Bank Net Worth and Frustrated Monetary Policy

Alexander K. Zentefis∗

Yale School of Management

(alexander.zentefis@yale.edu)

January 29, 2018

Abstract

This paper presents a theory in which monetary transmission depends on the state of bank net worth. When banks are flush with equity, changes in the policy rate pass through fully to bank lending rates. When banks have low equity, there is no pass-through. Banks in the model are local monopolists for borrowers near them. When they have lots of equity, they compete for customers at the edges of their markets. When they have little equity, they retreat and exploit their local monopoly power. In a dynamic setting, the economy can get trapped with no monetary transmission unless banks recapitalize.

JEL classification: E32, E44, E52, G21, L13

Keywords: banking, monetary policy, industrial organization, monopolistic competition, business fluctuations

∗I am deeply grateful to Doug Diamond, Stavros Panageas, John Cochrane, and Pietro Veronesi for many fruitful conversations about this topic and for their invaluable assistance. I also thank Amy Boonstra, Max Bruche (discussant), Will Cong, Jason Donaldson (discussant), Radha Gopalan, Gary Gorton, João Granja, Veronica Guerrieri, Zhiguo He, Yunzhi Hu, Jon Ingersoll, Anjini Kochar, Tara Levens, Asaf Manela, Gregor Matvos, Michael Minnis, Justin Murfin, Stefan Nagel, Paymon Khorrani, Aaron Pancost, Giorgia Piacentino, Jacopo Ponticelli, Raghu Rajan, Uday Rajan, Jung Sakong, Amit Seru, Amir Sufi, Chad Syverson, Fabrice Toure, Margdarita Tsoutsoura, Luigi Zingales, Eric Zwick, and seminar participants at the Chicago Booth Corporate Finance Reading Group, the Chicago Booth Finance Brownbag, the Chicago Booth Finance Lunch Seminar, UT Austin McCombs, Vanderbilt Owens, the Federal Reserve Board, OFR Treasury, Cleveland Fed, Yale SOM, Boston University Questrom, UCLA Anderson, the ECB, Stanford University, Copenhagen Business School, the 13th Annual Conference on Corporate Finance, OxFIT 2017, the WFA-CFAR and JFI Conference 2017, and AEA 2018 for very helpful comments.
1 Introduction

Following the global financial crisis of late-2008, there was concern among central bankers that the interest rate channel of monetary policy transmission was impaired. Although the Federal Reserve and central banks around the world held policy rates at historically low levels, interest rates on bank loans to firms were slow to respond, remaining persistently high. Meanwhile, measures of firm default risk in many bank credit markets had already declined to pre-crisis levels.

U.S. commercial and industrial (C&I) loans, which is bank credit to small- and medium-sized firms, was an example. The spread between the average C&I loan rate and the effective federal funds rate stayed elevated after the crisis, despite delinquency rates, the fraction of nonperforming loans, and net loan charge-offs for new C&I credit all subsiding to levels seen before the crisis. Figures 1(a)-1(b) present the relation. Meanwhile, credit spreads on firm debt issued outside the banking sector, such as in the corporate bond market, declined much faster. Figure (3) plots the C&I loan spread in comparison to the corporate bond spread as measured in Gilchrist and Zakrajšek (2012), which captures the difference between corporate bond yields and Treasury rates matched by bond maturity.

During this period, two other notable phenomena took place in the C&I bank loan market. First, bank concentration rose strongly, such that the number of banks extending this type of firm credit dropped sharply between 2008 and 2010. Figures 2(a)-2(b) illustrate the change. This spike in concentration coincided with the extraordinary wave of bank consolidation that took place following the financial crisis. From 2007 to 2013, 492 commercial and savings banks were put into FDIC receivership and sold at auction to acquiring banks (Granja et al. (2017)).

Second, banks expanded and contracted their small business loan portfolios across geographic areas in agreement with the health of their balance sheets. Banks that suffered asset losses from drops in real estate prices contracted their small business credit across regions, while banks that avoided the losses continued lending, and even entered new territories, expanding the geographic reach of their loans (Bord et al. (2017)).

The contribution of this paper is to present a theory demonstrating that the joint display of all three phenomena (the impairment of the interest rate channel, the rise in bank concentration, and the contraction of lending across localities by damaged banks) was no coincidence. All three can emerge after a financial crisis.

The paper presents a model of credit and the real economy to understand why. The model posits that aggregate equity capital in the banking sector provides an explanation. The central thesis connects the banking system’s industrial organization to its net worth: bank market power can impede the interest rate channel when bank net worth is low.

I argue that when banks are flush with equity, their required cost of equity is low, so they...
have incentive to compete across different parts of the loan market, because doing so is relatively cheap. In this case, competition compels banks to pass through changes in the central bank’s policy interest rate to their lending rates.

However, a severe contraction in bank equity sharply raises the cost of equity across banks, which forces them to consolidate for survival. Competing across each other’s territories is no longer profitable, so instead, banks contract their loan portfolios and turn into local monopolists in separate parts of the loan market.

No longer facing the same competitive pressure, banks do not pass through changes in the policy rate to their lending rates. Over time, as long as bank equity remains strained, there is no transmission via the interest rate channel. Monetary transmission, in fact, can be permanently choked off unless banks recapitalize. High bank loan rates consequently have real effects by reducing physical capital production and output.

In the model, banks lend money to firms that are managed by entrepreneurs who run industrial projects. These projects create physical capital for the production of output and are located around a circle. Locations on the circle represent geographic areas or productive industries. Banks are local monopolists for borrowers near them, but they can compete for customers at the edges of their markets.

Banks face a minimum equity capital requirement that is a function of their assets in place, part of which includes their loans to firms. Banks diversify the risk in their loan portfolios by extending credit to arcs of the circle. Diversification is increasingly expensive, however, in that the cost of liquidating projects in default grows in the size of the bank’s loan portfolio. Banks that focus on narrow parts of the loan market are more efficient at recovering value from distressed assets than are banks that operate over broad stretches. In equilibrium, each specializes in a class of industries or areas.

Bank market power arises out of an entrepreneur’s preference to contract with a bank that is more specialized in that firm’s project (i.e., closer to his or her location on the circle). Each bank can carve out a local monopoly market by offering credit at a price that entices entrepreneurs to start firms, borrow, and pursue their projects. With more aggressive pricing, the bank can try to lure borrowers away from a neighboring bank, igniting competition. Hence the local monopoly power of a bank can always be softened by another bank’s competitive entry.

When a bank lowers its lending rate to exactly match its neighbor’s, the local monopoly markets of the two banks just touch. At this price of credit, the bank observes a kink in its residual demand curve for loans. Charging any higher lending rate shrinks the bank’s local market, which the neighbor pays no attention to. Charging any lower lending rate expands the bank’s market into the neighbor’s territory. The amount the bank must offer as a price concession to get new customers doubles when it switches from a local monopolist to a competitor, which generates the
Figure 1: Commercial and Industrial Loans since the Crisis, All Commercial Banks

(a) C&I Loan Spread

(b) Delinquency, Nonperforming, Charge-off Rates

Notes: The C&I loan spread is the difference between the weighted-average effective annual loan rate on all commercial and industrial loans and the effective federal funds rate. Weights are by loan amount. The delinquency rate is the fraction of total C&I loans that are delinquent. Delinquent loans are those past due 30 days or more and still accruing interest, as well as those in nonaccrual status. The nonperforming rate is the fraction of total C&I loans that are nonperforming. Nonperforming loans are those that bank managers classify as 90 days or more past due or nonaccrual. The charge-off rate is the value of C&I loans removed from the books and charged against loss reserves divided by the total value of C&I loans. Charge-off rates are annualized, net of recoveries. Data are quarterly.

Sources: Board of Governors of the Federal Reserve System (C&I loan spread, delinquency rate, charge-off rate). Federal Financial Institutions Examinations Council (nonperforming rate). Data retrieved from FRED, Federal Reserve Bank of St. Louis.
Figure 2: Concentration in the C&I Loan Market since the Crisis

(a) Annual Percent Change in the Number of Banks with at Least 10% C&I Lending

<table>
<thead>
<tr>
<th>Year</th>
<th>Change in Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>2003</td>
<td>-3.2</td>
</tr>
<tr>
<td>2004</td>
<td>-3.0</td>
</tr>
<tr>
<td>2005</td>
<td>-2.6</td>
</tr>
<tr>
<td>2006</td>
<td>-1.3</td>
</tr>
<tr>
<td>2007</td>
<td>-1.4</td>
</tr>
<tr>
<td>2008</td>
<td>-4.7</td>
</tr>
<tr>
<td>2009</td>
<td>-6.0</td>
</tr>
<tr>
<td>2010</td>
<td>-7.4</td>
</tr>
<tr>
<td>2011</td>
<td>-3.1</td>
</tr>
</tbody>
</table>

(b) Number of Banks Constituting 50% of C&I Loan Market

Notes: Panel (a) depicts the annual percent change in the number of banks with at least 10% of their loan portfolio consisting of C&I lending. Panel (b) depicts the minimum number of banks required each year to amass 50% of the total market for C&I loans.

Source: Berger et al.

kink. The kink plays a key role in the analysis, because a bank that operates there does not adjust its lending rate to small changes in its marginal cost of financing.

This price rigidity at the kink is the reason for an obstructed interest rate pass-through. A central implication of the model is that all banks in the credit market collectively settle at the kink if aggregate bank equity capital is sufficiently depleted. A severe drop in net worth tightens the
Notes: The C&I spread is the same as in Figure 1(a). The GZ spread from Gilchrist and Zakrajšek (2012) is an unweighted cross-sectional average of the spreads between the yields of senior unsecured bonds of a sample of U.S. non-financial firms and synthetic risk-free securities that match the cash flows of those bonds. Each time series is normalized to equal its value in 2008Q2. The dashed lines start from those values and extend to the end of the two series in the figure.


capital requirement, raises the cost of equity, forces bank consolidation, and transitions the loan market to the kink in equilibrium. While there, each surviving bank maintains a local monopoly over a distinct segment of the loan market. As long as equity capital positions stay impaired, no bank finds it optimal to deviate its price of credit away from the kink and trigger competition, because doing so would further damage profits. Instead, banks act as if they tacitly collude to keep their lending rates fixed. Efforts by the central bank to get banks to pass through a low policy rate fail.

When the economy is dynamic, the central bank’s attempts to lower bank lending rates will be persistently thwarted in downturns because bank equity will be procyclical. A financial accelerator effect emerges in which an initial shock to bank equity propagates through time
because banks affect physical capital production. When banks have consolidated and turned into local monopolists following an aggregate shock that pushes them to the kink, their larger scale and weaker specialization hurt their ability to retrieve physical capital.

What ensues is a lower physical capital stock and lower output, which reduce bank net worth the following period. The banking sector persists at the kink where the price of firm credit is high and pass-through is obstructed. That initial shock to bank equity permeates the real economy through time. Indeed, if the lending market is pushed to the kink, monetary transmission can remain permanently shut off unless banks can recapitalize.

In summary, this paper provides a framework for studying how aggregate bank equity and the industrial organization of the banking sector affect the real economy and monetary policy. A novelty of the analysis is that bank net worth becomes an indicator for the degree of competition in the bank credit market and the effectiveness of the interest rate channel. Poor health of bank balance sheets leads the economy to persist in an equilibrium where efforts by a central bank to lower the cost of firm credit are repeatedly frustrated.

Literature

This paper combines the insights of several strands of literature to uniquely tie aggregate bank net worth to bank competition, monetary policy, and the real economy.

The first strand is the broad body of work exploring the effects of financial frictions on the macroeconomy. The papers most closely related to mine are Bernanke and Gertler (1989), Holmstrom and Tirole (1997), and Gertler and Kiyotaki (2010).

As in Bernanke and Gertler (1989), frictions in the financial market waste productive physical capital resources and affect aggregate output. A key distinction here is banks. Their efficiency at recovering value from defaulted projects influences the size of the physical capital stock.

Net worth in the banking sector plays a major role in the model, as in Gertler and Kiyotaki (2010). Here, the novelty is that net worth influences the degree of competition among banks and the effectiveness of monetary policy.

Holmstrom and Tirole (1997) analyze the effect of changes to the supply of intermediated financial capital on investment and credit spreads. A difference here is that the market for intermediated financial capital (bank loans) is imperfectly competitive.

A second strand is on bank diversification and specialization. Diamond (1984) stresses the benefits of bank diversification in lowering the uncertainty in a bank loan portfolio and shrinking costs of delegated monitoring. I adopt that insight.

Winton (1999) explores some hazards of bank diversification. Expansion into new industries may reduce the effectiveness of loan monitoring and increase the chance of bank failure. Acharya et al. (2006) find that a less diversified bank loan portfolio is associated with higher return on

Banks in the model specialize in different segments of the credit market rather than fully diversify because of the increasing costs of liquidating failed projects. The convex cost of diversification I use is similar to that in Gârleanu et al. (2015), who work in a setting of investor portfolios and asset pricing, rather than one with banks and competition.

A third strand is the industrial organization of banking. Berger et al. (2004) provides a survey. Matutes and Vives (1996) present a model in which banks rival each other in deposits. Whether a bank is a local monopolist or a competitor depends on the perceptions of that bank’s likelihood of failure. I fix depositor beliefs (banks cannot fail) to emphasize how bank net worth alters bank lending competition.

Matutes and Vives (2000) analyze how imperfect competition affects bank portfolio choice and whether deposit regulation intensifies or weakens risk-taking. Loans in my model all carry the same risk, so I can focus on the choice of pass-through rather than the choice of bank portfolio risk.

This paper owes a large debt to Salop (1979), whose structure of monopolistic competition on a circle I adopt. Other papers have also used the Salop framework to explore a variety of issues in banking:

Besanko and Thakor (1992) present a spatial model in which banks differentiate in loans and deposits and study the welfare implications of relaxed barriers to entry. To focus on bank lending, I have banks competing only in the credit market.

Chiappori et al. (1995) study the effects of deposit regulation on bank lending rates. In their model, the interest rate channel can also be hampered, but only when deposit rates are capped and deposits are bundled with credit services; otherwise, full transmission occurs. The deposit rate in my model is unregulated to put attention on bank lending.

Sussman and Zeira (1995) look at financial development across U.S. states and present a macroeconomic model in which costs of intermediation increase with the distance between the borrower and the bank. As in my model, greater bank specialization enhances physical capital production. Their economy displays no persistence.

Hauswald and Marquez (2006) feature bank-screening technology whose signal quality declines with the borrower’s distance from the bank. Their focus is on banks strategically screening borrowers to carve out different segments of the loan market and soften competition. In my model, all borrowers are identical prior to obtaining a loan, and my focus is on aggregate bank equity and the effects of bank competition on monetary policy and the real economy.

The final strand is the empirical and theoretical work on interest rate pass-through and its
relation to banks. One of the most robust empirical findings on impediments to interest rate pass-through is increased bank concentration (Cottarelli and Kourelis (1994); Borio and Fritz (1995); Mojon (2000); Sørensen and Werner (2006); van Leuvensteijn et al. (2008); Gigineishvili (2011)), which occurs in the model. Early work by Hannan and Berger (1991) and Neumark and Sharpe (1992) find a similar relation in deposit rates, as does Drechsler et al. (2017) in more recent work. Aristei and Gallo (2014) and Hristov et al. (2014) provide evidence that pass-through deteriorated in the Euro area during the financial crisis. Recently, Scharfstein and Sunderam (2016) find that higher mortgage lender concentration reduces the pass-through of declines in RMBS yields to mortgage rates.

Models of interest rate pass-through in the banking sector typically assign market power to banks, but they treat incomplete pass-through using either sticky prices (Hülsewig et al. (2009)) or menu costs (Hannan and Berger (1991); Cottarelli and Kourelis (1994); Scharler (2008); Hülsewig et al. (2009); Gerali et al. (2010); Günter (2011)). I micro-found the pass-through impairment from the kink in the demand curve for bank credit. The kink arises endogenously from the competitive market structure among banks.

2 Single Period Model

To reveal in a simple way the central mechanism of the model that links the industrial organization of the banking system and the state of bank equity capital to the effectiveness of monetary transmission, I distill the economy to a single period. In the next section, I make the economy dynamic.

2.1 Economy

The model setting is on a circle. Lining the circle are technologies called “projects” that initiate at the start of the period. In the dynamic economy, projects will create physical capital for the production of output; here, they simply return a random value of output for consumption at the end of the period. Banks finance these projects with loan contracts, and retain market power over borrowers, but engage in monopolistic competition à la Salop (1979). Banks will specialize in different segments of the loan market and finance themselves with deposits and equity.

2.2 Projects

Firms operate projects, and a project and a firm are synonymous in the model. A continuum of projects are uniformly distributed around the circle. I normalize the circumference of the circle to one, so projects take up unit measure. A project is identified by its unique location \( j \in [0, 1) \) on the circle. I interpret projects on different parts of the circle as belonging to different geographic areas or industries.
A project is indivisible and risky. All projects take a single unit of a homogeneous good as investment to initiate and can produce one of two possible returns as output at the end of the period: high and low. Let \( X_j \) denote the return of the project located at position \( j \) on the circle.

Denote the high return on the project \( \bar{\kappa} \) and the low return \( \kappa \). The returns are arranged

\[ 0 < \kappa < 1 < \bar{\kappa}, \]

making the low return a strict loss on investment (a failure), and the high return a strict gain (a success).

The random returns on projects are identically distributed and pairwise uncorrelated. For simplicity, I set the probability of success for each project to be \( \frac{1}{2} \). The distributional assumptions imply a weak law of large numbers (see Uhlig (1996)). Hence, there is zero aggregate risk in the economy, making total output always equal to \( \int_0^1 X_j \, dj = \frac{1}{2} (\bar{\kappa} + \kappa) \). In this economy, banks do not affect aggregate output; in the dynamic one, they will.

### 2.3 Entrepreneurs

Entrepreneurs are endowed with a project and an outside option worth \( w \) in utility. They are risk neutral and lack any wealth. An entrepreneur decides either to choose the outside option or start a firm, pursue the project, and consume the net return. Projects are non-tradeable and non-transferable between entrepreneurs. From the absence of personal wealth, entrepreneurs must rely entirely on bank financing to initiate their projects. Banks issue take-it-or-leave-it loan offers to entrepreneurs. When an entrepreneur borrows, the project is posted as collateral. If the entrepreneur defaults, control rights of the project transfer to the bank, and the entrepreneur receives nothing.

Entrepreneurs are identified by the locations of their projects \( j \in [0, 1) \). Consider an entrepreneur positioned at location \( j \) who thinks about taking out a loan from a bank at location \( i \) on the circle. Let \( R^i_L \) be the gross lending rate bank \( i \) charges on the loan. The expected utility of entrepreneur \( j \) who undertakes the project and borrows from bank \( i \) is

\[ U^{e}_{j,i} = \frac{1}{2} (\bar{\kappa} - R^i_L) - c|i - j|. \]  

Prior to financing, the entrepreneur expects the project to succeed and to repay the loan half the time. In the other half, he or she expects the project to fail, to default on the bank loan, and receive nothing. If the project is successful, the entrepreneur will receive the expected high return \( \bar{\kappa} \). The entrepreneur must also repay the bank loan with interest: \( R^i_L \).

Entrepreneurs have preference to borrow from a bank “nearby.” This preference for proximity is represented by the term \(-c|i - j|\), where \( c \) captures the strength of the preference and \(|i - j|\) is
the shortest arc length between $i$ and $j$. There are many interpretations of the proximity preference. An entrepreneur could prefer a closer bank because the bankers there speak the same language, or demand less paperwork, or because the bank has a reputation for specifically lending to the industry or area the entrepreneur’s project is in.

In fact, banks will end up specializing in different segments of the loan market. The preference for minimizing the distance will incline an entrepreneur to borrow from a bank with expertise in that entrepreneur’s industry or geographic area. Paravisini et al. (2015) provide empirical evidence that firms that export have a greater likelihood of borrowing from a bank that specializes in that firm’s exporting country.

2.4 Banks

The ability to evaluate projects involves expertise in writing financial contracts with entrepreneurs, collecting payments, and liquidating projects in the event of non-payment. Banks are unique in the economy in having this ability. Therefore, banks alone finance projects, and they do so with loan contracts.

The reason control rights transfer to the bank and the entrepreneur receives nothing in default can be justified by a variety of agency reasons. One is a costly state verification assumption similar to Townsend (1979) in which the entrepreneur alone observes the project return; a debt contract compels the entrepreneur to report the outcome of the project truthfully. In default, a project’s productivity after output is zero, so the bank’s only recourse is to liquidate the project and recover as much as possible. I assume banks can recover the full value of the project, namely $\kappa$.

I restrict a bank to only finance projects that are positioned along arcs centered at the bank’s home location (headquarters). Let $\Delta^i \in [0, 1]$ denote the arc length of the projects financed by a bank that is headquartered at position $i$ on the circle. With each project requiring a single unit of financing, the size of the total loan portfolio of bank $i$ is then $\Delta^i$. This length can also be considered the “size” of the bank. A visual depiction of a bank $i$'s portfolio is given in Figure 4.

A bank’s loan portfolio $\Delta^i$ signifies the class of industries or geographic areas to which it lends. Successfully extending credit to a specific set of industries or locations requires expertise in those markets. I interpret the arc length $\Delta^i$ to be an indicator of a bank’s expertise or specialization. A greater arc length means a greater “distance” between the implicit bankers who oversee individual loans and bank headquarters.

Banks know entrepreneurs have a preference for proximity, and they are aware of the preference structure $(-c|i - j|)$. However, the location of an entrepreneur is unobservable to any bank at the time the loan is contracted. Also at that time, all projects share identical potential returns per unit of investment and bear the same risk. For these reasons, a bank does not engage in any degree of price discrimination, but instead posts a single lending rate $R^i_L$, while taking
Figure 4: Bank Loan Portfolio Representation

Notes: Projects are uniformly distributed around the circle. Bank $i$ is headquartered at the bottom dot. The bank’s loan portfolio size $\Delta^i$ is the length of the arc centered at bank $i$’s headquarters. The three remaining dots represent other banks in the loan market.

into account the rates of all other banks on the circle. Entrepreneurs will self-select into banks according to their preferences and the posted lending rates. In Appendix B.2, I show that the results of the model go through even if banks can price discriminate.

Finally, banks must pay a fixed cost $f$ to enter and operate in the commercial loan market (i.e., costs of chartering, complying with regulations, building the organizational form, etc.). Market power without a fixed cost would encourage an unlimited number of banks to enter the lending market. I assume this fixed cost is capitalized into the assets of the bank, making the total assets of a typical bank $\Delta^i + f$.

2.5 Bank capital structure

Banks finance themselves using deposits and outside equity capital. The weak law of large numbers that applied to the entire circle also applies to individual bank portfolios. Any incremental loan portfolio is fully diversified and will earn an amount with certainty. Half of the entrepreneurs will succeed and repay the bank, while half will fail and the bank will recover the low output. The certainty in the bank’s loan portfolio returns implies that a bank could finance itself entirely using risk-free deposits. Bank diversification that shrinks bank portfolio risk is similar in spirit to the benefits of bank diversification identified in Diamond (1984).

Banks face an equity capital requirement. The requirement forces banks to issue a fraction $\lambda$
of their assets in equity. Letting $e^i$ denote the equity capital of bank $i$, the constraint is

$$e^i \geq \lambda (\Delta^i + f).$$

(2)

In Appendix B.1, I provide a micro-foundation for a minimum equity capital requirement.

Banks issue equity from the public equity capital market, but the supply of equity is limited to a fixed amount $E$. The reasoning for the limited supply is that only a set of outside sophisticated investors have the capacity to invest in banks and their capital is bounded. The supply of depositors is deep, On the other hand, so banks can issue deposits without limit.

The riskless rate on deposits is fixed at $R$, while the return on equity $R_E$ is endogenous and determined by equilibrium in the equity market. The riskless interest rate is the sole monetary policy tool in the model. A central bank adjusts $R$ to change the cost of bank debt financing in order to influence the price of bank credit to firms.

### 2.6 Bank decision

A typical bank $i$ chooses a lending rate $R^i_L$ and amount of equity capital $e^i$ to maximize profits. In doing so, it perfectly knows and takes as given (1) the demand curve for bank credit (described below), (2) the lending rates of other banks, (3) the number of banks $N$ on the circle, and (4) and the costs of debt and equity capital, $R$ and $R_E$, respectively.

Let $FC^i$ denote the financing function of the bank, representing the costs from repaying depositors and equity holders. Profits of the typical bank are

$$\Pi^i = \frac{1}{2} R^i_L \Delta^i + \frac{1}{2} R_E \Delta^i - FC^i.$$

(3)

As mentioned earlier, the bank knows that half the entrepreneurs who borrow will repay and half will default. The first term in (3) represents payments received from the fraction of projects that succeed. The second term is the proceeds from the fraction that fail. In this case, the bank recovers the low output.

The financing cost function $FC^i$ consists of the payments to depositors and equity holders. The function is

$$FC^i = R (\Delta^i + f - e^i) + R_E e^i,$$

(4)

The bank requires an amount $\Delta^i + f$ in financing. It will raise an amount $e^i$ in equity capital and finance the rest using deposits. The bank maximizes (3) subject to (2).

### 2.7 Demand curve for bank credit

The purpose of this section is to construct the demand curve for bank credit from the perspective of a typical bank. In doing so, I describe the industrial organization of the banking sector.
Entrepreneurs choose which bank to finance a project in order to maximize utility presented in (1). For an entrepreneur to borrow from a bank and undertake the project at all, the expected return from the project must exceed the outside option $w$. Because of a preference to be “close,” as measured by the distance cost $c$, an entrepreneur will always choose between three alternatives: the outside option, a loan from the bank to his or her “left” on the circle, and a loan from the bank to his or her “right.”

Because I will focus on symmetric equilibria, I derive the demand curve for a typical bank $i$ assuming (1) $N$ banks operate on the circle, located a distance $\frac{1}{N}$ from each other, and (2) all other banks on the circle charge the same fixed lending rate $R_L$, whereas bank $i$ decides on its lending rate $R^i_L$. The demand curve for credit from the typical bank will consist of a monopoly, competitive, and kinked component.

**Monopoly**

The monopoly portion of bank $i$’s demand curve consists of the set of lending rates the bank can charge and face no competition from its neighboring banks.

To begin, if bank $i$ sets $R^i_L > \kappa - 2w$, no entrepreneur on the circle would find it worthwhile to borrow from the bank. The price of the bank’s loan would be so high that even the entrepreneur located at the bank’s headquarters would rather work or borrow from a neighboring bank. This can be seen from the utility function in (1).

As bank $i$ lowers $R^i_L$, however, it will start attracting entrepreneurs whose surplus from the project exceeds the outside option value. Denote by $x$ the distance from the bank’s headquarters such that the entrepreneur located at that distance has a surplus from the project equaling the value from working. The entrepreneur’s surplus consists of the expected net return on the project $\frac{1}{2} (\kappa - R^i_L)$ less the distance cost $cx$. Formally, $x$ satisfies

$$\frac{1}{2} (\kappa - R^i_L) - cx = w.$$ 

The entrepreneur at distance $x$ is indifferent between managing the project and working. Solving for $x$ yields

$$x = \frac{\frac{1}{2} (\kappa - R^i_L) - w}{c}.$$ 

A typical bank will fund projects on either side of it, so the monopoly demand function for bank $i$, denoted $\Delta^{i,M}$, is

$$\Delta^{i,M} = \frac{\frac{1}{2} (\kappa - R^i_L) - w}{c/2}. \quad (5)$$

This quantity defines the potential local monopoly market of the typical bank. The monopoly
demand function is increasing in the high output return $\bar{\pi}$. It is declining in the lending rate $R^i_L$, the outside option value $w$, and distance cost $c$. Although a bank faces no competition from other banks in its local monopoly market, the bank implicitly competes with the outside option of entrepreneurs.

**Competitive**

The competitive part of bank $i$'s demand curve consists of the set of lending rates that would expand bank $i$'s loan portfolio into the lending market of a neighboring bank, igniting competition between the two.

If an entrepreneur is choosing between two banks, it must mean the expected return on the project exceeds the outside option value. The entrepreneur will borrow from the bank offering the lower financing and distance cost. Because the neighbor is located $\frac{1}{N}$ distance away on the circle, a typical bank will capture all projects within a distance $x$ satisfying

$$\frac{1}{2} R^i_L + cx \leq \frac{1}{2} R_L + c \left( \frac{1}{N} - x \right).$$

The entrepreneur who is indifferent between the two competing banks is at a distance $x'$ that satisfies the above relation with equality. Solving for $x'$ gives

$$x' = \frac{1}{2} \left( R_L - R^i_L \right) + \frac{c}{N}.$$  

Because the typical bank competes against the two neighbors on either side, its competitive demand function $\Delta^{i,C} = 2x'$, making

$$\Delta^{i,C} = \frac{1}{2} \left( R_L - R^i_L \right) + \frac{c}{N}. \quad (6)$$

Bank $i$'s competitive credit market shrinks the more its lending rate exceeds the rates of the neighbors. Additionally, the more banks on the circle, the closer every entrepreneur is to a potential bank, which narrows the competitive market of any one bank.\(^1\)

The size of bank $i$'s competitive lending market is determined by the marginal entrepreneur who is just indifferent between borrowing from bank $i$ and borrowing from the bank’s neighbors. Entrepreneurs located closer to bank $i$ will strictly prefer borrowing from it. Conversely, entrepreneurs located outside bank $i$'s competitive market strictly prefer borrowing from the competing neighbor. For these reasons, each entrepreneur will prefer funding the project using a

\(^1\)When competing with neighboring banks, bank $i$ could reduce $R^i_L$ enough to capture even those entrepreneurs residing at the neighbors' locations. Such a pricing strategy would drive the neighboring banks out of the market, and create a jump discontinuity in the demand curve for bank $i$'s credit. This predatory pricing would require posting a lending rate below marginal cost, which would necessarily lose money, so can be ruled out in equilibrium.
single bank. Petersen and Rajan (1994) document that small U.S. firms tend to concentrate their bank borrowing from one source. I assume the marginal entrepreneur flips a fair coin, and picks the bank according to the result.

**Kinked**

When bank \( i \) reduces its lending rate to exactly match the neighboring rate \( R_L \), its local monopoly market will just touch the monopoly markets of its two neighbors, and a kinked market arises. I denote this kinked lending rate \( R_{i,K}^L \).

The kinked market gets its name from the kink in the demand curve at the lending rate \( R_{i,K}^L \). If bank \( i \) set its lending rate just above \( R_{i,K}^L \), its local monopoly market would be segregated from that of its neighbor. The bank would lend according to the monopoly demand function in (5). The slope of the corresponding monopoly demand curve is \( \frac{dR_{i,M}^L}{d\Delta_i} = -c \). Alternatively, if the bank set a lending rate just below \( R_{i,K}^L \), its local monopoly market would cross the markets of the two neighboring banks, which sets off competition. Bank \( i \) would collect demand according to the competitive demand function in (6). The slope of the corresponding competitive demand curve is \( \frac{dR_{i,C}^L}{d\Delta_i} = -2c \). The difference in the slopes of the monopoly and competitive portions generates the kink in the demand for bank loans.

The slope of the competitive portion is twice that of the monopoly portion for the following reason. When the typical bank is a local monopolist that seeks to expand its market \( \Delta_i \) by an increment, it must offer a price concession in the amount \( c \) in order to entice the marginal entrepreneur to borrow from a more distant, less specialized bank.

But when the typical bank tries to expand in a competitive market, it must offer the same price concession as before plus an additional amount \( c \) because the marginal entrepreneur is now closer to a neighboring bank that is more specialized in the entrepreneur’s industry. The extra concession is meant to lure the entrepreneur away from the competition.

The kink in the demand curve for bank credit is a key feature of the lending market and critical for the main results. In the theory of kinked demand curves, prices under oligopoly may “stick” around a focal price. That price is sustainable in equilibrium out of each firm’s belief that undercutting will trigger a price war, but charging more leads no other firm to follow. The demand curve an individual firm faces will have a kink at the focal price.

The same economic reasoning applies here. If a bank reduced its lending rate below \( R_{i,K}^L \), it would expand its segment of the loan market into the territories of the neighboring banks, sparking competition and hurting profits. Alternatively, raising the lending rate simply reduces the breadth of that bank’s local monopoly market, which neighboring banks can safely ignore.

---

The novelty here is that aggregate net worth in the banking sector will determine whether banks settle at that focal price.

Figure (5) illustrates bank $i$'s demand curve for loans, stitching together the monopoly, kinked, and competitive lending markets.

Figure 5: Demand Curve for Bank Credit from a Typical Bank $i$.

\[ R^i_L \]

2.8 Equilibrium

I study a symmetric, pure-strategy, zero-profit, Nash equilibria. The equilibrium is characterized by the tuple $\mathcal{E} \equiv \{ R_L, N, R_E \}$, where $R_L$ is the single lending rate charged by all banks, $N$ is the positive-integer number of equally spaced banks on the circle, and $R_E$ is the required expected return on equity. The tuple is determined so that (1) every bank’s choice of lending rate $R_L$ is profit-maximizing, (2) this choice of lending rate earns zero expected profits, (3) the circle contains no gaps ($\Delta^i = \frac{1}{N}, \forall i$), and (4) the bank equity market clears.

The market for bank credit will be characterized by monopolistic competition, as in Chamberlin (1933), Robinson (1969), and Salop (1979). Entrepreneur preference for proximity will be a source of differentiation among banks that gives them market power in loan pricing, even when competing with one another to fund projects. Banks perfectly compete for deposits and bank equity.

I assume there is a limited supply of aggregate bank equity capital $E$ so that $R_E \geq R$. Because the cost of equity $R_E$ is at least as high as the cost of debt $R$, banks will issue the minimum equity capital necessary to meet their constraint. Hence, the equity capital requirement in (2) will hold.
Banks will finance the rest of their operations ($\Delta^i + f$) using deposits.

**Competitive and kinked equilibria**

Three types of equilibria are possible in the economy: monopoly, kinked, and competitive. These types correspond to the three parts of the demand curve for bank credit.

A convenient way to visualize the equilibrium of the economy is to plot the average revenue and average cost curves of banks, given a price $R_E$ that clears the equity market. The point of tangency between the two curves indicates the equilibrium. Tangency ensures all $N$ banks in the loan market jointly earn zero expected profits at the profit-maximizing lending rate $R_L$.

The average revenue curve is a simple affine transformation of the demand curve for bank loans. The part of the average revenue curve at which the average cost curve lies tangent indicates the equilibrium as monopoly, competitive, or kinked. If the average cost curve touches the kink in the average revenue curve, the first-order condition of the bank’s problem will hold as a strict inequality.

I focus on kinked and competitive equilibria rather than monopoly. The monopoly equilibrium does not add much to the main results, and the economy can be in a monopoly equilibrium at only one point on the average revenue curve.³ For these reasons, I ignore it.

Dividing the bank profit function in (18) by $\Delta^i$ gives the average revenue and average cost functions. To simplify notation, I define the weighted average cost of capital to be $R_\lambda = (1 - \lambda) R + \lambda R_E$. The average revenue and average cost functions $AR(\Delta^i)$ and $AC(\Delta^i)$, respectively, are then:

\[
AR(\Delta^i) = \frac{1}{2} \left( R^i_L + \kappa \right), \\
AC(\Delta^i) = R_\lambda \left( 1 + \frac{f}{\Delta^i} \right).
\]  

³The equilibrium requirement that $\Delta^i = \frac{1}{N}$ for all $i$ implies local monopoly markets will just touch in a monopoly equilibrium. However, a kinked equilibrium also features local monopoly markets just touching. The only point of tangency in a monopoly equilibrium, therefore, is located immediately before the kink (approaching from the left) in the average revenue curve.

Here, the lending rate $R^i_L$ is the inverse demand function for loans and a function of $\Delta^i$. An illustration of a kinked and competitive equilibrium is presented in Figure 6.

The average cost curve is downward sloping and convex because of the fixed cost of bank entry $f$. In both equilibria, banks specialize their lending over non-overlapping segments of the credit market. Local bank markets cannot overlap since entrepreneurs will only select a single bank in equilibrium given their preferences. In the kinked case, monopoly markets just touch and competition is threatened, whereas in the competitive case, competitive markets just touch and competition is active.
Figure 6: Kinked and Competitive Equilibria

Notes: The equilibrium lending rate $R_L$ and loan portfolio arc length $\Delta$ are determined at the point where the average revenue curve and average cost curve are tangent (competitive) or just touch (kinked). Kinked average revenue and average cost curves are solid; competitive are dashed.

A simple way to distinguish the two types of equilibria is to consider a deviation by a bank thinking to raise its lending rate. In a kinked equilibrium, if a bank were to raise the rate, its customers would elect to work rather than borrow. In a competitive equilibrium, its customers would still borrow, but from the neighboring bank. A positive deviation in the kinked equilibrium kicks entrepreneurs out of the credit market; in the competitive equilibrium, it relinquishes them to a competitor.

Market clearing

The condition of no gaps on the circle ensures the market for bank credit clears: the aggregate demand for project financial capital (1) will match the aggregate supply of bank loans $(N \times \frac{1}{N})$ at the equilibrium lending rate $R_L$.

The cost of bank equity $R_E$ is pinned down by market clearing in the equity market. Supply of equity is fixed at $E$, while the aggregate demand for equity is the sum of the individual minimum equity capital requirements in (2). Formally:

$$E = \lambda \left(1 + fN\right).$$  (9)
Banks are required to issue in equity a fraction of the amount to finance all projects on the circle plus the aggregate fixed costs of entry.

2.9 Monetary transmission

This section presents the results on interest rate pass-through for the single period model. In Proposition 1, I present the bank lending rates in both the competitive and kinked equilibria. The superscripts in the proposition signify different values of the endogenous objects (the number of banks $N$ and cost of equity $R_E$) across the two equilibria.

**Lemma 1. (Lending rates) The bank lending rate in a competitive equilibrium is**

$$R_L^C = 2R_C^C - \kappa + \frac{2c}{N_C},$$  \hspace{1cm} (10)

where the weighted average cost of capital $R_C^C = (1 - \lambda) R + \lambda R_E^C$.

The kinked equilibrium lending rate is

$$R_L^K = \bar{\kappa} - 2w - \frac{c}{N_K}.$$  \hspace{1cm} (11)

**Proof.** See Appendix A.1

**Perfect pass-through**

Consider first the competitive lending rate. The competitive lending rate reflects the marginal cost of doing business in the loan market plus a mark-up from bank market power. The first term is the marginal cost of financing passed onto entrepreneurs. The second term is the marginal recover value from a project in default. The more a bank can get from liquidation in the low state, the less it can charge the entrepreneur in the high state. These first two terms represent the cost of borrowing if entrepreneurs picked banks solely on price with no preference for those nearby ($c = 0$). In that case, perfect competition would drive banks to charge exactly the marginal cost of extending credit.

When banks can differentiate themselves by distance ($c > 0$), they charge a markup over marginal cost, which is the last term in (10). The slope of the competitive demand curve and the number of banks determine the size of this markup. The costlier it is for entrepreneurs to contract with banks at a distance from their industries or areas (large $c$), the faster their demand for loans from remote banks drops off. The imperfect competition allows banks to exploit this feature of the demand curve by charging a larger markup against those entrepreneurs who pick them. More banks competing in the market for loans (large $N_C$) lowers individual market power and shrinks...
Regarding monetary policy, suppose the supply of bank equity capital were so flush that $R_E$ in equilibrium declined to its lower bound of $R$. The economy is in a competitive equilibrium and the weighted average cost of capital would be $R_\lambda = R$. Monetary transmission in this case would be perfect: variation in the policy rate would pass-through fully to bank lending rates. Competition is the reason: if a bank observes its cost of funding drop, and does not lower its lending rate, it would lose customers to a neighboring bank. To remain in business, the bank must pass through any changes in the interest rate to its lending rate.

Now suppose the bank lending market is still competitive, but the supply of bank equity is lower, creating a positive wedge between the cost of equity and debt ($R_E > R$). The policy rate enters the bank lending rate linearly via the weighted average cost of capital

$$R_\lambda = (1 - \lambda) R + \lambda R_E.$$  

Also suppose the number of banks $N^c$ is fixed so that the cost of equity $R_E$ is fixed. This setting can be considered the “shorter run” when there are no changes to the industrial organization of the banking sector. In this case, interest rate pass-through again is perfect: changes to the policy rate pass-through fully to bank lending rates for the same reason.

It will turn out that over the “longer run”, when banks can enter or exit, monetary policy will be neutral. Any change to the policy rate will be met by an offsetting change to the cost of equity $R_E$ in equilibrium. The weighted average cost of capital $R_\lambda$ will stay unmoved and monetary policy will have no effect on the economy. The threat of entry or exit alone adjusts the cost of equity after a change to the policy rate, even though no bank actually enters or exits in equilibrium.

I provide an extension in Appendix B.1 that features the presence of monetary pass-through in the competitive equilibrium even in the longer run. That economy also displays the peculiar behavior of negative pass-through as part of a kinked equilibrium, in which banks raise lending rates in the longer run after a rate cut, making it contractionary.

**No pass-through**

In a kinked equilibrium, banks operate off the kink in the demand curve for loans, which means their profit-optimality condition will hold as a strict inequality. It does not determine the equilibrium lending rate. The lending rate instead is taken off the monopoly portion of the demand curve for bank credit, given by (11).

Banks in a kinked equilibrium are local monopolists in segmented industries of the credit market in which they specialize. They compete against the outside option of entrepreneurs rather than with each other. As seen in (11), a bank charges a higher lending rate if a project yields a higher successful output $\kappa$. The bank exploits the attraction to pursuing a project. On the other hand, the bank cuts back on the lending rate if the outside option $w$ is worth more. To entice an
entrepreneur to take out a loan, the bank has to lower the lending rate.

The kink in the demand curve generates a jump discontinuity in the marginal revenue curve of a bank. For this reason, if banks operate at the kink in equilibrium, small changes in the cost of providing a loan do not affect the cost of obtaining a loan. No component of the marginal financing cost function of the bank enters (11).

This result is critical to understanding why monetary policy is ineffective if the economy is in a kinked equilibrium. Any changes in the policy rate has no direct impact on the lending rate. Competition is missing to compel banks to adjust their loan rates after changes to their cost of funding. Banks act as if they tacitly collude to keep prices fixed. Each bank knows every other bank will not deviate from the kinked lending rate $R^K_L$. So no bank does. There is no transmission. Pass-through is absent in the kinked equilibrium even in the shorter run when the number of banks is held fixed.

The kink generates a sharp prediction of zero pass-through. Generally, a region of a demand curve that features higher concavity leads a monopolist to pass through less of any changes to marginal cost. The kink creates a sharp concavity in the demand curve for loans. In Appendix B.3, I provide one way of “smoothing” the kink and show that the limited monetary pass-through is preserved.

**Number of banks**

Banks will enter the circle until it is no longer profitable to do so. A condition of zero expected profits will determine the number of banks in equilibrium. A way to represent this condition is to equate the average revenue and average cost functions from (7) and (8), substitute the lending rate from either (10) or (11), and impose the no-gap condition $(\Delta^i = \frac{1}{N}, \forall i)$. Doing so gives

$$\frac{1}{2} (R_L(N) + \kappa) = R_\lambda (1 + fN). \quad (12)$$

The number of banks that operate in the lending market must be a positive integer. There is no guarantee, however, that the solution to (12) is an integer. Therefore, the equilibrium number of banks will be the largest previous integer to the solution of (12). If there is a difference between the largest previous integer and the solution of the zero profit condition, then banks on the circle earn positive profits in equilibrium. The next bank to enter, however, would earn negative profits, preventing the entry.

If there are multiple positive solutions to (12), the unique equilibrium number of banks will be the largest one. The reason is the following. Pick two adjacent positive solutions $N_1 < N_2$. Both positive roots signify an equilibrium number of banks such that profits are zero. If profits are zero for both a smaller number of banks $N_1$ and a larger number of banks $N_2$, it must be that positive profits can be made for some number of banks $N_1 < N < N_2$. Why else would banks enter when
Lemma 2 presents the equilibrium number of banks for the competitive and kinked case.

**Lemma 2.** (Entry) The number of banks in a competitive equilibrium is

\[ N^C = \sqrt{\frac{c}{f R^C}}. \]  

(13)

Assuming \( \frac{1}{2} (\kappa + \kappa) - (w + R^K) \geq \sqrt{2c f R^K} \), the number of banks in a kinked equilibrium is

\[ N^k = \frac{1}{2} (\kappa + \kappa) - (w + R^K) + \sqrt{\left[ \frac{1}{2} (\kappa + \kappa) - (w + R^K) \right]^2 - 2 c f R^K}. \]  

(14)

**Proof.** See Appendix A.2.

In the competitive case, an increase in either the weighted average cost of capital \( R^C \) or the fixed cost of entry \( f \) reduces the number of banks in the loan market. A rise in the distance cost, \( c \), on the other hand, increases differentiation among banks (entrepreneurs would prefer a closer, more specialized bank), which leads more banks to enter.

In the kinked case, a rise in either the marginal (\( R^K \)) or fixed (\( f \)) cost of extending a loan decreases the number of banks. However, a rise in the distance cost \( c \) leads to exit, opposite of the competitive case. The reason is from the effect on the kinked lending rate in (11): a lower distance cost reduces the price of credit, which reduces profit margins and curtails entry. Recall that banks in the kinked equilibrium are local monopolists. If entrepreneurs strongly prefer to contract with a close bank, then banks must reduce their lending rate in order to draw the entrepreneur into borrowing rather than pursuing the outside option.\(^4\)

### 2.10 Bank Net Worth

In this section, I discuss the equilibrium in the bank equity market and demonstrate how bank equity determines the kind of equilibrium in the credit market.

\[^4\text{The intuition behind the assumption } \frac{1}{2} (\kappa + \kappa) - (w + R^K) \geq \sqrt{2c f R^K} \text{ for the number of banks in the kinked equilibrium is the following. It turns out that the number of banks in a monopoly equilibrium is } N^M = \sqrt{\frac{c}{f R^K}}. \text{ If the loan market is a pure monopoly, in the sense that it can support only a single bank } N^M = 1 \text{ in equilibrium, it must be that } c/2 = f R^K. \text{ The marginal entrepreneur in this pure monopoly market pays a distance cost } c/2, \text{ which here would equal the fixed cost of entry gross of the bank financial capital cost } f R^K. \text{ Substituting this equality into the assumption of the Lemma leads it to be } \frac{1}{2} (\kappa + \kappa) - (w + c + R^K) \geq 0. \text{ This inequality states that the assumption for the existence of a kinked equilibrium is that the social net present value of the marginal project under a pure monopoly is positive.} \]
Equity capital market equilibrium again is given by

\[ E = \lambda (1 + fN(R_E)). \]

Substituting the number of banks from the competitive and kinked equilibria in Lemma 2 delivers the cost of equity capital \( R_E \) in each equilibrium, which are presented in Lemma 3.

**Lemma 3. (Cost of equity)** The cost of equity capital in a competitive equilibrium is

\[
R_E^C = \frac{cf\lambda}{(E - \lambda)^2} - \left( \frac{1 - \lambda}{\lambda} \right) R, \tag{15}
\]

while the cost of equity in a kinked equilibrium is

\[
R_E^K = \frac{1}{2} \left( \frac{\kappa + \kappa - 2w}{E} - \frac{\lambda fc}{E(E - \lambda)} \right) - \left( \frac{1 - \lambda}{\lambda} \right) R. \tag{16}
\]

In both equilibria, a decline in aggregate bank net worth raises the cost of equity capital so that \( \frac{\partial R_E}{\partial E} < 0. \)

**Proof.** See Appendix A.3. \( \square \)

A drop in the aggregate supply of equity capital \( E \) has the direct effect in both cases of increasing the cost of equity across banks. Higher operating costs in turn force banks to consolidate or exit the lending market. I show below that the amount of equity capital will influence both the industrial organization of the banking sector and the effectiveness of monetary transmission.

One can see in both (15) and (16) the offsetting effect of changes to the policy rate \( R \) on the cost of equity, making monetary policy neutral for both equilibrium in the longer run after accounting for bank entry and exit.

**Determining the equilibrium**

Sufficient changes in bank net worth \( E \) transition the economy between equilibria. Indeed, a sudden drop in bank equity positions can move the credit market away from competition. All banks in this situation shrink their loan portfolio, each exploits its local monopoly power over entrepreneurs, and all settle at the focal kinked lending rate. No bank thinks any other will deviate in pricing, so all act as if they tacitly agree to refrain from competing with one another.

Figure 7 presents an illustration of how a large enough decline in net worth raises the cost of equity \( R_E \) and the average cost curve of banks. The rise in bank funding costs transitions the credit market from a competitive to kinked equilibrium.
Notes: The economy starts in a competitive equilibrium with cost of equity $R_E$, represented by the dotted lines. A drop in aggregate bank net worth $E$ raises the cost of equity to $\bar{R}_E$. The average cost curve increases, leads to bank consolidation, and pushes the economy to a kinked equilibrium. The kinked equilibrium is represented by the solid lines.

The equilibrium of an economy is determined by a set of conditions that I explain in detail in Appendix A.4. Broadly speaking, the economy is in a kinked equilibrium if an entrepreneur at the edge of a bank’s market is indifferent between the outside option and borrowing at the equilibrium lending rate. The economy is in a competitive equilibrium if an entrepreneur at the edge of the market strictly prefers borrowing to the outside alternative.

The conditions that determine the equilibrium of the economy are expressible as conditions on two quadratic polynomials in bank net worth $E$. Proposition 1 presents those conditions.

**Proposition 1.** (Determining the equilibrium) Let $\eta_1$ and $\eta_2$ be the two positive roots of quadratic polynomials $H_1(E)$ and $H_2(E)$ that are defined over aggregate bank equity $E$. The economy is in a competitive equilibrium when $E > \eta_2$; the equilibrium is kinked when $E \in (\eta_1, \eta_2)$.

**Proof.** See Appendix A.4.

Figure 8 illustrates the two quadratic polynomials $H_1$ and $H_2$ over the domain $E > 0$. Values of bank net worth for which $H_2$ is positive (shaded in blue) constitute a competitive equilibrium, whereas values of net worth for which $H_2$ is negative and $H_1$ is positive (shaded in red) constitute...
a kinked equilibrium. Economies with values of $E$ outside the blue or red regions feature no equilibrium.

Figure 8: Bank Net Worth Determining the Equilibrium

Notes: The values of aggregate bank net worth $E$ for which $E > \eta_2$ (represented by the blue shaded area) constitute a competitive equilibrium. The values of bank net worth for which $E \in (\eta_1, \eta_2)$ (represented by the red shaded area) constitute a kinked equilibrium.

When bank net worth is high, the cost of equity is low and banks actively compete in the lending market. Following a severe enough drop in net worth $E$, the banking sector becomes impaired as the cost of equity spikes, and the economy shifts into the red kinked region. Here, banks constrict their loan portfolios, refrain from competing, and close the interest rate channel. If equity capital positions improve ($E$ increases), banks have incentive to break their fixed pricing, expand their loan breadth, enter the markets of other banks, and resume competition. The interest rate channel then opens.

2.11 Relation to the C&I Loan Market

Figure 8 can help us better understand the behavior of the C&I loan market during and after the financial crisis. Severe real estate losses impaired bank balance sheets and forced bank consolidation. The resulting sharp increase in bank concentration in the C&I loan market and reduction in net worth is in line with the bank credit market transitioning to the red region of a kinked equilibrium in the figure. At this stage, the Federal Reserve was limited in transmitting a low policy rate to bank lending rates. Hence the persistently high C&I loan spread despite declining measures of C&I
loan default risk. Meanwhile, banks with low equity positions constricted their C&I lending across regions, consistent with banks retreating to smaller, more local lending in a kinked equilibrium. Over time, as banks shored up their equity capital positions, they resumed more aggressive competition, which would correspond to the credit market moving into the blue, competitive equilibrium region. There the Federal Reserve would have more success in monetary transmission. The C&I loan spread gradually fell, moving more in tandem with the default-risk measures.

3 Dynamic Model

The dynamic model involves the economy now advancing through discrete time on the circle. Banks will continue engaging in monopolistic competition as before, but their net worth will persist over time. This persistence will generate extended periods of frustrated monetary transmission after a large negative shock to bank net worth, similar to what was observed in the C&I loan market. A closed interest rate channel could be permanent unless bank can recapitalize.

3.1 Production

Still lining the circle each period are industrial projects, but now they produce physical capital that contribute to the production of output. Investments in projects in period \( t \) produce physical capital that becomes available for use in \( t + 1 \). The main section features a single-unit scale for investment and fully depreciating physical capital each period. In Appendix B.4, I allow the scale of investment to vary. In that case, aggregate investment is influenced by bank competition and net worth.

At time \( t \), total output, denoted \( Y_t \), is produced by a perfectly competitive, representative firm. Output is produced using physical capital in linear production technology:

\[
Y_t = A_t K_t.
\]

Physical capital \( K_t \) is inherited from the previous period’s projects. The random productivity shock \( A_t \) is continuously distributed over a finite positive support \([A, \bar{A}]\), has mean \( \bar{A} \), and is i.i.d. over time. It is the only source of aggregate uncertainty in the economy.

3.2 Banks

Banks again finance projects along arc lengths \( \Delta i_t \) centered at their headquarters. After the project returns, banks collect the promised repayments from entrepreneurs with successful projects. They also liquidate failed projects, which involves selling the distressed assets in the physical capital market. After all payments are collected and sales are made, each bank repays its depositors and issues a liquidating dividend to its equity holders. Bank equity is therefore all externally raised with no retained earnings. In the next period, a fresh crop of banks enter the credit market.
**Liquidation costs**

I introduce one new friction that has the capacity by itself to generate cyclical persistence despite i.i.d. aggregate shocks, single unit investment, and fully depreciating physical capital. The friction limits how much a bank can recover from loans in default. In the event of borrower default, the bank recovers less than the full amount of low physical capital return $\kappa$, making liquidation costly.

I assume the total liquidation costs of a bank, denoted $g(\Delta^i_t)$, are increasing in a bank’s loan portfolio $\Delta^i_t$ at an increasing rate. I take the function $g$ to be

$$g(\Delta^i_t) = \gamma (\Delta^i_t)^2,$$

with $\gamma > 0$. Because liquidation costs are convex in $\Delta^i_t$, the bank suffers a diseconomy of scale.\(^5\)

Part of a bank’s expertise is intimate knowledge of the “soft” information about the industries or locations it lends to that is not easily communicated to others. This information includes the organizational structures of the firms, common production processes, the local customer markets, and importantly, the second-best use of the physical capital assets. Convex liquidation costs is a reduced-form representation of the economic reasoning in Stein (2002): bank lending that relies on soft information weakens the research incentives of the line managers in a large, hierarchical bank. These weakened incentives make bankers less capable of liquidating distressed assets at their full value.

**Bank decision**

Period-by-period, banks face the same credit demand curve constructed in the single period model and visualized in Figure 5. Their decision problem is identical as the one posed in the single period model, save for the consideration of the liquidation costs. Expected profits of the typical bank at time $t$ are

$$\Pi^i_t = \frac{1}{2} R^i_{L,t} \Delta^i_t + \frac{1}{2} (\kappa \Delta^i_t - g(\Delta^i_t)) - FC^i_t.$$  \hspace{1cm} (18)

The financing cost function $FC^i_t = R (\Delta^i_t + f - e^i_t) + R_{E,t} e^i_t$. The bank maximizes (18) subject to the minimum equity capital requirement $e^i_t \geq \lambda (\Delta^i_t + f)$.

**3.3 Equilibrium**

The dynamic equilibrium is defined as a sequence of the single-period Nash equilibria defined in the previous section. A single-period equilibrium is characterized by the tuple $\mathcal{E}_t \equiv \{ R_{L,t}, N_t, R_{E,t} \}$; the sequence of them will be linked together via bank net worth. I study only competitive and kinked equilibria again.

\(^5\)Having the liquidation costs be a function of the distance between the exact failed project and the bank headquarters greatly complicates the aggregation of physical capital. Instead, I make the liquidation costs a function of bank size, which is related to distance, because a larger bank will be farther from its average borrower.
Two markets require clearing: the market for bank credit and equity capital. No gaps on the circle ensures the first clears, just as in the single period model. The equity market clearing condition in the dynamic economy is slightly different than before.

I assume the supply of bank equity capital is a fixed fraction $\phi$ of total output $Y_t$. The equity market clearing condition is then

$$\phi Y_t = \lambda (1 + f N_t). \quad (19)$$

Tying the supply of equity to total output will make bank net worth procyclical. The point is to capture the idea that banks have weaker balance sheets in bad times. In a downturn, the equity position of banks will deteriorate, putting upward pressure on the cost of equity capital $R_{E,t}$.

### 3.4 Monetary transmission

Equilibrium lending rates and entry into the bank credit market for the competitive and kinked cases are virtually the same as in the single period model. The only adjustment is accounting for the quadratic liquidation costs. I present the results in Lemma 4.

**Lemma 4.** *(Lending rates, entry, and cost of equity, dynamic economy)* The bank lending rate in a competitive equilibrium is

$$R_{L,t}^C = 2R_{\lambda,t}^C - \kappa + \frac{2(\gamma + c)}{N_t^C}, \quad (20)$$

where the weighted average cost of capital $R_{\lambda,t}^C = (1 - \lambda) R + \lambda R_{E,t}^C$.

The kinked equilibrium lending rate is

$$R_{L,t}^K = \kappa - 2w - \frac{c}{N_t^K}. \quad (21)$$

The number of banks in a competitive equilibrium is

$$N_t^C = \sqrt{\frac{\gamma/2 + c}{f R_{\lambda,t}^C}}. \quad (22)$$

Assuming $\frac{1}{2}(\kappa + \kappa) - (w + R_{\lambda,t}^K) \geq \sqrt{2(\gamma + c)} f R_{\lambda,t}^K$, the number of banks in a kinked equilibrium is

$$N_t^K = \frac{1}{2}(\kappa + \kappa) - (w + R_{\lambda,t}^K) + \sqrt{\left[\frac{1}{2}(\kappa + \kappa) - (w + R_{\lambda,t}^K)\right]^2 - 2(\gamma + c) f R_{\lambda,t}^K}. \quad (23)$$
The cost of equity capital in a competitive equilibrium is

\[ R_{E,t}^C = \frac{f \lambda (\gamma/2 + c)}{(E_t^C - \lambda)^2} - \left( \frac{1 - \lambda}{\lambda} \right) R, \]  

while in a kinked equilibrium the cost of equity is

\[ R_{E,t}^K = \frac{1}{2} \left( \frac{1}{E_t^K} \left( \frac{\kappa + \kappa - 2w}{E_t^K (E_t^K - \lambda)} \right) - \frac{f \lambda (\gamma + c)}{E_t^K (E_t^K - \lambda)} \right) - \left( \frac{1 - \lambda}{\lambda} \right) R. \]  

**Proof.** Method of proof is identical to that of Lemmas 1, 2, and 3. \( \square \)

The results are akin to the single period case, but here featuring the marginal liquidation cost \( \gamma \). Higher \( \gamma \) increases the cost of extending loans, so banks pass on that higher marginal cost to their borrowers in a competitive equilibrium via a higher lending rate \( R_{L,t} \). The kinked equilibrium lending rate is unaffected. Again there is no monetary pass-through in the kinked equilibrium.

Quadratic liquidation costs create a source of differentiation among banks: banks that are closer to their average borrower are more efficient at liquidating distressed assets than those that are farther away. This differentiation is the reason why higher \( \gamma \) increases entry \( N_t \) in the competitive case, just as a higher proximity preference \( c \) does. The opposite is true in the kinked case, where higher \( \gamma \) shuns entry, because higher marginal liquidation costs eat away at potential profits because they do not pass through to lending rates.

Finally, the similarity of \( R_{E,t} \) to the single period counterpart reveals that declines in bank net worth also raise the cost of equity in the dynamic economy, forcing bank exit or consolidation.

### 3.5 Dynamics

In this section, I derive the dynamical system representing the equilibrium of the economy. The state variable of the economy is aggregate bank net worth \( E_t \). The economy will display non-trivial dynamics despite i.i.d. aggregate shocks, unit investment, and fully depreciating physical capital. Appendix B.4 provides an extension where dynamics also arise from time-varying investment.

**Physical capital evolution**

Total physical capital in the economy aggregates the high and low production from all projects. A successful project contributes its full quantity produced \( \bar{\kappa} \) to the physical capital stock. If a project produces a low return, the project liquidation costs lead physical capital recovery to be less than \( \bar{\kappa} \).

Aggregate project outcomes feature no uncertainty. At the end of each period, exactly half the projects will succeed and half will fail. Therefore, the aggregate quantity of physical capital produced and available for the next period will be known. For any period \( t \), the next period
physical capital stock $K_{t+1}$ in equilibrium is given by

$$K_{t+1} = \frac{1}{2} (\kappa + \kappa) - \frac{\gamma}{N_t}. \tag{26}$$

The first term of (26) comes from the real side of the economy. Half the projects will generate the high physical capital return $\kappa$ and half will generate the low return $\kappa$. The second term comes from the banking sector. It reflects the loss in the quantity of physical capital produced from the liquidation of failed projects.

A single bank will lose an amount $\gamma/N_t^2$ as a dead weight loss from all the projects in its portfolio that fail. In aggregate, a total of $N_t$ banks will lose this amount and a half the projects will fail. So the amount $\gamma/N_t^2$ will be lost from the physical capital stock across the banks in the credit market. A greater number of banks in the economy means each will lend to a more specialized set of industries, making the banking sector as a whole more efficient at recovering the low physical capital return from projects in default. Aggregate physical capital production will be higher, as well as output.

The capital formation equation in (26) also reveals how shocks to bank equity propagate through time. A negative shock to productivity $A_t$ will lower bank equity $E_t$, which will raise the cost of equity $R_{E,t}$ in either kind of equilibria. A high cost of funding will lower potential profits in the credit market and restrict the number of banks $N_t$.

In turn, fewer banks will worsen bank efficiency in the production of the physical capital stock that is usable the following period. A lower capital stock $K_{t+1}$ will lower output and aggregate bank equity $E_{t+1}$ the next period. So too will the cost of equity $R_{E,t+1}$ and the number of banks $N_{t+1}$ be affected, and so on and so forth in future periods. An initial contraction in bank equity persistently influences the evolution of the economy. And should the contraction be large enough to shift the economy to a kinked equilibrium, monetary transmission can be disrupted over prolonged periods.

**Bank net worth**

Substituting the equilibrium number of banks $N_t$ and cost of equity $R_{E,t}$ into (26) generates a dynamical system in aggregate bank equity $E_t$ that defines the state of the economy. As in the single period economy, the value of bank equity positioned in one or the other of two intervals determines the type of equilibrium.

The crucial distinction in the dynamic economy is the effects of monetary policy will persist through time. A negative shock to bank equity $E_t$, for instance, can push the banking sector into the kinked region where it can remain for multiple periods, disrupting monetary transmission for as long. Proposition 2 presents the results.
Proposition 2. (Dynamical system) Let $\delta_1$ and $\delta_2$ be the two positive roots of quadratic polynomials $D_1(E)$ and $D_2(E)$ that are defined over aggregate bank equity $E_t$. The economy is in a competitive equilibrium when $E_t > \delta_2$; the equilibrium is kinked when $E_t \in (\delta_1, \delta_2)$.

When the economy is in a competitive equilibrium, the evolution equation for bank equity is

$$E_{t+1} = \frac{1}{2} \phi A_{t+1} \left( \kappa + \kappa - \frac{\gamma f \lambda}{E_t - \lambda} \right).$$  \hspace{1cm} (27)

In a kinked equilibrium, the evolution equation is

$$E_{t+1} = \frac{1}{2} \phi A_{t+1} \left( \kappa + \kappa - \frac{\gamma}{N^K(E_t)} \right),$$  \hspace{1cm} (28)

where $N^K(E_t)$ is an explicit function for the number of banks in a kinked equilibrium.

Proof. See Appendix A.5. \hfill \Box

From Lemma 4, a negative shock to bank equity $E_t$ raises the cost of equity capital $R_{E,t}$ and reduces the number of banks $N_t$ in the credit market across both types of equilibria. Therefore, low equity in period $t$ predicts low equity the next period $t + 1$, generating positive persistence.

The economy evolves in the following way: given bank net worth $E_t$, if $E_t \geq \delta_2$ the equilibrium is competitive, so $E_{t+1}$ is determined by (27). Otherwise, if $E_t \in (\delta_1, \delta_2)$, the equilibrium is kinked, so $E_{t+1}$ is determined by (28).

With $E_{t+1}$ then pinned down, the economy in period $t + 1$ will be either in a competitive or kinked equilibrium depending on the relation between $E_{t+1}$ and the intervals defined by $\delta_1$ and $\delta_2$. The type of equilibrium will then determine the lending rate $R_{L,t+1}$ and the number of banks $N_{t+1}$, which will then influence the capital stock, output, and bank net worth the following period in $t + 2$.

The dynamical system involves two evolution equations in which one is employed depending on the state. When the shocks $\{A_t\}_{t \in \mathbb{N}}$ are shut off and the system is allowed to evolve from an initial bank net worth $E_0$, it turns out that a stable steady state exists in both types of equilibria under some conditions. Proposition 3 explains.

Proposition 3. (Steady states) Suppose $\frac{\phi}{4} (\kappa + \kappa) - \frac{\lambda}{2} > 0$. The competitive system in (27) has a unique, unstable steady state $E_{ss,0}$ if $\frac{\phi}{4} (\kappa + \kappa) - \frac{\lambda}{2} = \sqrt{\frac{1}{2} \phi \lambda \gamma f}$. If $\frac{\phi}{4} (\kappa + \kappa) - \frac{\lambda}{2} > \sqrt{\frac{1}{2} \phi \lambda \gamma f}$, the system has two steady states $E_{ss,1} < E_{ss,0} < E_{ss,2}$, with $E_{ss,1}$ unstable and $E_{ss,2}$ stable. If neither condition holds, the competitive system has no steady state.

The kinked system in (28) potentially has multiple steady states. When $\Gamma(\theta) \in (-1, 1)$, where
\( \Gamma \) is an explicit function of the parameters \( \theta \) of the model that is defined in Appendix A.6, at least one steady state exists and it is stable.

Proof. See Appendix A.6.

Both dynamical systems may feature stable steady states, though the characteristics of two separate economies that settle at each one are quite distinct. An economy settling at a competitive steady state is characterized by an open interest rate channel, lower spreads on bank firm credit, and robust competition with more banks in the credit market. An economy that settles at the kinked steady state features the exact opposite. Indeed, the existence of a stable kinked steady state implies an economy can get permanently trapped at the kink. Firm credit spreads are high, banks resist competition, and monetary policy is ineffective. Only if banks recapitalize could the economy push itself out of that equilibrium toward a competitive steady state.

4 Conclusion

This paper presents a model in which the industrial organization of the bank credit market affects the real economy and monetary policy. A driving force of that competitive structure is the net worth of banks. A sufficient drop in aggregate bank equity transitions the economy to an equilibrium where banks consolidate for survival, retreat to local monopoly markets, and act as if they tacitly collude not to compete. A wide commercial loan spread and impaired interest rate pass-through can persist, lowering physical capital production and output.

An important contribution of this paper is to tie bank net worth to the degree of competition and specialization in the banking sector, as well as to the functioning of the interest rate channel of monetary policy. Lower bank net worth raises the cost of equity and encourages local monopolies. This development obstructs the interest rate channel.

The number of banks \( N_t \) in the commercial credit market plays a significant role in the economy. The number proxies for the efficiency of the banking sector at recovering distressed physical capital. This efficiency is one source of cyclicality. One potential criticism is that the number of banks in the U.S. economy is not cyclical. Since the mid-1980s, the number of commercial banks has steadily fallen from around 14,000 to 5,000.\(^6\) This secular decline corresponds to the trend of bank consolidation that began with deregulation of interstate banking.

In the model, the number of banks is isomorphic to the specialization \( \Delta_t \) of bank lending. Changes to \( N_t \) can equivalently be interpreted as changes to the degree of specialization in the banking sector. Acharya et al. (2006) and Berger et al. document significant variation in bank specialization across industries in the cross section and through time. Paravisini et al. (2015) does the same for specialization across exporting countries.

\(^6\)Source: Federal Financial Institutions Examination Council.
Data available: https://fred.stlouisfed.org/series/USNUM
Regarding welfare, spatial models of firm competition with unit demand typically display excess entry in the decentralized equilibrium relative to the social optimum (see Vickrey (1964); Salop (1979); Anderson et al. (1992); Matsumura and Okamura (2006); Gu and Wenzel (2009)). Banks decide to enter based on the marginal borrower, while welfare is determined using the average one. Whether there is excess entry depends on the difference between the average borrower surplus and the surplus of the marginal borrower relative to the fixed cost of entry.

A novelty here is that a lot of banks has its benefits in physical capital production due to the evolution equation for physical capital in (26). More banks increases specialization in lending, and as a result, the physical capital stock, output, and consumption increase. Determining whether the equilibrium number of banks is too few, too many, or optimal will require a formal welfare analysis.

Regarding government policy, common tools to resolve a financial crisis are (1) reducing borrowing costs, (2) providing equity capital, and (3) purchasing distressed assets. In the model, the quality of assets does not enter the pricing decisions of banks, so purchasing assets will not resume transmission. Reducing borrowing costs is the same as lowering the interest rate. If the banking sector is in the kinked equilibrium, there will still be no pass-through despite accommodative monetary policy.

Providing equity will lower the cost of equity for banks and encourage the lending market to transition from a kinked to a competitive equilibrium and open the interest rate channel. Therefore, this tool seems particularly effective at resuming transmission after a negative shock to the equity of the banking sector.

Although the economic mechanism differs because of the element of imperfect competition, the model in this paper relates to the literature on intermediary asset pricing (e.g., He and Krishnamurthy (2013); Brunnermeier and Sannikov (2014)) in the following ways: (1) scarcity of bank equity makes the spread of risky assets (loans) over safe assets (deposits) rise after negative shocks to bank equity positions; (2) this elevated spread can persist as long as bank equity does not improve; and (3) equity capital injections are especially effective at reducing the risky asset spread. Key novelties here are that (1) the model can generate a stickiness in the spread despite improvements in equity positions, as long as banks continue to refrain from competing; and (2) the model features entry into the lending market, whereas the models in the intermediary asset pricing literature commonly do not.

In the model, banks enter or exit within the period. Typically bank exit (consolidation) occurs more rapidly than bank entry. If banks in the economy could only enter with delay—as in, after a certain number of periods—but could exit immediately, then existing banks in the lending market could earn positive profits until entry by competing banks. Allowing this asymmetry in entry and exit would allow banks to capture retained earnings and recapitalize their balance sheets. This
kind of “self-healing” is missing from the model, but such a process would hasten a transition away from the kink that would re-open the interest rate channel.

The effects of the industrial organization of banks on the real economy is a topic ripe for future research. Recent empirical work has explored the issue. Paravisini et al. (2015) find credit-supply shocks to a specialized bank have a disproportionate effect on exports to that bank’s country of expertise. Some unresolved questions are: What makes specialized bank debt difficult to replace? What distinguishes relationship skills from industry expertise? How does the failure of a specialized bank distort the allocation of credit? Much remains to be studied.
References


Berger, Philip G., Michael Minnis, and Andrew Sutherland. Commercial lending concentration and bank expertise: Evidence from borrower financial statements. *forthcoming Journal of Accounting and Economics*. [5], [8], [33]


Demsetz, Rebecca S. and Philip E. Strahan. Diversification, size, and risk at bank holding companies. *Journal of Money, Credit and Banking*, 29(3):300–313, August 1997. [57]


Salop, Steven C. Monopolistic competition with outside goods. *The Bell Journal of Economics*, 10(1):141–156, Spring 1979. [8], [9], [17], [34]


Townsend, Robert M. Optimal contracts and competitive markets with costly state verification. 


Leuvensteijn, Michielvan, Christoffer Kok Sørensen, Jacob Antoon Bikker, and Adrian A.R.J.M. Rixtelvan. Impact of bank competition on the interest rate pass-through in the euro area. Tjalling C. Koopmans Research Institute, Discussion Paper Series 08-08, March 2008. [9]


Winton, Andrew. Don’t put all your eggs in one basket? diversification and specialization in lending. September 1999. [7]
A Appendix A: Proofs (For Online Publication)

A.1 Proof of Lemma 1

In a competitive equilibrium, the first order condition for optimality using the profit function from (3) and (4) is

\[ \frac{1}{2} (R_L + \kappa) + \frac{1}{2} \Delta \left( \frac{dR_L}{d\Delta} \right) = R_C^\lambda. \]

Substituting the slope of the competitive demand curve \( \frac{dR_L}{d\Delta} = -2c \) and using the equilibrium condition \( \Delta = \frac{1}{N\sigma} \) gives the competitive equilibrium lending rate

\[ R_C^L = 2R_C^\lambda - \kappa + \frac{2c}{N\sigma}. \]

In a kinked equilibrium, the first order condition for optimality does not hold with equality, so the lending rate is instead the monopoly demand curve. Solving (5) for the lending rate and setting \( \Delta = \frac{1}{N\sigma} \) gives (11).

A.2 Proof of Lemma 2

In both the competitive and kinked cases, the number of banks in equilibrium is determined by a zero expected profit condition when the lending rate is set at its optimum and \( \Delta = \frac{1}{N} \). In the competitive case the expected profit function is

\[ \Pi (N) = \frac{1}{2} \left( R_C^L + \kappa \right) \frac{1}{N} - R_C^\lambda \left( \frac{1}{N} + f \right) \]

\[ = \frac{1}{2} \left( 2R_C^\lambda - \kappa + \frac{2c}{N} + \kappa \right) \left( \frac{1}{N} \right) - R_C^\lambda \left( \frac{1}{N} + f \right). \]

Setting expected profit function to zero and solving for \( N \) gives

\[ N^C = \sqrt{\frac{c}{f R_C^\lambda}}. \]

In the kinked case, the equilibrium lending rate is \( R^K_L = \bar{\kappa} - 2w - \frac{\kappa}{N} \). Substituting this lending rate into the profit function, setting the function to zero, and solving for \( N \) gives

\[ N^K = \frac{1}{2} \left( \bar{\kappa} + \kappa \right) - \left( w + R^K_L \right) + \sqrt{\left[ \frac{1}{2} \left( \bar{\kappa} + \kappa \right) - \left( w + R^K_L \right) \right]^2 - 2cf R^K_L}, \]

which is real-valued under the assumption of the Lemma. There are two roots to the profit function in the kinked case. The larger one is the only equilibrium number of banks by the reason given in
A.3 Proof of Lemma 3

The equilibrium cost of equity is found by inserting the number of banks given in Lemma 2 into the equity market clearing condition in (9).

In the competitive case, doing so yields

\[ E = \lambda \left( 1 + f N^C \right) = \lambda \left( 1 + f \sqrt{\frac{c}{f R^C}} \right) \]

Solving for \( R^C_E \) gives

\[ R^C_E = c f \lambda \left( \frac{E}{E - \lambda} \right)^2 - \left( \frac{1 - \lambda}{\lambda} \right) R. \]

A drop in the aggregate supply of equity \( E \) clearly increases the cost of equity.

In the kinked case, substituting the equilibrium number of banks into the market clearing condition yields the nonlinear relation

\[ \frac{E - \lambda}{f \lambda} = \frac{1}{2} \left( \bar{\kappa} + \kappa \right) - (w + R\lambda) + \sqrt{\left[ \frac{1}{2} \left( \bar{\kappa} + \kappa \right) - (w + R\lambda) \right]^2 - 2 fc R\lambda} \]

Solving for the weighted average cost of capital \( R\lambda \) gives

\[ R\lambda = \frac{\lambda}{2} \left[ \frac{(E - \lambda) \left( \bar{\kappa} + \kappa - 2w \right) - \lambda fc}{E (E - \lambda)} \right], \]

and then separating the cost of equity capital yields

\[ R_E = \frac{1}{2} \left[ \frac{\bar{\kappa} + \kappa - 2w}{E} - \frac{\lambda fc}{E (E - \lambda)} \right] - \left( \frac{1 - \lambda}{\lambda} \right) R. \]

Taking the first derivative with respect to \( E \) gives

\[ \frac{\partial R_E}{\partial E} = - \frac{(\bar{\kappa} + \kappa - 2w) (E - \lambda)^2 + cf \lambda (2E - \lambda)}{2E^2 (E - \lambda)^2}. \]

The derivative is negative provided the numerator is negative. The comparative static is negative provided

\[ - (\bar{\kappa} + \kappa - 2w) (E - \lambda)^2 + cf \lambda (2E - \lambda) < 0. \]

The expression on the left-hand side of the inequality above can be written as a quadratic polynomial
in $E$. Let $J$ denote that polynomial:

$$J(E) = -E^2(\kappa + \kappa - 2w) + 2E\lambda(\kappa + \kappa - 2w + cf) - \lambda^2(\kappa + \kappa - 2w + cf).$$

Because the parabola $J$ opens down and $J(0) < 0$, $J(E)$ is everywhere negative. Hence $\frac{\partial R^f}{\partial E} < 0$.

### A.4 Proof of Proposition 1

I present the conditions that separate equilibria. Those conditions are expressible as two quadratic polynomials defined over aggregate bank equity $E > 0$. The positive roots of these polynomials will then define intervals that separate equilibria.

Given a parameter vector $\theta$, a necessary and sufficient condition for a monopoly equilibrium is that the optimal lending rate $R^M_L$ is on the monopoly demand curve. Mathematically that condition is

$$R^M_L = \kappa - 2w - \frac{c}{N^M}. \tag{29}$$

A necessary and sufficient condition for a competitive equilibrium is that the competitive lending rate is below the monopoly demand curve, so that

$$R^C_L \leq \kappa - 2w - \frac{c}{N^C}. \tag{30}$$

Finally, for a kinked equilibrium, it must be that expected monopoly profits, when the monopoly lending rate is replaced with the monopoly demand curve, as in (29), are positive. This condition distinguishes a kinked from monopoly equilibrium. Mathematically the condition is

$$\Pi|_{R^M_L = \kappa - 2w - \frac{c}{N^M}} > 0, \tag{31}$$

where $\Pi$ is the bank profit function in (18), and the vertical line represents “conditional on.” The inequality in (31), however, is necessary, but not sufficient, for a kinked equilibrium. The inequality also holds under a competitive equilibrium. Therefore, a necessary and sufficient condition for a kinked equilibrium is (31) and the failure of (30). If the conditions fail for all equilibria, then no equilibrium exists under the parameter vector $\theta$.

To establish the conditions for the monopoly equilibrium, we require an optimal lending rate that equates to the monopoly demand curve. Since a lending rate implies a portfolio size $\Delta$, we can find the optimal $\Delta$ and then set the implied lending rate to the monopoly demand curve. The first order condition for maximizing the expected profit function $\Pi$ with respect to $\Delta$ is

$$\frac{\partial \Pi}{\partial \Delta} = \frac{1}{2} \left( R_L + \kappa - 2R^M_L - \frac{c}{2}\Delta \right) = 0,$$
where I have set $\frac{\partial R_L}{\partial \Delta} = -\frac{c}{2}$ as the slope of the monopoly demand curve. Substitute the monopoly demand curve $R_L = \kappa - 2w - c\Delta$ to get

$$\frac{\partial \Pi}{\partial \Delta} = \frac{1}{2} \left( \kappa - 2w - c\Delta + \kappa - 2R^M x - \frac{c}{2} \Delta \right) = 0.$$  

Solving for $\Delta$ gives

$$\Delta^* = \frac{1}{2} \left( \kappa + \kappa \right) - \frac{(w + R^M x)}{c}.$$  

Substitute the optimal $\Delta$ back into the profit function to obtain the value function:

$$V = \frac{1}{2c} \left[ \frac{1}{2} \left( \kappa + \kappa \right) - (w + R^M x) \right]^2 - fR^M x.$$  

Setting the value function to zero yields the condition.

$$\frac{1}{2} \left( \kappa + \kappa \right) - (w + R^M x) = \sqrt{2fcR^M x}.$$  

The monopoly equilibrium cost of equity is $R^M_E = \frac{cf\lambda}{2(E-\lambda)^2} - \left( \frac{1-\lambda}{\lambda} \right) R$. Substituting this into the condition above yields the equation

$$\left[ \frac{1}{2} \left( \kappa + \kappa \right) - w \right] (E - \lambda)^2 - cf\lambda (E - \lambda) - \frac{1}{2} cf\lambda^2 = 0. \quad (32)$$  

Let $h_1$ denote the left-hand side of this equation. The function $h_1$ is a quadratic polynomial defined over the domain $E - \lambda > 0$. Since $h_1(0) < 0$ and $\frac{1}{2} \left( \kappa + \kappa \right) - w > 0$, both roots are real. Let $\eta_1^-$ and $\eta_1^+$ denote the two roots of the $H_1$, where $\eta_1^- < \eta_1^+$. Multiplying the two roots gives

$$\eta_1^- \eta_1^+ = -\frac{\frac{1}{2} cf\lambda^2}{\frac{1}{2} \left( \kappa + \kappa \right) - w} < 0,$$

meaning one of the roots is negative while the other is positive.

Because monopoly expected profits are positive in a kinked equilibrium, one condition that defines the kinked equilibrium is $E - \lambda > \eta_1^+$. Over that region $h_1$ is positive. Let $H_1 (E)$ be be defined as a horizontal translation of $h_1 (E - \lambda)$ by $\lambda$ units. The positive root of $H_1$ is $\eta_1 = \eta_1^+ + \lambda$. Applying the quadratic formula to (32) demonstrates the positive root of $H_1$ to be

$$\eta_1 = \lambda \left( \frac{\kappa + \kappa - 2w + cf + \sqrt{cf \left( \kappa + \kappa - 2w + cf \right)}}{\kappa + \kappa - 2w} \right).$$
Therefore one condition on aggregate bank equity that defines the kinked equilibrium is

\[ E > \eta_1. \]

The condition for the competitive equilibrium can be determined by substituting the equilibrium lending rate and number of banks into (30). Doing so gives

\[ R_C^\lambda \leq \frac{1}{2} (\pi + \kappa) - w - \frac{3}{2} \sqrt{f c R_C^\lambda}. \]

Substituting the competitive equilibrium cost of equity from (15) gives the inequality

\[ \left[ \frac{1}{2} (\pi + \kappa) - w \right] (E - \lambda)^2 - \frac{3}{2} c f \lambda (E - \lambda) - c f \lambda^2 \geq 0. \]

Define \( h_2 \) as the quadratic polynomial in the left-hand side of the inequality above. Like \( h_1 \), it is defined over the positive interval \( E - \lambda > 0 \). For the same reasons as for \( h_1 \), both roots of \( h_2 \) are real, and one is negative and one is positive. Denote those roots as \( \eta_{2}^- \) and \( \eta_{2}^+ \), with \( \eta_{2}^- < \eta_{2}^+ \). Similar to before, define \( H_2 \) as the horizontal translation of \( h_2 (E - \lambda) \). The positive root of \( H_2 \) is

\[ \eta_2 = \eta_2^+ + \lambda. \]

That positive root is

\[ \eta_2 = \lambda \left( \frac{\pi + \kappa - 2w + \frac{3}{2} c f + \sqrt{2 c f (\pi + \kappa - 2w + \frac{9}{8} c f)}}{\pi + \kappa - 2w} \right). \]

Therefore, the equilibrium is competitive if \( E \geq \eta_2 \). From the definitions of \( \eta_1 \) and \( \eta_2 \), one can see that \( \eta_2 > \eta_1 \). Therefore, if the value of bank net worth \( E \in (\eta_1, \eta_2) \), then (31) is satisfied and (30) fails, making the equilibrium kinked.

### A.5 Proof of Proposition 2

The procedure for identifying the intervals that separate equilibria is the same as in the proof for Proposition A.4. The exception is substituting \( c \) for \( c + \gamma \). The positive root \( \delta_1 \) is

\[ \delta_1 = \lambda \left( \frac{\pi + \kappa - 2w + f (\gamma + c) + \sqrt{f (\gamma + c) (\pi + \kappa - 2w + f (\gamma + c))}}{\pi + \kappa - 2w} \right). \]

The quadratic \( d_1 (E_t - \lambda) \) whose positive root will delineate the competitive equilibrium is

\[ d_1 = \left[ \frac{1}{2} (\pi + \kappa) - w \right] (E_t - \lambda)^2 - \left[ f \lambda \left( \gamma + \frac{3}{2} c \right) \right] (E_t - \lambda) - f \lambda^2 \left( \frac{\gamma}{2} + c \right). \]
As in the single period economy, both roots are real and one is positive. Solving for the positive root and applying a horizontal translation by $\lambda$ units gives

$$\delta_2 = \lambda \left( \frac{\kappa + \kappa - 2w + f (\gamma + \frac{3}{2}c) + \sqrt{f^2 (\gamma + \frac{3}{2}c)^2 + f (\kappa + \kappa - 2w) (\gamma + 2c)}}{\kappa + \kappa - 2w} \right).$$

Note that $\delta_2 > \delta_1$. The economy is competitive when $E_t \geq \delta_2$, while the economy is in a kinked equilibrium when $E_t \in (\delta_1, \delta_2)$.

The dynamical system for the economy is derived as follows. Suppose the economy is competitive so that $E_t \geq \delta_2$. The equilibrium number of banks is given by $N^C_t = \sqrt{\frac{\gamma}{2 + c}}$, with the weighted average cost of capital given by

$$R^C_{\lambda,t} = f (\frac{\gamma}{2} + c) \left( \frac{\lambda}{E^C_t - \lambda} \right)^2.$$

Substitute $R^C_{\lambda,t}$ into $N^C_t$ to get the relation

$$\frac{1}{N^C_t} = \frac{f \lambda}{E^C_t - \lambda}.$$

Now substitute this expression into the physical capital evolution equation from (26) to get

$$K_{t+1} = \frac{1}{2} \left( \kappa + \kappa - \frac{\gamma f \lambda}{E_t - \lambda} \right).$$

Now use the relation $E_{t+1} = \phi A_{t+1} K_{t+1}$ to get

$$E_{t+1} = \frac{1}{2} \phi A_{t+1} \left( \kappa + \kappa - \frac{\gamma f \lambda}{E_t - \lambda} \right),$$

which matches (27).

The evolution equation for a kinked equilibrium can be expressed explicitly, but doing so loses the intuition. Both the kinked equilibrium number of banks and cost of equity are given explicitly in (23) and (25), respectively. Substituting (25) into (23) would give an explicit function, denoted $N^K (E_t)$. Inserting this function into the physical capital evolution equation from (26) and using the relation between aggregate bank equity and physical capital delivers the evolution equation for bank equity in a kinked equilibrium:

$$E_{t+1} = \frac{1}{2} \phi A_{t+1} \left( \kappa + \kappa - \frac{\gamma}{N^K (E_t)} \right).$$
A.6 Proof of Proposition 3

I solve for the competitive steady states first and then the kinked ones second.

Competitive

Where a steady state exists, I assume it is in the interval $E \geq \eta_2$, with $\eta_2$ defined in previous section. Setting both $E_{t+1} = E_t = E$ in the competitive system (27) and re-arranging terms delivers the equation

$$2E^2 - (2\lambda + \phi (\overline{\kappa} + \kappa)) E + \phi \lambda (\overline{\kappa} + \kappa + \gamma f) = 0.$$ 

Let $U(E)$ be the quadratic polynomial on the left-hand side of the above equation. Because $U(0) > 0$, $U'(0) < 0$ and the parabola of the quadratic opens upward, the quadratic has either no real roots, a unique positive real root, or two positive real roots.

The roots of the quadratic are

$$E = \frac{\phi (\overline{\kappa} + \kappa) + 2\lambda \pm \sqrt{(\phi (\overline{\kappa} + \kappa) - 2\lambda)^2 - 8\phi \gamma f \lambda}}{4}$$

$$= \frac{\phi}{4} (\overline{\kappa} + \kappa) + \frac{\lambda}{2} \pm \sqrt{\left(\frac{\phi}{4} (\overline{\kappa} + \kappa) - \frac{\lambda}{2}\right)^2 - \frac{1}{2} \phi \lambda \gamma f}.$$  \hspace{1cm} (33)

If $\left(\frac{\phi}{4} (\overline{\kappa} + \kappa) - \frac{\lambda}{2}\right)^2 - \frac{1}{2} \phi \lambda \gamma f < 0$, the quadratic has no real roots, and so the competitive system has no steady state.

If $\left(\frac{\phi}{4} (\overline{\kappa} + \kappa) - \frac{\lambda}{2}\right)^2 - \frac{1}{2} \phi \lambda \gamma f \geq 0$, the system has at least one steady state. The steady state is unique if the condition holds with equality. The unique steady state is $E_{ss,0} = \frac{\phi}{4} (\overline{\kappa} + \kappa) + \frac{\lambda}{2}$. If the inequality is strict, then the system has two steady states $E_{ss,1} < E_{ss,0} < E_{ss,2}$ given in (33).

Any steady state of the competitive system is stable if the derivative $\frac{dE_{t+1}}{dE_t} \in (0, 1)$ when it is evaluated at the steady state value. The first derivative is

$$\frac{dE_{t+1}}{dE_t} = \frac{\frac{1}{2} \phi \gamma f \lambda}{(E_t - \lambda)^2}.$$ 

At the steady state $E_{ss,0}$ the derivative is

$$\frac{dE_{t+1}}{dE_t} \bigg|_{E_t = E_{ss,0}} = \frac{\frac{1}{2} \phi \gamma f \lambda}{\left(\frac{\lambda}{2} + \frac{\phi}{4} (\overline{\kappa} + \kappa) - \lambda\right)^2} = \frac{\frac{1}{2} \phi \gamma f \lambda}{\left(\frac{\phi}{4} (\overline{\kappa} + \kappa) - \frac{\lambda}{2}\right)^2}.$$
Under the assumption of the unique steady state: \((\frac{\phi}{4} (\kappa + \kappa) - \frac{\lambda}{2})^2 = \frac{1}{2} \phi \lambda \gamma f\), the derivative equals one, and hence the steady state is unstable. When \((\frac{\phi}{4} (\kappa + \kappa) - \frac{\lambda}{2})^2 > \frac{1}{2} \phi \lambda \gamma f\), there are two steady states. Substituting the larger steady state \(E_{ss,2}\) into the derivative yields
\[
\frac{dE_{t+1}}{dE_t} \bigg|_{E_t = E_{ss,2}} = \frac{\frac{1}{2} \phi \gamma f \lambda}{\left( \frac{\phi}{4} (\kappa + \kappa) + \frac{\lambda}{2} - \sqrt{ \left( \frac{\phi}{4} (\kappa + \kappa) - \frac{\lambda}{2} \right)^2 - \frac{1}{2} \phi \lambda \gamma f - \lambda} \right)^2} = \frac{\frac{1}{2} \phi \gamma f \lambda}{\left( \frac{\phi}{4} (\kappa + \kappa) - \frac{\lambda}{2} + \sqrt{ \left( \frac{\phi}{4} (\kappa + \kappa) - \frac{\lambda}{2} \right)^2 - \frac{1}{2} \phi \lambda \gamma f} \right)^2}.
\]
The slope is less than one provided
\[
\left( \frac{\phi}{4} (\kappa + \kappa) - \frac{\lambda}{2} + \sqrt{ \left( \frac{\phi}{4} (\kappa + \kappa) - \frac{\lambda}{2} \right)^2 - \frac{1}{2} \phi \lambda \gamma f} \right)^2 > \frac{1}{2} \phi \gamma f \lambda.
\]
Expanding the left-hand side of the inequality and re-arranging terms gives
\[
\left( \frac{\phi}{4} (\kappa + \kappa) - \frac{\lambda}{2} \right)^2 - \frac{1}{2} \phi \lambda \gamma f > - \left( \frac{\phi}{4} (\kappa + \kappa) - \frac{\lambda}{2} \right) \left( \sqrt{ \left( \frac{\phi}{4} (\kappa + \kappa) - \frac{\lambda}{2} \right)^2 - \frac{1}{2} \phi \lambda \gamma f} \right),
\]
which holds by the assumption. Hence \(E_{ss,2}\) is stable.

Substituting the smaller root into the first derivative gives
\[
\frac{dE_{t+1}}{dE_t} \bigg|_{E_t = E_{ss,1}} = \frac{\frac{1}{2} \phi \gamma f \lambda}{\left( \frac{\phi}{4} (\kappa + \kappa) - \frac{\lambda}{2} - \sqrt{ \left( \frac{\phi}{4} (\kappa + \kappa) - \frac{\lambda}{2} \right)^2 - \frac{1}{2} \phi \lambda \gamma f} \right)^2}.
\]
The root is unstable if the numerator exceeds the denominator. Comparing the two after expanding the denominator gives
\[
\left( \frac{\phi}{4} (\kappa + \kappa) - \frac{\lambda}{2} \right)^2 - \frac{1}{2} \phi \gamma f \lambda < \left( \frac{\phi}{4} (\kappa + \kappa) - \frac{\lambda}{2} \right) \left( \sqrt{ \left( \frac{\phi}{4} (\kappa + \kappa) - \frac{\lambda}{2} \right)^2 - \frac{1}{2} \phi \lambda \gamma f} \right).
\]
Square both sides of the inequality to get

\[
\left(\frac{1}{2} \phi \gamma f \lambda\right)^2 < \left(\frac{1}{2} \phi \gamma f \lambda\right) \left(\frac{\phi}{4} (\bar{\kappa} + \kappa) - \frac{\lambda}{2}\right)^2
\]

\[
\frac{1}{2} \phi \gamma f \lambda < \left(\frac{\phi}{4} (\bar{\kappa} + \kappa) - \frac{\lambda}{2}\right)^2,
\]

which is the assumption. Hence the smaller root is unstable.

**Kinked**

Where a steady state exists, I assume it is in the interval \( E \in (\delta_1, \delta_2) \), with \( \delta_1 \) and \( \delta_2 \) defined in previous section. Setting both \( E_{t+1} = E_t = E \) in the kinked system (28), substituting the cost of equity \( R_{E_t}^K \) from (25) and the number of banks \( N^K_t \) from (23) and re-arranging terms delivers the equation

\[
E = \frac{\alpha_2 (E) E^2 - \alpha_1 (E) E + \alpha_0}{\beta_2 (E) E^2 - \beta_1 (E) E + \beta_0},
\]

(34)

where

\[
\alpha_2 (E) = \phi (\bar{\kappa} + \kappa) (\bar{\kappa} + \kappa - 2w + \varphi_1 (E))
\]

\[
\alpha_1 (E) = \lambda \phi \left[2 (\bar{\kappa} + \kappa)^2 + (\bar{\kappa} + \kappa) (\varphi_1 (E) - 4w) + 2f \gamma (\bar{\kappa} + \kappa - 2w)\right]
\]

\[
\alpha_0 = \phi \lambda^2 \left[2f^2 \gamma (\gamma + c) - 4wf \gamma + ((3\gamma + c) f - 2w) (\bar{\kappa} + \kappa) + (\bar{\kappa} + \kappa)^2\right]
\]

\[
\beta_2 (E) = 2 (\bar{\kappa} + \kappa - 2w + \varphi_1 (E))
\]

\[
\beta_1 (E) = 4 \lambda \left(\bar{\kappa} + \kappa - 2w + \frac{1}{2} \varphi_1 (E)\right)
\]

\[
\beta_0 = 2 \lambda^2 (\bar{\kappa} + \kappa - 2w + f (\gamma + c))
\]

\[
\varphi_1 (E) = \frac{\sqrt{\omega_4 E^4 - \omega_3 E^3 + \omega_2 E^2 - \omega_1 E + \omega_0}}{E (E - \lambda)},
\]

and where

\[
\omega_4 = (\bar{\kappa} + \kappa - 2w)^2
\]

\[
\omega_3 = 4 \lambda \left((\bar{\kappa} + \kappa - 2w)^2 + f (\gamma + c) (\bar{\kappa} + \kappa - 2w)\right)
\]

\[
\omega_2 = 2 \lambda^2 \left(3 (\bar{\kappa} + \kappa - 2w)^2 + 5f (\gamma + c) (\bar{\kappa} + \kappa - 2w) + 2 (f (\gamma + c))^2\right)
\]

\[
\omega_1 = 4 \lambda^3 \left((\bar{\kappa} + \kappa - 2w) + f (\gamma + c))^2\right)
\]

\[
\omega_0 = \lambda^4 \left((\bar{\kappa} + \kappa - 2w) + f (\gamma + c))^2\right).
\]
All the coefficients are positive. It turns out that there are up to four solutions that satisfy (34), and that also can reside within the interval \( E \in (\delta_1, \delta_2) \). Those solutions are

\[
E_{1,2} = \frac{\phi (c(\kappa + \kappa) + 2\gamma w) \pm \sqrt{\left(\phi (c(\kappa + \kappa) + 2\gamma w))^2 + 8\phi \lambda \gamma (\gamma + c) ((\kappa + \kappa - 2w) + f (\gamma + c))}\}}{4 (\gamma + c)},
\]

\[
E_{3,4} = \frac{\phi (\kappa + \kappa) + 2\lambda \pm \sqrt{\left(\phi (\kappa + \kappa) - 2\lambda)^2 - 8f \gamma \lambda \phi}\}}{4}.
\]

I will put attention on the solution

\[
E_3 = \frac{\phi}{4} (\kappa + \kappa) + \frac{\lambda}{2} + \sqrt{\left(\phi (\kappa + \kappa) - 2\lambda)^2 - 8f \gamma \lambda \phi}.
\]

I have found this solution to be the most frequent steady state value that arises from numerical analysis of the system.

Let the right-hand side of (34) be defined as

\[
\Omega (E) \equiv \frac{\alpha_2 (E) E^2 - \alpha_1 (E) E + \alpha_0}{\beta_2 (E) E^2 - \beta_1 (E) E + \beta_0}.
\]

The steady state \( E_3 \) is stable provided

\[
\Omega (E) \bigg|_{E=E_3} \in (-1, 1).
\]

Substituting the solution \( E_3 \) into \( \Omega (E) \) generates a function \( \Gamma (\theta) \) of the parameters \( \theta \). The steady state \( E_3 \) is stable if \( \Gamma (\theta) \in (-1, 1) \). With some tedious algebra, the function \( \Gamma (\theta) \) can be explicitly expressed and is given below.

\[
\Gamma (\theta) = \frac{\phi}{\sigma_3^2} \left( 128 \gamma \left( 2f \lambda^2 (\sigma_1 + f (\gamma + c)) - 2f \lambda \sigma_1 \sigma_6 \right) \left( 2\lambda \sigma_1 - \frac{1}{2} \sigma_4 \sigma_8 + 16 \lambda \sigma_5 + \frac{\sigma_2 (\sigma_8^2 - 16 \lambda \sigma_6)}{128 \sigma_5} \right) \right)
- \frac{\phi}{\sigma_3} (16f \gamma \lambda \sigma_1),
\]
where

\[ \sigma_1 = \kappa + \kappa - 2w \]
\[ \sigma_3 = 16\lambda^2 (\sigma_1 + f (\gamma + c)) + \sigma_8 (\sigma_4 \sigma_8 - 8\lambda (\sigma_1 + 8\sigma_5)) \]
\[ \sigma_4 = \sigma_1 + 16\sigma_5 \]
\[ \sigma_6 = \frac{\sigma_8}{4} \]
\[ \sigma_7 = (\sigma_8^2 - 4\lambda \sigma_8)^2 \]
\[ \sigma_8 = 2\lambda + \phi (\kappa + \kappa) + \sigma_9 \]
\[ \sigma_9 = \sqrt{(\phi (\kappa + \kappa) - 2\lambda)^2 - 8f\gamma\lambda\phi}, \]

and

\[ \sigma_5^2 \sigma_7 = \frac{\sigma_4^2 \sigma_1^2}{256} \]
\[ - \frac{\lambda\sigma_3^2 \sigma_1 (\sigma_1 + f (\gamma + c))}{16} \]
\[ + \frac{\lambda^2 \sigma_8^2 (3\sigma_1^2 + 2f^2 (\gamma + c)^2 + 5f (\gamma + c) \sigma_1)}{8} \]
\[ - 4\lambda^3 \sigma_6 (\sigma_1 + f (\gamma + c))^2 \]
\[ + \lambda^4 (\sigma_1 + f (\gamma + c))^2, \]

and

\[ \sigma_2 = \frac{t (\theta)}{\sigma_7} + \left( \frac{\sigma_8 (2\lambda (4\lambda - 3\sigma_8) + \sigma_8^2)}{4\sigma_7^2} \right) r (\theta), \]

with

\[ t (\theta) = -4\sigma_3^3 \sigma_1^2 \]
\[ + 48\lambda \sigma_3^2 \sigma_1 (\sigma_1 + f (\gamma + c)) \]
\[ - 256\lambda^2 \sigma_6 (3\sigma_1^2 + 2f^2 (\gamma + c)^2 + 5f (\gamma + c) \sigma_1) \]
\[ + 256\lambda^3 (\sigma_1 + f (\gamma + c))^2, \]
and

\[ r(\theta) = 16\sigma_8^4 \sigma_1^2 \\
- 256\lambda \sigma_8^2 \sigma_1 (\sigma_1 + f(\gamma + c)) \\
+ 512\lambda^2 \sigma_8^2 (3\sigma_1^2 + 2f^2(\gamma + c)^2 + 5f(\gamma + c)\sigma_1) \\
- 16384\lambda^3 \sigma_6 (\sigma_1 + f(\gamma + c))^2 \\
+ 4096\lambda^4 (\sigma_1 + f(\gamma + c))^2. \]
B Appendix B: Extensions (For Online Publication)

B.1 A model with an endogenous equity capital requirement

I provide a micro-foundation for a minimum equity capital requirement as a way to justify the constraint in 2. The setting features project outcomes that are now correlated. This way the size of a bank’s portfolio $\Delta^i$ will directly influence its level of diversification as well as how much equity capital it must issue. I retain the liquidation costs from the dynamic model, but the economy here exists over a single period.

An economy with this equity capital constraint will also feature monetary pass-through in a competitive equilibrium even after accounting for entry and exit. It also displays the peculiar behavior of negative pass-through in a kinked equilibrium in the longer run. In that case, banks raise their lending rates after a policy rate cut, making it contractionary.

The economy exists for a single period. Projects still produce one of two possible returns $\pi$ or $\kappa$. However, the probability that a project produces the high return now takes a special form. This form allows all projects to bear the same expected probability of success prior to financing, but different probabilities after initiation. At the beginning of the period, the probability that project $j$ on the circle reaches the high state at the end of that period is random. This random probability takes the form:

$$\tilde{Pr}(H|j, \tilde{u}) = \frac{1}{2} \left(1 + \cos \left(2\pi \left(j + \tilde{u}\right)\right)\right),$$  \hfill (35)

where $\tilde{u} \sim U[0, 1]$. The object in (35) is a random measure that maps a realization of the uniform shock $\tilde{u}$ to a probability distribution over the two states at a location $j$. I call (35) the success probability of a project.

The periodicity of the cosine function guarantees $\tilde{Pr} : [0, 1] \mapsto [0, 1]$. The shock $\tilde{u}$ is realized in the middle of the period, making the success probability measurable as of the middle of the period. Properties of a project’s success probability are presented in Lemma 5.

**Lemma 5.** The success probability in (35) satisfies the following properties:

1. *(Distributional symmetry)* The probability density function of a project’s success probability is the same at all locations.
2. *(Mean and variance)* Each project is expected to succeed half the time, with variance $\frac{1}{8}$.
3. *(Distance-dependent covariance)* The covariance between projects $j$ and $k$ in their success probabilities is $\frac{1}{8} \cos (2\pi (j - k))$.

**Proof.** See Appendix B.1.8.

The form in (35) is a way to make a project’s probability of reaching the high state invariant to its location. Prior to the realization of $\tilde{u}$, a project’s outcome distribution cannot be distinguished
from its neighbors’, because all projects bear the same uncertainty of success. As a result, projects at every location share the same expected probability of generating the high return $\pi$—namely, $\frac{1}{2}$.

Another important feature of physical capital production is that the covariance of success probabilities between projects depends exclusively on the distance between those projects rather than on their locations. From the Lemma, the correlation between the success probabilities of projects located at positions $j$ and $k$ on the circle is

$$\text{corr} \left( \tilde{\Pr} (H|j, \tilde{u}), \tilde{\Pr} (H|k, \tilde{u}) \right) = \cos (2\pi (j - k)).$$

The above expression implies projects located near one another on the circle have more positively correlated probabilities of success than those located farther apart. Projects positioned opposite one another on the circle have the lowest correlated probability. This correlation structure is meant to capture the notion of integrated industries (e.g., metals and automobiles) or nearby geographic areas (e.g., neighboring cities) sharing more correlated production outcomes than more “distant” ones.

Figures 9(a) - 9(b) present an illustration of project uncertainty. At the start of the period, each project around the circle bears the same uncertainty of project success, having one-half chance of yielding the high return. Once the shock $\tilde{u}$ is drawn in the middle of the period, projects bear different probabilities of success according to their locations, with those close to one another on the circle sharing similar likelihoods of yielding the high return.

A project’s life follows this sequence: at the beginning of the period, the project is financed and the investment is made. In the middle of the period, $\tilde{u}$ is realized, which determines the project’s actual probability of the high return, denoted $\Pr (H|j, u)$. No action related to project financing or the project itself can be made at that time. Finally, at the end of the period, the project produces either the high or low amount of physical capital.

**B.1.1 Bank diversification**

The new form of project uncertainty prevents banks from perfectly diversifying their portfolios with infinitesimal lending arcs. With correlated project returns, the degree of bank diversification will depend on the size of the arc. For a given $\Delta^i$, the bank’s average probability per project of receiving payment $R_i^L$, prior to the realization of $\tilde{u}$, is

$$\tilde{\Pr} (H|j, \tilde{u}) = \frac{1}{\Delta^i} \int_{-(\Delta^i)/2}^{(\Delta^i)/2} \left( 1 + \cos \left( 2\pi (i + j + \tilde{u}) \right) \right) dj$$

$$= \frac{1}{2} + \frac{\sin (\pi \Delta^i)}{\Delta^i} \cos \left( \frac{2\pi (i + \tilde{u})}{2\pi} \right).$$

(36)
Notes: At the beginning of the period, all projects share the same expected success probability of one-half. This common probability of success in expectation is represented in Figure 9(a) by the color yellow along the entire circle. In the middle of the period, the shock $\tilde{u}$ is realized. The example in the figure has a realized value of $u = 0$. At that moment, projects differ in their success probabilities according to (35). In Figure 9(b), arcs of the circle with projects having high success probability are colored green. Arcs with projects having low success probability are colored red. The four numbers positioned around the circle are the success probabilities of the projects located at those positions.

I call (36) the repayment rate of bank $i$'s loan portfolio, as it is the fraction of projects whose owners can repay the bank.

Two components comprise the repayment rate of a portfolio: diversification and residual uncertainty. The diversification component captures the reduction in the uncertainty of a bank loan portfolio’s payoff from choosing a larger arc length around the circle. The residual uncertainty component reflects the risk that remains in a loan portfolio that is imperfectly diversified.

Important properties of the repayment rate are presented in Lemma 6.

**Lemma 6.** The repayment rate of bank $i$'s portfolio satisfies the following properties:

1. *(Common mean)* The expected repayment rate is always $\frac{1}{2}$, no matter the choice of $\Delta^i$.
2. *(No diversification)* As $\Delta^i \downarrow 0$, the bank’s repayment rate approaches the same probability that a single project succeeds, given in (35).
3. *(Declining variance)* As $\Delta^i$ increases, the variance of the repayment rate declines.
4. *(Perfect diversification)* When $\Delta^i \uparrow 1$, the repayment rate approaches $\frac{1}{2}$, no matter the realization of $\tilde{u}$. 
Proof. See Appendix B.1.9.

The repayment rate of an imperfectly diversified bank is a random variable prior to the realization of \( \tilde{u} \). No matter the bank’s arc length, though, the expected repayment rate on its portfolio is always \( \frac{1}{2} \) : the bank expects half its loan portfolio to repay and half to default.

As a bank lends to more and more entrepreneurs around the circle, it reduces the variability of its repayment rate by diversifying its loan portfolio. Eventually, if a bank lends the circumference of the circle, its portfolio becomes risk-free, being immune to the random realization of \( \tilde{u} \). In this case, half the portfolio will succeed and half will fail.

### B.1.2 Bank capital structure

Bank equity holders also perfectly observe the uniform shock \( \tilde{u} \), and thus the realized profits of their bank’s loan portfolio; depositors, on the other hand, do not. Depositors do know the loan repayment rate function in (36), however, and hence they are certain of their bank’s minimum possible repayment rate. The minimum profit on the loan portfolio is the maximum amount depositors can prove and recover from the bank in bankruptcy court. Depositors are only willing to finance their bank up to this amount. Depositor preference for safe assets can also generate this contract, as bank debt, unlike bank equity, will bear no risk. Deposits can be considered fully collateralized by the minimum loan portfolio return of the bank.

The minimum\(^7\) of (36) over the shock \( \tilde{u} \) is

\[
\Pr_{\min}\left( R_L^i | \Delta^i \right) = \frac{1}{2} \left( 1 - \frac{\sin \left( \frac{\pi \Delta^i}{\pi \Delta^i} \right)}{\pi \Delta^i} \right).
\]

Denote the minimum loan profits for bank \( i \) as \( \Pi_{\text{min}}^i \). Depositors are willing to lend an amount up to the discounted face value of \( \Pi_{\text{min}}^i \). Because deposits are safe, the deposit rate is the risk-free interest rate \( R \). Let \( d^i \) be the amount of deposits a bank chooses. Banks can raise deposits up to the amount

\[
d^i \leq \frac{\Pi_{\text{min}}^i}{R}.
\]

I call the maximum amount a bank can raise in deposits the debt capacity of the bank. Whatever additional outside financial capital a bank requires to finance its operations, it obtains from the equity market at the required expected equity return \( R_E \). In equilibrium, equity will be at least as expensive as debt \((R_E \geq R)\).

The minimum repayment rate of a bank determines its debt capacity, which influences its leverage. As a bank increases \( \Delta^i \), it expands its lending operations to more and more industries

---

\(^7\)Bank \( i \) suffers its minimum repayment rate if the shock lands at location \( |\frac{1}{2} - i| \) if \( i \in \left[ 0, \frac{1}{2} \right] \) or location \( 1 - |\frac{1}{2} - i| \) if \( i \in \left( \frac{1}{2}, 1 \right) \). If the shock lands at some other location for all \( i \in N_t \), every bank’s realized repayment rate exceeds the minimum.
or areas across the circle, and diversifies its portfolio. Depositors, in turn, are then willing to lend more to the bank. The minimum repayment rate of the bank is increasing in $\Delta^i$, and so too will its debt capacity and leverage. Liang and Rhoades (1991), McAllister and McManus (1993) and Demsetz and Strahan (1997) give empirical evidence of a positive correlation between bank diversification and leverage.

The debt capacity condition in (37) can equivalently be considered a minimum equity capital requirement that is imposed by the market rather than an outside rule as in the single period model of the main text. Denote the total assets of the bank by $a^i = \Delta^i + f$. Substituting the balance sheet identity $a^i \equiv d^i + e^i$, the constraint can be written as $e^i \geq \Delta^i + f - \frac{\Pi^i_{\min}}{R}$. So rather than choosing an amount $d^i_t$ in deposits, the bank instead chooses an amount $e^i_t$ in equity, provided its choice satisfies a minimum amount.

**B.1.3 Bank decision**

A typical bank $i$ chooses a lending rate $R^i_L$ and quantity of deposits $s^i_t$ to maximize expected profits over a single period, perfectly knowing and taking as given (1) the demand curve for bank credit from the main text, (2) the lending rates of other banks, (3) the number of banks $N$ on the circle, and (4) the costs of debt and equity capital, $R$ and $R^E$, respectively. Let $FC^i$ denote the financing cost function for bank $i$.

Expected profits of the typical bank at time $t$ are

$$\Pi^i = \frac{1}{2} R^i_L \Delta^i + \frac{1}{2} \left( \kappa \Delta^i - g \left( \Delta^i \right) \right) - FC^i. \quad (38)$$

Prior to the realization of $\tilde{u}_{t+1}$, the bank expects half the projects in its loan portfolio to repay and half to default. The first term in (38) represents expected payments received from the fraction of projects that succeed. The second term is the expected proceeds from the fraction that fail. In this case, the bank recovers the low returns on physical capital net of the liquidation costs.

The financing cost function $FC^i$ consists of the payments to depositors and equity holders. The function is

$$FC^i = Rd^i + R^E \left( \Delta^i + f - d^i \right). \quad (39)$$

The minimum loan profits that determines the constraint of (37) are

$$\Pi^i_{\min} = Pr_{\min} \left( R^i_L | \Delta^i \right) R^i_L \Delta^i$$

$$+ \left( 1 - Pr_{\min} \left( R^i_L | \Delta^i \right) \right) \left( \kappa \Delta^i - g \left( \Delta^i \right) \right).$$

The bank maximizes (38) subject to (37).
B.1.4 Monetary transmission

Proposition 4 presents the bank lending rates in both the competitive and kinked equilibria. I then discuss how the pass-through depends on the amount of bank equity and the type of equilibrium.

**Proposition 4. (Lending rates)** The bank lending rate in a competitive equilibrium is

\[
R^C_L = \frac{R^C_E + \frac{1}{2} \left[ g' \left( \frac{1}{N^C} \right) - \kappa + \frac{2c}{\kappa N^C} \right] + \left( \frac{R^C_E}{R} - 1 \right) \phi \left( \frac{1}{N^C} \right)}{\frac{1}{2} + \left( \frac{R^C_E}{R} - 1 \right) \psi \left( \frac{1}{N^C} \right)},
\]

where the functions \( \phi \) and \( \psi \) are defined in Appendix B.1.10 by equations (46) and (47), respectively.

The kinked equilibrium lending rate is

\[
R^K_L = \kappa - 2w - \frac{c}{N^K}.
\]

**Proof.** See Appendix B.1.10. \( \square \)

B.1.5 Perfect pass-through

Consider first the competitive lending rate. Suppose the supply of equity were so large the equity market cleared at the lower-bound price \( R^C_E = R \). In this situation, deposits and equity are perfect substitutes, so the bank faces a single cost of financial capital \( R \). Because the bank could finance itself entirely with equity, the constraint (37) would be slack. The functions \( \phi \) and \( \psi \) in (40) reflect a bank’s debt capacity. They enter the lending rate if the constraint binds. Here, they are set to zero.

The lending rate in such a competitive equilibrium would be

\[
R^C_L = 2R + g' \left( \frac{1}{N^C} \right) - \kappa + \frac{2c}{N^K}.
\]

The competitive lending rate is virtually the same as the one in the main text given in (10), reflecting the marginal cost of financing plus a markup. The interest rate enters the bank lending rate linearly, so perfect pass-through occurs, again because of competition.

B.1.6 Imperfect pass-through

Now suppose equity were scarce, so that the equity market cleared at price \( R^C_E > R \). A bank would have strict preference for cheaper deposit financing, so the credit constraint would bind. The debt capacity of the bank now becomes important when the bank chooses the competitive loan rate.

The loan rate is now (40). The rate reflects the blend of debt and equity in the bank’s financial capital structure. The functions \( \phi \) and \( \psi \) capture this blend. They adjust the marginal cost of
financing from an incremental growth in the loan portfolio. This fact can be seen clearly from Lemma 7, which presents a typical bank’s marginal financing cost function $FC’(\Delta^i)$ as it increases its portfolio size $\Delta^i$.

**Lemma 7.** The marginal financing cost function of a typical bank $i$ is

$$\frac{\partial FC^i}{\partial \Delta^i} = R_E + \left( \frac{R_E}{R} - 1 \right) \left[ \phi (\Delta^i) - \psi (\Delta^i) \right] R_L^i. \tag{43}$$

**Proof.** See Appendix B.1.10.

A marginal increase in a bank’s loan portfolio has two effects on its cost of funding. The first effect is a higher financing cost from the need for more equity to fund the portfolio at price $\hat{R}_{E,t+1}$. The second effect is a decrease to the cost of funding as the bank tilts its capital structure to cheaper debt financing because of greater diversification.

The second term in (43) are cost savings from diversification. They reflect changes to the minimum possible loan profit of the bank $\Pi^i_{min}$, which determines its debt capacity. For a fixed lending rate $R_L^{i,t}$, an expansion in $\Delta^i$ increases the debt capacity of the bank at rate $\psi$. The higher debt capacity generates financing cost savings at rate $\left( \frac{R_E}{R} - 1 \right)$, which are passed onto entrepreneurs, in the form of a lower lending rate in equilibrium.

Greater diversification decreases the bank’s minimum failure rate on its portfolio $(1 - Pr_{min})$. A lower failure rate means the bank will receive payment $R_L^i$ on more of its loans. It also means the bank will retrieve the low physical capital return $\kappa$ net of the liquidation costs on less of its loans, as fewer will default. Less recovery values from fewer defaults reduces the minimum loan profit of the bank and its debt capacity. The function $\phi$ is the rate at which debt capacity decreases with $\Delta^i$. Lower debt capacity increases marginal financing costs, raising the competitive lending rate.

When the market’s constraint on bank financing binds, the degree of pass-through now relies on bank financial capital structure. The relation is non-linear and imperfect. Interest rate pass-through depends on the functions $\phi (\Delta^i)$ and $\psi (\Delta^i)$, which reflect the bank’s debt capacity and its diversification. The pass-through also depends on the cost of equity capital $R_E$, which is a function of aggregate bank net worth.

**B.1.7 Negative pass-through**

In a kinked equilibrium, the interest rate channel is closed completely in the “shorter run” when the number of banks $N^K$ is held fixed, for the same reasons as in given in the main model. The only effect of a lower interest rate on bank lending rates in a kinked equilibrium will be indirect through adjustments in the number of banks $N^K$ over the “longer run”. Because of the endogenous
constraint in (37), monetary policy will not be neutral even in the longer run, in contrast to the economy of the main text.

A lower interest rate will reduce the average cost of operating a bank, increase profits, and encourage entry into the lending market. An important perverse feature of the kinked lending rate in (11), however, is that more banks in the credit market actually leads all of them to raise their loan rates.

In a kinked credit market, banks are local monopolists. More banks on the circle means that an entrepreneur can find one that specializes in an industry or area “closer” to the entrepreneur’s. A bank takes advantage of its greater local monopoly power by charging a higher lending rate.

Conversely, fewer banks lead all of them to reduce their loan rates. When a bank exits the lending market, the average entrepreneur needs to “travel” a longer distance on the circle, contracting with a bank that is less specialized in his or her particular industry or location than before. Undertaking the project becomes less attractive to the entrepreneur relative to the outside option of working. Because the typical bank in a kinked market is competing against its borrowers’ outside options, it needs to lower the lending rate to encourage the entrepreneur to borrow instead.

By encouraging bank entry, an accommodative monetary policy has the unintended effect of increasing the cost of bank credit to firms and worsening the commercial loan spread. I call a decrease to the interest rate that leads to an increase in the bank loan rate (and vice versa) “negative pass-through.”

The expected profit function of the bank can be written as

$$\Pi^i = \frac{1}{2} \left( R^i_L + \kappa \right) \Delta^i - \frac{1}{2} g(\Delta^i) - R_E (\Delta^i + f) + (R_E - R) \frac{\Pi^i_{\min}}{R}.$$ 

The direct effect of a decline in $R$ is to encourage bank entry because of higher expected profits from lower funding costs. Another first order effect works in the same direction: a lower policy rate increases the debt capacity $\frac{\Pi^i_{\min}}{R}$ of a bank, allowing it to raise cheaper debt financing over equity capital. This effect can be considered an “asset pricing” channel of the monetary policy because it increases the value of the collateral which banks post to raise cheaper debt, hence increasing expected bank profits. These direct effects lead to higher entry and negative pass-through.

One second order general equilibrium effect works in the same direction: higher debt capacities reduce demand for equity capital, lowering the cost of equity $R_E$, further increasing expected profits and entry. Another second order effect comes from the reaction of the equity market and works in the opposing direction: more banks in the lending market increases the demand for equity capital, which puts upward pressure on $R_E$, dampening entry. A final second order effect also works in the opposite direction: more banks on the circle reduces the arc length of each one and the minimum possible profits $\Pi^i_{\min}$, limiting the capacity to raise cheaper debt and dampening
entry.

Provided the positive forces for entry dominate, the kinked equilibrium features negative pass-through once accounting for entry. Figure (10) illustrates the effect of a lower interest rate on the average cost curve that encourages bank entry and negative pass-through. The average cost curve shifts inward from the lower funding costs, increasing profits of existing banks, giving reason for other banks to enter the lending market.

Figure 10: Bank Entry from Lower Interest Rate, Kinked Equilibrium

Notes: The policy rate declines from \( R \) to \( \bar{R} \), lowering the average cost curve from the dotted to solid line. This change leads the number of banks to increase from \( N \) to \( \bar{N} \), which pushes the average revenue curve inward from the dotted to solid line.

The negative interest rate pass-through over the longer run in a kinked equilibrium presents a dilemma for the central bank. On the one hand, the government has reason to restrict entry (or encourage consolidation) among banks when aggregate equity capital is low to prevent bank loan rates from rising. On the other hand, fewer specialized banks reduce the physical capital stock and output. A central bank must trade-off these effects when choosing optimal policy.

B.1.8 Proof of Lemma 5

The success probability of a project at location \( j \) from (35) is

\[
\tilde{\Pr} (H|j, \bar{u}) = \frac{1}{2} \left( 1 + \cos \left( 2\pi (j + \bar{u}) \right) \right)
\]
The success probability can be treated as a transformation of a uniform random variable \( x_j = 2\pi (j + \tilde{u}) \), which has support \([2\pi j, 2\pi (j + 1)]\). For ease of notation, define

\[
Y_j \equiv \frac{1}{2} (1 + \cos (x_j)).
\]

For \( y \in [0, 1] \), the equation

\[
y = \frac{1}{2} (1 + \cos (x_j))
\]

has two solutions in \([2\pi j, 2\pi (j + 1)]\). Therefore, the transformed density is

\[
f_{Y_j}(y) = 2 \times \frac{2}{\sqrt{1 - y^2}} \times \frac{1}{2\pi}
\]

\[
= \frac{\pi}{\sqrt{1 - y^2}},
\]

for \( y \in [0, 1] \) and zero otherwise for all \( j \). The leading factor of 2 accounts for the two solutions in the support. Thus, the density of the success probability is the same at all locations.

The expected probability of success for a single project is \( \frac{1}{2} \). To see this, integrate the success probability over the unit interval since \( \tilde{u} \sim U [0, 1] \) to get

\[
E \left[ \tilde{\Pr} (H|j, \tilde{u}) \right] = \int_0^1 \frac{1}{2} \left( 1 + \cos \left( 2\pi (j + \tilde{u}) \right) \right) d\tilde{u}
\]

\[
= \frac{1}{2} + \frac{1}{2\pi} [\sin (2\pi (j + 1)) - \sin (2\pi j)]
\]

\[
= \frac{1}{2} + \frac{1}{2\pi} [\sin (2\pi j) \cos (2\pi) + \cos (2\pi j) \sin (2\pi) - \sin (2\pi j)]
\]

\[
= \frac{1}{2} + \frac{1}{2\pi} [\sin (2\pi j) - \sin (2\pi j)]
\]

\[
= \frac{1}{2},
\]

where the third equality follows from the sum-difference formula for sine.

The variance of the success probability is

\[
\sigma^2 \left[ \tilde{\Pr} (H|j, \tilde{u}) \right] = E_t \left[ \tilde{\Pr}_{t+} (H|j, \tilde{u})^2 \right] - E_t \left[ \tilde{\Pr}_{t+} (H|j, \tilde{u}) \right]^2
\]

\[
= \frac{1}{4} E_t \left[ \left( 1 + \cos \left( 2\pi (j + \tilde{u}) \right) \right)^2 \right] - \frac{1}{4}
\]

\[
= \frac{1}{4} \int_0^1 \cos^2 \left( 2\pi (j + \tilde{u}) \right) d\tilde{u}.
\]
Using the half-angle trigonometric formula $\cos^2 u = \frac{1 + \cos(2u)}{2}$, the variance can be written as

$$\sigma^2 \left[ \tilde{\Pr} (H|j, \tilde{u}) \right] = \frac{1}{8} \int_{0}^{1} [1 + \cos 4\pi (j + \tilde{u})] \, d\tilde{u}$$

$$= \frac{1}{8} \left[ 1 + \frac{1}{4\pi} \left[ \sin (4\pi (j + 1)) - \sin (4\pi j) \right] \right]$$

$$= \frac{1}{8}.$$

Finally, the covariance of success probabilities between projects located at $j$ and $k$ on the circle is

$$\text{cov} \left( \tilde{\Pr} (H|j, \tilde{u}), \tilde{\Pr} (H|j, \tilde{u}) \right) = E \left[ \tilde{\Pr} (H|j, \tilde{u}) \tilde{\Pr} (H|j, \tilde{u}) \right] - \frac{1}{4}$$

$$= \frac{1}{4} E_t \left[ \cos \left( 2\pi (j + \tilde{u}) \cos \left( 2\pi (k + \tilde{u}) \right) \right) \right].$$

Using the cosine product formula $\cos (a) \cos (b) = \frac{1}{2} \left[ \cos (a + b) + \cos (a - b) \right]$ gives

$$\text{cov} \left( \tilde{\Pr} (H|j, \tilde{u}), \tilde{\Pr} (H|j, \tilde{u}) \right) = \frac{1}{8} \cos \left( 2\pi (j - k) \right) + E_t \left[ \cos \left( 2\pi (j + k + 2\tilde{u}) \right) \right]$$

$$= \frac{1}{8} \cos \left( 2\pi (j - k) \right) + \frac{1}{8} \int_{0}^{1} \cos \left( 2\pi (j + k + 2\tilde{u}) \right) \, d\tilde{u}$$

$$= \frac{1}{8} \cos \left( 2\pi (j - k) \right).$$

### B.1.9 Proof of Lemma 6

The repayment rate of bank $i$ conditional on its chosen arc length $\Delta^i$ is

$$\tilde{\Pr} \left( R^i_L | \Delta^i, \tilde{u} \right) = \frac{1}{2} + \frac{\sin (\pi \Delta^i)}{\Delta^i} \frac{\cos \left( 2\pi (i + \tilde{u}) \right)}{2\pi}.$$

As $\Delta^i \downarrow 0$, the bank’s lending arc reduces to its home location alone. Apply L’Hôpital’s rule to get

$$\lim_{\Delta^i \downarrow 0} \tilde{\Pr} \left( R^i_L | \Delta^i, \tilde{u} \right) = \lim_{\Delta^i \downarrow 0} \left\{ \frac{1}{2} + \pi \cos (\pi \Delta^i) \frac{\cos \left( 2\pi (i + \tilde{u}) \right)}{2\pi} \right\}$$

$$= \frac{1}{2} \left( 1 + \cos \left( 2\pi (i + \tilde{u}) \right) \right)$$

$$= \tilde{\Pr} (H|j, \tilde{u}),$$

matching the probability of a single project generating the high physical capital return from (35).

Next, the expected repayment rate of a bank’s portfolio is always $\frac{1}{2}$, no matter its arc length.
Integrate the repayment rate over the unit interval to get

\[
E \left[ \tilde{\Pr} \left( R_i^L | \Delta^i, \tilde{u} \right) \right] = \int_0^1 \left[ \frac{1}{2} + \sin \left( \frac{\pi \Delta^i}{\Delta^i} \right) \cos \left( \frac{2\pi (i + \tilde{u})}{\Delta^i} \right) \right] d\tilde{u}
\]

\[
= \frac{1}{2} + \frac{\sin \left( \frac{\pi \Delta^i}{\Delta^i} \right)}{\left( \frac{2\pi}{\Delta^i} \right)^2} \left[ \sin \left( 2\pi (i + 1) \right) - \sin \left( 2\pi i \right) \right]
\]

\[
= \frac{1}{2}.
\]

The variance of the repayment rate is

\[
\sigma^2 \left[ \tilde{\Pr} \left( R_i^L | \Delta^i, \tilde{u} \right) \right] = \int_0^1 \left( \tilde{\Pr} \left( R_i^L | \Delta^i, \tilde{u} \right) - E \left[ \tilde{\Pr} \left( R_i^L | \Delta^i, \tilde{u} \right) \right] \right)^2 d\tilde{u}
\]

\[
= \int_0^1 \left( \tilde{\Pr} \left( R_i^L | \Delta^i, \tilde{u} \right) - \frac{1}{2} \right)^2 d\tilde{u}
\]

\[
= \int_0^1 \left( \frac{\sin \left( \frac{\pi \Delta^i}{\Delta^i} \right) \cos \left( \frac{2\pi (i + \tilde{u})}{\Delta^i} \right)}{\Delta^i} \right)^2 d\tilde{u}
\]

\[
= \left[ \frac{\sin \left( \frac{\pi \Delta^i}{\Delta^i} \right)}{\frac{2\pi}{\Delta^i}} \right]^2 \int_0^1 \cos^2 \left( 2\pi (i + \tilde{u}) \right) d\tilde{u}
\]

Using the half-angle formula, the variance can be written as

\[
\sigma^2 \left[ \tilde{\Pr} \left( R_i^L | \Delta^i, \tilde{u} \right) \right] = \frac{1}{2} \left[ \frac{\sin \left( \frac{\pi \Delta^i}{\Delta^i} \right)}{\frac{2\pi}{\Delta^i}} \right]^2 \int_0^1 \left[ 1 + \cos \left( 4\pi (i + \tilde{u}) \right) \right] d\tilde{u}
\]

\[
= \frac{1}{2} \left[ \frac{\sin \left( \frac{\pi \Delta^i}{\Delta^i} \right)}{\frac{2\pi}{\Delta^i}} \right]^2 + \frac{1}{8\pi} \left[ \frac{\sin \left( \frac{\pi \Delta^i}{\Delta^i} \right)}{\frac{2\pi}{\Delta^i}} \right]^2 \left[ \sin \left( 4\pi (i + 1) \right) - \sin \left( 4\pi i \right) \right]
\]

\[
= \frac{1}{2} \left[ \frac{\sin \left( \frac{\pi \Delta^i}{\Delta^i} \right)}{\frac{2\pi}{\Delta^i}} \right]^2,
\]  

where the third equality follows after using the sum-difference formula like before.

The variance of the repayment rate is strictly decreasing for \( \Delta^i \in (0, 1) \). Taking the first
derivative of (44) with respect to \( \Delta^i \) gives

\[
\frac{\partial \sigma^2}{\partial \Delta^i} \left[ \tilde{\Pr} \left( R_{iL}^\Delta, \tilde{u} \right) \right] = \frac{\sin \left( \pi \Delta^i \right)}{2 \pi \Delta^i} \frac{\partial \sin (\pi \Delta^i)}{\partial \Delta^i} = \frac{\pi \Delta^i \cos (\pi \Delta^i) - \sin (\pi \Delta^i)}{2 \pi (\Delta^i)^2} \frac{\sin (\pi \Delta^i) - \sin^2 (\pi \Delta^i)}{4 \pi^2 (\Delta^i)^3}.
\]

(45)

The sign of \( \frac{\partial \sigma^2}{\partial \Delta^i} \) is determined by the numerator of (45), as the denominator is always positive. The variance is non-increasing in \( \Delta^i \) if

\[
\pi \Delta^i \cos (\pi \Delta^i) - \sin (\pi \Delta^i) \leq 0.
\]

For \( \Delta^i = 0 \) or \( \Delta^i = 1 \), the numerator is zero, so the above inequality holds. For \( \Delta^i \in (0, 1) \), \( \sin (\pi \Delta^i) \neq 0 \), so the expression can be written as

\[
\pi \Delta^i \cos (\pi \Delta^i) \leq \sin (\pi \Delta^i).
\]

Perform a change of variable \( \theta = \pi \Delta^i \). The aim is to show

\[
\theta \cos \theta < \sin \theta
\]

over the domain \( \theta \in (0, \pi) \). Over the upper-half of the interval \( \theta \in \left[ \frac{\pi}{2}, \pi \right) \), since \( \cos \theta \leq 0 \) and \( \sin \theta > 0 \), the relation holds. Now define the function

\[
f (\theta) \equiv \sin \theta - \theta \cos \theta.
\]

Note that \( \lim_{\theta \downarrow 0} f (\theta) = 0 \) and \( f' (\theta) = \theta \sin \theta > 0 \) for \( \theta \in (0, \frac{\pi}{2}) \). Therefore, \( f (\theta) > 0 \) over the lower half of the interval. This proves the numerator of (45) is negative for \( \Delta^i \in (0, 1) \) and that the variance of the repayment rate is non-increasing for \( \Delta^i \in [0, 1] \) and strictly decreasing over the open unit interval.

Finally, as \( \Delta^i \uparrow 1 \), the repayment rate has the following limit:

\[
\lim_{\Delta^i \uparrow 1} \tilde{\Pr} \left( R_{iL}^\Delta, \tilde{u} \right) = \frac{1}{2} + \sin (\pi) \frac{\cos \left( \frac{2 \pi (i + \tilde{u})}{2 \pi} \right)}{2 \pi} = \frac{1}{2}.
\]
Thus, the repayment rate becomes a constant $\frac{1}{2}$, no matter the realization of the random variable $\tilde{u}$.

**B.1.10 Proof of Proposition 4 and Lemma 7**

In a competitive equilibrium, the first order condition for optimality is

$$\frac{1}{2} \left[ R^i_L + \kappa \right] + \frac{1}{2} \Delta^i \left( \frac{dR^i_L}{d\Delta^i} \right) = \frac{1}{2} g'(\Delta^i) + \frac{\partial FC^i}{\partial \Delta^i}. $$

Using (39) and a binding constraint (37), the marginal financing cost function is

$$\frac{\partial FC^i}{\partial \Delta^i} = R^C_E + \left( 1 - \frac{R^C_E}{R} \right) \frac{d\Pi^i_{\text{min}}}{d\Delta^i}. $$

Computing $\frac{d\Pi^i_{\text{min}}}{d\Delta^i}$ gives for the marginal financing cost function:

$$\frac{\partial FC^i}{\partial \Delta^i} = \left( 1 - \frac{R^C_E}{R} \right) \left( \kappa - g'(\Delta^i) \right) - \left( 1 - \frac{R^C_E}{R} \right) \Pr_{\text{min}}(\Delta^i) \left( \kappa - g'(\Delta^i) \right) $$

$$+ \left( 1 - \frac{R^C_E}{R} \right) \Pr_{\text{min}}(\Delta^i) \left[ R^i_L + \Delta^i \left( \frac{dR^i_L}{d\Delta^i} \right) \right] $$

$$+ \left( 1 - \frac{R^C_E}{R} \right) \frac{d\Pr_{\text{min}}(\Delta^i)}{d\Delta^i} \left[ (R^i_L - \kappa) \Delta^i + g(\Delta^i) \right] $$

$$+ R^C_E \left[ \kappa - g'(\Delta^i) \right] $$

Substituting the slope of the competitive demand curve $\frac{dR^i_L}{d\Delta^i} = -2c$ and re-arranging terms gives

$$\frac{\partial FC^i}{\partial \Delta^i} = \left( 1 - \frac{R^C_E}{R} \right) [\Pr_{\text{min}}(\Delta^i) (R^i_L - 2c\Delta^i) + (1 - \Pr_{\text{min}}(\Delta^i)) (\kappa - g'(\Delta^i))] $$

$$+ \left( 1 - \frac{R^C_E}{R} \right) \frac{d\Pr_{\text{min}}(\Delta^i)}{d\Delta^i} \left( [R^i_L - \kappa] \Delta^i + g(\Delta^i) \right] + R^C_E$$

The first line in the above expression is the marginal financing cost savings from greater diversification and larger debt capacity, holding the minimum probability fixed. The first term of the second line is the cost savings from a higher minimum repayment rate after a marginal increase in the loan portfolio breadth $\Delta^i$. The bank gets repaid on more of its loans, recovers less of the low project returns and saves on liquidation costs from fewer projects in default. The final term $R^C_E$ of the second line is the marginal cost of equity financing.

The marginal financing cost function can be conveniently represented by separating terms involving $R^i_L$. Doing so gives
\[
\frac{\partial FC^i}{\partial \Delta^i} = -\left(\frac{R_E^C}{R} - 1\right) \left[ Pr_{\min} (\Delta^i) + \frac{d Pr_{\min} (\Delta^i)}{d \Delta^i} \Delta^i \right] R^i_L \\
+ \left(\frac{R_E^C}{R} - 1\right) \left[ Pr_{\min} (\Delta^i) 2c \Delta^i + (1 - Pr_{\min} (\Delta^i)) (g' (\Delta^i) - \kappa) \right] \\
+ \left(\frac{R_E^C}{R} - 1\right) \frac{d Pr_{\min} (\Delta^i)}{d \Delta^i} (\kappa \Delta^i - g (\Delta^i)) \\
+ R_E^C
\]

Define the functions

\[
\phi (\Delta^i) \equiv Pr_{\min} (\Delta^i) 2c \Delta^i + (1 - Pr_{\min} (\Delta^i)) (g' (\Delta^i) - \kappa) \\
+ \frac{d Pr_{\min} (\Delta^i)}{d \Delta^i} (\kappa \Delta^i - g (\Delta^i)) \tag{46}
\]

\[
\psi (\Delta^i) \equiv Pr_{\min} (\Delta^i) + \frac{d Pr_{\min} (\Delta^i)}{d \Delta^i} \Delta^i \tag{47}
\]

and re-write \(\frac{\partial FC^i}{\partial \Delta^i}\) as

\[
\frac{\partial FC^i}{\partial \Delta^i} = R_E^C + \left(\frac{R_E^C}{R} - 1\right) \left[ \phi (\Delta^i) - \psi (\Delta^i) R^i_L \right]
\]

The function \(\psi > 0\). Provided \(\kappa\) is not too large, then also \(\phi > 0\). The terms in \(\phi\) are those that increase the marginal cost of financing for the bank, while those in \(\psi\) decrease the marginal cost of financing.

Using the representation of \(FC'^i\) in the last expression, the optimality condition becomes

\[
\frac{1}{2} \left[ R_L^i + \kappa \right] - c \Delta^i = \frac{1}{2} g' (\Delta^i) + R_E^C + \left(\frac{R_E^C}{R} - 1\right) \left[ \phi (\Delta^i) - \psi (\Delta^i) R^i_L \right]
\]

Solving for \(R_L^i\) and using the equilibrium condition \(\Delta^i = \frac{1}{Nc}\) gives for the competitive equilibrium lending rate

\[
R_L^C = \frac{R_E^C + \frac{1}{2} \left[ g' \left( \frac{1}{Nc} \right) - \kappa + \frac{2c}{N\kappa} \right] + \left(\frac{R_E^C}{R} - 1\right) \frac{\phi \left( \frac{1}{Nc} \right)}{\frac{1}{2} + \left(\frac{R_E^C}{R} - 1\right) \psi \left( \frac{1}{Nc} \right)}}{\frac{1}{2} + \left(\frac{R_E^C}{R} - 1\right) \psi \left( \frac{1}{Nc} \right)}.
\]

In a kinked equilibrium, the first order condition for optimality does not hold with equality, so the lending rate is instead the monopoly demand curve. Just as in the main text, solving (5) for the lending rate and setting \(\Delta^i = \frac{1}{Nc}\) gives (41).
B.2 Price discriminating banks

In this section, I assume that banks can identify the location of any prospective borrower. They are free to offer a loan rate that depends on the borrower’s location. I do so to demonstrate the robustness of the interest rate pass-through results to price discrimination.

I consider first-degree price discrimination in that a bank can capture the entire consumer surplus. A simple way to insert price discrimination is to allow banks to charge a personalized fixed premium to each entrepreneur for taking out a loan but keep all other ingredients of the model the same. The fixed premium could be a loan application or closing fee.

The premium would need to depend on the borrower’s distance from the bank. It would be highest for those closest to the bank, because these borrowers would retain the largest surplus under uniform pricing, as in the baseline model.

The personalized fixed premium is a two-part tariff or affine pricing schedule. It is equivalent to a system of personalized prices with each borrower paying a sum equal to his willingness to pay.

Under both the kinked and competitive cases, let \( S(x) \) be the net surplus for an entrepreneur located a distance \( x \) from bank \( i \), which is charging lending rate \( R^i_L \). The total amount of money \( T^i(x) \) the entrepreneur pays for the loan from bank \( i \) would then be

\[
T^i(x) = S(x) + R^i_L.
\]

B.2.1 Kinked case

In the kinked case, the indifference condition for the entrepreneur located a distance \( x \) from bank \( i \) was

\[
\frac{1}{2} (q\kappa - R^i_L) - cx = wL^e.
\]

Without price discrimination, the equilibrium kinked lending rate was \( R_L = q\kappa - 2wL^e - \frac{c}{N} \).

Substituting the lending rate into the indifference condition gives the surplus for the borrower:

\[
S(x) = c\left(\frac{1}{N} - x\right).
\]

The surplus is positive for \( x \leq \frac{1}{N} \). The upper bound is the edge of bank \( i \)'s potential local monopoly market before it reaches the headquarters of a neighboring bank. The personalized premium is decreasing in the borrower’s distance from bank \( i \). The bank has the most market power over those borrowers nearest to it and so it can charge the largest premium without them rejecting the loan.

Since the kinked interest rate with price discrimination does not depend on the interest
rate (holding fixed the number of banks), the absence of interest rate pass-through in a kinked equilibrium holds again.

B.2.2 Competitive case

For the competitive case, it is easiest to assume a very simple expected profit function for the bank:

$$\Pi = \frac{1}{2} R^i L \Delta^i - R \left( \Delta^i + f \right).$$

I have removed benefits from diversification, recovery value and liquidation costs. The equilibrium competitive lending rate in this case would be

$$R_L = 2R + \frac{2c}{N}.$$  

This lending rate is very similar to the perfect pass-through lending rate in (10). The indifference condition for a borrower located a distance \(x\) from bank \(i\) is

$$\frac{1}{2} R^i L + cx = \frac{1}{2} R_L + c \left( \frac{1}{N} - x \right).$$

In the competitive case, an entrepreneur minimizes costs between the two neighboring banks. The surplus from borrowing from bank \(i\) is the cost savings of doing so:

$$S(x) = \frac{1}{2} \left( R_L - R^i_L \right) + c \left( \frac{1}{N} - 2x \right).$$

The lower bank \(i\) sets the lending rate, the more surplus goes to the entrepreneur; the closer the entrepreneur is to bank \(i\), the more surplus he or she receives.

In equilibrium, the lending rates match, so the surplus comes to

$$S(x) = c \left( \frac{1}{N} - 2x \right).$$

This surplus is positive for \(x \leq \frac{1}{2N}\). An entrepreneur located that distance \(x = \frac{1}{2N}\) is in between the two banks. That entrepreneur is marginal, so he or she will not be charged a personalized premium. Everyone else will be charged the fixed premium \(S^i(x) = c \left( \frac{1}{N} - 2x \right)\) according to their distance from the bank. Compared to the kinked case, the surplus in the competitive case declines twice as fast due to the entrepreneur’s credible alternative of contracting with a competitor.

B.3 Smoothing the kink

In this section I provide one way to “smooth” the kink in the demand curve for bank credit (make the region differentiable). I do so in order to demonstrate that the limited pass-through at the kink
is robust even after smoothing it.

**B.3.1 General Case**

Generally, the pass-through of marginal costs to prices is lower at points of a downward sloping demand curve that feature greater concavity. In the model, the kink is a sharp way of creating concavity in the demand curve for loans when consumer preferences would otherwise imply a linear demand curve (and perfect pass-through). As long as the smoothing procedure preserves the highest concavity at the smoothed kink, then the pass-through will be lowest there, just as when the kink is sharp.

One way to smooth the kink is to assume that banks are unsure about which demand curve they are on when setting a price. Since banks set prices at the margin, they may equivalently be unsure about the slope of the demand curve at a given price.

The two possible slopes a bank faces is either \(-c\) or \(-2c\). I assume the bank assigns a probability \(h\) that the slope is \(-2c\), which is to say that it believes with probability \(h\) that it is competing with a neighboring bank. The probability with which the bank believes it is a local monopolist is then \((1 - h)\).

I assume that \(h\) is an increasing function of the bank’s market share \(\Delta\), is continuous, and is three times differentiable over the domain I specify below. This kind of uncertainty can be rationalized by a bank not knowing the precise boundary of its neighbors’ market, but knowing that it has increasingly likely penetrated that boundary as it expands its market share. I assume the bank knows the number of banks operating in the lending market \(N\).

For simplicity, I also assume the bank cannot recover any value from a loan in default \((\kappa = 0)\) and that it bears no liquidation costs \((g = 0)\). I also assume the bank is flush with equity so that its cost of capital is the interest rate \(R\), which I take as a parameter.

The expected profit function of a bank is then

\[
\Pi = \frac{1}{2} R_L \Delta - R (\Delta + f).
\]

The bank will chose a market share \(\Delta\) that satisfies the first order condition

\[
\Omega \equiv R_L (\Delta) + \Delta R'_L (\Delta) - 2R = 0.
\]
By the implicit function theorem, the quantity pass-through pass-through $\frac{\partial \Delta}{\partial R}$ is

$$
\frac{\partial \Delta}{\partial R} = -\frac{\partial Q}{\partial \Delta} \frac{\partial \Delta}{\partial R}
= \frac{2}{R'_L + \Delta R''_L (\Delta) + R'_L}
= \frac{2}{2R'_L + \Delta R''_L (\Delta)}.
$$

Now by the chain rule, the interest rate pass-through is

$$
\frac{\partial R_L}{\partial R} = \frac{\partial R_L}{\partial \Delta} \frac{\partial \Delta}{\partial R}.
$$

Substituting for the quantity pass-through gives

$$
\frac{\partial R_L}{\partial R} = R'_L \left( \frac{2}{2R'_L + \Delta R''_L} \right).
$$

Assuming symmetry in market shares and a completely served circle gives $\Delta = \frac{1}{N}$. Substituting and re-arranging terms gives

$$
\frac{\partial R_L}{\partial R} = \frac{2R'_L}{2R'_L + \frac{1}{N} R''_L}
= \frac{2}{2 + \frac{1}{N} R''_L / R'_L}.
$$

Define the concavity of the demand curve as

$$
\omega (\Delta) \equiv \frac{R''_L (\Delta)}{R'_L (\Delta)}.
$$

The interest rate pass-through is then

$$
\frac{\partial R_L}{\partial R} = \frac{2}{2 + \omega (\Delta) / N}.
$$

With a downward sloping demand curve, a larger concavity implies a lower interest rate pass-through. Also, as the number of banks tends to infinity $N \to \infty$, the market reaches perfect competition and features perfect pass-through.
Returning to a bank’s uncertainty over the slope of its demand curve, we have

\[ R'_L (\Delta) = -(2c \times h (\Delta) + c \times (1 - h (\Delta))) \]

\[ = -(c + ch (\Delta)) \]

\[ = -c (1 + h (\Delta)) \]

This object is the slope of the bank’s subjective demand curve given its beliefs about being a competitor or local monopolist. The second derivative gives

\[ R''_L (\Delta) = -ch' (\Delta). \]

The concavity of the demand curve for loans is then

\[ \omega (\Delta) = \frac{h' (\Delta)}{1 + h (\Delta)}. \]

Because \( h \) is a probability, I require \( h (\Delta) \geq 0 \) for all \( \Delta \). I also assume that \( h' \geq 0 \), making the bank increasingly believe it is competing as it expands. These assumptions make \( \omega (\Delta) \geq 0 \). Furthermore, a bank knows that its headquarters is located at \( \Delta = 0 \). It also knows that its loan portfolio reaches the neighboring bank’s headquarters when \( \Delta = \frac{2}{N} \). Therefore, it is reasonable to assume that \( \lim_{\Delta \downarrow 0} h (\Delta) = 0 \) and \( \lim_{\Delta \uparrow \frac{2}{N}} h (\Delta) = 1 \). I define the domain of \( h \) to be the closed interval \( [0, \frac{2}{N}] \).

If the concavity is uniquely highest at the point midway between headquarters of banks, then the lowest pass-through would occur at the location of the kink in the baseline model. The function \( \omega \) should be globally maximized at \( \frac{1}{N} \) and hence satisfy

\[ \omega' \left( \frac{1}{N} \right) = 0 \]  
(48)

\[ \omega'' \left( \frac{1}{N} \right) < 0. \]  
(49)

A simple way to ensure the global maximum is uniquely reached when \( \Delta = \frac{1}{N} \) is to assume that the function \( \omega (\Delta) \) is strictly concave along the entire interval. A sufficient but not necessary condition for the global maximum is

\[ \omega'' (\Delta) < 0, \quad \forall \Delta \in \left[ 0, \frac{2}{N} \right]. \]  
(50)

The restrictions on the belief function \( h \) imposed by the conditions (48)-(49) will smooth the
kink in the demand curve in a way to minimize the interest rate pass-through at that point. If condition (50) is also imposed, then it is guaranteed that that point will uniquely minimize the pass-through.

**B.3.2 Example for** $h$

Given the assumptions for $h$, a cumulative distribution function with non-negative bounded support that satisfies (48)-(49) and where the maximum is unique would deliver an appropriate example.

I use the beta distribution $h(x) = \text{Beta}(x; \alpha, \beta)$, where I make the linear transformation $x = \frac{\Delta}{(2/N)}$ that adjusts the support to $[0, 1]$. Thus, a bank that has a portfolio size of $x = 1$ unit believes its loan portfolio has reached the headquarters of the two neighboring banks.

Choosing parameters $\alpha$ and $\beta$ that deliver a global maximum for $\omega$ at $x = 1/2$ would give what is needed, which is the highest curvature at $\Delta = 1/N$, the location of the kink in the baseline model. I find those parameters computationally. I search for the parameter vector $(\alpha, \beta)$ that minimizes the function $|\arg\max_{x(\alpha, \beta)} \omega(x(\alpha, \beta)) - \frac{1}{2}|$ over the unit interval and confirm graphically that the solution associates with a global maximum of $\omega(x)$. I use 10,000 starting points for the optimization routine.

The optimal solution from the search is $\alpha = 1059$ and $\beta = 1046$. The differences between the smooth and original demand curves under these parameters, however, are difficult to see. Therefore, in the figures below, I use the parameters $\alpha = 10.59$ and $\beta = 10.46$, which make the differences clearer and delivers a concavity function $\omega$ that approximately is maximized at $x = \frac{1}{2}$.

Figures 11(a)-11(b) plot the probability function $h(\Delta)$ and the concavity function $\omega(\Delta)$. 

73
Notes: The function $h(\Delta)$ represents the probability a bank believes it is competing with a neighboring bank. This uncertainty affects the weight a bank places on the two possible slopes in the demand curve for loans. The function $\omega(\Delta)$ is the concavity of the subjective demand curve implied by the bank’s beliefs. The support of the beta distribution has been transformed from the unit interval to $[0, 2/N]$ in the figures. The scale parameters for the beta distribution are $\alpha = 10.59$ and $\beta = 10.46$. 

74
The figures reveal that all the uncertainty is concentrated around $\frac{1}{N}$, which is the location a bank would expect the boundary of a neighboring bank’s market to be in equilibrium. This probability distribution leads the concavity of the subjective demand curve to be maximized around the location of the kink.

Figure 12 illustrates the original sharp kink featuring no uncertainty about the demand curve and the corresponding smoothed kink in which there is uncertainty. For $\Delta < \frac{1}{N}$, the smooth curve begins to deviate from the original curve once the bank starts assigning positive probability to competing with a neighbor. The bank reduces its lending rate faster in order to attract the marginal borrower because the bank believes it might now be competing for that customer. As $\Delta$ approaches $\frac{1}{N}$, the original kink is entirely “rounded out” and the smooth demand curve displays the most concavity. Interest rate pass-through will be lowest in that region, though not zero.

As the bank extends its loan portfolio further past $\frac{1}{N}$, it becomes more confident that it is competing with the neighbor, and so the bank puts more weight on the slope of the demand curve being $-2c$. The original and smooth demand curves converge.

### B.4 Variable Investment Scale

The baseline model featured a single unit of investment to initiate a project. Aggregate investment was fixed and the only dynamics originated from persistence in the banking sector’s efficiency at liquidating failed projects. Here, the scale of project investment can vary. Importantly, this change allows a productivity shock to affect aggregate investment and to propagate through time from both the real side of the economy and the banking sector. Also, an impairment to interest rate pass-through now directly obstructs the central bank from affecting investment and influencing real output.

A simple way to allow variable investment is to insert concavity into the production function of a project. Investment thus depends on the cost of bank credit. Rather than a constant returns-to-scale technology, the production function is now

$$\kappa (\iota^j_t) = \tilde{\kappa} \left(1 - \exp \left(-\iota^j_t\right)\right),$$

where $\tilde{\kappa} = \kappa$ in the high state and $\tilde{\kappa} = \kappa$ in the low state. The probability of the high state is again given by (35).

The function in (51) is continuously differentiable, strictly increasing, strictly concave, satisfies $\kappa (0) = 0$, and has a first derivative that vanishes as $\iota^j_t \to \infty$. All these attributes are appropriate for a production function. This production function will generate results that are analogous to the unit investment case, which is why I use it.

Entrepreneurs choose an amount of output to invest into the project at time $t$. Investment decisions are independent of an entrepreneur’s location. So I denote by $\iota^j_t$ an investment by an
Notes: The dashed curve is the original demand curve that features a kink at $\Delta = \frac{1}{N}$. The solid curve is the smooth demand curve that is derived from a bank being uncertain about the slope of the demand curve it faces. The uncertainty is captured by a beta distribution with scale parameters $\alpha = 10.59$ and $\beta = 10.46$.

entrepreneur who considers a loan from bank $i$ at rate $R_{L,t}^i$. That investment decision satisfies

$$V_{j,i} \equiv \max_{\iota_t^i} E_t \left[ \pi \left( \iota_t^i \right) \right] - c|i - j|$$

with

$$E_t \left[ \pi \left( \iota_t^i \right) \right] = \frac{1}{2} \left[ \kappa \left( 1 - \exp \left( -\iota_t^i \right) \right) - R_{L,t}^i \iota_t^i \right].$$

The optimal investment is

$$\iota_t^i = \log \left( \frac{\kappa}{R_{L,t}^i} \right). \quad (52)$$

Investment is now decreasing in the lending rate and increasing in the expected relative price of physical capital. To ensure positive investment, the high return of the project $\kappa$ must be large enough so that

$$\kappa > R_{L,t}^i.$$
So long as $\kappa$ is sufficiently above 1, this relation will hold for all reasonable values of the gross lending rate $R_{i,L,t}^i$.

**B.4.1 Demand curve for loans**

Exactly as in the unit investment case, the demand curve for bank credit will consist of monopoly, kinked, and competitive components.

The indifference condition of the marginal entrepreneur who defines the size of the local monopoly market is

$$E_t \left[ \pi \left( \iota_t^i \right) \right] - cx - w = 0.$$  

Substituting the entrepreneur’s expected profit at the optimal level of investment from (52), solving for $x$, and multiplying by 2 gives the size of the local monopoly market:

$$\Delta^{i,M}_{t} = \frac{1}{2} \left[ \kappa - R_{i,L,t}^i (1 + \iota_t^i) \right] - \frac{w}{c/2}. \quad (53)$$

The monopoly demand curve here is virtually the same as the one in unit investment case. The only difference is an adjustment for the optimal investment $\iota_t^i$.

When banks compete and an entrepreneur has a choice between two banks for a loan, the entrepreneur will choose the lowest-cost financing. The indifference condition for the marginal entrepreneur is

$$\frac{1}{2} \left[ R_{L,t}^i (1 + \iota_t^i) \right] + cx = \frac{1}{2} \left[ R_{L,t} (1 + \iota_t) \right] + c \left( \frac{1}{N_t} - x \right),$$

making the competitive demand curve

$$\Delta^{i,C}_{t} = \frac{1}{2} \left[ R_{L,t} (1 + \iota_t) - R_{i,L,t}^i (1 + \iota_t^i) \right] + \frac{c_x}{N_t}. \quad (54)$$

Here I denoted the investment of an entrepreneur who contracts with the competing bank by $\iota_t^i$. As in the monopoly case, the only adjustment in the competitive demand curve compared to the unit scale case, is the inclusion of the investment scale.

The different slopes of the monopoly and competitive demand curves generate the kink. The slopes of the monopoly and competitive demand curves are:

$$\frac{\partial R_{L,t}^{i,M}}{\partial \Delta_t^i} = -\frac{c}{\iota_t^i},$$

$$\frac{\partial R_{L,t}^{i,C}}{\partial \Delta_t^i} = -\frac{2c}{\iota_t^i}. $$
Both slopes now reflect the investment scale, and again, the competitive demand curve slope is twice that of the monopoly.

### B.4.2 Bank problem

Banks again are restricted to finance projects that are positioned along arcs $\Delta_i^t$ centered at the headquarters. But now, each loan has size $\iota_i^t$, rather than a single unit. Because of these two dimensions to lending, I refer to $\Delta_i^t$ as the “breadth” of bank $i$’s loan portfolio at time $t$, and $\iota_i^t$ as the “depth” of the portfolio. The size of the total loan portfolio of bank $i$ is then $\Delta_i^t\iota_i^t$.

The bank diversifies into new industries that require new expertise from expansions in $\Delta_i^t$ rather than $\iota_i^t$. For that reason, I augment the total liquidation cost function $g$ to be linear in $\iota_i^t$, but again quadratic in $\Delta_i^t$:

$$g(\Delta_i^t, \iota_i^t) = \gamma \iota_i^t (\Delta_i^t)^2.$$  

Expected profit of the typical bank now includes the scale of project financing:

$$\Pi_i^t = \frac{1}{2} R_{L,t}^i \Delta_i^t \iota_i^t + \frac{1}{2} (\kappa (\iota_i^t) \Delta_i^t - g(\Delta_i^t, \iota_i^t)) - FC_i^t.$$  

So too does the financing cost function:

$$FC_i^t = R(\Delta_i^t \iota_i^t + f - e_i^t) + R_{E,t} e_i^t.$$  

And the equity capital requirement:

$$e_i^t \geq \lambda (\Delta_i^t \iota_i^t + f).$$  

The bank chooses $R_{L,t}^i$ to maximize expected profits subject to the equity capital constraint, costs of financial capital $R$ and $R_{E,t}$, the number of banks, the loan-pricing strategies of those banks, and the demand curve for bank credit as given.

### B.4.3 Equilibrium

I study the same types of Nash equilibria as in the main text. Because I look at symmetric equilibria, lending rates are the same across banks, so all entrepreneurs will make the same investment choices. Thus, $\iota_i^t = \iota_t$ for all $i$. Therefore, the equilibrium loan portfolio of a typical bank $i$ can be represented graphically as the surface area of a sector of a cylinder. This representation is depicted in Figure 13.

### B.4.4 Bank lending rates

Given the costs of financial capital $R$ and $R_{E,t}$, the equilibrium lending rate $R_{L,t}$ and number of banks $N_t$ again are jointly determined by the point of tangency between the average revenue and
average cost curves of a typical bank.

Dividing the expected bank profit function in (18) by $\Delta^i_t$ gives the average revenue and average cost functions:

\[
AR\left(\Delta^i_t, \iota^i_t\right) = \frac{1}{2} \left( R^i_{L,t} \left( \iota^i_t - \frac{\kappa}{\rho} \right) + \kappa \right)
\]

\[
AC\left(\Delta^i_t, \iota^i_t\right) = \frac{1}{2} \gamma \Delta^i_t \iota^i_t + R_{L,t} \left( \iota^i_t + \frac{f}{\Delta^i_t} \right).
\]

The average revenue and average cost of a typical bank are functions of the bank’s loan portfolio breadth $\Delta^i_t$ and depth $\iota^i_t$. Tangency between the average revenue and average cost curves, along with all markets clearing, will pin down the number of banks and the lending rate in the economy.

In the kinked equilibrium, the lending rate is available in closed form by re-arranging (53) and setting $\Delta^i_t = \frac{1}{N_t}$. Doing so gives

\[
R^K_{L,t} = \frac{\kappa - 2w - \frac{c}{N_t}}{1 + \iota_t}.
\]

The kinked lending rate is virtually the same as the case with unit investment, save for the adjustment for the investment level $\iota_t$ in the denominator. A greater level of project investment is associated with a lower lending rate. Because the investment $\iota_t$ is also a function of $R^K_{L,t}$, equation (55) expresses the lending rate implicitly. Note the cost of bank financing again does not enter the kinked lending rate.
B.4.5 Dynamics

The most significant change to the dynamical system compared to the fixed-investment case is the formation equation of physical capital. Substituting optimal investment (52) into the production function (51), and using the capital formation (26) gives the evolution equation for physical capital:

\[
K_{t+1} = \frac{1}{2} (\kappa + \kappa) - \frac{1}{2} \left(1 + \frac{z}{\pi}\right) R_{L,t} - \frac{\gamma/2}{N_t} \log \left(\frac{\kappa}{R_{L,t}}\right).
\]

The first term of (56) is the physical capital production from projects if they required a single unit of investment. Half will succeed and produce the high return \(\kappa\), and half will fail and produce the low return \(\kappa\). The second term is new. It reflects the adjustment to the physical capital stock from the scale of project investment \(\iota_t\). Investment is decreasing in the bank lending rate \(R_{L,t}\). The final term is the aggregate bank liquidation costs, which depress the physical capital stock. Because liquidation costs are proportional to investment, they now are a function of the lending rate and the expected relative price of physical capital.

B.4.6 Monetary policy and the real economy

When investment can vary, the industrial organization of the banking sector and impairment to the interest rate channel has a direct effect on the production of physical capital from the real side of the economy. The equilibrium lending rate \(R_{L,t}\) will differ depending on whether banks actively compete across industries or act as local monopolists in segregated areas of the lending market.

If the economy is in a competitive equilibrium, an accommodative monetary policy through a decrease in the interest rate \(R\) will pass through to bank lending rates and spur investment. In the kinked equilibrium, however, a lower interest rate will have neither effect on the cost of bank credit or investment.

Indeed, a lower interest rate would have the perverse effect of decreasing investment in the kinked equilibrium. A decline in the interest rate would lower the financing costs of banks and encourage entry. In the kinked equilibrium, entry increases the lending rate, which lowers investment. Again, the perverse effect is in the longer run after time has elapsed for banks to enter the lending market.

A secondary effect of a lower interest rate is on the liquidation costs. A greater number of banks \(N_t\) and lower investment \(\iota_t\) narrow bank specialization and lowers aggregate liquidation costs. So the total effect of an accommodative monetary policy in the kinked equilibrium is a reduction in aggregate investment but also a reduction in liquidation costs.

With investment dependent on the cost of bank credit, the economy now features cyclical persistence that originates both from investment fluctuations out of the productive sector as well
as changes in the efficiency of projection liquidation out of the banking sector.