieved in the absence of MAP.

**Proposition 1** When \( \lambda_s = 1 \), in the optimal MAP-based discriminatory price scheme the manufacturer sets \( (w^*, T^*) \) as in Equations (1) and (2)

\[
\begin{align*}
 w^* &= \frac{(1 + \sigma)l - h(1 - \sigma)(1 - \lambda_N)}{2\sigma + (1 - \sigma)\lambda_N}; \quad (1) \\
 T^* &= \frac{1 - \sigma^2 (h - l)(1 - \lambda_N)}{2\sigma + (1 - \sigma)\lambda_N} \quad (2)
\end{align*}
\]

and the MAP price is set at or above \( h \). \( R_1 \) sets \( p = l \) and \( R_2 \) sets \( p = h \).\(^{27}\) Under this scheme, the manufacturer earns

\[
(1 - \lambda_N)(1 - \sigma)h + \frac{1 + \sigma}{2}l. \tag{3}
\]

\(^{26}\lambda_s = 1 \) for expositional ease. This characterization is used later in the section to construct examples and illustrative cases. Asker and Bar-Isaac (2016) provide a characterization of the scheme with \( \lambda_s \leq 1 \) and heterogeneity in the marginal costs of retailing.\(^{27}\)Of course, there is another equilibrium in which \( R_2 \) sets \( p = l \) and \( R_1 \) sets \( p = h \) Allowing for heterogeneity in retailers’ costs is a simple means of extending the model to resolve a coordination issue in selecting roles in these asymmetric equilibria.
Proof. Given that $R_1$ sets a price of $l$, $R_2$ must prefer setting a price of $h$ and selling to half of the high value non-searchers (of whom there are $(1 - \lambda_N)(1 - \sigma)$) who randomly visit $R_2$ to setting a price just below $l$ and selling to all searchers as well as to half of all consumers who visit randomly; that is:

$$
\frac{(1 - \lambda_N)(1 - \sigma)}{2}(h - w^*) \geq \left[\frac{1 - \sigma}{2} + \sigma\right](l - w^*).
$$

Similarly, $R_1$ must prefer to set a price of $l$ rather than one just below $h$ when $R_2$ sets a price of $h$. Equivalently,

$$
\left[\frac{1 - \sigma}{2} + \sigma\right](l - w^*) \geq \frac{(1 - \sigma)(1 - \lambda_N)}{2}(h - w^*). \quad (4)
$$

These inequalities are both satisfied when they hold with equality, yielding the expression for $w^*$, where both retailers earn the same profits (net of the fixed fee). The manufacturer sets the fixed fee $T^*$ to extract this profit.

The manufacturer profits come from collecting $T^*$ from each of the two retailers, and a margin of $w^*$ on every unit sold, where sales are from all consumers apart from a fraction $\frac{(1 - \sigma)\lambda_N}{2}$ corresponding to high value consumers who do not search and randomly find themselves at the firm charging a high price.

With $\lambda_s = 1$, a MAP-based scheme as described in Proposition 1 can be sustained for retailers that are identical.\(^{28}\) When $\lambda_s < 1$ an extra term is added in the right-hand-side of equation 4 for the high value searches that are captured by a price just below a price of $h$. As a result of this, if $\lambda_s < 1$, both inequalities can not hold in the proof above. This indicates that, for a MAP scheme to exist when some high value consumers search, there needs to be some heterogeneity in the marginal costs of retailing across retailers. When this is present, the low cost retailer prices low, and the high cost retailer prices high. That is, the retailers coordinate pricing according to their relative marginal costs.

We illustrate the optimality (over a subset of the parameter space) of the MAP-based discriminatory scheme with a transparent example in which $\lambda_S = 1$ and $\lambda_N = 0$, such that all low value consumers costlessly search both retailers, and the high value consumers will only visit one retailer. This is intentionally stark; the discriminatory scheme can be sustained without such perfect negative correlation between valuations and the fraction of searchers.

\(^{28}\)Kaplan and Menzio (2016) provide empirical support for an assumption that searchers are more likely to have low valuations than non-searchers. Proposition 1 is expressed leveraging an extreme version of such a view.
Consider the profit that a manufacturer that sets the MAP price at $p^{MAP} = h$ can extract. It is clear that for MAP to be strictly profitable, it must lead one retailer (denoted $R_H$) to charge a retail price equal to $h$ and the other, $R_L$, to set the retail price to $l$.

Figure 1 illustrates this scenario with $\lambda = 0.5$. Of the high value consumers, due to the identical advertised price imposed by MAP and their inability to search, half go to $R_H$ and half to $R_L$. All the low value consumers (who are assumed to be able to search costlessly between retailers) go to $R_L$. Hence, the profit (ignoring $T$) made by $R_H$ is $\frac{1}{2}(h - w)(1 - \lambda)$, the sum of rectangles A and B. Similarly, the profit made by $R_L$ is $(l - w)(\lambda + (1 - \lambda)\frac{1}{2})$, the diagonally hatched rectangle corresponding to the sum of C and D. By setting $w$ such that the two profits are equal, then using $T$, the manufacturer can capture all the surplus other than area $E$.

It is easy to find parameterizations such that this equilibrium exists and this discriminatory pricing policy is optimal for the manufacturer, as shown in Example 1, below.

**Example 1** Let $\lambda_S = 1$ and $\lambda_N = 0$, $h = 2$, $l = 1$ and $\sigma = 0.5$. Following Proposition 1, $(w^*, T^*) = (\frac{1}{2}, \frac{3}{8})$ and $p^{MAP} = 2$. Further, the manufacturer’s profit from this pricing scheme is $1.25$. The maximal profit from a two part tariff, absent MAP, is 1. Thus, the MAP policy, in this example, leads to strictly greater profit for the manufacturer.

All price discrimination operates by (usually imperfectly) homogenizing con-
sumers so that targeted prices extract more surplus.\footnote{Sometimes the emphasis is tilted toward better targeting the prices (in the extreme, Type 1 price discrimination), while other times the emphasis is on homogenizing the consumers (bundling, and Type 3 price discrimination, being obvious examples).} The MAP policies explored here are no exception. By introducing an information friction, the manufacturer is able to isolate at least some of the high value consumers and extract surplus from them via the high price charged by $R_H$. By contrast, RPM does little to segregate consumers and so, here, would serve only to solve a possible double margin problem (which arises if $T$ is restricted to 0). This illustrates the sense in which MAP has elements analogous to the much more invidious market division schemes that invite criminal sanction under antitrust laws, while RPM is similarly analogous to price fixing.\footnote{To be clear, we do not argue that MAP and RPM should invite the same sanctions as market division and price fixing. Indeed, as much of this paper shows, both often serve to make markets more efficient and more consumer friendly.} Merely agreeing on a common price does not help retailers, in this setting, to extract more surplus. However, if retailers could allocate high value consumers to $R_2$ and low value consumers to $R_1$, as might be done in some ideal market division scheme, then they would be able to extract all available gains from trade. MAP generates an imperfect implementation along these lines, albeit ultimately to the benefit of the upstream manufacturer.\footnote{Interestingly, contingent on IC constraints being satisfied, as more retailers are used in a market, surplus extraction via MAP will approach perfect (type 1) price discrimination in this setting.}

As is standard for price discrimination, welfare consequences are ambiguous and depend on quantity effects. If in the absence of MAP, the optimal scheme involves a retail price of $h$ that excludes $l$-consumers then MAP would raise welfare; instead if the unrestricted case involved a price of $l$ and sales to all customers then the MAP scheme (in which not all $h$-type consumers are served) would reduce it.

### 2.3 Price discrimination and simple service

We extend the model in Section 2, to allow for service in a simple way. Here, we suppose that service is a discrete choice: either a retailer provides it or not.\footnote{This could correspond, for example, to a fixed cost to participate in the market.} Moreover, both retailers need to provide service for demand to be non-zero.\footnote{This is an (extreme) way of capturing the free-riding effect explored in Matthewson and Winter (1984) and many subsequent papers.} Thus, the timing in Section 2.1.1 is extended so that at stage 3, retailers also have a decision of whether to incur a fixed cost $F$ to undertake service. It is noteworthy that this service consideration implies that MAP may be optimal for the manufacturer even
when the manufacturer is restricted to linear tariffs.\footnote{This is immediate, if for example it happens to be that $F = T^*(w^*)$ where $T^*(w^*)$ is as characterized in Proposition 1; but, of course, is true much more generally. Conversely, if $F = 0$ and the manufacturer is restricted to linear tariffs, it can be shown that the manufacturer would never gain through the use of a MAP restriction.}

In this model of discrete service provision, where both retailers must be active, the market breaks down in the absence of vertical restrictions. This is due to the Bertrand competition between the retailers – at least one retailer will earn zero profits, before any fixed costs are deducted, and hence, anticipating this outcome, retailers have no incentive to provide service at cost $F > 0$.\footnote{This will be true even if retailers have positive and different (constant) marginal costs of retailing.} As a result, there is no demand for the product, and the product is not offered to the market.

**Figure 2: Service with MAP and RPM**

Panel (a) of Figure 2 illustrates how a price discrimination-based MAP program can be used to generate service. Building on the model illustrated in Figure 1, Panel (a) shows that, rather than capturing surplus using the lump-sum part of a two part tariff, the manufacturer may leave surplus with the retailers and the retailers can use this to fund service. That is, when the wholesale unit price is $w_{MAP}$, the rectangles with area $F$ will be sufficient to provide service. The profit to the manufacturer in Panel (a) is equal to $w_{MAP}$. Note that all customers get served and some consumer surplus is generated for those high value consumers that find the low price retailer.

Panel (b) of Figure 2 illustrates the outcome when $p_{RPM} = h$. Given a cost of service $F$, the optimal wholesale unit price is equal to $w_{MAP}$. This is easy to see, once it is noted that the rectangles $F$ in Panel (b) must be the same size as the rectangle...
The key difference between Panel (b) and Panel (a) is that, while service is provided in both instances, in Panel (b) a quantity distortion exists. This is, by virtue of MAP allowing more effective price discrimination, more consumers are served under MAP than RPM. Further, some consumer surplus is generated under MAP, but not when using RPM. Finally, the profit of the manufacturer using RPM is half that obtained when using MAP ($w$ is the same, but half the consumers are served). Thus, MAP dominates RPM for both consumers and the manufacturer, despite the fact that service is provided under both policies.

Panel (c) shows what happens when the optimal RPM policy is $p_{\text{RPM}} = l$.\(^{37}\) Note that $w_{\text{RPM}} < w_{\text{MAP}}$, since, under RPM, the retailers are sharing the entire market at the low price: this means consumers have more surplus, and to give the retailer sufficient rents to fund service, the manufacturer has to drop the wholesale unit price. Thus, consumers are better off under RPM, service is provided under both RPM and MAP, and no distortions exist. It still remains that, since $w_{\text{MAP}} > w_{\text{RPM}}$, the MAP policy is the more profitable way for the manufacturer to provide service.

The dominance, from the point of view of the manufacturer, of MAP policies as a way to incentivize service is not surprising, as it has been already established that, in this setting, MAP is a superior method of surplus extraction. What is perhaps more notable, is that when gains from trade are ‘low’, consumers also prefer MAP policies to RPM, as MAP does not generate a quantity restriction.\(^{38}\)

We end by providing a parametrized example in which the possibility results and intuitions provided in this section are easily verified.

**Example 2** Let $\lambda_S = 1$, $\lambda_N = 0$, $h = 1 + \epsilon$, $l = \epsilon$ and $\lambda = 0.5$. Let $F = \frac{3}{8}$. For $\epsilon < \frac{1}{2}$, neither RPM nor MAP will provide sufficient surplus to retailers to induce them to provide service and allow the manufacturer to make profits. For $\epsilon \in \left(\frac{1}{2}, 1\right)$, the optimal RPM price is $h$, $w = h - 4F$ and service is provided, but only the high value consumers are served and a deadweight loss exists. By contrast, with a MAP policy in which $p_{\text{MAP}} = h$ and $w = h - 4F$, all consumers are served, and no deadweight loss occurs. Further, half the high value consumers receive some consumer surplus. For $\epsilon > 1$, there is no deadweight loss under either MAP or RPM, service is provided and RPM generates the greater consumer surplus ($p_{\text{RPM}} = l$). For any $\epsilon > \frac{1}{2}$ the manufacturer’s profits from MAP will be strictly higher than that using RPM.

\(^{36}\)It is also immediate from this observation that, in this model, if service can be provided under MAP, there exists an RPM policy that will also induce service.

\(^{37}\)This will occur after some point as $h$ and $l$ are increased by the same additive constant.

\(^{38}\)That is, when $p_{\text{RPM}} = h$. 

15
3 MAP and service provision

In this section, we develop the idea that a service component might lead a MAP arrangement to be optimal, independently of the usefulness of MAP as a price discrimination device (as in Section 2.3).

We focus on retailer cost heterogeneity, and suppose that consumers are homogeneous. This shuts down price discrimination. To further homogenize consumers, and to highlight the role of search costs, we set $\sigma = 0$. That is, all consumers have high costs of search and so visit only one store. As above, the goods sold at the different stores are homogeneous. The model is enriched, such that consumers have downward-sloping, rather than unit, demand, with each consumer’s demand given by $q(p)$.

This means that retailers with different marginal costs have different optimal retail prices.

On the retailer side, we enrich the scope of service at stage 3 in the timing of the core model described in Section 2.1.1. Specifically, retailer $j$ sets a service level $s_j \in [0, 1]$ at cost $I(s_j)$, which is continuously differentiable and increasing and $I'(s)$ is sufficiently high that equilibrium investments are strictly less than 1. Thus, service is now a continuous choice variable for each retailer. It acts to increase consumer awareness of the product.

A consumer that is aware of the product can purchase from either retailer, as in Matthewson and Winter (1984). An investment of $s_j$ will expose a measure of consumers of size $s_j$ to the product. The probability that a consumer is exposed to the investment of one retailer is independent of whether it is exposed to the other, so the total measure of consumers that are aware of the product is equal to $S = 1 - (1 - s_1)(1 - s_2)$.

Finally, we suppose that each retailer’s marginal cost of retailing is independently drawn from an ex ante known distribution. This marginal cost takes the value $c_H$ with probability $\alpha$, and is $c_L = 0$ otherwise. Retailer costs remain private information; that is, retailers do not know each others’ realizations. Further, this implies that the relevant equilibrium concept in this section is Perfect Bayesian Equilibrium. We suppose that retailers’ cost are realized only after entering into contracts with the manufacturer; that is, after stage 2 and before stage 3 in the timing of Section 2.1.1. Hence, the two-part tariff extracts surplus by setting the fixed fee equal to a retailer’s marginal cost.

---

$3^9 q'(p) \in (-\infty, 0), q''(p) \in [0, \infty)$.

$4^0$ As pointed out in Asker and Bar-Isaac (2014) service need not be pro-competitive, and (at least in their model) can lead to exclusion when retailers are gatekeepers to a market. All the arguments made in Asker and Bar-Isaac (2014) can be transferred to environments with MAP.

$4^1$ At a technical level, this provides ex post price dispersion, which is needed for RPM and MAP to have a meaningful role.
expected net profit.\footnote{This is implied by the timing in Section 2.1.}

This model shows how MAP can lead to pro-competitive service when the price discrimination channel outlined above is not operative. That is, the focus is shifted from MAP’s efficacy in leveraging consumer heterogeneity, to its efficacy in environments dominated by retailer heterogeneity.

3.1 No vertical restrictions

Conditional on a realization of $S$, the measure of aware consumers, profit for $R_1$ is given by

$$
\pi(p_1, p_2) = \begin{cases} 
S q(p_1)(p_1 - w - c_1) & \text{if } p_1 < p_2 \\
\frac{1}{2}S q(p_1)(p_1 - w - c_1) & \text{if } p_1 = p_2 \\
0 & \text{if } p_1 > p_2 
\end{cases}
$$

Profit for $R_2$ is similarly defined.

In considering the equilibrium of the pricing subgame (stage 3 in the timing in Section 2.1.1), note that this is a discrete analog to Spulber (1995) which points out that Bertrand with privately known costs mirrors the equilibrium in a first price auction with risk aversion. Equilibrium of the pricing subgame then follows Proposition 2 in Spulber (1995) which is, itself, an adaptation of Theorem 2 in Maskin and Riley (1984). Equilibrium pricing is characterized as follows.

**Proposition 2** There exists a unique symmetric equilibrium pricing strategy such that, for $j \in \{1, 2\}$:

1. if $c_j = c_H$, $p_H = w + c_H$

2. if $c_j = 0$, price ($p_L$) is drawn from the distribution $F_L(p) = \frac{(p-w)q(p) - \alpha(p-w)q(p)}{(1-\alpha)(p-w)q(p)}$ with the support $[\underline{p}, \bar{p}]$, where $\bar{p} = \min\{p_H, p_L^M(w)\}$ and $p_L^M$ is the monopoly price a low cost retailer would charge; i.e. $p_L^M = \arg\max(p-w)q(p)$; and $\underline{p}$ is implicitly defined by $q(p) (\underline{p} - w) = \alpha(\bar{p} - w)q(\bar{p})$.

**Proof.** See the appendix. □

Next, we proceed by backward induction to consider the equilibrium of the investment subgame. Attention is restricted to symmetric equilibria where a retailer with a low cost realization chooses an interior investment $s_L > 0$. Note that given
the pricing equilibrium in Proposition 2; it is immediate that a high cost retailer earns no (net of investment) profits and, therefore, that there is no investment at a high cost realization; that is, $s_H = 0$.

In choosing $s_L$, each retailer solves the following problem (illustrated here for Retailer 1):\[ \max_{s_1} E(s_2) \tilde{\pi}(c_L) - I(s_1) \] (6)

where $E(S) = 1 - (1 - s_1)(1 - \alpha s_H - (1 - \alpha) s_L) = 1 - (1 - s_1)(1 - (1 - \alpha) s_L)$ and $\tilde{\pi}(c_L) = \alpha(\bar{p} - w)q(\bar{p})$ (note, by construction, in the mixed strategy equilibrium profits adjusted by the probability of winning is constant over $p$). This yields the following best response function in the investment subgame that arises from the first order condition: \[ I'(s_1(c_L)) = (1 - (1 - \alpha) s_L)\alpha(\bar{p} - w)q(\bar{p}). \] (7)

By imposing symmetry, the equilibrium advertising levels are implicitly defined as follows:\[ I'(s_L) = (1 - (1 - \alpha) s_L)\alpha(\bar{p} - w)q(\bar{p}). \] (8)

Finally, we can turn to the problem of the manufacturer. The manufacturer sets $T$ equal to a retailer’s expected profits so that\[ T = (1 - \alpha) [(1 - (1 - s_L)(1 - (1 - \alpha) s_L))\alpha(\bar{p} - w)q(\bar{p}) - I(s_L)], \]

where this expression follows on noting that only the low cost retailer earns profits and expected size of market and expected profits per customer follow from the expressions that appear below (6). The manufacturer chooses $w$ to maximize:\[ 2T + w \left[ 2\alpha(1 - \alpha)s_L \int_{\bar{p}}^{\bar{p}} q(p)f_L(p)dp + (1 - \alpha)^2(1 - s_L)^2 \int_{\bar{p}}^{\bar{p}} q(p)^2f_L(p)(1 - F_L(p))dp \right], \]

where the term in square brackets corresponds to expected sales. To understand this term, first note that when both retailers have high costs then there is no investment in advertising and so there are no sales; with probability $2\alpha(1 - \alpha)$ there is just one low cost retailer, a mass of $s_L$ consumers aware of the product and each consumer purchases $\int_{\bar{p}}^{\bar{p}} q(p)f_L(p)dp$ in expectation; finally, the second term inside the square brackets describes the event in which both retailers get low cost realizations. When both retailers are low cost, consumers aware of the product will purchase from the

\footnote{Imposing specific functional forms allows for a closed form solution, for example when $I(s) = \frac{s^2}{2}$ then $s_L = \frac{\alpha(\bar{p} - w)q(\bar{p})}{1 + (1 - \alpha)\alpha(\bar{p} - w)q(\bar{p})}$.}
lower of two price drawn independently from \( F_L(.) \); thus, the final integral reflects the expected quantity corresponding to a price distributed as the second order static of \( F_L(.) \).

### 3.2 RPM

From the perspective of the manufacturer, resale price maintenance can strictly improve on the outcome with no restraints. The channels that allow this are standard. First, minimum RPM can soften price competition between retailers, and hence, by allowing the retailers higher expected profits, can induce greater investment. For example, consider the case where \( \alpha \) is close to 0, so that both retailers are almost certainly low cost retailers. Retailer (net) profits, and hence advertising investments, would be negligible. In this case, a RPM provision that guaranteed retailers some net profits and induced some investment would clearly help induce service. Second, retail prices (for retailers with low cost realizations) may be too high due to a standard double mark-up problem; maximum RPM can help solve this. Here, the focus is on the case of a binding minimum RPM, set at a level denoted by \( P \), reflecting the need for service.\(^{44}\)

First, note that if minimum RPM is binding for both high and low cost retailers, the retail price (regardless of the retailers’ cost realizations) is given by \( P \). Instead, if the restriction is binding only for retailers with low cost realizations, then the pricing equilibrium will, again, involve a mixed strategy for any retailer with a low cost realization. In order to characterize it, it is useful to define an upper limit for the support of the mixing distribution as follows:

\[
\bar{p}_{\text{min}} = \min \{c_H + w, p^M_L(w)\}.
\]

Then the pricing equilibrium can be characterized as follows:

**Proposition 3** Suppose that the minimum RPM price is \( P < w + c_H \) (so that it does not bind given a high cost realization) but binds given the low cost realization, there exists a unique symmetric equilibrium pricing strategy such that for \( j \in \{1, 2\} \):

1. if \( c_j = c_H \), \( p_H = w + c_H \);

2. if \( c_j = 0 \), and \( p_{\text{min}} < \bar{p}_{\text{min}} \) then price, \( p_L \), is drawn from the distribution \( F_{\text{min}}(p) \) with support \( P \cup (p_{\text{min}}, \bar{p}_{\text{min}}) \) and an atom at \( P \) where

\(^{44}\)While the double margin issues are not the primary focus, it should be noted that the problem is limited by \( p_H = w + c_H \).
(a) \( p < \bar{p}_{\text{min}} \), \( F_{\text{min}}(p) \) is characterized by \( \alpha(\bar{p}_{\text{min}} - w)q(\bar{p}_{\text{min}}) = q(p)(p - w)(\alpha + (1 - \alpha)(1 - F_{\text{min}}(p))) \)

(b) \( p_{\text{min}} \) is implicitly defined by \( q(P)(P - w)\left[\alpha + (1 - \alpha)\frac{F_{\text{min}}(p_{\text{min}})}{2}\right] + (1 - \alpha)(1 - F_{\text{min}}(p_{\text{min}}))\right] = q(p_{\text{min}})(p_{\text{min}} - w)(\alpha + (1 - \alpha)(1 - F_{\text{min}}(p_{\text{min}}))) \);  

3. if \( c_j = 0 \), and \( p_{\text{min}} \geq \bar{p}_{\text{min}} \) then \( p_L = P \) with probability 1.

**Proof.** See the appendix. ■

The characterization of the pricing equilibrium suggests that in case the optimal minimum RPM induces price dispersion, it cannot aid the manufacturer through inducing service provision.\(^{45}\)

**Corollary 1** Suppose that the minimum RPM price is \( P < w + c_H \) (so that it does not bind given a high cost realization) and that a low cost retailer plays a mixed strategy in pricing, then the level of investment in service is identical to the level of investment in the absence of RPM or any other restrictions—that is the solution to (6).

**Proof.** This is immediate on noting that a retailer with a high cost realization makes no net profits and that \( \bar{p}_{\text{min}} = \bar{p} \) so that a retailer’s profit per aware consumer is identical to the case of no restrictions. Thus, the low cost retailer faces the identical maximization problem (6) in its choice of investment in service. ■

It follows that RPM can only play a role in inducing service provision in one of two cases (i) either it binds for both low and high cost realizations; that is, if \( P > w + c_H \); or (ii) it leads a firm with a low cost realization to choose \( P \) with certainty (which requires \( p_{\text{min}} > \bar{p}_{\text{min}} \) as defined in Proposition 3). We provide a complete characterization of the manufacturer’s problem with RPM, analogous to our analysis of the case with no restrictions, above, in Appendix B. Here, it is worth noting that in the former case, where both high and low cost firms set a price equal to \( P > w + c_H \), demand for a retailer irrespective of its costs is given by \( q(P) \). Service for low cost and high cost realization can be easily implicitly characterized through

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\(^{45}\)Interestingly, minimum RPM (and not only maximum RPM) can play a role in affecting “double mark-up” concerns by changing the price distribution and so may still dominate no restriction from the manufacturer’s perspective even if it has no impact on service provision.
the first order conditions as follows:

\[ I'(s_L(P)) = (1 - \alpha s_H - (1 - \alpha) s_L) \frac{q(P)}{2} (P - w), \] and (9)  

\[ I'(s_H(P)) = (1 - \alpha s_H - (1 - \alpha) s_L) \frac{q(P)}{2} (P - w - c_H). \] (10)

Note that these equations must be satisfied simultaneously; in comparison, in the case of no restrictions solving for a high cost retailer’s investment is trivial which allows for a simple characterization of a low cost retailer’s investment in (7). In particular, here, investment by the high cost and low cost retailers are strategic substitutes.\(^{46}\)

It is straightforward to find instances where the manufacturer might benefit from imposing such a price restriction; in particular, this will be the case for \(\alpha\) close to 0 or 1.

Summarizing the discussion above, we obtain Proposition 4.

**Proposition 4**: Minimum RPM set at \(P\) can strictly improve the manufacturer’s profits relative to the case of no restrictions in the case of smooth service investments. It can only do so through raising retailers’ advertising investments when it leads retailers to choose pure pricing strategies.

**Proof.** See the appendix. □

### 3.3 MAP

MAP can have the benefit of the minimum RPM scheme described above in dulling competition between retailers and allowing them higher profits, thereby encouraging investment in advertising. However, MAP has the benefit that it allows some retailers to drop actual price below the advertised minimum. The reason that they may want to do this is that their monopoly price may be below the MAP price. Moreover, this can be in the interest of the manufacturer as it increases the sales per customer for customers who are aware relative to the minimum RPM policy, and can increase the low cost retailer’s level of investment and the overall number of customers aware of the product.

Specifically, MAP at the MAP price \(P\) plays a role that might be different from RPM when the high cost retailer charges this price, and the low cost retailer charges

\(^{46}\)For specific parameterizations unique closed form solutions are assured. For example, for quadratic investment costs \(I(s) = \frac{s^2}{2}\), \(s_L(P) = \frac{(P - w)q(P)}{2 + (P - w - \alpha c)q(P)}\) and \(s_H(P) = \frac{(P - w - \alpha c)q(P)}{2 + (P - w - \alpha c)q(P)}\).
a different (lower) price. It is clear that the low cost retailer would charge \( p^m(w) \) which maximizes \( \frac{q(p^m(w))}{2}(p^m(w) - w) \).

Under MAP, half of the consumers who are aware of the good purchase from either retailer (irrespective of its costs), since the retailers advertise the same price. These consumers then respond to the price that they observe at the retailer. Consequently, service for low cost and high cost realization can be easily implicitly characterized through the first order conditions as follows:

\[
I'(s_L(P)) = (1 - \alpha s_H - (1 - \alpha) s_L) \frac{q(p^m(w))}{2}(p^m(w) - w), \quad \text{and} \quad (11)
\]

\[
I'(s_H(P)) = (1 - \alpha s_H - (1 - \alpha) s_L) \frac{q(P)}{2}(P - w - c_H). \quad (12)
\]

These differ from the equations that characterize investment under RPM at the same price \( P \), in (9) and (10) since the low cost retailer sells at a price \( p^m(w) \) rather than \( P \) and sells \( q(p^m(w)) \) to each consumer who arrives at the retailer rather than \( q(P) \). Given that retailers’ investments are strategic substitutes, this affects the equilibrium investments of both retailers. Thus, although greater investment from a low cost retailer might be anticipated as a result of the imposition of MAP, a consequence would be a force that dampens a high cost retailer’s investment. As for the case of no restrictions, the manufacturer optimally \( T \) equal to a retailer’s expected profits and chooses \( w \) to maximize expected profits. A formal treatment can be found in Appendix B.

We argue that the different economics of MAP as compared to RPM, by allowing the low cost retailer to charge a lower price than the MAP price, lead to higher industry profits and more effective investments. We do so both by example and in the Proposition below.

**Proposition 5** *In the case of smooth service investments, MAP can earn the manufacturer strictly higher profits than minimum RPM. When investment costs are sufficiently convex or if the firm is unlikely to be low cost (\( \alpha \) is high enough), then MAP can never earn less than minimum RPM.*

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47 At a sufficiently high MAP price, both a high cost retailer and a low cost retailer would charge a lower price than the MAP price. However, since a high cost retailer’s price can be predicted—it is simply its monopoly price \( (\text{given } w) \)—the manufacturer could set the MAP price at this level. Consequently, it is without loss of generality to suppose that the MAP price is always binding for a high cost retailer in equilibrium. For expositional simplicity we assume \( P \leq p^M_H(w) \).

48 For example, with quadratic investment costs \( I(s) = \frac{s^2}{2} \), a unique closed form solution is assured. Specifically, \( s_L(P) = \frac{(P-w)q(P)}{2+\alpha(\text{given } P)} \) and \( s_H(P) = \frac{(P-w-c_H)q(P)}{2+\alpha(\text{given } P)} \).
Proof. Suppose that the optimal RPM involves a two-part tariff \((T, w)\) and the
level of the price minimum at \(P\). Rather than considering the optimal MAP scheme,
suppose that MAP is imposed at \(P\) and with a wholesale price of \(w\), it is immediate
that a retailer with a high cost realization would choose a retail price equal to \(P\).
However, it is possible that the monopoly price for a retailer with a low cost real-
ization is below \(P\). In this case, under MAP, a retailer with a low cost realization
would charge this monopoly price \(p^m_L(w) = \arg \max(p - w)q(p) < P\).

This potentially affects \(M\)'s profits in several ways:

First, through sales directly, by increasing the sales per aware consumer (since a
low cost retailer charges a lower price). Secondly, it affects sales by changing retailers’
investments: Holding fixed the service level of a high cost retailer, it is clear that
a lower cost retailer would invest more. However, the higher investment of a low
cost retailer reduces a high cost retailer’s gains from investment, as discussed below
Equation (12). Overall, the combined effect is ambiguous. With sufficiently convex
costs (or high \(\alpha\)), the effects on the low cost retailer’s investment is negligible leading
to a positive overall effect on profits.

Finally, there is an effect through a (potentially) higher fixed fee \(T^{MAP}\) which
might also reflect different service levels and so, in principle, the sign of the effect
might be ambiguous. Again, when the investment cost function is sufficiently convex
or \(\alpha\) high enough, the impact on service levels is small, so that this effect is also
positive. Thus, overall \(M\)'s profits rise.

Finally, to easily observe that it indeed possible to have \(p^m_L(w) < P\) and, thereby,
prove the first statement of the proposition, consider a case where \(\alpha\) is close to 1 and
\(c_H > p^m_L(0)\).

It is worth noting that MAP may under-perform RPM from the perspective of the
manufacturer. This is (trivially) the case when all consumers can search costlessly
(\(\sigma_l = \sigma_h = 1\)) and MAP serves no useful function.

That said, the earlier Example 2 and Example 3, below, which provides a nu-
merical counterpoint to Proposition 5, make it clear that settings do exist in which,
by allowing for greater investment in efficient service and simultaneously allowing
more sales to realized, MAP can be pro-competitive (whether judged on a consumer
surplus or total surplus criterion).

Example 3 Let \(q(p) = 1 - p\), \(\alpha = 0.9\) and \(c_H = 0.4\). Table 1 shows the charac-
teristics of the optimal polices in the no restrictions, RPM and MAP cases. MAP
generates the highest consumer and total surplus, and the highest expected manufac-
turer and retailer profits.\(^{49}\)

\(^{49}\)Matlab code for generating the results in Table 1 can be found at

23
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4 MAP and collusion

Vertical restrictions, and RPM in particular, have long been regarded as potentially collusive mechanisms. That is, the binding of retailers, by the manufacturer, to a common price, can look, at first glance, like a price fixing agreement.\textsuperscript{50} In the modern era, the concern that RPM facilitates collusion continues to shape policy, supported by research, most notably the central contribution of Jullien and Rey (2007).\textsuperscript{51} Jullien and Rey (2007) demonstrate that RPM facilitates collusion by making deviations more transparent, at the cost of reducing responsiveness to local idiosyncratic demand fluctuations.\textsuperscript{52,53} In the analysis below, it is shown that MAP can similarly soften competition, again by inducing demand patterns akin to a market division.

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\textsuperscript{50}See Overstreet (1983) for a comprehensive historical survey.

\textsuperscript{51}Jullien and Rey (2007) provide a formal foundation for ideas that have existed, albeit in only heuristic form, since at least Telser (1960) and Yamey (1954).

\textsuperscript{52}Jullien and Rey (2007) note other elements of the tradeoff, notably that RPM can make punishment more severe.

\textsuperscript{53}In a recent paper Sugaya and Wolitzky (2016) highlight that even costless transparency might not always be beneficial for sustaining a cartel; although in many standard settings it is.
scheme, without reducing responsiveness to local market demand fluctuations. This results in MAP facilitating a more profitable and stable cartel.

The idea can be seen heuristically in the following simple example. Two manufacturers sell to a market via dedicated (non-shared) retailers. Consider a market in which there is always a high value consumer who would pay \( h \). In addition, with probability \( \mu \), there is an additional consumer who has a low valuation \( l \). This additional consumer is local to one of the two manufacturers’ retailers (with equal probability). That is with probability \( 1 - \mu \) there is only a high value consumer, and with probability \( \mu/2 \) there is also a low value consumer available to manufacturer 1 and with probability \( \mu/2 \) a low value consumer available to manufacturer 2. Suppose that \( 2l > h \) so that if a low value consumer is available, the industry would earn higher profits by selling to both consumers than to the high value consumer alone.\(^{54}\)

Jullien and Rey (2007) argue that RPM allows the manufacturers to more easily monitor compliance with a collusive scheme at the cost of less responsiveness to demand (or cost shocks). In the simple environment outlined above, RPM does not help manufacturers collude (with this simple unit demand specification, the manufacturers can agree on a common (high) retail price by setting \( w_1 = w_2 = h \), and observing retail prices allows manufacturers to monitor their compliance with this scheme. MAP accommodates heterogeneity in this case by allowing for responses to local demand shocks but without intensifying competition (since, for a retailer, there is always a 50% chance of selling to the high value consumer and if there is a low cost consumer then the relevant retailer will want to sell to him and does so). Hence, here, MAP can increase the scope for collusion relative to no vertical arrangements or, indeed, relative to RPM.

In what follows we embed this logic in a formal model that builds on the Jullien and Rey (2007) framework. The modeling objective is to allow price advertising to have an explicit role in the market, while retaining tractability and enriching the economic forces illustrated in Jullien and Rey (2007). The resulting framework is one in which RPM facilitates a more stable cartel (in the sense of admitting collusion for a wider range of discount rates than a cartel without restraints), at the cost of flexibility. MAP, by contrast, can facilitate a yet more stable cartel, and does not sacrifice flexibility, leading to higher cartel profits.

\(^{54}\)Note that this also implies that each manufacturer would rather sell with certainty at a price \( l \) than sell at a price of \( h \) with probability \((1/2)\).
4.1 Model details

There are two manufacturers, each of whom distributes to the local market via a dedicated (non-shared) retailer; that is there is an $M_1$ that sells through $R_1$ and an $M_2$ that sells through $R_2$. In this section, as in Section 2.1, we suppose that consumers have unit demand. However, instead of assuming that the two goods are homogeneous, we suppose that consumers value retailer $R_j$’s good at $v + \xi_j$, where $\xi_j \sim U[0, 1]$ is a manufacturer-retailer specific demand shock reflecting idiosyncratic local market factors. It follows that consumers will choose to visit retailer $R_j$ if

$$v + \xi_j - E(p_j|p^a_j) \geq 0$$

and

$$v + \xi_j - E(p_j|p^a_j) > v - \xi_k - E(p_k|p^a_k),$$

where $p^a_j$ is $R_j$’s advertised price and $E(p_j|p^a_j)$ is the expectation of the retail price, conditional on the advertised price.\(^{55}\)

The first inequality requires that the consumer must expect non-negative surplus, and the second that the consumer expects greater surplus from visiting $R_j$ than $R_k$. If the first inequality holds and the second condition holds with equality, then we suppose that the consumer purchases from whichever retailer offers a higher $\xi$. To keep exposition simple, $\sigma = 0$ so consumers visit at most only one store (search costs are high).

Furthermore, we suppose that $\xi_j$ is not observed by any manufacturer nor by the rival retailer; it is only privately observed by $R_j$ after contracting with the manufacturer and before setting the retail price. Each manufacturer sells to its dedicated retailer using a two-part tariff $\{w_j, T_j\}$ (which is not publicly observed).\(^{56}\) Lastly, $v > 1$, which ensures that $w_j$ is always set such that all $\xi_j$ realizations are served. Advertised prices are observed by all—including rival manufacturers.

Timing within a stage is otherwise as in Section 2.1.1. There is an infinite repetition of such stages where manufacturers discount the future at a common rate $\delta$ and retailers and consumers are myopic in their behavior.

\(^{55}\)In the absence of a MAP restraint, $E(p_j|p^a_j) = p_j$. With a MAP restraint, this expectation will be a determined by the equilibrium described in section 4.4.

\(^{56}\)These assumptions reflect those in Julien and Rey (2007). The assumptions on $\xi_j$ give a reason to give some pricing discretion to the retailer. The assumptions on $\{w_j, T_j\}$ make the cartel’s monitoring of compliance non-trivial (and keeps IR constraints relatively simple).
4.2 Collusion with no vertical restrictions

With no vertical restraints, deriving the equilibrium of this game is somewhat involved. Here we sketch the elements, and provide a characterization that is fleshed out in the appendix. Once characterized, collusion in the absence of vertical restraints provides a benchmark against which to compare the much simpler RPM- and MAP-facilitated schemes.

We begin by presenting a preliminary result that will prove useful. Specifically, we begin by examining the pricing game between retailers, supposing (as will turn out to be the case in equilibrium) that manufacturers choose the same input price; that is, supposing that \( w_1 = w_2 = w \). In this case, the pricing game between retailers is a minor variant on the standard IPV symmetric first price sealed bid auction in which each retailer solves:

\[
p_j = \arg \max_{w \leq p \leq v + \xi_j} (p - w) \Pr(\xi_j > p > \xi_k - p_k)
\]

Equilibrium pricing is stated in Proposition 6 and derived following the steps in, for instance, Krishna (2002).

**Proposition 6** Assume that \( w_j = w_k = w \). In a monotone perfect Bayesian equilibrium, \( p_j^a = p_j = w + \frac{\xi_j}{2} - \frac{w-v}{2} I_{v<w} \).

**Proof.** See the appendix. ■

Proposition 6 means that if \( w = 0 \), as would arise in the static version of the game, then retailers price using the rule \( p_j = \frac{\xi_j}{2} \). That is, prices fluctuate by half the actual fluctuations in local demand. The retailers’ expected profit, which is then transferred to the manufacturer through the fixed fee, \( T_j \), is equal to the expectation of \( \frac{\xi_j^2}{2} \), which is \( \frac{1}{6} \).

Instead, the manufacturer-optimal scheme in this setting involves both manufacturers setting \( w = v \) and \( T = \frac{1}{6} \). That is, the manufacturers set \( w \) as high as possible without excluding any consumers,\(^{58}\) and then retailers compete accounting for idiosyncratic market conditions.\(^{59}\) The manufacturers extract the remaining expected surplus from retailers using the lump sum component of the two part tariff.

In a collusive outcome, this scheme is enforced using a variant of a grim trigger strategy, in which manufacturers set \( w_t = v \) if all past advertised pricing has been in

\(^{57}\)That is, the retailer’s expected profit is \( \int_0^1 \frac{\xi_j}{2} \Pr(\xi_j > \xi_k) d\xi_j \).

\(^{58}\)This follows from the assumption that \( v > 1 \).

\(^{59}\)We assume retailers are not parties to the cartel agreement and always play statically optimal equilibrium strategies.
the interval \([v, v + \frac{1}{2}]\), and \(w_t = 0\) otherwise. This results in an expected per period collusive profit, for each manufacturer, of \(\frac{v}{2} + \frac{1}{6}\). The characterization is complicated by the fact that determining the optimal deviation is not immediate. In particular, if a manufacturer charges a slightly lower wholesale price in some period \(t\), because of the local demand \(\xi_j\) demand variations, this may lead to only a marginal gain but will have only a marginal probability of detection. Alternatively, a manufacturer might prefer a more drastic deviation that leads to a substantive gain albeit at the cost of a higher probability of detection. These two forms of deviation can, in principle, lead to two locally optimal deviations, which then need to be compared. Full details appear in the appendix.\(^{60}\) The following proposition provides the lower bound on the discount factor, \(\delta\), required for this collusive scheme to be supportable in equilibrium.

**Proposition 7** A collusive equilibrium in which manufacturers set \(w_t = v\) if \(p^a_j \in [v, v + \frac{1}{2}]\) for all \(j\) in all past periods, and \(w_t = 0\) otherwise, is supportable if \(\delta > \frac{6v-2}{6v-1}\). In such an equilibrium, the per period profit earned by each manufacturer, \(\pi_{NR}\), is \(\frac{v}{2} + \frac{1}{6}\).

**Proof.** See the appendix. \(\blacksquare\)

### 4.3 Collusion with RPM

When RPM is available, the manufacturers can set the price at which the retailers sell to customers. As discussed in Section 2.1, this will also be the advertised price. The value of RPM is that it removes any uncertainty on the part of the cartel members as to whether there has been a deviation or not, albeit at the expense of losing some flexibility in adjusting to local market conditions (that is, adjusting for \(\xi_j\) realizations). Thus, collusion can now take the form: Set \(w_t = p_{RPM} = v\) if the same advertised pricing has been observed in all past periods, and \(w_t = 0\) otherwise.\(^{61}\) Proposition 8 describes this much simpler, collusive environment.

**Proposition 8** A collusive equilibrium in which manufacturers set \(w_t = p_{RPM} = v\) as long as \(p^a_j,t = v\) for all \(j\) in all past periods, and \(w_t = 0\) otherwise, is supportable if \(\delta \geq \delta_{RPM} = \frac{3v}{6v-1}\). In such an equilibrium, the per period profit earned by each manufacturer, \(\pi_{RPM}\), is \(\frac{v}{2}\).

\(^{60}\)Jullien and Rey’s central proof is aimed at eliminating exactly the same types of deviations in their setting.

\(^{61}\)An alternative punishment phase may involve using RPM and setting \(w_t = p_{RPM} = 0\). This is a more extreme form of punishment. With this punishment stage equilibrium, \(\pi^p = 0\) in the notation of the proof of Proposition 8. Thus, in Proposition 8, the threshold would adjust to \(\delta > \frac{1}{2}\).
Proof. The proof is standard. It involves solving for \( \delta \) such that \( \frac{\pi_{cRPM}^c}{1-\delta} \geq \pi_{RPM}^D + \frac{\delta}{1-\delta} \pi^p \), where \( \pi_{RPM}^c = \frac{v}{2}, \pi_{RPM}^D = v \) and \( \pi^p = \frac{1}{6} \). \( \square \)

4.4 Collusion with MAP

Collusion, facilitated by MAP, involves coordinating on the advertised price, but allowing the transaction price (as usual under MAP) to only be constrained to be less than or equal to that advertised price. The value of this to the colluding manufacturers is that it obfuscates pricing in the market for consumers, effectively introducing a form of market division, while retaining flexibility on the part of consumers to adjust prices to local idiosyncratic conditions; that is it is a scheme that partially accommodates the realized demand heterogeneity. In this environment, the optimal collusive strategy is to set \( p_{j,t}^a = p_{k,t}^a = v + 1 \), resulting in the market being split equally. Retailers, having retained the flexibility to adjust prices downward, now are free from competition and can extract all consumer surplus by setting \( p_{j,t} = v + \xi_{j,t} \) (in contrast, with no restraints there is competition so that \( p_{j,t} = v + \frac{\xi_{j,t}}{2} \)). Thus the MAP provision, in this environment allows the retailer to extract all available surplus from consumers they service (assuming a rationing rule in which consumers purchase the good for which they have the highest value, given their indifference between goods after expected pricing is taken into account).\(^{62}\) Proposition 9 formalizes this intuition.

The proposition supposes that if the cartel agreement breaks down, the manufacturers revert to play the static equilibrium policies.\(^{63}\)

**Proposition 9** A collusive equilibrium in which manufacturers enforce \( p_{j,t}^{MAP} = v + 1 \) as long as \( p_{j,t}^a = v + 1 \) for all \( j \) in all past periods, and \( w_t = 0 \) without MAP otherwise, is supportable if \( \delta \geq \delta^{MAP} = \frac{1}{2} \). In such an equilibrium, the per period profit earned by each manufacturer, \( \pi_{MAP}^c \), is \( \frac{v}{2} + \frac{1}{3} \).

Proof. The proof is standard, requiring solving for \( \delta \) such that \( \frac{\pi_{cMAP}^c}{1-\delta} \geq \pi_{MAP}^D + \frac{\delta}{1-\delta} \pi^p \), where \( \pi_{MAP}^c = \frac{v}{2} + \frac{1}{3}, \pi_{MAP}^D = v + \frac{1}{2} \) and \( \pi^p = \frac{1}{6} \). \( \square \)

\(^{62}\)In any other rationing rule, the surplus extraction is inefficient, in that by retarding competition, consumers are not directed toward those products that maximize gains from trade. In the case where consumers who are indifferent are equally likely to visit either firm, the corresponding expressions for those that appear in Proposition 9 are \( \pi_{MAP}^c = \frac{v}{2} + \frac{1}{4} \) and \( \delta^{MAP} = \frac{6v + 3}{12v + 4} \).

\(^{63}\)An alternative punishment phase may still involve using MAP and setting \( w = p^{MAP} = 0 \). This is a more extreme form of punishment. With this punishment stage equilibrium, \( \pi^p = 0 \). In this case, Proposition 9 would instead require \( \delta > \frac{1}{2} \).
4.5 Vertical restraints can facilitate collusion

A comparison of Propositions 7, 8 and 9 makes it clear that the collusive profits with MAP dominate those without restrictions, which in turn dominate those from employing RPM. That is, \( \pi_{c \text{MAP}} > \pi_{c \text{NR}} > \pi_{c \text{RPM}} \).

Further, employing a vertical restraint (whether MAP or RPM) for the cartel can increase stability, in the sense that the force of future retribution for a deviation coupled with the prospect of losing collusive gains becomes a stronger incentive relative to the gains from deviating. This increase in the strength of dynamic incentives is captured by comparing the lower bound of the range of discount factors under which collusion can be supported. It can be easily verified that for \( v \) high enough (a sufficient condition is that \( v > \frac{2+2\sqrt{2}}{3} \), so that \( \frac{3v}{6v-1} < \frac{6v-2}{6v-2} \)) a cartel can be more easily sustained with RPM than with no restraints. MAP, by contrast, increases stability relative to no restraints for a wider range (or, indeed, relative to RPM). This follows as it is easier to monitor a manufacturer deviation from the cartel arrangement, when this arrangement involves a vertical restraint. When \( v \) is low then an RPM arrangement may be sacrificing too much in terms of profits, just as in Jullien and Rey (2007), and the flexibility to respond to local demand conditions is relatively valuable. MAP, on the other hand, maintains flexibility and so does not demand the same trade-off.

Finally, as mentioned above, \( \delta_{\text{MAP}} < \delta_{\text{RPM}} \) suggesting that MAP policies may help in stabilizing cartel agreements relative to RPM in addition to providing higher cartel profits.\(^{64}\) This follows since, although in both arrangements it is equally easy to monitor deviations from the cartel agreements, the MAP arrangement allows for a higher cartel profit since it accommodates the heterogeneity associated with local demand variation.

Thus MAP policies can allow a manufacturers’ cartel to attain greater surplus extraction than a cartel that does not use MAP or RPM, and strengthen the dynamic incentives that give the cartel stability, such that it is at least as stable as RPM. As such, in this setting with search frictions, MAP appears a more effective cartel facilitation device than RPM.

5 Discussion

This paper has discussed both pro- and anti-competitive features of MAP policies. MAP policies are distinct from RPM in that they can serve as a more effective way

\(^{64}\)As already noted above, in Footnotes 59 and 61, if MAP or RPM is used in the punishment stage, then \( \delta_{\text{MAP}} = \delta_{\text{RPM}} \).
to incentivize service, and as a more effective way to facilitate collusion. As such it suggests that, to the extent that controversy surrounds the appropriate policy treatment of vertical price restraints, the same, or greater, controversy should surround vertical information restraints, of which MAP is a prominent example. At the very least, carefully considering the nature and impact of information constraints in markets where search is a central feature seems warranted.

Of course, another way of restraining information is a ban on retail price advertising. This may be an interesting restriction in its own right. Formally, in the MAP as price discrimination discussion of Section 2, such a ban on advertising would lead to equivalent retail pricing decisions and economic outcomes. Similarly, in the case of upstream collusion in Section 4, a ban on price advertising would lead retailers to charge their monopoly prices (which manufacturers can extract through fixed fees) leading to similar profits and conditions to sustain a collusive agreement to ban price advertising. In richer environments with downward-sloping demand rather than unit demand, as in Section 3, the level of the advertising restriction can be used to affect the double marginalization concern raised by Spengler (1950) in addition to the other considerations outlined. Further in the presence of additional upstream and retail (imperfect) competitors, advertising directed at inter-brand competition would still be valuable and would lead the “monopoly” prices we focus on to be interpreted as best responses to such competitors’ strategies. By contrast, when faced with an advertising ban and sufficiently high search costs (still retaining inter-brand competition), retailers could not commit to charge anything other than their monopoly price.\footnote{Note advertising is a way to commit to not do this, due to the requirement that price be at or below the advertised price.} Thus, a MAP policy can allow a manufacturer to reassure consumers that they will not suffer this kind of hold-up, and as a consequence help the manufacturer in engaging in inter-brand competition.

This paper leaves at least three areas of enquiry open. First, aside from the collusion model, the frameworks presented here do not consider the impact of competition at the manufacturer level. Given that information restraints will likely change the cross-price elasticities between competitors, even absent collusion, this seems a rich area for further investigation.

Second, the models presented here limit the role of advertising to pure price advertising. This is analytically helpful in creating a clear mapping between the advertised price and the transaction price, but suppresses aspects that may be important in a richer model. An obvious issue in the price discrimination and collusion models is that unit demand means that prices do not influence quantities to the extent they might in a real market. A more delicate, and interesting, issue is that advertised
prices may signal quality—an issue that is claimed to be a concern in some MAP policy statements.

Lastly, the extent to which MAP, like other vertical restraints, can generate exclusion of a rival is not explicitly investigated, although it is conjectured that, as elsewhere, this effect can be generated.
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Appendix A: Proofs

Proposition 2 There exists a unique symmetric equilibrium pricing strategy such that, for \( j \in \{1, 2\} \):

1. if \( c_j = c_H, p_H = w + c_H \)

2. if \( c_j = 0 \), price \( (p_L) \) is drawn from the distribution \( F_L(p) = \frac{(p-w)q(p)-\alpha(p-w)q(\bar{p})}{(1-\alpha)(p-w)q(p)} \)
   with the support \([p, \bar{p}]\), where \( \bar{p} = \min\{p_H, p^M_L(w)\} \) and \( p^M_L \) is the monopoly price a low cost retailer would charge; i.e. \( p^M_L = \arg \max(p-w)q(p) \); and \( p \) is implicitly defined by \( q(p) (p - w) = \alpha(\bar{p} - w)q(\bar{p}) \).

Proof. By standard Bertrand reasoning, if \( c_j = c_H \) then \( \pi(p_H) = 0 \). This implies \( p_H = c_H + w \).

For \( c_j = 0 \), the equilibrium price distribution resembles that in Stahl (1989). The maximal price is the minimum of the monopoly price given \( w \) and \( c_j = 0 \), denoted \( p^M_L(w) \), and \( c_H + w - \varepsilon \) such that \( \varepsilon \to 0 \). That is, if \( p^M_L(w) \geq c_H + w \) then the maximal price of the price distribution will be arbitrarily close to \( c_H + w \). Hence, if \( p^M_L(w) \geq c_H + w, \bar{p} = c_H + w \). Otherwise \( \bar{p} = p^M_L(w) \).

Since a mixed strategy requires that \( \alpha(\bar{p} - w)q(\bar{p}) = [\alpha + (1-\alpha)(1-F_L(p))] (p-w)q(p) \) for all \( p \) in the support, it follows that \( F_L(p) = \frac{(p-w)q(p)-\alpha(p-w)q(\bar{p})}{(1-\alpha)(p-w)q(p)} \).

The lower bound of the support is found by setting \( F_L(p) = 0 \). \( \blacksquare \)

Proposition 3 Suppose that the minimum RPM price is \( P < w + c_H \) (so that it does not bind given a high cost realization) but binds given the low cost realization, there exists a unique symmetric equilibrium pricing strategy such that for \( j \in \{1, 2\} \):

1. if \( c_j = c_H, p_H = w + c_H \);

2. if \( c_j = 0 \), and \( p_{\min} < \bar{p}_{\min} \) then price, \( p_L \), is drawn from the distribution \( F_{\min} p(p) \) with support \( P \cup (\bar{p}_{\min}, \bar{p}_{\min}) \) and an atom at \( P \) where

   (a) \( p < \bar{p}_{\min}, F_{\min} p(p) \) is characterized by \( \alpha(\bar{p}_{\min} - w)q(\bar{p}_{\min}) = q(p)(p - w)(\alpha + (1-\alpha)(1-F_{\min} p(p)) \)

   (b) \( \bar{p}_{\min} \) is implicitly defined by \( q(P)(P - w) \left[ \frac{\alpha + (1-\alpha) F_{\min} p(p_{\min})}{2} \right] + (1-\alpha)(1-F_{\min} p(p_{\min})) = q(p_{\min})(p_{\min} - w)(\alpha + (1-\alpha)(1-F_{\min} p(p_{\min})) \);
3. if $c_j = 0$, and $\bar{p}_{\text{min}} \geq \bar{p}_{\text{min}}$ then $P_L = P$ with probability 1.

**Proof.** The logic is similar to the proof of Proposition 2.

That there must be a mass point at $P$ follows from the fact that it binds. Bertrand reasoning determines the behavior of retailers with high cost realization.

Indifference (so that profits are the same at all prices that a low cost retailer chooses in the pricing equilibrium) determines both properties 3 and 5. ■

**Proposition 6** Assume that $w_j = w_k = w$. In a monotone perfect Bayesian equilibrium, $p_j = w + \frac{\xi_j}{2} - \frac{w-v}{2} 1_{v<w}$.

**Proof.** The proof follows the proof of equilibrium in a IPV symmetric first price seal bid auction. See Krishna (2002) p.16ff for omitted details, easily adapted to this setting. The retailers solve

$$p_j(\xi_j) = \arg \max_{w \leq p \leq v+\xi_j} (p-w) \Pr(\xi_j - p > \xi_k - p_k)$$

Let $u_k(\xi_k) = \xi_k - p_k(\xi_k)$ and denote the equilibrium strategy to be $\beta_k(\xi) = u(\xi_j)$, and $\beta^{-1}(u_k) = \xi_k$. Assume that this strategy is monotone and increasing. To begin assume that $w \leq v$. Recall that $\xi$ is uniformly distributed on $[0,1]$. The retailers’ problem is then:

$$p_j = \arg \max_p (p-w) \beta^{-1}(\xi_j - p)$$

(17)

First order conditions yield

$$\beta^{-1}(\xi_j - p) - (p-w) \frac{\partial \beta^{-1}(\xi_j - p)}{\partial (\xi_j - p)} = 0$$

(18)

Imposing the equilibrium condition that $\beta(\xi_j) = u(\xi)$, yields

$$\xi_j - (p(\xi_j) - w) \frac{1}{1-p'(\xi)} = 0$$

(19)

$$p(\xi) + p'(\xi) \xi = \xi + w$$

(20)

$$\frac{\partial}{\partial \xi} (p(\xi)\xi) = \xi + w$$

(21)

$$p(\xi)\xi = \int (\xi + w) d\xi$$

(22)

Noting that $u(0) = 0$, allows the integral to be evaluated, yielding

$$p(\xi) = \frac{\xi}{2} + w$$

(23)
It remains to deal with the case where \( w \geq v \). In this case, some measure of \( \xi \) are excluded. The measure of included realizations is now \( U[w - v, 1] \). This implies that the boundary condition is now \( u(w - v) = 0 \), yielding the pricing part of the proposition. That \( p_j^a = p_j \) follows from Lemma 1.

**Proposition 7** A collusive equilibrium in which manufacturers set \( w_t = v \) if \( p_j^a \in [v, v + \frac{1}{2}] \) for all \( j \) in all past periods, and \( w_t = 0 \) otherwise, is supportable if \( \delta > \frac{6w - 2}{9w - 2} \). In such an equilibrium, the per period profit earned by each manufacturer, \( \pi_{NR}^c \), is \( \frac{v}{2} + \frac{1}{6} \).

**Proof.** There are three possible deviations. The first is \( w \in [0, v - 1] \), denoted D1. The second is \( w \in (v - 1, v - \frac{1}{2}) \), denoted D2. The third is \( w \in [v - \frac{1}{2}, v) \), denoted D3.

Given these three types of deviation, there are three conditions that are necessary for collusion to be able to be sustained. These are:

\[
\frac{\pi_{NR}^c}{1 - \delta} \geq \pi^{D1} + \Pr(p_j < v | w^{D1}) \frac{\delta}{1 - \delta} \pi^p + \left(1 - \Pr(p_j < v | w^{D1})\right) \frac{\delta}{1 - \delta} \pi_{NR}^c; \quad (24)
\]

and

\[
\frac{\pi_{NR}^c}{1 - \delta} \geq \pi^{D2} + \Pr(p_j < v | w^{D2}) \frac{\delta}{1 - \delta} \pi^p + \left(1 - \Pr(p_j < v | w^{D2})\right) \frac{\delta}{1 - \delta} \pi_{NR}^c; \quad (25)
\]

and

\[
\frac{\pi_{NR}^c}{1 - \delta} \geq \pi^{D3} + \Pr(p_j < v | w^{D3}) \frac{\delta}{1 - \delta} \pi^p + \left(1 - \Pr(p_j < v | w^{D3})\right) \frac{\delta}{1 - \delta} \pi_{NR}^c; \quad (26)
\]

where \( \pi_{NR}^c \) denotes the per-period collusive profit, and following discussion in the text \( \pi_{NR}^c = \frac{v}{2} + \frac{1}{6} \); and in case the collusion breaks down, manufacturers earn the one-shot profit which is denoted by \( \pi^p = \frac{1}{6} \).

Note that it is immediate that deviating to \( w > v \) cannot be optimal.

The proof proceeds by establishing properties of the optimal deviation of each type, first by characterizing the retailers pricing when \( w_j < w_k = v \) and then examining the manufacturers problem in setting \( w_j \) for each type of deviation. Finally, a \( \delta \) that is sufficient for none of the deviations to be attractive is derived.

First, consider the pricing of a retailer with a wholesale unit price of \( w_j \) facing a rival that prices in line with the cartel rule, such that \( p_k = v + \frac{\xi_k}{2} \). The retailer’s pricing problem is

\[
p_j^*(\xi_j) = \arg \max_p (p - w_j) \Pr(\xi_j - p > \xi_k - v - \frac{\xi_k}{2}) \quad (27)
\]

\[\text{66Recall that retailers are not part of any cartel agreement and compete in a static game.}\]
Given that $\xi_k$ is uniformly distributed on \([0, 1]\), this amounts to maximizing

$$
(p - w) (2\xi_j + 2v - 2p)
$$

in the region where \((2\xi_j + 2v - 2p) \in [0, 1]\) (outside of this region, either there is no chance of winning, or the retailer wins for certain and is merely forgoing revenue by dropping price). Ignoring this constraint results in the pricing rule $p^*(\xi_j) := \frac{v+w}{2} + \frac{\xi_j}{2}$. If $2\xi_j + 2v - 2p^* > 1$, then the pricing rule is $p = v - \frac{1}{2} + \xi_j$.\(^{67}\) Further, if $2\xi_j + 2v - 2p^*(\xi_j) > 1$, then the same inequality holds for all $\xi > \xi_j$. This results in the following retailer pricing rule when $w_j < w_k = v$: $p_j = \max \left\{ \frac{v+w}{2} + \frac{\xi_j}{2}, v - \frac{1}{2} + \xi_j \right\}$.

Next we turn to consider the manufacturer’s strategy. The first style of deviation ($D1$), involves choosing the optimal deviation in the interval $w \in [0, v-1]$. In this interval, by inspection of the retailer’s pricing rule derived above, the probability of detection is equal to 0.5 regardless of $w$. Hence, the optimal deviation can be derived by maximizing the combined retailer-manufacturer-pair deviation payoff (since the manufacturer can extract the retailer’s profit through the fixed fee). It is clear that this is exactly what the retailer will do when $w_j = 0$. Hence, the profit maximizing $D1$ deviation arises when the retailer sets prices such that $p_j = v - \frac{1}{2} + \xi_j$. From the manufacturers point of view, this retailer pricing policy will arise for any $w \in [0, v-1]$ and so can be implemented in a variety of ways, all resulting in the same deviation profit of $v$ (the lump-sum component of the manufacturer’s two part tariff will be used to extract remaining expected profits from the retailer). Thus the $D1$ deviation yields $\pi_{D1} = v$ and $\Pr(p_j < v | w^{D1}) = \frac{1}{2}$.

Given this, we can solve for the minimum $\delta$ such that a $D1$ deviation is not attractive. The $D1$ condition requires that

$$
\left( \frac{v}{2} + \frac{1}{6} \right) \frac{1}{1-\delta} \geq v + \frac{1}{2} \frac{\delta}{1-\delta} \frac{1}{6} + \frac{1}{2} \frac{\delta}{1-\delta} \left( \frac{v}{2} + \frac{1}{6} \right)
$$

implying that when $\delta \geq \frac{6v-2}{9v-2}$ a $D1$ deviation is not attractive.

The second style of deviation ($D2$), leaves the probability of detection unchanged at 0.5. To see this note that for all $w \in (v-1, v - \frac{1}{2})$, $p_j = \max \left\{ \frac{v+w}{2} + \frac{\xi_j}{2}, v - \frac{1}{2} + \xi_j \right\} = v$ when $\xi = \frac{1}{2}$ and that $p$ is monotonic in $\xi$ and so the deviation is detected for $\xi \leq \frac{1}{2}$ and undetected otherwise. Also, recall that when $w = 0$ the retailer chooses $p_j = v - \frac{1}{2} + \xi_j$. Hence, by setting $w \in (v-1, v - \frac{1}{2})$, the manufacturer diminishes stage profits, with no compensating return in terms of adjusting the probability.

\(^{67}\)Recall, $v > 1$, making this event relevant. In this event, the pricing rule is derived by raising $p$ until the chance of winning is equal to 1.
around detection (that is, leaving the continuation value unchanged). Hence, a \( D_2 \) deviation must always be dominated by a \( D_1 \) deviation.

The third style of deviation \( (D_3) \), involves choosing the optimal deviation in the interval \( w \in [v - \frac{1}{2}, v) \). In this interval, a change in \( w \) will affect the probability of detection; specifically, in this range, \( \Pr(p_j < v|w) = \Pr(\frac{w+w_j}{2} + \frac{\xi_j}{2} < v) = \Pr(\xi_j < v - w) = v - w \). Trivially, \( \frac{d\Pr(p_j < v|w_j)}{dw_j} = -1 \). Given that \( w + 1 \) is the value of \( \xi_j \) such that \( \frac{w+w_j}{2} + \frac{\xi_j}{2} = v - \frac{1}{2} + \xi_j \), the deviation profit can be written as:

\[
\pi^{D_3} = \int_{0}^{w_j+1-v} \left( \frac{v+w_j}{2} + \frac{x}{2} \right) (v+x-w_j) \, dx + \int_{w_j+1-v}^{1} \left( v - \frac{1}{2} + x \right) \, dx. \tag{30}
\]

It will be useful to note that \( \frac{\partial \pi^{D_3}}{\partial w_j} = -w_j (w_j + 1 - v) \) and that equation (30) also describes \( \pi^{D_2} \).

The optimal \( D_3 \) deviation is the solution to

\[
\max_{w_j} D_3(w_j) \equiv \max_{w_j} \pi^{D_3} + \Pr(p_j < v|w_j) \frac{\delta}{1-\delta} \pi^p + \Pr(1 - p_j < v|w_j)) \frac{\delta}{1-\delta} \pi^c \tag{31}
\]

which, taking the derivative with respect to \( w_j \), yields

\[
\frac{\partial D_3(w_j)}{\partial w_j} = \frac{\partial \pi^{D_3}}{\partial w_j} - \frac{\partial \Pr(p_j < v|w_j)}{\partial w_j} \frac{\delta}{1-\delta} (\pi^c - \pi^p) = -w_j (w_j + 1 - v) + \frac{\delta}{1-\delta} \frac{v}{2} \tag{32}
\]

where the last equality follows on substituting for the two derivatives, as calculated above. Note that the second derivative is \( \frac{\partial^2 D_3(w_j)}{\partial w_j^2} = v - 1 - 2w_j < 0 \) in the range \( w \in [v - \frac{1}{2}, v) \).

Necessary conditions for the existence of an optimal \( D_3 \) deviation (i.e. an interior solution in \( [v - \frac{1}{2}, v) \)) are that \( \frac{\partial D_3(w_j)}{\partial w_j} \bigg|_{w_j=v-\frac{1}{2}} \geq 0 \) and \( \frac{\partial D_3(w_j)}{\partial w_j} \bigg|_{w_j=v} < 0 \).

From equation (33), \( \frac{\partial D_3(w_j)}{\partial w_j} \bigg|_{w_j=v-\frac{1}{2}} \geq 0 \) implies that \( \delta \geq \frac{2v-1}{4v-1} \). Similarly, \( \frac{\partial D_3(w_j)}{\partial w_j} \bigg|_{w_j=v} < 0 \) implies \( \delta < \frac{2}{3} \). Hence, for a \( D_3 \) deviation to exist it must be that \( \delta \in \left[ \frac{2v-1}{4v-1}, \frac{2}{3} \right) \).

It remains to establish that a \( D_1 \) deviation will always dominate a \( D_3 \) deviation. It is established above that the optimal \( D_1 \) deviation can be implemented by setting \( w = v - 1 \). In the region \( w \in (v-1, v-\frac{1}{2}) \) (a \( D_2 \) deviation), deviation profits,
\( \pi^{D2} \), decrease. Hence, adapting equation (33), and noting that \( \frac{\partial \pi^{D3}}{\partial w_j} = \frac{\partial \pi^{D2}}{\partial w_j} = -w_j (w_j + 1 - v) \) for a D3 deviation to dominate a D1 deviation it must be that

\[
\int_{v-1}^{w^{D3}} -w (w + 1 - v) \, dw + \int_{v-\frac{1}{2}}^{w^{D3}} \frac{\delta}{1 - \frac{\delta}{2}} \frac{v}{\delta} \, dw > 0 \quad (34)
\]

where \( w^{D3} \) is the optimal \( w \) for a D3 deviation. Note that, on the assumption that equation (34) is true,

\[
\int_{v-1}^{w^{D3}} -w (w + 1 - v) \, dw + \int_{v-\frac{1}{2}}^{w^{D3}} \frac{\delta}{1 - \frac{\delta}{2}} \frac{v}{\delta} \, dw
\]

\[
\leq \int_{v-1}^{v-\frac{1}{2}} -w (w + 1 - v) \, dw + \int_{v-\frac{1}{2}}^{v} \frac{v}{2} - \frac{3}{4} \, dw + \int_{v-\frac{1}{2}}^{v} \frac{\delta}{1 - \frac{\delta}{2}} \frac{v}{\delta} \, dw \quad (35)
\]

\[
\leq \int_{v-1}^{v-\frac{1}{2}} -w (w + 1 - v) \, dw + \int_{v-\frac{1}{2}}^{v} -\frac{3}{4} \, dw + \int_{v-\frac{1}{2}}^{v} \frac{\delta}{1 - \frac{\delta}{2}} \frac{v}{\delta} \, dw \quad (36)
\]

Where the first inequality arises from noting that \( \frac{\partial^2 \pi^{D3}(w)}{\partial w_j^2} < 0 \) and substituting in \( \left. \frac{\partial \pi^{D3}(w)}{\partial w_j} \right|_{w=v-\frac{1}{2}} \). The second inequality comes from noting that the second and third integrals are linear in \( w \), and so their difference, under the maintained assumption, is increasing in the limits of integration. Setting, \( \delta = \frac{2v+2}{3v+2} \) (to maximize the third integral) and evaluating yields

\[
\left( \frac{1}{12} - \frac{3v}{8} \right) + \left( -\frac{v}{4} - \frac{3}{8} \right) + \left( \frac{v}{2} \right) < 0, \quad (37)
\]

which, by contradiction, establishes that D1 is the more profitable deviation.
Appendix B: Additional material: Not for publication

Strategic Consumers and Advertising

In the text, we do not allow consumers to be strategic, but simply suppose that high-search cost consumers choose to purchase from the retailer with the lowest advertised price, unless prices are the same in which case they randomize. In this appendix, we argue that such behavior is an equilibrium of a game that allows for strategic consumers that engage in reasoning consistent with a perfect Bayesian equilibrium.

Specifically, a retailer’s advertised retail price is denoted $p^a_j$. We assume that this is flexible, and so $p^a_j$ is set contemporaneously with the retailers’ transaction price. We make the following assumption. It ensures that the advertised price is not simply cheap talk.

A1. Advertising cannot be fraudulent in the following sense: the advertising price can be no lower than the retail transaction price $p_j$, i.e. $p^a_j \geq p_j$.

In all other respects we keep the game form the same as in Section 2.1. Given that some consumers can visit only a single retailer, beliefs as to $p_j$ given $p^a_j$ need to be considered. This results in the relevant equilibrium concept being (a refinement of) perfect Bayesian equilibrium.

In principle, there is considerable flexibility in assigning off-equilibrium beliefs and, thereby, inducing perverse equilibrium behavior. However, Assumption A1 specifies that a retailer cannot advertise a price below the price that it charges imposes real constraints. For example, suppose that a retailer charging a price of $4$ or charging a price of $6$ were expected to advertise the same price. The advertised price would have to be a price at or above $6$ to conform with this restriction.

Suppose that this advertised price is $20$ and that these are the only two kinds of retailer expected to advertise at $20$ in equilibrium. Then consumers expect on average some price above $4$ (if it was equally likely that retailers charge $4$ and $6$, consumers anticipate $5$ on average).\(^{68}\) Suppose that the retailer charging $4$ instead of advertising a price of $20$ advertises a price of $4$. Given the restriction that retailers cannot advertise at prices below what they charge, it must be that consumers even if this is an off-equilibrium advertised price anticipate paying a price $\ldots$

\(^{68}\)Recall that retailers are indistinguishable to consumers, so that if $p^a_j = p^a_k$ then $E(p_j | p^a_j) = E(p_k | p^a_k)$.

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at or below $4; it follows that this advertised price attracts more consumers, as their expected surplus must be strictly greater, and hence would be more profitable.\footnote{If, in equilibrium, competing retailers advertise and charge a price of $6, but the retailer of interest has an actual price of $4, any advertised price in the interval \([4, 6]\) is considered equivalent to advertising a price of $4.}

When there are no advertising restrictions, the simple logic in the example above has considerable bite. The underlying point is that for every advertised price arising in equilibrium there is a unique retail transaction price. This creates advertised prices that are unambiguous, and are equivalent to requiring that retailers will advertise the price that they actually charge.\footnote{Two mappings from advertised prices to transaction prices are equivalent if changing the set of prices from which advertised prices can be drawn makes no difference to the realized transaction prices or consummated transactions. That is, the language does not matter so long as the message is the same.} The following result makes this precise for pure strategy equilibria.\footnote{The result can easily be extended to mixed strategy equilibria, albeit with much more notation.}

**Proposition 10** Suppose that there are no advertising restrictions. Then in any (pure strategy) perfect Bayesian equilibrium, on the equilibrium path, each $p^a$ maps to a unique $p$.

**Proof.** Suppose, toward a contradiction, that there is an equilibrium where this is not the case. Then the equilibrium must involve (at least) two retailers, who charge different transaction prices, choosing the same advertised price $a$. The other possibility is that one retailer sets the same advertised price for two different transaction prices, but this is ruled out by virtue of each retailer being able to set only one price.

First, it must be that all retailers advertising at $a$ get some customers. If not, then they can always set their advertised price equal to a level that does get customers and still make a profit, since the manufacturer cannot discriminate at the wholesale level.

Now consider all retailers who advertise at $a$. Of this set, let $p_{\text{min}}(a)$ denote the price of the retailer with the lowest actual price. Note that since there is more than one retailer who advertises at $a$, it must be the case that consumers anticipate a non-zero probability of an actual price strictly greater than $p_{\text{min}}(a)$, and hence expect surplus smaller than that generated by receiving $p_{\text{min}}(a)$ with certainty.\footnote{Recall that, from the point of view of consumers, retailers are identical (aside from their advertised prices). Hence, consumers cannot condition their expectations of $p_j$ on $j$.}

Further, for all retailers that advertise at $a$, high search cost consumers randomize over these retailers.

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The restriction that a retailer cannot charge a price higher than its advertised price implies that, if the retailer charging \( p_{\text{min}}(a) \) advertised its actual price \( p_{\text{min}}(a) \), consumers could not put any probability on the retailer charging a higher price than \( p_{\text{min}}(a) \) (and might even put some probability on a lower price). That is they anticipate an actual price that is equal to, or lower than \( p_{\text{min}}(a) \). Expected surplus is similarly strictly increased.

Putting together the observations in each of the three paragraphs above, it is immediate that setting the advertised price at \( p_{\text{min}}(a) \) would attract strictly greater demand as compared to choosing the advertised price \( a \). This generates the required contradiction. ■

In particular, trivially, with no advertising restrictions, there is no loss in supposing that equilibria will have the intuitive property that consumer beliefs are monotone in the following sense.

**Definition 1** Consumers’ beliefs are **monotone** if, \( \forall p_j^a > \hat{p}_j^a, E(p_j|p_j^a) > E(p_j|\hat{p}_j^a) \).

**Definition 2** A **monotone perfect Bayesian equilibrium** is a perfect Bayesian equilibrium in which consumers’ beliefs are monotone.

In case of advertising restrictions, a retailer may not be able to set the advertised price equal to its actual price. Moreover, when the MAP price is high, the logic in the example above that underlies Proposition 10 can have little bite. Returning to the example above, suppose that there was a MAP price of $10, and consumers expected that retailers advertising a price of $20 were equally likely to actually charge $4 or $12, supported by the (off-equilibrium) beliefs that any retailer advertising anything between $10 and $20 actually charged $9. Here, since the MAP restriction prevents the retailer charging $4 from advertising at a sufficiently low price, this retailer has little to gain from advertising at a price below $20 and differentiating itself from the retailer charging $12. A focus on monotone perfect Bayesian equilibrium rules out such perverse outcomes. Restricting equilibrium to requiring monotone beliefs leads retailers to advertise at as low a price as possible. We obtain the following result.

**Proposition 11** In a MAP regime, in all monotone perfect Bayesian equilibria, each retailer advertises its actual price unless the MAP restriction binds, and in this case it advertises the MAP price; i.e. \( p_j^a = \max \{ p_{\text{MAP}}^-, p_j \} \).

In the paper we assume that firms advertise a price \( p_j^a = p_j \), or, if a MAP restriction exists, \( p_j^a = \max \{ p_{\text{MAP}}^-, p_j \} \). In this appendix, in the absence of fraudulent
advertising (that is under Assumption A1) we have shown that doing so can capture all the economic richness that a richer strategy space would deliver with only minimal (reasonable) restrictions on the set from which consumer beliefs are drawn. Further, these restrictions are only required when MAP policies bind.

Additional Material for Section 3

Section 3.2: The manufacturer’s problem with RPM

As described in the text, RPM can only play a role in inducing service provision in one of two cases (i) either it binds for both low and high cost realizations; that is, if $P > w + c_H$; or (ii) it leads a firm with a low cost realization to choose $P$ with certainty (which requires $p_{\text{min}} > \bar{p}_{\text{min}}$ as defined in Proposition 3). In the latter case, it is immediate that a firm with a high cost realization makes no investment, and the level of the investment of a firm with a low cost realization satisfies:

$$I'(s_L(P)) = \left(\alpha + \frac{(1 - \alpha)(1 - s_L(P))}{2}\right)(P - w)q(P);$$

(39)

the manufacturer sets $T$ equal to the retailer’s expected profits

$$T = (1 - \alpha) \left[\left(\alpha s_L + \frac{(1 - \alpha)(1 - (1 - s_L)^2)}{2}\right)(P - w)q(P) - I(s_L)\right],$$

(40)

and $w$ to maximize expected profits

$$2T + wq(P) \left[2\alpha(1 - \alpha)s_L + (1 - \alpha)^2(1 - (1 - s_L)^2)\right] + w\alpha^2 q(w + c_H).$$

(41)

In the former case, where both high and low cost firms set a price equal to $P > w + c_H$, demand for a retailer irrespective of its costs is given by $\frac{q(P)}{2}$. Service for low cost and high cost realization can be easily implicitly characterized through the first order conditions described in the main text in (10) and (9). Finally, we can turn to the problem of the manufacturer who sets $T$ equal to a retailer’s expected profits so

$$T = \alpha \left[(P - w - c)\frac{q(P)}{2}(1 - (1 - s_H)(1 - \alpha s_H - (1 - \alpha)s_L)) - I(s_H)\right]$$

(42)

$$+ (1 - \alpha) \left[(P - w)\frac{q(P)}{2}(1 - (1 - s_L)(1 - \alpha s_H - (1 - \alpha)s_L)) - I(s_L)\right],$$

and chooses $w$ and $P > w + c_H$ to maximize

$$2T + wq(P) \left[\alpha^2(1 - (1 - s_H)^2) + 2\alpha(1 - \alpha)(1 - (1 - s_L)(1 - s_H)) + (1 - \alpha)^2(1 - (1 - s_L)^2)\right].$$

(43)
Section 3.3: The manufacturer’s problem with MAP

The manufacturer set $T$ as follows

$$T = \alpha \left[ (P - w - c) \frac{q(P)}{2} (1 - (1 - s_H)(1 - \alpha s_H - (1 - \alpha)s_L)) - I(s_H) \right]$$

$$+ (1 - \alpha) \left[ (p^m(w) - w) \frac{q(p^m(w))}{2} (1 - (1 - s_L)(1 - \alpha s_H - (1 - \alpha)s_L) - I(s_L) \right],$$

and chooses $w$ and $P > w + c_H$ to maximize expected profits—the sum of the fixed fee $T$ and expected revenue from the per unit fee:

$$2T + w \left[ \alpha^2(1 - (1 - s_H)^2)q(P) + (1 - \alpha)^2(1 - (1 - s_L)^2)q(p^m(w)) \right]$$

$$+ \alpha(1 - \alpha)(1 - (1 - s_L)(1 - s_H))(q(P) + q(p^m(w)) \right].$$