Bail-Ins, Optimal Regulation, and Crisis Resolution

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Abstract

We provide a contracting framework to understand bank bail-in regimes. In the presence of a monitoring problem, the optimal bank capital structure combines standard debt, which induces liquidation and provides strong incentives, and bail-in debt, which restores solvency but provides weaker incentives. Socially optimal policy increases use of bail-in debt when fire sales make liquidation socially costly. The social optimum can also be implemented using an ex post resolution authority. Although bail-ins replace bailouts of existing debt, they can destabilize bank refinancing during market stress, giving a role for temporary guarantees of new long-term debt in the crisis resolution toolkit.

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1 Introduction

In the aftermath of the 2008 financial crisis, the question of orderly bank resolution has received significant attention on both sides of the Atlantic. In many advanced economies, governments employed bailouts to stem financial turbulence in late 2008 and early 2009.1 Bailouts were arguably very effective at stabilizing financial markets, but have been criticized for leading to moral hazard and perverse redistribution.2 As a result, the US (Title II of the Dodd-Frank Act) and the EU (Bank Recovery and Resolution Directive) have introduced “bail-ins,” which allow the government to impose haircuts on (long-term) debt holders. The goals of bail-in regimes include ensuring that “creditors and shareholders will bear the losses of the financial company” and that “[n]o taxpayer funds shall be used to prevent the liquidation of any financial company under [Title II]” (Dodd-Frank Act Sections 204, 214). Nevertheless, important concerns arise from the introduction of bail-ins. If bank solvency can be improved by introducing state contingencies into debt contracts, then what prevents banks from efficiently doing so using private contracts?3 Moreover, why are bail-ins a preferable instrument to other liability instruments, such as (outside) equity, from a regulatory perspective? Lastly, is it indeed optimal for bail-ins to replace bailouts as the resolution tool for banks?

Studying these issues requires a framework in which debt is part of an optimal liability structure, so as to understand the impact of a regulator changing that liability structure. We provide such a framework in an optimal contracting model based on an incentive problem in the tradition of Innes (1990). Banks must monitor the quality of their loans both at the onset of the lending relationship and in its continuation. Because monitoring effort is not contractible at either stage, the optimal contract must incentivize monitoring, which involves the bank keeping a sufficient stake (“agency

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1Two examples in the US are the Troubled Asset Relief Program (TARP), which authorized the government to buy toxic bank assets, and the Temporary Liquidity Guarantee Program (TLGP), which provided guarantees of bank debt.

2The Dodd-Frank Wall Street Reform and Consumer Act (Dodd-Frank Act) lists “protect[ing] the American taxpayer by ending bailouts” as one of its main objectives, and lists “minimiz[ing] moral hazard” (Section 204) as one of the purposes of bail-ins.

3For example, banks could use contingent convertible (CoCo) securities that have gained traction in Europe, which are an internal recapitalization instrument with a trigger event (for example, the bank’s capital ratio falling below some threshold) for either a principal write-down or a conversion into equity.
rent”) in its loan performance not only ex ante but also in continuation. Banks write optimal liability contracts in a complete markets setting, but the contracts they write must respect the underlying incentive problem.

Our first main result is that the privately optimal bank contract can be implemented with a combination of two debt instruments: standard debt and bail-in debt. Standard debt has a face value that does not depend on the bank’s return, and leads to insolvency and liquidation when bank returns are low. This provides strong incentives to the bank for initial monitoring effort because liquidating the bank eliminates the agency rent that the bank would receive in the continuation monitoring problem. Standard debt therefore ensures the bank receives no payoff, but requires costly liquidation. In contrast, bail-in debt provides weaker incentives by transferring the entire cash flows to investors, except for the continuation agency rent. However by doing so, it avoids the resource costs of liquidation. Bail-in debt is thus useful because although it cushions against liquidation, it still provides maximal cash flow transfer on the downside. Both instruments retain the upside for the bank. Instruments such as outside equity transfer cash flows from the bank to investors when returns are high, and so discourage monitoring effort. As a result, the bank finds it optimal not to use such instruments.

We then use our model to study the design of socially optimal bank regulation in the presence of a fire sale externality – more bank liquidations reduce the recovery value to any individual bank in liquidation. The social planner uses a complete set of Pigouvian wedges to influence the bank’s choice of contract structure, internalizing the fire sale externality. In principle, this means that the social planner can incentivize the bank to write any other feasible alternative contract. For example, the planner could incentivize banks to issue outside equity.

Our second main result is that the social planner also finds it optimal for banks to write contracts that can be implemented with a combination of standard and bail-in debt. In other words, the planner chooses not to require banks to introduce other liability instruments such as outside equity.

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4Our model does not differentiate between standard short-term debt and ( uninsured) deposits, and standard debt could be interpreted as a deposit. It could also be interpreted as a repurchase agreement, where insolvency arises when the value of collateral falls sufficiently far that it no longer covers the debt.
equity into their capital structures. However, relative to the private optimum the social planner increases the use of bail-in debt and decreases the use of standard debt, in order to mitigate the fire sale.

We further show that the social optimum can be implemented with an ex post bail-in resolution authority, rather than ex ante contractual bail-in debt. In particular, the social optimum can be implemented with an ex ante capital structure that uses short-term senior non-bail-inable debt and long-term junior bail-inable debt, along with an ex-post resolution authority whose ex post objective is to choose Pareto efficient write downs of bail-inable debt that maximize total creditor recovery. We show that this authority implements the same write downs as arose under the ex ante contractual implementation, so that it achieves the same social optimum. This structure closely resembles the architecture of existing resolution regimes such as Title II, and so rationalizes their existing design.

The model helps to understand the role of bail-ins in the regulatory regime, as opposed to other forms of liability regulation such as equity requirements. Bail-in debt is less efficient at addressing the bank’s incentive problem than standard debt but avoids the social cost of bank failures. By contrast, (outside) equity worsens the bank incentive problem even further by giving away the upside of the bank to investors. This motivates a role for bail-in debt in the regulatory regime.

Our model rationalizes bail-ins as a socially desirable method of resolving distressed banks. However, the core optimal contracting framework of our model can also be applied to non-financial corporates. We provide an interpretation of bail-ins in our model as a Chapter 11 bankruptcy reorganization process, with liquidations corresponding to Chapter 7. If a planner were concerned about the possibility of fire sales arising from excessive liquidations of non-financials, our model can also be viewed as rationalizing intervention to promote reorganization under Chapter 11.

Finally, we study whether bailouts are complementary to bail-ins for recapitalizing insolvent banks in our framework. We first introduce taxpayer-financed bailouts in the model and ask whether a planner would find it desirable to commit to resolve some banks with bailouts. We show that the socially optimal contract is exactly the same as before, and the planner commits to never engage in bailouts. Bail-ins fully replace bailouts as the desired recapitalization instrument. The intuition
is that bail-ins and bailouts introduce the same state contingencies into bank debt contracts, with bail-ins achieving it through internal recapitalization by investors and bailouts achieving it through external recapitalization. This means that bailouts are simply resource transfers to banks and their investors and thus not Pareto efficient. This coincides with a core principle of post-crisis resolution, that the costs of bank resolution should be borne by bank investors and not by taxpayers.\(^5\)

However, we also show that the design of bail-in regimes leaves banks exposed to sunspot rollover crises. A rollover crisis is a self-fulfilling prophecy in which bail-in debt purchasers believe they are about to be bailed in and become unwilling to purchase newly issued debt, leading to bank failure and liquidation. Since the bail-in regime subordinates new issuances of bail-in debt to outstanding stocks of standard debt, bail-in debt receives no payoff in liquidation, justifying the equilibrium beliefs. Bail-ins give rise to refinancing instability for banks that are fundamentally solvent and can thus lead to inefficient bank failures. We show that an extension of temporary guarantees to new long-term debt during crisis times can help stabilize the market and prevent rollover crises, even though guarantees are not paid out on the equilibrium path. These proposals have precedent in programs used by the US government during the 2008 financial crisis; for example, the *Temporary Liquidity Guarantee Program* (TLGP), which extended debt guarantees to new issuances of long-term debt.

Taken together, our results rationalize bail-ins as an optimal regulatory regime and a replacement for bailout protections of legacy debt. However, they also imply that bail-ins can create instabilities for banks during times of market stress. As a result, “bailout” policies that protect new debt issuances are a desirable complement to a well-functioning bail-in regime, even while bailout policies that protect existing debt are not.

**Related Literature.** First, we relate to a growing literature on bail-ins.\(^6\) Keister and Mitkov (2021)

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\(^5\)See Sections 204 and 214 of the Dodd-Frank Act as cited above. The Dodd-Frank Act further states that “[t]axpayers shall bear no losses from the exercise of any authority under this title” (Section 214). See also e.g. French et al. (2010).

\(^6\)There are also related literatures on contingent debt instruments (Flannery (2002), Raviv (2004), Sundaresan and Wang (2015), with Flannery (2014) providing a broader overview) and optimal derivatives protection (Biais et al. (2016,2019)).
show that banks may not write down their (deposit) creditors if they anticipate government bailouts, motivating mandatory bail-ins, but that some bailouts may be optimal for insurance purposes. Chari and Kehoe (2016) use a costly state verification model and show that bail-ins are not required in the optimal regulatory regime. In their model, costly state verification implies that standard debt contracts are the only renegotiation-proof contracts, so that the possibility of bail-ins leads to a reduction in standard debt issuance but not the use of bail-in debt. Pandolfi (2021) studies a similar incentive problem to ours, based on Holmstrom and Tirole (1997), but takes standard debt contracts as given and has a different focus. Their paper shows that the prospect of bail-in resolution can sufficiently weaken bank monitoring incentives that the credit market collapses and banks cannot raise financing, with optimal policy being either commitment to liquidation or commitment to partial bailouts. Mendicino et al. (2018) numerically studies the optimal composition of bail-in debt and equity in the presence of both private benefit taking and risk shifting, taking contracts as given and with a regulatory objective of protecting insured deposits. Walther and White (2020) show that precautionary bail-ins of long-term debt can signal adverse information about a bank’s balance sheet and cause a bank run, leading to an overly weak bail-in regime and motivating bail-in rules based on public information. Bailouts that reduce required debt rollover alleviate the information friction and complement discretionary bail-ins. Colliard and Gromb (2018) study how government intervention via bail-ins and bailouts affects the negotiation process of distressed bank restructuring. Bolton and Oehmke (2019) studies the trade-off between single- and multi-point-of-entry in the resolution of global banks. Dewatripont and Tirole (2018) study how bail-ins can complement liquidity regulation. Berger et al. (2020) provides a quantitative study of bailouts versus bail-ins. Our main contribution to this literature is to provide a contracting framework based on an incentive problem in which the privately optimal contract can be implemented with a combination of standard and bail-in debt. We use this framework to rationalize an ex-post bail-in regime as an optimal policy when there are fire sales. We also show that temporary guarantees for new long-term (bail-inable) debt can be desirable, even while bailouts of existing debt is not desirable.

More broadly, we relate to a vast literature on theories of debt in both the banking and corporate
finance contexts. One agency view of debt is that debt provides incentives by transferring cash flows to investors when returns are bad, signaling low effort by the firm manager (Jensen and Meckling (1976), Innes (1990), Dewatripont and Tirole (1994), Hébert (2018)). A second view is that debt provides a threat from investors to withdraw funds and force liquidation (Calomiris and Kahn (1991), Diamond and Rajan (2001)). Our paper combines these two in a multi-period agency framework which builds upon Innes (1990), in which effort incentives are provided both by cash flow transfers and by threats to liquidation. Our paper is also related to Bolton and Scharfstein (1996), where default can be both strategic and non-strategic due to non-verifiable cash flows and cash flow diversion. In their model, debt that is easy to renegotiate leads to inefficient strategic defaults but prevents non-strategic defaults, whereas debt that is hard to renegotiate prevents strategic defaults but leads to inefficient non-strategic defaults. In our model, standard and bail-in debt differ in ex ante incentive effects rather than in ex post strategic default decisions.

A large literature studies macroprudential regulation in the presence of pecuniary externalities (Bianchi and Mendoza (2010,2018), Caballero and Krishnamurthy (2001), Dávila and Korinek (2018), Farhi et al. (2009), Lorenzoni (2008)), aggregate demand externalities (Farhi and Werning (2016), Korinek and Simsek (2016), Schmitt-Grohé and Uribe (2016)), and fiscal externalities (Chari and Kehoe (2016), Farhi and Tirole (2012)), which motivate ex ante interventions such as leverage requirements. Our optimal contracting model rationalizes government intervention which promotes use of bail-in debt (or an orderly resolution regime) over other loss absorbing instruments such as equity, and centers around the importance of the composition of debt (standard versus bail-in) rather than the overall debt level. This literature also finds regulation of debt can be complementary to bailouts, which introduce state-contingencies in otherwise non-contingent contracts (Bianchi (2016), Jeanne and Korinek (2020)). In our model bail-in debt allows for the same state contingent bank recapitalizations as bailouts, and the planner cannot achieve Pareto improvements through use of bailouts.

7There are many additional theories of debt, such as costly state verification (Townsend (1979)), liquidity provision (Diamond and Dybvig (1983)), and asymmetric information (Myers and Majluf (1984), Nachman and Noe (1994)). We also connect in particular to the related literature that emphasizes the monitoring role of banks (Diamond (1984), Holmstrom and Tirole (1997)).
Finally, a long literature has studied multiplicity in debt rollover and policy interventions to prevent multiplicity, such as deposit insurance. This includes self-fulfilling bank runs (Diamond and Dybvig (1983), Keister (2016)), self-fulfilling high interest rates (Calvo (1988)), and self-fulfilling maturity shortenings (He and Milbradt (2016)). Our rollover crisis equilibrium builds upon the Calvo (1988) tradition in that it results from high bail-in debt interest rates being self-fulfilling due to the combination of default costs and subordination of bail-in debt to standard debt. Our main contribution is to show how the properties of a bail-in regime generate this crisis, and to rationalize temporary guarantees of new issuances of long-term (bail-in) debt during times of market stress, rather than protections for only short-term debt or demand deposits.

2 Model

We develop a three-period model with three economic agents: banks, investors, and firms. Banks are run by their owners (inside equity). Banks sign contracts with investors to raise investment funds, which they lend to firms. We tailor the model to address the core trade-off of bail-ins: between standard debt and bail-in debt. Our baseline model will have no role for instruments such as outside equity, or for other trade-offs that affect the use of debt (e.g. tax benefits). We consider such extensions in the appendix.

The three-period economy, $t = 0, 1, 2$, has a unit continuum of banks and investors. Banks invest in a firm of variable scale $Y_0 = A_0 + I_0 > 0$ by using their own funds, $A_0 > 0$, and funds $I_0 \geq 0$ from (date 0) investors. Investors are deep-pocketed at date 0 and can finance any investment scale. Firms are penniless and have an outside option of zero, and we allocate the entire value of the bank-firm lending relationship to the bank.

Banks and investors are risk-neutral and do not discount the future. We denote bank consumption by $(c_0, c_1, c_2)$, so that bank expected utility is given by $E_0[c_0 + c_1 + c_2]$. We denote the payments to investors by $(x_1, x_2)$. $x_t$ is the actual amount received by investors, and is distinct from

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8A separate literature studies multiplicity that arises in contingent debt when the debt-equity conversion trigger is based on market value (Sundaresan and Wang (2015), Pennacchi and Tchisty (2019)).
the face value of liabilities (that is, promised repayment). Investor expected utility from the bank contract is 
\[ E_0 [-I_0 + x_1 + x_2]. \] Contracts are subject to limited liability constraints for banks, given by

\[ c_0, c_1, c_2 \geq 0. \] (1)

Limited liability is not required for investors, with \( x_1 < 0 \) denoting investors making a payment to the bank. However, the optimal contract of the model will result in non-negative investor payoffs.

The economy features idiosyncratic uncertainty, but no aggregate uncertainty.\(^9\)

### 2.1 Bank Projects

Banks extend financing to firms, thereby establishing a lending and monitoring relationship with those firms. When first extending funds to firms, banks monitor their borrowers, ensuring that the projects undertaken are of good quality. In doing so, banks develop specialized knowledge of that firm, and are uniquely able to monitor and collect from the firm in continuation. This relationship is the foundation of banking in our model. Because we allocate all value of the lending relationship to the bank, we omit firms going forward and refer to the relationship as bank projects.

Our model proceeds similarly to a multi-period version of Innes (1990). The bank project experiences a stochastic quality shock \( R \) at date 1, adjusting its scale to \( Y_1 = RY_0 \), at which point uncertainty is resolved. The shock \( R \) is idiosyncratic with a density \( f_e(R) \) that has support over \([R, \overline{R}]\). The state \( R \) is contractible, but the distribution of \( R \) depends on the bank’s non-contractible monitoring effort \( e \in \{H, L\} \), where \( e = H \) is high monitoring effort and \( e = L \) is low monitoring effort. \( f_e(R) \) satisfies the monotone likelihood ratio property (MLRP), that is \( \frac{\partial}{\partial R} \left[ \frac{f_H(R)}{f_L(R)} \right] < 0 \). MLRP is a standard assumption in generating debt contracts, and implies that high (low) returns are a signal that the bank exerted high (low) monitoring effort. However, if a bank exerts low monitoring effort, it receives a private benefit \( BY_0 \), with \( B > 0 \). We assume throughout the paper that the bank

\(^9\)See Appendix B.2 for aggregate uncertainty.
finds it optimal to write a contract that induces high monitoring effort, that is \( e = H \).\(^{10}\)

Because monitoring effort is non-contractible, the bank chooses \( e \) to maximize its utility after contracts have been signed. Given the consumption profile \((c_1, c_2)\) under the contract signed, the bank exerts high monitoring effort if

\[
\mathbb{E}[c_1(R) + c_2(R) | e = H] \geq \mathbb{E}[c_1(R) + c_2(R) | e = L] + B Y_0.
\]

We rearrange this incentive compatibility constraint to obtain the representation

\[
\mathbb{E} \left[ (c_1(R) + c_2(R)) (1 - \Lambda(R)) \right]_{e = H} \geq B Y_0, \tag{2}
\]

where \( \Lambda(R) \equiv \frac{f_L(R)}{f_H(R)} \) is the likelihood ratio. Higher payoffs \( c_1(R) + c_2(R) \) in states where the likelihood ratio \( \Lambda(R) \) is low relax incentive compatibility because these states signal that monitoring effort was high. Recall that monotone likelihood implies that \( \Lambda'(R) < 0 \).

Although the quality shock \( R \) is realized at date 1, the project does not mature until date 2 and yields no dividend at date 1. If the project survives to date 2, it generates 1 unit of the consumption good per unit of final scale, \( Y_2 = Y_1 \). However, the bank must again exert high monitoring effort to ensure that the project survives until date 2. If instead the bank shirks, it receives a private benefit \( bY_1 \), with \( b > 0 \), but the project fails and generates no cash flow. Therefore, at date 1 the bank must maintain an agency rent \( bY_2 \), with only the residual \( (1 - b)Y_2 \) being pledgeable to investors. Holding projects to maturity therefore implies a maximum pledgeability constraint,

\[
c_2 \geq bY_2. \tag{3}
\]

Because of limited liability \( c_1 \geq 0 \), this implies that a bank that continues to date 2 must receive total payoff of at least \( c_1 + c_2 \geq bY_2 \), meaning that total payoff to investors must satisfy \( x_1 + x_2 \leq (1 - b)Y_2 \).

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\(^{10}\)A sufficient condition for banks to find it optimal to write a contract inducing high effort is

\[
\mathbb{E}[R | e = L] + B < 1 < \mathbb{E}[R | e = H],
\]

in which case the project is NPV negative under low effort and NPV positive under high effort. If optimal contracts induced low effort, incentive compatibility would either not bind or would lead the bank to adopt a capital structure that rewarded the bank for low returns and punished it for high returns.
Banks can liquidate projects prematurely at date 1, in which case they yield $\gamma Y_1 < Y_1$ units of the consumption good at date 1 and nothing at date 2, with the proceeds accruing entirely to investors.\textsuperscript{11} We assume that $\gamma < 1 - b$, so that liquidating the project is not desirable from an investor repayment perspective and so are not desirable ex post. However, liquidations may be ex ante efficient for incentive reasons, since the bank can be paid 0 when liquidated but must be paid the agency rent $bY_2$ when not liquidated.

2.2 Bank Liabilities

In order to raise investment $I_0 \geq 0$, banks pledge state-contingent liabilities to investors at date 0, which promise a face value of $L_t(R)$ to be repaid to investors in period $t$. If $L_t(R) < 0$, then this is a payment promised to the bank from investors. Banks can issue debt at date 1 at a price of 1 to roll over a liability claim $L_1(R) > 0$. If banks pledge a total face value of liabilities in excess of pledgeable income, that is $L_1(R) + L_2(R) > (1 - b)Y_2$, then the bank is then unable to repay its liabilities in full after realizing return $R$. It therefore enters bankruptcy at date 1 and liquidates its assets.\textsuperscript{12}

Given a liability structure $L_t$, the resulting payoff profiles of banks and investors are

$$c_1(R) + c_2(R) = \begin{cases} RY_0 - L_1(R) - L_2(R), & \text{if } L_1(R) + L_2(R) \leq (1 - b)RY_0 \\ 0, & \text{otherwise} \end{cases}$$

(4)

$$x_1(R) + x_2(R) = \begin{cases} L_1(R) + L_2(R), & \text{if } L_1(R) + L_2(R) \leq (1 - b)RY_0 \\ \gamma RY_0, & \text{otherwise} \end{cases}$$

(5)

To understand this payoff profile, if $L_1(R) + L_2(R) \leq (1 - b)RY_0$, the bank can roll over its face

\textsuperscript{11}We think of the liquidation discount as arising from selling projects to second-best users, who have not developed the knowledge of the firm lending relationship that the bank has. In this sense, we also assume the banker is not severable from the bank.

\textsuperscript{12}We could obtain the same results by assuming the bank can both pledge liabilities and make a state-contingent commitment to liquidation. In fact, the proof of Proposition 2 is derived assuming banks can make state-contingent commitments to liquidation, and then deriving the liability structure that implements these state-contingent liquidations.
value of liabilities $L_1(R)$ by raising money from date 1 investors, who break even at the same face value $L_1(R)$. Date 0 investors receive $x_1(R) = L_1(R)$ and $x_2(R) = L_2(R)$, while the bank receives $c_1(R) + c_2(R) = Y_2 - L_1(R) - L_2(R)$. If instead $L_1(R) + L_2(R) > (1 - b)RY_0$, the face value of liabilities exceeds pledgeable income and the bank is liquidated, yielding payoffs $x_1(R) + x_2(R) = \gamma(s)RY_0$ and $c_1(R) + c_2(R) = 0$.

The voluntary investor participation constraint states that investors must at least break even in expectation on the contract they signed. It is given by

\[ Y_0 - A_0 \leq \mathbb{E}[x_1(R) + x_2(R) | e = H]. \]  

where $I_0 = Y_0 - A_0$ is the amount financed by investors.

Finally, we assume that total liabilities $L_1(R) + L_2(R)$ must be monotone, that is

\[ R \geq R' \Rightarrow L_1(R) + L_2(R) \geq L_1(R') + L_2(R'). \]  

Monotonicity is a common assumption in many settings of optimal contracts or security design, although in Appendix B.3 we characterize optimal contracts without monotonicity and argue our insights for optimal policy still hold.\(^{13}\) It generates the flat face value of liabilities in high-return states. Note that monotonicity does not preclude a bank from issuing an individual instrument whose payoff profile is non-monotone, but rather states that the overall liability structure summed across instruments must be monotone.

### 2.3 Bank Optimal Contracting

The bank signs a contract with investors, which specifies initial funds provided $I_0$ in exchange for liabilities $L_t$. The contract must be feasible, which we now define.

\(^{13}\)For example, one justification offered is that banks would be incentivized to pad their returns, for example by secretly borrowing from a third party (Nachman and Noe (1990,1994)).
**Definition 1** (Feasible Contracts). A bank contract $C = (L_t, I_0, c_1, x_t, Y_0)$ is *feasible* if it satisfies: (1) limited liability; (2) incentive compatibility; (3) limited pledgeability; (4) determination of $c_1 + c_2$; (5) determination of $x_1 + x_2$; (6) investor participation; and, (7) monotonicity.

Banks choose a feasible contract $C$ to maximize their own expected utility,

$$E[c_1(R) + c_2(R)|e = H].$$

**Liquidations and Ex Post Renegotiation.** Our model has ruled out ex post liability renegotiation at date 1 in cases where liabilities exceed assets and force liquidations. In our model, banks can write complete (state-contingent) contracts, and so can implement the outcome of any feasible ex post renegotiation. Thus, renegotiation could not increase ex ante welfare. However, renegotiation can create a time consistency problem: since $\gamma < 1 - b$, liquidations are *ex post* Pareto inefficient, meaning both the bank and investors have an incentive to renegotiate. Thus, if a bank ex ante finds it optimal to pledge a liability structure that results in liquidations, it would also prefer to rule out (or make as difficult as possible) renegotiation. For example, if debt is an optimal contract because it liquidates the bank, a bank might implement it using runnable demand deposits dispersed over many creditors in order to make renegotiation difficult. This is in keeping with parts of the banking literature that emphasize the value of demand deposits as a threat to liquidation (Calomiris and Kahn (1991), Diamond and Rajan (2001)). This idea is also consistent with the design of the Title II process, which focuses debt write-downs on long-term debt and not on short-term debt or deposits, due to a concern that “the threat of a restructuring may cause clients to flee and short-term creditors to withdraw their capital” (French et al. (2010)).

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14Moreover, Title II resolution includes a “clean holding company” requirement, which bars the top tier holding company (the target of resolution) from issuing any short-term debt to external investors (12 CFR §252.64).
3 Privately Optimal Contracts

In this section, we characterize the privately optimal contract written by banks in this environment. We show that the optimal contract can be implemented by a combination of two debt instruments. The first, which we call standard debt, has a fixed face value that does not depend on $R$, and liquidates the bank in low-return states. The second, which we call bail-in debt, has a face value that can be written down based on $R$, and restores bank solvency when total debt exceeds the pledgeable income.

We begin by characterizing the privately optimal bank contract in terms of two thresholds, $R_l$ and $R_u$. We then associate these two thresholds with the two debt instruments. These thresholds are sufficient statistics for the privately optimal liability structure of the bank.

**Proposition 2.** A privately optimal bank contract has a liability structure

$$L_1(R) + L_2(R) = \begin{cases} (1-b)R_l Y_0, & R \leq R_l \\ (1-b)R Y_0, & R_l \leq R \leq R_u \\ (1-b)R_u Y_0, & R_u \leq R \end{cases}$$

where $0 \leq R_l \leq R_u \leq \bar{R}$. The bank is liquidated if and only if $R \leq R_l$. These thresholds, when interior and not equal,\(^\text{15}\) are given by

$$\mu b (\Lambda(R_l) - 1) = b + \lambda (1 - b - \gamma)$$

\hspace{2cm} Incentive Provision \hspace{2cm} Liquidation Costs

$$0 = \mathbb{E} \left[ \lambda - 1 - \mu (1 - \Lambda(R)) \right] \quad R \geq R_u, e = H$$

\hspace{2cm} Investor Repayment \hspace{2cm} Incentive Provision

\(^\text{15}\)For the remainder of the paper, we assume that the thresholds are interior and not equal, except when explicitly stated otherwise. Generally speaking, $R_l$ will be interior when the likelihood ratio $\Lambda(\bar{R})$ is sufficiently large, that is when $\bar{R}$ is a sufficiently good signal of low effort. $R_u$ will be interior when $\Lambda(\bar{R})$ is sufficiently small and $\mu > \lambda - 1$, that is when $\bar{R}$ is a sufficiently good signal of high effort.
where \( \mu > 0 \) is the Lagrange multiplier on incentive compatibility (2) and \( \lambda > 1 \) is the Lagrange multiplier on investor participation (6).

All proofs are contained in Appendix A.\(^{16}\) The optimal liability contract is not unique in the sense that there are many maturity structures \((L_1, L_2)\) with the same total liabilities \(L_1 + L_2\) that implement the optimal contract. For example, one optimal contract is to make use only of short-term liabilities, in which case \(L_2(R) = 0\). We revisit this later when discussing practical implementations.

Before unpacking the properties of the privately optimal contract, we associate these two thresholds \(R_l\) and \(R_u\) with the two debt instruments that we discussed before. We associate \(R_l\) with standard debt and \(R_u\) with bail-in debt. To understand this terminology consider Figure 1, which illustrates the optimal liability contract \(L_1 + L_2\). There are three regions of the liability structure. The first region, where \(R \leq R_l\), is one where the total face value of liabilities is constant, but exceeds the pledgeable income of the bank. In this region, the bank is liquidated, and investors only receive partial repayment on the face value of their debt contracts. This is a standard debt contract.

In the third region, above \(R_u\), investors receive a constant payoff equal to the total face value of liabilities, so that \((1 - b)(R_u - R_l)Y_0\) corresponds to the total level of bail-in debt. What distinguishes bail-in debt from standard debt is that in the second region, where \(R_l \leq R \leq R_u\), the face value of bail-in debt is written down to \((1 - b)(R - R_l)Y_0\). This recapitalizes the bank and allows it to continue operating, rather than being liquidated. We refer to this as bail-in debt because its face value can be written down (“bailed in”) based on the idiosyncratic state.

**Corollary 3.** The privately optimal contract can be implemented with a combination of standard debt with face value \((1 - b)R_lY_0\), which cannot be written down contingent on the idiosyncratic state \(R\), and bail-in debt with face value \((1 - b)(R_u - R_l)Y_0\), which can be written down contingent on the idiosyncratic state.

\(^{16}\)In the proof of this proposition, see Appendix A.1.1 for a comment on non-uniqueness of total face value of liabilities \(L_1(R) + L_2(R)\) below the threshold \(R_l\). Non-uniqueness arises in this region because any face value of liabilities above \((1 - b)RY_0\) results in bank liquidation. We have chosen the face value of liabilities that correspond to standard debt, which seems most natural in the context of banks and bail-ins. Moreover, uniqueness is restored if there is an \(\varepsilon \to 0\) premium for standard debt, for example due to tax benefits of debt. The face value of liabilities is unique above \(R_l\).
For the remainder of the paper, we associate standard and bail-in debt with the thresholds $R_l$ and $R_u$, respectively, rather than writing out their associated (face value) liabilities.

The core property of standard debt is that it forces liquidations in low-return states. Equation (9) describes the marginal trade-off the banker faces in replacing a unit of bail-in debt with a unit of standard debt. On the one hand, liquidating the bank results in a total resource loss $b + \lambda (1 - b - \gamma)$ to the bank and investors. On the other hand, pledging to liquidate the bank provides higher-powered monitoring incentives at date 0, reflected in the term $\mu b (\Lambda(R) - 1)$, by depriving the bank of the non-pledgeable income $b R Y_0$. In particular, the liquidation threshold features $\Lambda(R_l) > 1$, that is at $R_l$ the likelihood ratio $\frac{f_L(R_l)}{f_H(R_l)}$ is greater than 1. This implies that the state $R_l$ provides a stronger signal that low effort may have been exerted.

By contrast, for a given level of standard debt, an additional unit of bail-in debt does not change liquidations but does transfer value from banks to investors. The marginal trade-off is summarized in equation (10). On the one hand, the binding investor participation constraint implies this transfer is valuable ($\lambda - 1 > 0$), as it allows the bank to increase project scale. On the other hand, increasing the total debt level reduces bank consumption in high-return states, where the likelihood ratio $\Lambda(R)$ is low and the signal of high effort is stronger. This weakens bank monitoring incentives and tightens the incentive compatibility constraint (2). The optimal level of bail-in debt equalizes these two effects on the margin.

**Alternate Contract Implementations.** Proposition 2 and Corollary 3 express an implementation of the privately optimal contract using a combination of (short-term) standard and (short-term) bail-in debt, but this is not the unique implementation of the optimal contract in our model. First, the maturity structure of the optimal contract is not unique. The implementation closest to bail-in regimes in practice involves issuing $R_l$ in short-term standard debt ($L_1(R)$) and $R_u - R_l$ in

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17 Bail-in debt can also be interpreted as a contingent convertible (CoCo) debt instrument, which is a contractual bail-in instrument that has gained prominence in Europe. See Avdjiev et al. (2017) and Flannery (2014) for more background on CoCos. The most natural interpretation in this context is that bail-in debt in our model is a principal write-down CoCo debt security that applies at the point of non-viability. However, because uncertainty resolves at date 1, bail-in debt can also be interpreted as a CoCo with a debt-equity conversion, rather than principal write-down.
long-term bail-in debt \((L_2(R))\),\(^{18}\) but the privately optimal liability structure could also, for example, be implemented entirely using short-term debt. Second, the privately optimal contract of Proposition 2 can also be implemented using different instruments entirely. Two such examples are: (i) using a combination of standard debt and outside equity, with a managerial compensation scheme that implemented the same payoff profile \(c_1 + c_2\) as arose under Proposition 2.\(^{19}\); (ii) using a single partially bail-inable debt instrument, which has face value \(R_u\) and can only be written down to \(R_l\).

3.1 The Role of Agency Problems and Costly Liquidation

Our model features three ingredients that are jointly necessary to generate contracts that consist of combinations of standard and bail-in debt: the ex ante incentive problem \((B > 0)\), limited pledgeability \((b > 0)\), and costly liquidations \((\gamma < 1)\). In the absence of any one of these elements, contracts in our model would not combine standard and bail-in debt.

**Proposition 4.** The privately optimal contract can be implemented with a single liability instrument if \(B = 0\), \(b = 0\), or \(\gamma = 1\). In particular,

1. If \(B = 0\), then the privately optimal contract can be implemented with (outside) equity.

2. If \(b = 0\), then the privately optimal contract can be implemented with bail-in debt.

3. If \(\gamma = 1\), then the privately optimal contract can be implemented with standard debt.

When \(B = 0\), a standard Modigliani-Miller logic applies. The bank can ensure incentive compatibility with any monotone consumption policy for the bank, and in particular does not need to use liquidations as a disciplining device. As a result, without loss of generality the bank can employ entirely (outside) equity financing. In this case, limited pledgeability serves only to restrict the amount of outside equity that can be pledged, that is a fraction \(b\) of total equity must be retained.

\(^{18}\)For example, in the US the top-tier bank holding company is subject to a “clean holding company” requirement, which bars it from issuing short-term debt to external investors. See 12 CFR §252.64.

\(^{19}\)Under this implementation, outside equity would have the same payoff profile bail-in debt did in Corollary 3.
by the bank. As a result, a date 0 incentive problem, with $B > 0$, is necessary in the current model to generate an optimal contract that combines standard and bail-in debt.

The second and third cases of Proposition 4 show that $B > 0$ alone is not sufficient to generate a privately optimal contract that combines standard and bail-in debt. When $B > 0$, the privately optimal contract employs some debt instrument for ex ante incentive reasons. In the second case with $b = 0$, all income is pledgeable to investors, and the bank can guarantee zero consumption, $c_1(R) = c_2(R) = 0$, without having to liquidate prior to maturity. As a result, the bank finds it optimal to only use bail-in debt. In contrast in the third case, with $\gamma = 1$ but $b > 0$, there is a limit to pledgeable income, but no bankruptcy costs from liquidation. Banks can repay any amount $L_1(R) \leq RY_0$ by liquidating bank projects, and the pledgeability constraint ceases to be relevant. Banks use only standard debt.

In both the second and third cases, the key property of debt is the full cash flow transfer from the bank to investors in low-return states. This corresponds to a common understanding of debt in the optimal contracting literature: the core property of debt is its payoff profile $x_1(R) = \min\{RY_0, R_uY_0\}$ to investors (see e.g. Hébert (2018)). In the absence of pledgeability limitations, this value transfer is achieved with bail-in debt. In the absence of bankruptcy costs, this value transfer is achieved with standard debt.

However, if there are both limited pledgeability and bankruptcy costs, then bail-in debt cannot enact a full cash flow transfer, while standard debt enacts a full cash flow transfer at a resource cost. A role emerges for both forms of debt in the optimal contract.

4 Optimal Policy

In this section, we study optimal policy. We do so in the context of a fire sale externality, which is a common motivation for studying government intervention in regulation and bailouts. The planner of our model has a complete set of regulatory wedges, and so can incentivize the bank to adopt any feasible capital structure, for example requiring issuance of outside equity as a loss
absorbing instrument. Nevertheless, we show that the social planner finds it optimal for banks to write contracts that combine standard and bail-in debt, so that bail-in debt is the socially optimal loss absorbing instrument. Relative to private banks, the planner prefers greater use of bail-in debt to mitigate the fire sale. We use our results to rationalize the current design of ex post bail-in resolution.

4.1 Arbitrageurs and Liquidation Prices

We introduce to the economy a unit continuum of arbitrageurs, who at date 1 can purchase assets liquidated by banks at date 1. In particular, at date 1 arbitrageurs can convert bank products into the consumption good using a production technology \( F(\Omega)Y_0 \), where \( \Omega \) is the economy-wide fraction of bank projects purchased relative to the initial scale. Arbitrageur surplus at date 1 from purchasing projects is \( F(\Omega)Y_0 - \gamma \Omega Y_0 \), yielding an equilibrium liquidation price

\[
\gamma = \gamma(\Omega) = \frac{\partial F}{\partial \Omega}, \quad \Omega = \int_R \alpha(R)f_H(R)dR
\]  

(11)

where \( \alpha(R) \in \{0, 1\} \) indicates whether or not a bank that realizes the idiosyncratic state \( R \) liquidates its assets, with \( \alpha(R) = 1 \) denoting liquidation. From the setup in Section 2, we have \( \alpha(R) = 1_{L_1(R) + L_2(R) > (1 - b)R_i Y_0} \). Note that the baseline model can be interpreted as the case where \( \frac{\partial F}{\partial \Omega} = \gamma \) is a constant which does not depend on \( \Omega \). By contrast when \( \frac{\partial \gamma}{\partial \Omega} = \frac{\partial^2 F}{\partial \Omega^2} < 0 \), there is a fire sale spillover from bank liquidations: more liquidations reduce the liquidation value.

Arbitrageurs have initial wealth \( \overline{A} - A_0 \), but cannot borrow against future income. Their total date 0 welfare is \( u(\overline{A} - A_0) + (F(\Omega)Y_0 - \gamma \Omega Y_0) \), with \( u'(\overline{A}) > 1 \) so that the borrowing constraint binds. The intertemporal borrowing constraint gives rise to a distributive externality (Dávila and Korinek (2018)) that makes fire sales Pareto inefficient (see Appendix B.1). The inefficient distributive externality arises because the borrowing constraint creates means arbitrageurs have a higher marginal value of wealth at date 0 than at date 1.\(^{20}\)

\(^{20}\)This externality is similar to the case where there are multiple date 1 aggregate states, and incomplete markets prevent arbitrageurs from equating the marginal value of wealth across date 1 states.
4.2 Social Optimum

Banks solve the same problem as in Section 3, taking as given the equilibrium price $\gamma$. Private contracts follow the same optimal form as in Proposition 2. By contrast, we now consider optimal regulation of a social planner. The social planner possesses a complete set of “Pigouvian” regulatory wedges (or taxes) $\tau$ on the contract terms $C = (L_t, I_0, c_t, x_t, Y_0)$ of banks. Wedges are fully contingent, so that for example $\tau^L_t(R)$ is the wedge placed on liability claim $L_t(R)$. Wedge revenues are remitted lump-sum to banks at date 0, that is the project scale of banks can be written as $Y_0 = A_0 + I_0 + T^* - \tau C$, where $T^* = \tau^* C^*$ is equilibrium revenue collected and remitted. We have adopted inner product notation to simplify exposition, where for example $\tau^L_t L_t = \mathbb{E}[\tau^L_t(R)L_t(R)|e = H]$. Given the planner possesses a complete set of Pigouvian wedges, the planner can incentivize banks to write any feasible contract (satisfying Definition 1) through appropriate choice of wedges $\tau$. We solve directly for the optimal contract $C$ chosen by the social planner, rather than its decentralization $\tau$.

The social welfare function of the planner is bank welfare (8), that is a welfare weight of 0 is assigned to arbitrageurs. In Appendix B.1, we show that the qualitative properties of the social optimum are the same even with positive welfare weights on arbitrageurs.\footnote{\textit{In particular, fire sale spillover term in equation (12) is still positive but is lower in magnitude due to arbitrageur surplus from liquidations. To achieve Pareto efficiency, the social optimum combines a reduction in standard debt with a lump sum transfer from banks to arbitrageurs at date 0 to compensate them for losses on purchases.}} The planner’s problem is to choose a feasible contract $C$ to maximize social welfare, internalizing the equilibrium pricing relationship (11).

In principle, the planner can choose a contract of any feasible form, even if that contractual form differs from that chosen privately by banks. For example, the planner might prefer banks to choose a contract featuring outside equity. The following result characterizes the socially optimal contract.
Proposition 5. A socially optimal bank contract has a liability structure

\[ L_1(R) + L_2(R) = \begin{cases} 
(1-b)R Y_0, & R \leq R_l \\
(1-b)R Y_0, & R_l \leq R \leq R_u \\
(1-b)R_u Y_0, & R_u \leq R 
\end{cases} \]

The threshold \( R_l \) is given by

\[ \mu b (\Lambda(R_l) - 1) = b + \lambda (1-b - \gamma) + \lambda \left| \frac{\partial \gamma(\Omega)}{\partial \Omega} \right| \Omega \]

where \( \omega^* \equiv 1 - \frac{1}{u'(A-A_0)} > 0 \). The threshold \( R_u \) is given by equation (10).

Therefore, the socially optimal contract can be implemented with a combination of standard and bail-in debt.

Even though the social planner has the ability to write any feasible contract \( \mathcal{C} \), such as requiring some issuance of outside equity, Proposition 5 shows that the social planner finds it optimal to write a contract of the same structural form as private banks chose. That is to say, the socially optimal contract, like the privately optimal contract, can be implemented with a combination of standard and bail-in debt. The planner thus agrees with the bank that the optimal capital structure should make use of these two debt instruments, and not other instruments such as outside equity.

Even though the planner uses the same debt instruments as the bank, the fire sale spillover results in an additional social cost of liquidation in the planner’s optimality condition for \( R_l \): the project liquidations of one bank increase the resource loss to all other banks that liquidate projects at the same depressed prices. This liquidation cost term represents the only difference between the private and social optimality conditions in equations (9) and (12), respectively. By contrast, there is no additional wedge in the determination of \( R_u \), since a greater total debt level arising from more bail-in debt does not change total liquidations. Relative to private banks, the planner does not disagree with the bank over the optimal total amount of debt \( R_u \), but rather disagrees with the
bank on the optimal composition, preferring the bank replace some of its standard debt instead with bail-in debt.

4.3 Ex-post (Bail-in) Resolution as Optimal Policy.

Proposition 5 shows that the planner, designing ex-ante optimal regulation, implements an optimal contract for banks that combines standard and bail-in debt. Bail-in debt in Proposition 5 involves contractual provisions for write-downs, that is write-downs are pre-specified at the time the contract is written. Bail-ins are commonly implemented in practice via an ex post resolution authority: the planner takes a debt contract with a fixed face value, and writes down that face value ex post. Both forms of authority are used in practice, with the US emphasizing ex-post resolution and the EU being more accommodating of contractual recapitalization.\footnote{In the US, banks are required to maintain a certain level of total loss-absorbing capital (TLAC), principally long-term debt and equity, to safeguard the bank against poor returns. Debt used to satisfy TLAC requirements must be plain-vanilla, implying a fixed face value, while debt with contractual contingencies cannot generally be used to satisfy TLAC requirements. In particular, “eligible external LTD [is] prohibited from including contractual triggers for conversion into or exchange for equity.” 82 FR 8266. See Avdjiev et al. (2017) for background on the European case.}

We now show that there is an implementation of the social optimum which makes use an of ex-post bail-in resolution authority. We define the ex-post bail-in authority as follows. The ex-post bail-in authority has discretion to impose write downs on liability contracts that are designated “bail-inable,” but cannot impose write downs on contracts that are designated “non-bail-inable.” The objective of the ex-post bail-in authority is, at the level of the individual bank (i.e. taking equilibrium prices as given), to maximize total recovery value to creditors, subject to write downs being Pareto efficient. In the implementation that follows, it will be necessary to allow for debt seniority.

Corollary 6. \textit{The social optimum can be implemented by establishing an ex post bail-in authority, and by using a combination of }\( R_l \) of non-bail-inable senior short-term debt and \( R_u - R_l \) of bail-inable junior long-term debt, where \( R_l \) and \( R_u \) are given as in Proposition 5.\textit{ }

Corollary 6 provides an implementation of the social optimum using a bail-in regime using an
ex-post resolution authority. To understand why this implements the outcome of Proposition 5, any insolvent bank $R_l \leq R < R_u$ is resolved by the bail-in authority by imposing write downs on the bail-inable debt, to the level $R - R_l$. These are the minimal write downs that recapitalize the bank, and so maximizes creditor recovery. Moreover, this policy is Pareto efficient since bail-inable debt is junior to non-bail-inable debt, and hence all creditors and the bank benefit from avoiding liquidation. Thus, the bail-in authority implements the same outcome ex post as contractual bail-in debt. By contrast if $R > R_u$ then the bail-in authority’s objective is maximized without imposing write downs, whereas if $R < R_l$ the bail in authority lacks the ability to write down sufficient bail-inable debt to recapitalize the bank.\(^{23}\)

The implementation of optimal policy in Corollary 6 is consistent with the design of bail-in regimes in practice, for example Title II. First, bail-ins are applied to long-term debt, rather than to short-term debt. Second, bail-in regimes subordinate bail-inable long-term debt to non-bail-inable long-term debt, that is short-term debt enjoys absolute priority in bankruptcy, liquidation, and resolution.\(^{24}\) Finally, the objective of Pareto efficiency is consistent with the No Creditor Worse Off principle of bank resolution (BRRD Article 73).

### 4.4 Bail-in Debt or Equity?

Proposition 2 highlights why standard debt can be a valuable loss-absorbing instrument for banks, relative to equity. Bail-in debt combines the incentive properties of standard debt with the loss-absorbing properties of equity. It generates a cash flow transfer below $R_u(s)$ and a flat investor payoff above $R_u(s)$, similar to standard debt, but does so without liquidating the bank. By contrast, equity transfers the upside of the bank to investors, which worsens incentives. Bail-in debt therefore

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\(^{23}\)In our model, bank fundamentals $R$ are common knowledge, so there is no informational time consistency problem as in Walther and White (2020).

\(^{24}\)In practice, short-term debt priority has three implementations. The first is contractual: bail-in debt is junior to short-term debt. The second is organizational: short-term debt is issued at the operating subsidiary, whereas long-term debt is issued at the top-tier holding company. The third is legal: national bankruptcy law confers priority to short-term debt in the case of banks. The US induces seniority through organizational form, and we could implement the US approach under Corollary 6 by assuming that bail-inable debt is held at a resolvable holding company, whereas non-bail-inable is held at a non-resolvable operating subsidiary.
achieves a capital structure that standard debt and equity combined cannot. Under the incentive problem of the baseline model, banks prefer bail-in debt to equity as a loss-absorbing instrument.

As in the privately optimal contract, bail-in debt, rather than equity, is used as the loss absorbing instrument in the social optimum of Proposition 5. Although the social planner and the bank disagree about the costs of bankruptcy, they agree about the underlying incentive problem. As a result, the planner replaces standard debt with bail-in debt, which addresses the underlying incentive problem that standard debt was designed to solve. This provides a role for bail-in debt rather than equity.25

4.5 Relationship to Chapter 7 and Chapter 11 Bankruptcy

In practice, bail-in debt is generally associated with banks. Our model helps to understand why the government would set up a controlled resolution process for banks when it is concerned out externalities resulting from fire sales. However, the core optimal contracting model of the paper could also be applied to non-financial corporates. In principle, this suggests that non-financial corporates might also wish to use bail-in debt. One interpretation in this spirit can be provided in the context of Chapter 11 of the US Bankruptcy Code. Chapter 11 provides a reorganization and debt restructuring process for non-financials, allowing them to avoid liquidation under Chapter 7. The ex post bail-in regime of Corollary 6 could also be interpreted as a Chapter 11 process. Thus if a government were also concerned about fire sales resulting from excessive liquidations of non-financials under Chapter 7, our model could also be viewed as rationalizing government intervention to promote firm reorganization under Chapter 11. In particular, our model would rationalize intervention in the non-financial corporate bankruptcy process rather than other interventions such as equity capital requirements.26

One important consideration is that the design of Chapter 11 may simply not be appropriate for banks due to the financial nature of their activities (French et al. (2010)). Reflecting this, the US

25In Appendix B.6 we add a role for outside equity in the model by incorporating risk aversion and risk shifting. We show that the core trade-off between standard debt and bail-in debt exists as in the baseline model.

26See Antill and Clayton (2021) for a related analysis of optimal intervention in the insolvency process for non-financials.
Treasury Department has adopted a proposal for a Chapter 14 bankruptcy process, with the aim of creating a process in the spirit of Chapter 11 that is tailored to banks (Scott and Taylor (2012), US Department of Treasury (2018)). Our model predicts that banks would privately under-utilize Chapter 14 relative to the social optimum by over-issuing standard (non-resolvable) debt, leaving a role for government intervention to promote resolvability by increasing use of bail-in (resolvable) debt.

5 Bail-Ins or Bailouts?

One of the goals of policymakers in designing bail-in regimes has been to replace bailouts as the method of preventing costly bank liquidations. Reflecting this, post-crisis regulatory reforms have included restrictions on governments’ ability to engage in bailout or fiscal backstop measures employed during the crisis. This has raised concerns that the prospect of bail-ins may have destabilizing effects on markets during times of crisis, absent such backstop measures.27

In this section, we study the interplay between bail-ins and bailouts from these two policy perspectives. Concretely, we ask whether bailouts can be complementary to bail-ins both in preventing costly fire sales and in preventing market instabilities that can arise during times of crisis.28 We first show that a planner with the ability to commit to bailout policies chooses never to bail out banks to prevent costly liquidations, instead relying solely on bail-ins, despite the presence of fire sales. This helps to rationalize the goal of bail-in regimes to replace bailouts with bail-ins. We next show that the optimal bail-in regime can trigger a rollover crisis for banks during times of market stress. A rollover crisis is a sunspot equilibrium in the date 1 rollover problem that results in inefficient liquidation of otherwise solvent banks. Temporary guarantees of new issuances of long-term debt during a crisis can rule out the sunspot equilibrium and complement bail-in

27See Geithner (2016) for a policy discussion of both of these points, for example noting that “Congress...took away the FDIC’s discretion to guarantee the broader liabilities of banks and bank holding companies. This guarantee was critical in the fall of 2008 to limiting the run on the U.S. banking system.”

28See the related prior work studying complementarities between macroprudential regulation and bailouts (e.g. Bianchi (2016), Jeanne and Korinek (2020)).
resolution of insolvent banks. Our results imply an important distinction between existing debt, which should be bailed in rather than bailed out, and new debt issuances during a crisis, which should be protected by guarantees.

5.1 Preventing Fire Sales

In this subsection, we study whether bailouts are complementary to bail-ins in preventing costly liquidations and fire sales. To do so, we introduce the possibility of bailouts into the model of Section 4. We show that the planner prefers to rely solely on bail-ins to resolve distressed banks, meaning that bail-ins fully replace bailouts.

Formally, we extend the model of Section 4 by assuming the planner can commit to bail out insolvent banks in order to prevent liquidation. Commitment over bailouts is consistent with commitment over the liabilities contract, that is both bail-ins and bailouts are designed under commitment. The bailout that recapitalizes an insolvent bank with return \( R \) is

\[
T_1(R) = L_1(R) + L_2(R) - (1 - b)RY_0,
\]

which allows the bank to meet its liabilities and pay its agency rent. Bailout funds are raised from taxpayers. Taxpayers have utility

\[
u_0(c_{T0}) + u_1(c_{T1}),
\]

where

\[
c_{T0} = A_{T0} + T_0 + B_0
\]

and

\[
c_{T1} = A_{T1} - B_0 - \int R T_1(R) f_H(R) dR.
\]

\( B_0 \) is taxpayer savings, and \( \int R T_1(R) f_H(R) dR \) is the total tax burden resulting from bailouts. \( T_0 \) is a possible lump sum transfer at date 0 from banks to taxpayers, which may be required to achieve a Pareto improvement if the social optimum features bailouts.

We study the Pareto efficient problem characterized by choosing a feasible contract \( C \) and bailout rules \( T_1 \) to maximize social welfare, with a welfare weight \( \omega^T \) placed on taxpayers. Thus, the problem is the same as in Section 4, except for the addition of the bailout instrument and the presence of taxpayers in the social welfare function. The following Proposition shows that the outcome is the same as Proposition 5.

**Proposition 7.** In the model with bailouts, the socially optimal contract is the same as in Proposition 5. No bailouts occur, that is \( T_1(R) = 0 \) for all \( R \).

Proposition 7 shows that a planner with commitment always chooses to use bail-ins, rather than
bailouts, in order to resolve distressed banks. This result can be understood as follows. Consider the 
marginal insolvent bank, $R \uparrow R_l$. Suppose first that this bank is resolved with a bail-in, which entails 
a marginal reduction in $R_l$ while leaving $R_u$ fixed. However, the characterization of Proposition 5 
has already specified the optimal bail-in threshold at $R_l$. As a result, by construction there is no 
value to making the marginal insolvent bank solvent via bail-in.

Now, suppose instead that the planner resolved this bank with a bailout. There are two effects 
of a bailout. First, the insolvency threshold is lowered, that is $R_l$ effectively falls. But as just argued, 
a reduction in $R_l$ is not welfare enhancing at the optimum of Proposition 5, so this does not produce 
welfare gains from the bailout. Second, a bailout also generates a resource transfer from taxpayers to 
banks. The marginal social benefit of the resource transfer is the value of relaxing the participation 
constraint, given by the Lagrange multiplier $\lambda$. The marginal social cost of the resource transfer is 
the burden to taxpayers, $\omega^T u_1'(c_1^T)$. However, at a Pareto efficient allocation, the date 0 value of 
resources is equalized across agents, that is $\lambda = \omega^T u_0'(c_0^T)$, while taxpayer optimization implies 
that $u_0'(c_0^T) = u_1'(c_1^T)$. Combining these, we have $\lambda = \omega^T u_1'(c_1^T)$, meaning that the net welfare gain 
from the resource transfer is also zero. As a result, there is no social gain from engaging in a bailout 
of the marginal inslvent bank, and the social optimum is achieved at $T_1(R_l) = 0$. The remainder 
of the argument then follows simply by noting that for all banks $R < R_l$, the first effect is negative 
whereas the second effect is 0.

**Optimal Bailouts.** In Proposition 7, bailouts are not optimal because: (i) bail-ins can achieve 
the same liquidation rule as bailouts; and, (ii) ex-post resource transfers from taxpayers to 
banks/investors do not improve welfare. However, bailouts can be efficient if banks and their 
investors have a greater value of consumption at date 1 than taxpayers, but incomplete markets 
prevents a contract between them. Formally, we can write this as assuming that taxpayers are 
borrowing constrained, $B \leq 0$, in which case $u_0'(c_0^T) \geq u_0'(c_1^T)$. In the case of strict inequality, a 
Pareto improvement can be attained by coupling ex-post bailouts with an ex-ante lump sum transfer 
from banks to taxpayers, effectively circumventing the borrowing constraint. The purpose here of
bailouts is intertemporal consumption smoothing (insurance), rather than preventing fire sales. This connects with results in several papers that rationalize bailouts as arising from insuring risk averse investors who cannot directly contract with taxpayers, and can rationalize instruments like insured deposits (Dewatripont and Tirole (2018), Farhi and Tirole (2020), Keister and Mitkov (2021)). In conjunction with this, in Appendix B.8 we show that a social planner who allows for insured deposits may also find it optimal to engage in partial bailouts in order to reduce costs to the deposit insurance scheme. In this setting, we show that even if bailouts are used to protect insured deposits, all other bank debt is bailed in. In other words, bailouts only protect insured deposits.

5.2 Preventing Rollover Crises

In this subsection, we study whether bailouts are complementary to bail-ins in preventing market instabilities during a crisis. We show that the prospect of bail-ins can lead to a sunspot rollover crisis: long-term bail-in debt holders believe they will be bailed in, and so are unwilling to extend financing to a bank nearing distress, resulting in a run. Guarantees of new issuances of long-term debt during a crisis can rule out the sunspot equilibrium and complement bail-ins in safeguarding financial stability.

We study the refinancing problem of a bank at date 1, which in the baseline model we assumed was frictionless. To generate rollover crises, we introduce a notion of fragility in the date 1 economy. In particular, we introduce a shock and insolvency cost at date 2, which will motivate the bank to replace its outstanding standard debt with bail-in debt when refinancing at date 1. Formally, banks experience a second and independent quality shock at date 2, \( R_2 \sim F_2 \) with support \([0, \bar{R}]\) and \( \mathbb{E}[R_2] = 1 \). If the bank is solvent at date 2, it pays off \( Y_2 = R_2 Y_1 \), while if it is insolvent it pays off \( \gamma Y_2 \). As before, limited pledgeability at date 1 means that banks must maintain at least a share \( bY_1 \) of the final expected project value, so that \((1 - b)Y_1\) is maximum pledgeable repayment to investors. For simplicity there is no fire sale. This model admits the same optimal contract as under Proposition 2 in the investor repayment space.\(^{29}\)

\(^{29}\)Given the stochastic shock in continuation, one possible implementation of bail-ins here is through a debt-equity
We consider the refinancing problem at date 1 of the threshold solvent bank, \( R = R_l \). This bank’s long-term bail-in debt has been fully written down at date 1, but it still must roll over its standard short-term debt \( D_1 = (1 - b)R_lY_0 \). Given the insolvency cost and that the support of \( R_2 \) extends to 0, there is a unique securities package \((D_1, L_1) = (0, L_1^*)\) of short-term debt and bail-in debt that can recapitalize the bank while maintaining the banks’ agency rent. The market quotes the bank a price for this securities package, \( Q_1 \).

We now show that there are two equilibria associated with this securities package issuance: a good equilibrium with successful refinancing, and a rollover crisis equilibrium that results in bank failure.

**Proposition 8.** There are two equilibria for the marginal solvent bank \( R_l \):

1. A good equilibrium with \( Q_1 = D_0 \), in which the bank successfully refines itself at date 1.

2. A rollover crisis equilibrium with \( Q_1 = 0 \), in which bank refinancing at date 1 is unsuccessful and the bank is forced to liquidate.

Proposition 8 documents the existence of both a good equilibrium and a rollover crisis equilibrium for the marginal solvent bank. Under the good equilibrium with successful refinancing, the bank is quoted the fair price for its securities issuance of bail-in debt, allowing it to recapitalize itself. However, there is also a sunspot equilibrium in which markets quote a price of 0 for the package \((0, L_1^*)\). In this equilibrium, prospective purchasers of new bail-in debt expect to be bailed in, and so quote a price of 0 for new bail-in debt. Being unable to raise new financing, the bank is forced into liquidation by its existing short-term standard debt creditors. In the liquidation, new bail-in debt is subordinated to the existing stock of standard debt due to the bail-in regime (Corollary 6), meaning that it receives no recovery value in liquidation. This rationalizes the equilibrium price of 0, completing the equilibrium. The rollover crisis equilibrium arises because the bail-in regime conversion for bail-in debt at date 1, rather than a principal write-down as in the baseline model. We could also express this model as assuming that the project does not mature until date 3, with final payoff \( Y_3 = Y_2 \) at date 3 or early liquidation payoff \( \gamma Y_2 \) at date 2, and obtain the same results.
confers explicit priority to standard debt over bail-in debt. The conferred priority applies not only under bail-in resolution, but also under liquidation, leading to the instability.

**Private Solutions to Rollover Crises.** There are at least two possible private solutions to rollover crises in our model. The first is to implement the optimal contract by using a method other than short-term debt, which must be rolled over, to enforce liquidation after low returns. For example, the optimal contract could in theory be implemented using entirely long-term debt, along with a contractual specification of when liquidation would occur. The second would be for the bank to use a line of credit. The rollover crisis equilibrium arises because the rollover price at date 1 cannot be contracted upon, meaning one solution is to contractually prespecify the price at which rollover occurs. This resembles a line of credit, which would specify that in states $R \geq R_l$, the bank can borrow the prices and quantities of the good equilibrium. Importantly although the line of credit can rule out rollover crises, the availability and terms of the line of credit are state contingent.

### 5.2.1 Debt Guarantees as a Solution to Rollover Crises

Proposition 8 showed that the optimal bail-in regime is susceptible to rollover crises at date 1. We now show that temporary guarantees of new issuances of bail-in debt at date 1 can rule out the sunspot equilibrium, while not being paid out on the equilibrium path. This policy has precedent in the Temporary Liquidity Guarantee Program (TLGP) instituted in 2008, under which the US government provided guarantees to new issuances of senior unsecured debt with the goal of “preserving confidence in the banking system and encouraging liquidity” (12 CFR 370, RIN 3064–AD37). These guarantees were “temporary” in that they expired no later than June 2012. Our results can help to rationalize the use of such policies in conjunction with bail-in policies applied to outstanding long-term debt.

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30Notably, both early triggers and covenants at date 1 cannot rule out the rollover crisis. Early triggers do not rule it out because the bail-in debt is already wiped out under Proposition 8, in other words an early trigger was already applied. Covenants are a common solution to dilution problems (e.g. Brunnermeier and Oehmke (2013), He and Milbradt (2016)), but covenants at date 1 do not help in this case because the rollover crisis arises for the equilibrium package $(0, L^*_1)$, which uses no new standard debt, because new bail-inable debt is subordinated to existing standard debt, and existing standard debt was part on an optimal contract.
We model guarantees as follows: at the beginning of date 1, the government extends a temporary guarantee to the new issuance $L^*_1$ of bail-in debt. The guarantee expires at the end of date 1, but requires the government to cover any losses relative to face value during the guarantee period. In particular, this means the government cannot apply a bail-in to this debt at date 1. Guarantees expire at the end of date 1, after which the debt is once again subject to the bail-in regime. This guarantee extension eliminates rollover crises.

**Proposition 9.** A temporary guarantee of new bail-in debt $L^*_1$ that expires at the end of period 1 eliminates the rollover crisis equilibrium. Guarantees are not paid out on the equilibrium path.

Proposition 9 is closely related to the rationale for deposit insurance. Since debt is guaranteed, investors cannot expect a price of 0, ruling out the rollover crisis and allowing the bank to refinance itself. Guarantees are never paid out on the equilibrium path because they expire the end of date 1, consistent with the principle of TLGP. Naturally, it is too strong in practice to assume that guarantees can be timed perfectly to never be paid out. Some guarantees would be paid out on the equilibrium path, leading to moral hazard concerns.\(^{31}\) Proposition 9 is an idealized result that helps explain why debt guarantee programs such as TLGP may be a valuable part of a crisis resolution toolkit.

### 5.3 Policy Implications: Bailouts versus Debt Guarantees

This section has studied two different perspectives on bailouts and bail-ins – preventing costly liquidations and preventing market instabilities – and shown these perspectives offer different policy conclusions for the relative roles of bailouts and bail-ins. Proposition 7 shows that the social planner prefers to rely solely on bail-ins to prevent fire sales resulting from costly liquidations, and makes no use of bailouts. Proposition 9 shows that ex-post debt guarantees can be desirable to prevent sunspot rollover crises. The key difference is in the type of debt that is being protected: during times of crisis, outstanding debt is not protected by bailouts (Proposition 7) whereas new debt issuances

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\(^{31}\)The Debt Guarantee Program (DGP) portion of TLGP, which involved guarantees of new loans, at its peak guaranteed approximately $345 billion in debt. Approximately $153 million in guarantees were paid out. These numbers are reported by the FDIC as of February 2019 (https://www.fdic.gov/regulations/resources/tlgp/index.html).
are protected by guarantees (Proposition 9). Bail-ins replace bailouts as the optimal tool for bank resolution, while guarantees serve a different function and can be a valuable stabilization tool during a crisis. A commitment against bailouts of existing debt needs not preclude protection of new debt during a crisis.

6 Extensions

In Appendix B, we provide several extensions.

Appendix B.1 allows for positive arbitrageur welfare weights, and shows that the privately optimal contract of Proposition 2 is Pareto inefficient even though arbitrageurs benefit from asset purchases. The social optimum combines a reduction in standard debt with a lump sum transfer from banks to arbitrageurs at date 0 to ensure Pareto efficiency.

In Appendix B.2, we allow for the possibility of aggregate risk. Optimal contracts take the same form, but both instruments are contingent on the aggregate state, implying the addition of a dual price trigger or aggregate risk hedge in the optimal contract. Banks inefficiently limit contingencies on aggregate risk when there are fire sales or bailouts.

In Appendix B.3, we relax the assumption of liability monotonicity, and show that the privately optimal contract involves a “live or die” feature (Innes (1990)). It can be implemented with a combination of standard debt, bail-in debt, and an insurance contract that pays out to the bank when \( R \geq R_u \). As in the baseline model, optimal policy is characterized by increasing issuance of bail-in debt relative to standard debt, but has no impact on the insurance contract written for \( R \geq R_u \).

In Appendix B.4, we consider the interaction between macroprudential (asset-side) and liability-side regulation by introducing multiple investment projects. While bail-ins still constitute optimal liability regulation, asset-side regulation is also required to implement the social optimum because the asset composition affects the probability of bank failure and liquidation.

In Appendix B.5, we study the allocation of bail-in securities among heterogeneous investors with different risk tolerances and different exposures to the banking sector. We show that retail
investors, who maintain greater exposures to individual banks, and institutional investors who experience spillovers from fire sales should hold safer (non-bail-inable) claims.

In Appendix B.6, we incorporate a role for (outside) equity-like claims into the bank’s capital structure by incorporating bank risk aversion and risk shifting. There is still a role for bail-in debt in addressing the incentive problem of the baseline model. We show that although the bank and the planner disagree over the quantity of standard debt versus total loss-absorbing capital (bail-in debt + equity), they do not disagree over the composition of total loss-absorbing capital.

In Appendix B.7, we allow for standard debt to command a premium over other instruments, including bail-in debt. This increases use of standard debt and helps to explain why, in practice, the level of standard debt banks employ is so high. The marginal trade-off for banks is still influenced by the incentive problem and leads to use of bail-in debt. In absence of the incentive problem, there would be no reason to use bail-in debt over equity.

In Appendix B.8, study the trade-off between bailouts and bail-ins in protecting insured deposits when banks are allowed to issue insured deposits as part of their standard debt. The planner faces a trade-off between greater deposit insurance (i.e. taxpayer) losses when liquidating the bank, and worse bank incentives when bailing out the bank. Bailouts may be desirable to lessen the taxpayer burden of deposit insurance. All non-deposit investors are fully bailed in whenever the planner bails out the bank. This motivates the possibility of having a deposit guarantee scheme, even in the absence of bailouts of other forms of debt.

7 Conclusion

We characterize optimal bank contracts under a monitoring incentive problem. The privately optimal bank contract can be implemented by a combination of standard debt and bail-in debt. However, banks privately under-use bail-in debt in the presence of fire sale externalities from bank failures, motivating the government to set up a bail-in resolution regime. Bail-ins are desirable from a regulatory perspective relative to greater equity capital requirements because bail-in debt is better
suited than outside equity to address the incentive problem that motivated banks to issue standard debt in the first place. Optimal regulation replaces bailouts with bail-ins, but does not preclude issuance of debt guarantees for new long-term debt during times of market stress. This helps to understand the introduction and design of bail-in regimes in the US and EU.

Our model simplified the continuation (date 1) agency problem to a required agency rent. We conjecture that the key forces in this paper would arise in a model with multiple periods of Innes (1990) effort choice: liquidations would be costly but strong incentive devices, while maximal cash flow transfers (“bail-ins”) would be less costly but weaker incentive devices. Moreover, it is possible that pledging contracts that induce future shirking (low effort) after low returns could be an optimal form of money burning, similar to liquidation, by reducing continuation agency rents. This might take the form of optimal debt overhang. A full dynamic model of this form would be an interesting avenue for future research.

Our model also assumes that the banker is not severable from the bank, that is the banker cannot be fired without also liquidating the bank. In this sense, standard debt can be viewed as enforcing a transfer of control rights to investors, whose best option is to liquidate the bank. If the banker were severable but firings were costly, and if there were a trade-off between costly liquidation and costly firing, we might expect the government to prefer use of costly firings and bail-ins rather than liquidations. Another interesting avenue for future research would be to consider socially optimal punishment schemes in this context.32

References


See Zentefis (2021) for analysis in this direction, which studies whether accompanying bailouts with manager equity dilution serve as a disciplining device.

32


Figure 1: This figure provides an illustration for the privately optimal contract. Up to a threshold $R_l$, bank liabilities are constant and exceed pledgeable income, leading to liquidations ("standard debt"). Between $R_l$ and $R_u$, the face value of liabilities is written down to coincide with pledgeable income ("bail-in" or "write down"). Above $R_u$, the face value of liabilities is constant ("bail-in debt").
Internet Appendix

A Proofs

A.1 Proof of Proposition 2

Consider the program

\[
\max_{\mathcal{E}} E \left[ c_1(R) + c_2(R) | e = H \right]
\]

Subject to

\[
E \left[ (c_1(R) + c_2(R)) \left( 1 - \frac{f_L(R)}{f_H(R)} \right) | e = H \right] \geq BY_0
\]

\[
Y_0 - A = \int_{R|\alpha=1} \gamma(s)RY_0 f_H(R) dR + \int_{R|\alpha=0} (L_1(R) + L_2(R)) f_H(R) dR
\]

\[
R \geq R' \Rightarrow L_1(R) + L_2(R) \geq L_1(R') + L_2(R')
\]

\[
c_2(R) \geq (1 - \alpha(R)) bRY_0
\]

\[
c_1(R), c_2(R) \geq 0
\]

and recall the second to last constraint is limited pledgeability. Notice that we can set \(c_1(R) = 0\) without loss of generality. It is helpful to redefine the problem in the investor payoff space, and then to define the implementing liability structure. Total investor payoff \(x(R)\) is given by

\[
x(R) = \alpha(R) \gamma RY_0 + (1 - \alpha(R)) RY_0 - c_2(R)
\]

where \(\alpha(R) \in \{0, 1\}\) is the liquidation rule. We treat \(\alpha(R)\) as a choice variable, and then back out the liability structure that implements it. Note that because banks are repaid 0 when \(\alpha(R) = 1\), it is irrelevant whether we multiply \(c_2(R)\) by \(1 - \alpha(R)\). Given this characterization, investor voluntary
participation can be rewritten as

\[ Y_0 - A = E \left[ \alpha \gamma R Y_0 + (1 - \alpha) R Y_0 - c_2 | e = H \right]. \]

We begin by studying the optimization problem not subject to liability monotonicity, and show that it generates a non-monotone contract. The Lagrangian of this relaxed problem is

\[
\mathcal{L} = E \left[ c_2 | e = H \right] + \mu \left[ E \left[ c_2 \left( 1 - \frac{f_L(R)}{f_H(R)} \right) | e = H \right] - BY_0 \right] + \lambda \left[ E \left[ \alpha(R) \gamma(s) R Y_0 + (1 - \alpha) R Y_0 - c_2 | e = H \right] + A - Y_0 \right] + E \left[ \chi(c_2 - (1 - \alpha) b R Y_0) | e = H \right] + E \left[ \zeta((\alpha \gamma R Y_0 + (1 - \alpha) R Y_0 - c_2)) | e = H \right]
\]

From here, first order condition for bank consumption as

\[
0 = f_H(R) + \mu \left( 1 - \frac{f_L(R)}{f_H(R)} \right) f_H(R) - \lambda f_H(R) + \chi(R) f_H(R) - \zeta(R) f_H(R)
\]

\[
= \left[ 1 - \lambda + \mu \left( 1 - \frac{f_L(R)}{f_H(R)} \right) \right] f_H(R) + \chi(R) - \zeta(R)
\]

By MLRP, there is a threshold \( R^\ast \) such that \( \chi(R) > 0 \) for \( R \leq R^\ast \) and \( \zeta(R) > 0 \) for \( R \geq R^\ast \). This threshold is given by

\[
1 - \lambda + \mu \left( 1 - \frac{f_L(R^\ast)}{f_H(R^\ast)} \right) = 0.
\]

(13)

However, this contract violates liability monotonicity unless \( L_1(R) + L_2(R) \) is constant for all \( R \). Therefore, we have an upper pooling region in the optimal contract, where liabilities and investor repayment are constant.\(^{33}\)

It is worth remarking that monotonicity therefore binds in the optimal contract. We relax the assumption of monotonicity in Appendix B.3, and show that the optimal contract takes a live-or-die

\(^{33}\)If \( L_1(R) \) is constant, then the entire contract is pooled. If \( R^\ast = \bar{R} \), then the results that follow apply setting \( R_u = \bar{R} \) to be the pooling threshold.
form (Innes (1990)). We interpret the optimal contract as a combination of standard debt, bail-in debt, and an “insurance” contract that pays off to banks in a high-return states.

We now characterize the optimal contract using the following strategy. First, we conjecture pooling thresholds $R_u$ with corresponding liabilities $x_u \equiv x(R_u) = L_1(R_u) + L_2(R_u)$, so that $x(R) = x_u$ for all $R \geq R_u$. The live-or-die result of the contract not subject to monotonicity implies such a pooling threshold exists.\(^{34}\) We then solve for the optimal contract below $R_u$, taking as given $R_u$ and $x_u$, subject to a relaxed monotonicity constraint $x(R) \leq x_u \forall R \leq R_u$, and verify that the resulting contracting is monotone. In doing so, we characterize the space of implementable contracts (that satisfy monotonicity). Finally, we optimize over the choice of $R_u$ and $x_u$.

Conjecture pooling thresholds $R_u$ with liabilities $x_u$. The associated Lagrangian is given by

$$
\mathcal{L} = E[c_2 | e = H] + \mu \left[ E \left[ c_2 \left( 1 - \frac{f_L(R)}{f_H(R)} \right) | e = H \right] - BY_0 \right] \\
+ \lambda \left[ E \left[ \alpha \gamma RY_0 + (1 - \alpha) RY_0 - c_2(R) | e = H \right] + A - Y_0 \right] \\
+ E \left[ \chi \left( c_2 - (1 - \alpha) bRY_0 \right) | e = H \right] \\
+ E \left[ \nu \left( x_u - (\alpha \gamma RY_0 + (1 - \alpha) RY_0 - c_2) \right) | e = H \right]
$$

where the final line is the relaxed monotonicity constraint, and where we have anticipated that limited liability $x(R) \geq 0$ does not bind below $R_u$ for feasible contracts. Taking the derivative in consumption $c_2(R)$ for $R \leq R_u$, we obtain

$$
0 = 1 + \mu \left( 1 - \frac{f_L(R)}{f_H(R)} \right) - \lambda + \chi(R) + \nu(R).
$$

Observe that the resulting contract is non-monotone if $R_u > R^*$ (we would have $\zeta(R) > 0$ so that $x(R) = 0$), by the same logic as above. Therefore, we can discard candidate contracts with $R_u > R^*$.

This implies that $1 + \mu \left( 1 - \frac{f_L(R_u)}{f_H(R_u)} \right) - \lambda < 0$ among the set of viable contracts.

\(^{34}\)Note that this is without loss, since the pooling threshold could be $R_u = \bar{R}$ if $R^* = \bar{R}$.  

42
Now, consider the derivative in liquidations $\alpha(R)$, given by\footnote{Implicitly, we are treating $\alpha(R)$ as a continuous variable in performing the differentiation. To do so, we implicitly incorporate the constraint $\alpha(R)(1-\alpha(R)) = 0$, which ensures that implementable contracts must set $\alpha(R) \in \{0, 1\}$. The logic below is unaffected.}

\[
\frac{\partial \mathcal{L}}{\partial \alpha(R)} \propto \lambda (\gamma - 1) + \chi(R)b + v(R)(1-\gamma)
\]

When $\alpha(R) = 1$, $v(R) = 1$ is possible at at most a single point, in particular at $\gamma RY_0 = x_u$. $\alpha(R) = 1$ therefore generically implies $\chi(R) > 0$ and $v(R) = 0$. From the FOC for $c_2(R)$, we have (almost everywhere) that when $\alpha(R) = 1$

\[
\chi(R) = \lambda - 1 - \mu \left( 1 - \frac{f_L(R)}{f_H(R)} \right)
\]

which, combined with the liquidation rule, yields

\[
\frac{\partial \mathcal{L}}{\partial \alpha(R)} \propto \lambda (\gamma - 1) + \left( \lambda - 1 - \mu \left( 1 - \frac{f_L(R)}{f_H(R)} \right) \right) b.
\]

By MLRP, there is a threshold rule $R \leq R_l$ for liquidations.

Finally, in the region (if non-empty) between $R_l$ and $R_u$, by MLRP we have

\[
1 + \mu \left( 1 - \frac{f_L(R)}{f_H(R)} \right) - \lambda < 1 + \mu \left( 1 - \frac{f_L(R_u)}{f_H(R_u)} \right) - \lambda < 0
\]

so that we have either $\chi(R) > 0$ or $v(R) > 0$. This implies that $x(R) = \min\{(1-b)RY_0, x_u\}$ for all $R_l \leq R \leq R_u$.

As a result, the optimal contract is a three-part liability structure. First, there is a threshold $R_l$ such that $\alpha(R) = 1$ and $x(R) = \gamma RY_0$ for $R \leq R_l$, and $\alpha(R) = 0$ for $R \geq R_l$. Second, there is a threshold $R_u \geq R_l$ such that $x(R) = \min\{(1-b)RY_0, x_u\}$ for $R \leq R_u$ and $x(R) = x_u$ for $R \geq R_u$. Note finally that there cannot be a discontinuity in liabilities at $R_u$. If there were a discontinuity, we
would have

\[ x_u > \lim_{R \uparrow R_u} x(R) = (1 - b)R_uY_0 \]

and liabilities would exceed pledgeable income at \( R_u \). The capital structure is therefore continuous at \( R_u \).

Finally, the above capital structure can be implemented by a liabilities contract:

\[ L_1(R) + L_2(R) = (1 - b)R_lY_0 \quad \text{for} \quad R \leq R_l \quad \text{and} \quad L_1(R) + L_2(R) = x(R) \quad \text{for} \quad R > R_l. \]

This liability structure is monotone, and so we have implementable contracts.

In sum, the optimal contract lies within a class of contracts characterized by thresholds \( R_l \) and \( R_u \) and corresponding liability structure above. This proves the first part of the proposition.

Now, we characterize the optimal thresholds \( R_l \) and \( R_u \). Considering the case where these thresholds are interior, \( R < R_l \leq R_u \leq R \) we have the optimization problem

\[
\max_{R_l, R_u, Y_0} \int_{R_l}^{R_u} bRy_0 f_H(R) dR + \int_{R_l}^{R_u} [R - (1 - b)R_u] y_0 f_H(R) dR
\]

subject to

\[
\int_{R_l}^{R_u} bRy_0 (f_H(R) - f_L(R)) dR + \int_{R_u}^{R} [R - (1 - b)R_u] y_0 (f_H(R) - f_L(R)) dR \geq BY_0
\]

\[
Y_0 - A = \int_{R_l}^{R} RY_0 f_H(R) dR + \int_{R_l}^{R_u} (1 - b)Ry_0 f_H(R) dR + \int_{R_u}^{R} (1 - b)R_u y_0 f_H(R) dR
\]

Under the same multiplier convention, the optimality condition for \( R_l \) is

\[
0 = -bR_lY_0 - \mu bR_lY_0 \left( 1 - \frac{f_L(R_l)}{f_H(R_l)} \right) + \lambda (\gamma - (1 - b)) R_lY_0,
\]

which reduces to

\[
\mu b \left( \frac{f_L(R_l)}{f_H(R_l)} - 1 \right) = b + \lambda (1 - b - \gamma).
\]
Similarly, the optimality condition for $R_u$ is

$$0 = \int_{R_u}^\infty \left[-(1-b)Y_0 f_H(R) - \mu (1-b) Y_0 (f_H(R) - f_L(R)) + \lambda (1-b)Y_0 f_H(R) \right] dR,$$

which reduces to

$$0 = E \left[ \lambda - 1 - \mu \left(1 - \frac{f_L(R)}{f_H(R)} \right) \right] \bigg| R \geq R_u, e = H.$$

This completes the proof.

A.1.1 A Remark on Contract Uniqueness

The optimal contract is not generally unique in the following sense. In the region $R \leq R_l$, the bank only needs a liability face value that is sufficient to liquidate the bank, and so any contract with monotone face value $L_1(R) + L_2(R) > (1-b)Y_0$ in this region is optimal. We selected the contract with a flat face value below $R_l$ due to its correspondence to standard debt. The face value of liabilities above $R_l$ is uniquely determined. Moreover, in the presence of an $\epsilon \to 0$ premium for standard debt (e.g. as in Appendix B.7), the implementation using standard debt becomes uniquely optimal.

A.2 Proof of Corollary 3

Consider the proposed liability structure. The amount $(1-b)R_l Y_0$ of standard debt liquidates the bank when $R \leq R_l$, generating the lower region. $(1-b)(R_u - R_l)$ is written down in the region $R_l \leq R \leq R_u$, so that the bank is always held to the agency rent over this region. The full debt level $(1-b)R_u Y_0$ is repaid above $R_u$. Therefore, we replicate the contract in Proposition 2.

A.3 Proof of Proposition 4

We split the proof into the different cases.

Case 1: Suppose first that $B = 0$, but $b > 0$ and $\gamma(s) < 1 - b$. We impose $(1-b)E[R] < 1$ to obtain
a finite solution.

The result is a Modigliani-Miller type result. Incentive compatibility is now

\[ E \left[ c_2(R) \left( 1 - \frac{f_L(R)}{f_H(R)} \right) \mid e = H \right] \geq 0. \]

Let \( c_2(R) \) be some monotone consumption rule. We have

\[ E \left[ c_2(R) \left( 1 - \frac{f_L(R)}{f_H(R)} \right) \mid e = H \right] = \text{cov} \left( c_2(R), 1 - \frac{f_L(R)}{f_H(R)} \right) \geq 0 \]

where the inequality follows from MLRP. As a result, any monotone consumption rule is implementable. We can span the frontier of expected repayment splits between banks and investors, with \( \Pi \in [0, (1-b)E[R]] \) to investors and \((1-b)E[R] - \Pi \) to bankers, with monotone consumption rules (e.g. equity). Because all agents are risk-neutral, all that matters is the expected revenue division, and there is no need to liquidate the bank. Equity allocation rules \( E \in [0, 1-b] \), with investors receiving \( E \) and banks retaining equity \( 1 - E \), generate monotone consumption profiles and so are incentive compatible. They also span the range of possible surplus divisions. As a result, pure equity constitutes an optimal contract.

**Case 2:** Consider next \( b = 0 \). The RHS of (9) then collapses to \( \lambda(1-\gamma) \) while the LHS collapses to 0, and so banks never choose to liquidate. Optimal contracts use only bail-in debt.

**Case 3:** Consider finally \( \gamma = 1 \). Any face value \( L_1(R) \leq RY_0 \) can then be repaid by liquidating assets, so that bank consumption is \( c_2(R) = RY_0 - L_1(R) \) for any \( L_1(R) \leq RY_0 \). Therefore for any liability structure \( L_1(R) \), we can define

\[
(c_2(R), x(R)) = \begin{cases} 
(RY_0 - L_1(R), L_1(R)), & L_1(R) \leq RY_0 \\
(0, RY_0), & L_1(R) \geq RY_0 
\end{cases}
\]

where the relevant liquidation function \( \alpha(R) \in [0,1] \) is defined from the liability structure. For
example, without loss of generality we could define $\alpha(R) = \frac{x(R)}{R_Y}$. As a result, minimum pledgeability never binds.

Defining the problem in the repayment space, we then have

$$\max \int_R [R_Y - x(R)] f_H(R) dR,$$

subject to

$$\int_R [R_Y - x(R)] (f_H(R) - f_L(R)) dR \geq BY_0$$

$$Y_0 - A = \int_R x(R) f_H(R) dR$$

$$R \geq R' \Rightarrow x(R) \geq x(R')$$

with $0 \leq x(R) \leq R_Y$. Relaxing monotonicity, the FOC for $x(R)$ is given by

$$\frac{\partial L}{\partial x(R)} = \left[-1 - \mu \left(1 - \frac{f_L(R)}{f_H(R)}\right) + \lambda\right] f_H(R)$$

yielding a threshold rule $R^*$ such that $x(R) = R_Y$ for $R \leq R^*$ and $x(R) = 0$ for $R \geq R^*$. This results in an upper pooling region $R_u$ with liabilities $x_u$. Because $R_u < R^*$ as in the proof of Proposition 2, we have $x(R) = R_Y$ for all $R \leq R_u$. Continuity implies $L_1(R) = R_u Y_0$ for all $R$, and so the contract is standard debt.

### A.4 Proof of Proposition 5

Consider the program of the social planner, where for simplicity we assume $c_1 = L_2 = 0$, without loss of generality due to irrelevance of maturity structure:

$$\max_{L_1, x_0} E \left[c_2 | e = H\right]$$
subject to
\[ E \left[ c_2 \left( 1 - \frac{f_L(R)}{f_H(R)} \right) \left| e = H \right. \right] \geq BY_0 \]

\[ Y_0 - A = \int_{R|\alpha=1} \gamma(\Omega) R Y_0 f_H(R) dR + \int_{R|\alpha=0} L_1(R) f_H(R) dR \]

\[ R \geq R' \Rightarrow L_1(R) \geq L_1(R') \]

\[ c_2 \geq (1 - \alpha(R)) b R Y_0 \]

\[ \Omega = \int \alpha(R) R f_H(R) dR \]

The proof follows as in the proof of Proposition 2. Redefine the payoff space over \( x(R) \) and solve for the optimal contract without imposing monotonicity. The first order condition for \( c_2(R) \) is the same as in the proof of Proposition 2, since \( c_2(R) \) does not directly affect \( \Omega \). This implies as before that we obtain a pooling region at the top.

As before, take \( R_u \) and \( x_u \) as given, and solve for the optimal contract for \( R \leq R_u \). The same steps imply that implementable contracts must satisfy \( R_u < R^* \). The FOC for optimal liquidations \( \alpha(R) \) is now

\[
\frac{\partial \mathcal{L}}{\partial \alpha(R)} \propto \lambda \left( \gamma(\Omega) - 1 \right) R Y_0 f_H(R) + \chi(R) b R Y_0 f_H(R) + \nu(R) (1 - \gamma) R Y_0 f_H(R) \\
+ \frac{\partial \gamma(\Omega)}{\partial \Omega} \frac{\partial \Omega}{\partial \alpha(R)} \lambda \int_{R'} \alpha(R') R Y_0 f_H(R') dR' 
\]

Substituting in the derivative \( \frac{\partial \Omega}{\partial \alpha(R)} = R f_H(R) \), we obtain

\[
\frac{\partial \mathcal{L}}{\partial \alpha(R)} \propto \lambda (\gamma - 1) + \chi(R) b + \nu(R) (1 - \gamma) + \frac{\partial \gamma(\Omega)}{\partial \Omega} \lambda \int_{R'} \alpha(R') R f_H(R') dR' 
\]

The additional wedge \( \frac{\partial \gamma(\Omega)}{\partial \Omega} \lambda \int_{R'} \alpha(R') R f_H(R') dR' \) is negative and independent of \( R \). The same steps apply as in the proof of Proposition 2, yielding a liquidation threshold rule \( R_l \). Because as before \( R_u < R^* \), we have \( x(R) = \min \{ (1 - b) R Y_0, x_u \} \) in the region \( R_l \leq R \leq R_u \). Thus, the set of candidate optimal contracts is the same as in the private equilibrium, and the implementation of
Corollary 3 holds.

Lastly, we characterize the optimal choices of $R_l$ and $R_u$ for interior solutions. The optimality condition for $R_u$ is identical to the private optimality condition, since it does not affect the liquidation value. By contrast, the social optimality condition for $R_l$ satisfies

$$b + \lambda ((1 - b) - \gamma) = \mu b \left( \frac{f_L(R_l)}{f_H(R_l)} - 1 \right) + \frac{\lambda}{R_l Y_0 f_H(R_l)} \frac{\partial \gamma(\Omega)}{\partial \Omega} \int_R^{R_l} R Y_0 f_H(R) dR.$$

Substituting in $\frac{\partial \Omega}{\partial R_l} = R_l f_H(R_l)$ and rearranging, we obtain

$$\mu b \left( \frac{f_L(R_l)}{f_H(R_l)} - 1 \right) = b + \lambda ((1 - b) - \gamma) - \lambda \frac{\partial \gamma(\Omega)}{\partial \Omega} \int_R^{R_l} R f_H(R) dR.$$

This completes the proof.

A.5 Proof of Corollary 6

We need only to verify that the ex post bail-in authority achieves the same outcome as the contractual liabilites of the social optimum. In the region $R < R_l$, the non-bail-inable senior debt exceeds asset values, and the bail-in authority is unable to resolve the bank. The bank is liquidated.

In the region $R_l \leq R < R_u$, if the bail-in authority does not intervene then the bank gets 0, senior non-bail-inable debt gets $x^S(R) = \min \{(1 - b)R_l, \gamma R\}$, and junior bail-inable debt gets $\max \{\gamma R - x^S(R), 0\}$. If by contrast the bail-in authority intervenes, it recapitalizes the bank with any haircut $R' - R_l \leq R - R_l \leq R_u - R_l$ to bail-inable junior debt. Senior non-bail-inable debt gets fully repaid and is weakly better off. The bank gets payment $bR'$ and is better off. Junior bail-inable debt gets $(1 - b)(R' - R_l)$, and is better off local to $R' = R$ because $\gamma < 1 - b$. Therefore, there is a Pareto efficient haircut. The haircut that maximizes total recovery value to creditors is $R' = R$, which is the same outcome as contractual bail-in debt.

In the region $R_u \geq R$, the bank is solvent, and all debt is repaid in full. A haircut on bail-inable debt is not Pareto efficient, and the bail-in authority does not act.
Hence, the bail-in authority implements the social optimum.

A.6 Proof of Proposition 7

We adopt the following proof strategy. We will consider a contract that results in bailouts, and show that it is equivalent to a contract that: (1) features bail-ins (rather than bailouts) ex post; and, (2) implements an ex ante lump sum transfer from taxpayers to the bank. Thus, all contracts with bailouts are equivalent to contracts without bailouts combined with lump sum transfers. We finally show that the social optimum does not feature ex ante lump sum transfers.

Suppose that there is a liability structure with a bailout at return $R$, so that $L_1(R) + L_2(R) > (1 - b)RY_0$ and $T_1(R) = L_1(R) + L_2(R) - (1 - b)RY_0$. This generates consumption profile $c_2(R) = bRY_0$ and a repayment to investors $x_1(R) + x_2(R) = L_1(R) + L_2(R) = (1 - b)RY_0 + T_1(R) = \hat{x}_1(R) + \hat{x}_2(R) + T_1(R)$, where $\hat{x}_1(R) + \hat{x}_2(R)$ is repayment out of bank resources. Substituting into the participation constraint, we have

$$Y_0 - A_0 \leq \mathbb{E}[\hat{x}_1(R) + \hat{x}_2(R) + T_1(R)].$$

The problem is otherwise identical. Hence, from the bank perspective the bailout $T_1(R)$ is equivalent to a bail-in contract combined with a lump-sum transfer of equal expected value from taxpayers to the bank at date 0. Moreover, taxpayer optimization of $B_0$ implies the change in contract also has no impact on taxpayer welfare, since the taxpayer simply adjusts $B_0$ in response to maintain the same path of consumption.

Finally, we need merely to show that lump sum transfers are not Pareto efficient. Defining the social welfare weight on taxpayers to be $\omega^T = \frac{\lambda}{u'_0(c'_0)}$, then the social planner is indifferent to transfers between banks and taxpayers at date 0. Thus, there are no Pareto improving transfers, meaning there are no Pareto improving bailouts, meaning that the contract of Proposition 5 is optimal.
A.7 Proof of Proposition 8

Define $L_1^*$ by

$$(1-b)Y_1 = \int_0^{L_1^*/Y_1} R_2 Y_1 f_2(R_2) dR_2 + \int_{L_1^*/Y_1}^\infty L_1^* f_2(R_2) dR_2$$

for $Y_1 = R_1 Y_0$. Clearly this is the only viable refinancing package for the bank given the liquidation cost at date 2. First, suppose that the market quotes $Q_1^* = (1-b)R_1 Y_0 = D_0$. Then, the bank successfully refines itself using this package, giving the first equilibrium.

Second, suppose that the market instead quotes $Q_1^* = 0$. Then, the bank fails. The liquidation value of the bank is

$$\gamma Y_1 < (1-b)Y_1 = D_0$$

so the outstanding stock of standard debt exhausts the bank liquidation value. Since bail-in debt is junior to standard debt, this justifies the equilibrium price $Q_1^* = 0$, completing the equilibrium.

Finally, there is no other equilibrium $(D_1, L_1)$ with $D_1 > 0$ and $D_1 + L_1 \leq L_1^*$. If $D_1 + L_1 > L_1^*$, the bank’s agency rent is not maintained, thus $D_1 + L_1 \leq L_1^*$. If $D_1 > 0$ but $D_1 + L_1 \leq L_1^*$, then total investor repayment is

$$Q_1 = \int_0^{D_1/Y_1} \gamma R_2 Y_1 f_2(R_2) dR_2 + \int_{D_1/Y_1}^{(D_1+L_1)/Y_1} R_2 Y_1 f_2(R_2) dR_2 + \int_{(D_1+L_1)/Y_1}^\infty (D_1 + L_1) f_2(R_2) dR_2$$

$$< \int_0^{(D_1+L_1)/Y_1} R_2 Y_1 f_2(R_2) dR_2 + \int_{(D_1+L_1)/Y_1}^\infty (D_1 + L_1) f_2(R_2) dR_2$$

$$= (1-b)Y_1$$

$$= D_0.$$ 

Hence, no other package can both achieve investor repayment and pay the bank its agency rent.
A.8 Proof of Proposition 9

Suppose that there is a debt guarantee of $L_1^*$. The good equilibrium still exists, and moreover in the good equilibrium the bank continues to date 2, so that no guarantees are paid out. The bad equilibrium is ruled out trivially: if $Q_1^* = 0$, then the bank fails. But then debt $L_1^*$ is fully repaid, a contradiction.

Finally, the bank cannot issue any other securities package $(D_1, L_1)$ with $D_1 > 0$. Consider any other package with $D_1 > 0$ and with $D_1 + L_1 \leq L_1^*$, and suppose it has price $Q_1 = D_0$. Under the conjectured equilibrium, the bank rolls over its debt and survives to date 2, at which point guarantees expire. The total recovery value is then lower than $(1 - b)Y_1$ due to the liquidation cost, contradicting that the price was $D_0$ and hence contradicting it was an equilibrium.

B Extensions

In this Appendix, we provide a number of extensions, as previewed in Section 6. Appendix B.1 allows for positive arbitrageur welfare weights. Appendix B.2 allows for aggregate risk. Appendix B.4 considers the interaction between macroprudential (asset-side) and liability-side regulation. Appendix B.5 studies the allocation of bail-in securities among heterogeneous investors with different risk tolerances and different exposures to the banking sector. Appendix B.6 incorporates a role for (outside) equity-like claims into the bank’s capital structure by incorporating bank risk aversion and risk shifting. Appendix B.7 allows for standard debt to command a premium over other instruments, including bail-in debt. Appendix B.8 studies the trade-off between bailouts and bail-ins in protecting insured deposits when banks are allowed to issue insured deposits as part of their standard debt.

B.1 Pareto Efficiency

We now study whether the socially optimal contract in Section 3 is indeed Pareto efficient relative to the privately optimal contract. Recall that we have assumed that $u'(\bar{A}) > 1$. We obtain the following
Proposition 10. Let $\frac{\partial \gamma}{\partial \Omega} < 0$. The socially optimal contract features $R_L$ given by

$$
\mu b \left( \frac{f_L(R_l)}{f_H(R_l)} - 1 \right) = b + \lambda \left( (1 - b) - \gamma \right) + \lambda \left( 1 - \frac{1}{u'(A - A_0)} \right) \left| \frac{\partial \gamma}{\partial \Omega} \right| \Omega.
$$

As a result, the privately optimal contract is not Pareto efficient given $u'(\bar{A}) > 1$.

Pareto efficient improvements arise because arbitrageurs are borrowing constrained, so that their marginal utility at date 0 exceeds that at date 1. Efficiency is achieved by transferring resources to arbitrageurs at date 0 in order to compensate them for resource losses from lower surplus from bank liquidations.

When we take $\omega^A \rightarrow 0$, we have the optimal allocation has $A_0 \rightarrow \bar{A}$ and $u'(\bar{A} - A_0) \rightarrow +\infty$, and we obtain the first order condition of Proposition 5.

B.1.1 Proof of Proposition 10

We can characterize a Pareto efficient contract by adopting the welfare function

$$
E [c_2 | e = H] + \omega^A \left( u (\bar{A} - A_0) + (\mathcal{F}(\Omega) - \gamma(\Omega)\Omega) Y_0 \right),
$$

where $\omega^A$ is the welfare weight on arbitrageurs. The optimality of standard and bail-in debt follows the same steps as in the proofs of Propositions 2 and 5. However, in writing the optimal choice of threshold $R_l$, we now account for arbitrageur surplus and obtain

$$
b + \mu b \left( \frac{f_L(R_l)}{f_H(R_l)} - 1 \right) = b + \lambda \left( (1 - b) - \gamma \right) + \left( \lambda - \omega^A \right) \left| \frac{\partial \gamma}{\partial \Omega} \right| \Omega.
$$

Finally, the optimality condition for $A_0$ is given by $\lambda = \omega^A u'(\bar{A} - A_0)$, so that we obtain

$$
b + \mu b \left( \frac{f_L(R_l)}{f_H(R_l)} - 1 \right) = b + \lambda \left( (1 - b) - \gamma \right) + \lambda \left( 1 - \frac{1}{u'(\bar{A} - A_0)} \right) \left| \frac{\partial \gamma}{\partial \Omega} \right| \Omega.
$$
which completes the proof when $u'(\bar{A}) > 1$.

### B.2 Aggregate Risk

To incorporate aggregate risk into the model, we add an aggregate state $s \in S$ of the economy at date 1. For expositional simplicity, we assume that $S$ is a finite set, with probability measure $\pi(s)$.

The aggregate state $s$ affects the return distribution, so that we have $f_e(R|s)$. All contracts can be written on the aggregate state. MLRP now applies contingent on the aggregate state, and liability monotonicity is also contingent on the aggregate state.

From here, the characterization of privately optimal contracts follows almost identically to before.

**Proposition 11.** A privately optimal bank contract has a liability structure

$$L_1(R,s) + L_2(R,s) = \begin{cases} 
(1-b)R_l(s)Y_0, & R \leq R_l(s) \\
(1-b)RY_0, & R_l \leq R \leq R_u(s) \\
(1-b)R_u(s)Y_0, & R_u(s) \leq R 
\end{cases}$$

where $0 \leq R_l(s) \leq R_u(s) \leq \bar{R}$ are aggregate-state-contingent thresholds. The bank is liquidated if and only if $R \leq R_l(s)$. These thresholds, when interior and not equal, are given by

$$\mu b \left( \frac{f_L(R_l(s)|s)}{f_H(R_l(s)|s)} - 1 \right) = b + \lambda \left( 1 - b - \gamma(s) \right) \tag{14}$$

$$0 = E \left[ \left( \lambda - 1 - \mu \left( 1 - \frac{f_L(R|s)}{f_H(R|s)} \right) \right) \mid R \geq R_u(s), s, e = H \right] \tag{15}$$

where $\mu > 0$ is the Lagrange multiplier on incentive compatibility (2) and $\lambda > 1$ is the Lagrange multiplier on investor participation (6).

**Proof.** The proof follows the same steps as the proof of Proposition 2. $\blacksquare$
In contrast to the baseline model, both instruments are contingent on the aggregate state, reflecting that the terms of bank contracts adjust to verifiable events that are beyond a bank’s control. For example, if all else equal a state $s$ has lower returns due to an aggregate (TFP) shock, equation (9) implies it should have a lower liquidation threshold.\textsuperscript{36}

In the context of CoCos, conditioning the level of bail-in debt on both the idiosyncratic state (i.e. individual bank health) and aggregate state (i.e. banking sector health) can be thought of as a dual price trigger.\textsuperscript{37} In this context, there is $R_I(s_{\text{min}})$ of fully non-contingent debt, and $R_u(s_{\text{max}})$ of bail-in debt with a dual-price trigger. The dual price trigger writes down bail-in debt automatically to $R_u(s)$ based on the aggregate state $s$, and allows for it to be additionally written down to $R_I(s) - R_I(s_{\text{min}})$ to restore bank solvency. The dual price trigger thus conditions recapitalization of banks on the aggregate state as well as the idiosyncratic state.\textsuperscript{38}

From here, the results on the socially optimal contract proceed identically, with the state contingency. Similarly, the bailout results can also be derived, where the result is that no bail-in debt is issued for state $s$ whenever there are bailouts in state $s$.

This helps to understand the limits of bank contingencies on verifiable aggregate risk. Although aggregate risk is verifiable and not a result of bank shirking, banks neglect fire sales and expect to receive bailouts in bad aggregate states. This limits the extent to which they write contingencies on aggregate risk.

\textbf{B.2.1 Bail-in Equivalence with Aggregate Risk}

As highlighted above, the degree of bail-inability of debt depends on the aggregate state. Such rules either must be contractually pre-written into debt contracts, or must be written into the rules governing the operations of the bail-in authority. Provided that such rules are specified to govern ex-post resolution, the equivalence between ex-ante contractual provisions and ex-post resolution follows as in the baseline model.

\textsuperscript{36}See Dewatripont and Tirole (2012) for a related argument.

\textsuperscript{37}See e.g. Allen and Tang (2015) and McDonald (2013).

\textsuperscript{38}Alternatively, we could consider it a combination of $R_I(s_{\text{min}})$ of fully non-contingent debt, $R_u(s_{\text{max}})$ of debt with a dual price trigger but no automatic write-down, and an aggregate risk hedge that mimicked the automatic write-down.
In the US, such rules could be implemented using the organizational structure of the bank. Bank holding companies are required to maintain an amount of loss-absorbing debt at the level of the top-level holding company. The goal is to resolve the top-level holding company while allowing operating subsidiaries to continue operations without being affected by the resolution of the holding company. In principle, however, if a full write-down of the liabilities at the holding company level is not sufficient to recapitalize the bank, recapitalization would require bail-ins of debt at the operating subsidiaries. One could structure the governing rules of the bail-in authority to condition the ability of that authority to resolve operating subsidiaries based on the state of the economy. Operating subsidiaries could be resolved by the bail-in authority in crises, but not in normal times.

It is not clear whether aggregate state contingent rules governing the bail-in authority could credibly be implemented and followed. A bail-in authority is likely to be tempted to recapitalize a bank if there is enough long-term debt available to do so, suggesting the potential for time inconsistency in bail-ins.

### B.3 Optimal Contract Without Monotonicity

In the main text, we impose that liability contracts must be monotone. In this appendix, we characterize the optimal contract without monotonicity.

In order to do so, we will need to bound investor payoffs below. In particular, we impose the following limited liability constraint on investors,

$$x_1 + x_2 \geq -x,$$

where $x \geq 0$ is therefore the minimum payment that can be made to the bank by investors. If $x = 0$, then this is a limited liability constraint in the standard sense that investors cannot be forced to pay money into the bank after date 0.

We obtain the following characterization of a privately optimal contract.
Proposition 12. A privately optimal bank contract has a liability structure

\[
L_1(R) + L_2(R) = \begin{cases} 
(1 - b)R_iY_0, & R \leq R_l \\
(1 - b)RY_0, & R_l \leq R \leq R_u \\
-x, & R_u \leq R
\end{cases}
\]

where \( R_l \) and \( R_u \) are given by

\[
\mu b (\Lambda(R_l) - 1) = b + \lambda (1 - b - \gamma)
\]  
Incentive Provision  Liquidation Costs \hspace{1cm} (16)

\[
\mu (1 - \Lambda(R_u)) = \lambda - 1
\]  
Incentive Provision  Investor Repayment \hspace{1cm} (17)

The optimal contract of Proposition 12 is a form of live or die contract (Innes (1990)). As in the model with monotonicity (Proposition 2), in the region \( R \leq R_l \) the bank is liquidated, while in the region \( R_l \leq R \leq R_u \) the bank is held to its agency rent via “bail ins.” The key difference is the upper region. In the model with monotonicity, liabilities were \( L_1(R) = (1 - b)R_uY_0 \) in this region, corresponding to debt. Here instead, investors make the largest payment possible to banks in this region, \( L_1(R) = -x \leq 0 \), in order to best incentivize effort. Note that taking \( x \to \infty \) results in \( R_u \to \bar{R} \), that is the bank will receive an arbitrarily large repayment in only the highest return state.\(^{39}\)

The following corollary provides a simple implementation of this contract, using one additional instrument relative to Corollary 3.

Corollary 13. Absent monotonicity, the privately optimal contract can be implemented by a combination of three instruments:

1. Standard debt with face value \( (1 - b)R_iY_0 \).

2. Bail-in debt with face value \( (1 - b)R_uY_0 \).

\(^{39}\)This follows necessarily from the participation constraint of investors.
3. An insurance contract (or option) that pays out \((1 - b)R_u Y_0 - x\) to the banker in the event that \(R \geq R_u\), and pays out 0 otherwise.

Corollary 13 shows that removing monotonicity simply requires the addition of an insurance contract to the bank’s capital structure. The insurance contract pays off a fixed amount \((1 - b)R_u Y_0 - x\) in “success” states of high returns, \(R \geq R_u\). This allows the banker to repay all debtholders, as well as receive the maximum payment \(x\) from investors possible.

From here, regulatory results of Section 4 proceed as before. A social planner with a complete set of regulatory instruments implements the same contract structure as Proposition 12, but sets \(R_l\) lower to account for the fire sale, thus increasing the use of bail-in debt. However, the planner agrees with the bank over choice of \(R_u\) and use of the insurance contract. Thus, the qualitative insights for bail-in policies remain in this case.

### B.4 Macroprudential Regulation and Bail-ins

In the baseline model, the fact that banks have a single investment project means that liability-side regulation is sufficient. In practice, banks asset allocations also affect their risk profiles. We now show that macroprudential (asset-side) regulation is a necessary complement to bail-ins when banks can affect risks using both sides of their balance sheet.

We augment the model as follows. Banks choose a contractible vector \(\theta = (\theta_1, \ldots, \theta_N)\) of asset allocations. The total return \(R\) on bank scale \(Y_0\) follows a density \(f_e(R|\theta)\), which depends on the allocation \(\theta\). \(f_e(R|\theta)\) satisfies MLRP (conditional on \(\theta\)) over the relevant range of allocations \(\theta\). To simplify exposition, the support of \(R\) is an interval \([R, R]\) that does not depend on \(\theta\). Otherwise, the setup is the same as before.\(^{40}\)

As before, optimal liability contracts combine contingent and standard debt, and the trade-off

\(^{40}\)In Appendix B.4.1, we show how a standard asset allocation problem generates a density function of this form. If the shirking benefit \(B(\theta)\) depended on the allocation, e.g. because riskier assets are more difficult to monitor, the planner and banker would agree on how \(\theta\) affects \(B\). Assets in our model all sell at the same discount and generate the same fire sale spillover. If they differed in terms of liquidation discounts and fire sale spillovers, there would be an additional regulatory incentive on this margin.
between standard and bail-in debt reflects the same forces as before.\textsuperscript{41} We now characterize the optimal asset allocation rule under the socially optimal contract.

**Proposition 14.** The socially optimal contract has FOC for $\theta_n$

\[
0 = E \left[ \lambda x(R) + c_2(R) \left( 1 + \mu \left( 1 - \frac{\partial f_L(R|\theta)}{\partial \theta_n} \right) \right) \right] \frac{\partial f_H(R|\theta)}{\partial \theta_n} \\
+ \lambda E \left[ \frac{\partial \gamma}{\partial \Omega} \mathcal{Y}_0 \cdot \left[ \int_{R_l}^{R_f} R \frac{\partial f_H(R|\theta)}{\partial \theta_n} dR \right] \right]
\]

The first line of Proposition 14 reflects the private trade-off to banks of a change in asset composition, corresponding to changes in the return distribution. These changes are weighted by the (weighted) sum of payoff to investors in those states, and to banks in those states, where the weighting reflects both the direct value of payoffs, and the incentive value of payoffs. The second line of Proposition 14 reflects the social cost of changes in asset composition. The social cost arises when changes in the return distribution affect the magnitude of the fire sale spillover, by altering the measure $\Omega$ of bank liquidations. When an asset increases the probability that the banks’ total return is lower than $R_l$, larger allocations to that asset result in more severe fire sale spillovers. The social cost term penalizes investment in such assets. The social cost term exists whenever $R_l > R$, that is whenever liability-side regulation has not completely eliminated bank failures.

Proposition 14 illustrates that macroprudential (asset) regulation is a necessary complement to bail-ins (liability regulation). Macroprudential regulation and bail-ins co-exist in the regulatory regime because they control fire sales in different manners. For a given level of asset risk, bail-ins mitigate fire sales by reducing the liquidation threshold. For a given liquidation threshold, macroprudential regulation mitigates fire sales by reducing the probability that a bank will fall below that threshold.\textsuperscript{42} These two aspects of regulation are not generally perfect substitutes, so they

\textsuperscript{41}Given that $\theta$ is contractible, the proof follows the same steps as Proposition 2.

\textsuperscript{42}Macroprudential regulation in our model closely risk weights on loss-absorbing capital.
co-exist under the optimal regulatory regime.

Even though macroprudential regulation and bail-ins are not perfect substitutes, Proposition 14 suggests that bail-ins are a partial substitute for macroprudential regulation. Stronger liability regulation pushes the magnitude of the additional wedge in the asset allocation decision towards zero, by reducing the size of the liquidation region.

B.4.1 Multiple Assets Density Function

Suppose that there are $N + 1$ assets between which the bank allocates its funds. Denote $\omega \in [\underline{\omega}, \overline{\omega}]$ to be the underlying idiosyncratic state of the bank, with associated density $f_{e}^{\omega} (\omega)$, where $e \in \{H, L\}$. Suppose that $f_{e}^{\omega} (\omega)$ satisfies MLRP, so that $\frac{\partial}{\partial \omega} \left( \frac{f_{H}^{\omega} (\omega)}{f_{L}^{\omega} (\omega)} \right) > 0$.

Asset $n \in \{1, ..., N + 1\}$ generates a return $R_{n} (\omega)$ per unit. Let $\theta = (\theta_{1}, ..., \theta_{N+1})$ be a vector that determines the asset allocations $\theta_{1}Y_{0}, ..., \theta_{N+1}Y_{0}$. Allocations $\theta$ satisfy a technological restriction $F(\theta) = 0$, for example there may be a concave technology. Note that to coincide with the previous parts, we assume the technology is linear in the scale $Y_{0}$, and only (potentially) concave in the asset weights. If $F(\theta) = \sum_{n=1}^{N+1} \theta_{n} - 1$, we have a simple linear technology with equal cost of investment across assets.

We invert $\theta_{N+1}$ from $(\theta_{1}, ..., \theta_{N})$ via $F$, so that we can internalize the constraint. We denote the total return to the bank, given an asset allocation vector $\theta$, by

$$R(\omega \theta) = \sum_{n=1}^{N+1} \theta_{n}R_{n}(\omega)Y_{0}$$

where $\theta_{N+1}$ is derived from the technology $F(\theta) = 0$, given $\theta_{1}, ..., \theta_{N}$.

Suppose that conditional on $\theta$, there is an injective mapping between $\omega$ and $R$. In this case, $R$ identifies $\omega$, given $\theta$, and we can write contracts on $R$. We assume that the mapping is injective over the relevant range of asset allocations $\theta$. For example, this will be the case if asset allocations are non-negative ($\theta_{n} \geq 0$) and individual asset returns are monotone in $\omega$. Without loss of generality, we assume the injective mapping is monotone increasing: high states $\omega$ identify high returns $R$,
consistent with the interpretation of $e = H$ as “high effort.”

Denote $R^{-1}(R|\theta)$ to be the inverse function mapping the total return $R$ into the idiosyncratic state $\omega$. The inverse function does not depend directly on $e$, but rather the density will depend on $e$. We now derive the density of $R$, conditional on $\theta$. We have

$$F_e(R|\theta) = \Pr(R(\omega|\theta) \leq R|e) = \Pr(\omega \leq R^{-1}(R|\theta)|e) = F_{e|\theta}^{\omega}(R^{-1}(R|\theta)).$$

Differentiating in $R$, we obtain the density function:

$$f_e(R|\theta) = f_{e|\theta}^{\omega}(R^{-1}(R|\theta)) \frac{\partial R^{-1}(R|\theta)}{\partial R}$$

We impose the simplifying assumption that the support $[R, \bar{R}]$ of the density is invariant to the allocation $\theta$. If the support depended on the portfolio allocation, we would have boundary terms in derivatives. The principal term of relevance would be how the lower boundary of the support moves in the asset allocation, which reflects changes in the measure of the liquidation region. These effects are qualitatively the same as the direct effects of changing the measure from changes in the density. For simplicity, we keep the support fixed.

Finally, we can show that this function satisfies monotone likelihood. Differentiating the likelihood ratio in $R$, we obtain

$$\frac{d}{dR} \left( \frac{f_H(R|\theta)}{f_L(R|\theta)} \right) = \frac{d}{dR} \left( \frac{f_H^{\omega}(R^{-1}(R|\theta))}{f_L^{\omega}(R^{-1}(R|\theta))} \right)$$

$$= \frac{\partial f_H^{\omega} \partial R^{-1}(R|\theta)}{\partial \omega} f_L^{\omega} - \frac{\partial f_L^{\omega} \partial R^{-1}(R|\theta)}{\partial \omega} f_H^{\omega}$$

$$= \frac{\partial}{\partial \omega} \left( \frac{f_H^{\omega}}{f_L^{\omega}} \right) \frac{\partial R^{-1}(R|\theta)}{\partial R}$$

$$> 0$$

where in the last line, we have used MLRP on $f_{e|\theta}^{\omega}$ combined with monotonicity of $R^{-1}$. 
As a result, we obtain a representation of the problem as a density \( f_e(R|\theta) \). Implicitly, we differentiate in \((\theta_1, \ldots, \theta_N)\), where we have internalized \( \theta_{N+1} \) as arising from the technology.

### B.4.2 Proof of Proposition 14

Consider the optimal contract of the social planner. Holding fixed the debt levels \( R_l \) and \( R_u \), the derivative of the planner’s Lagrangian in \( \theta_n \) is given by

\[
0 = E \left[ c_2 \frac{\partial f_H(R|\theta)}{f_H(R|\theta)} \right] + \mu E \left[ c_2 \left( \frac{\partial f_H(R|\theta)}{f_H(R|\theta)} - \frac{\partial f_L(R|\theta)}{f_H(R|\theta)} \right) \right] \\
+ \lambda E \left[ \sum \frac{\partial f_H(R|\theta)}{f_H(R|\theta)} \right] \\
+ \lambda E \left[ \int_{R_l}^{R_u} \left( \frac{\partial \gamma(\Omega)}{\partial \Omega} \frac{\partial \Omega}{\partial \theta_n} f_H(R|\theta) \right) dR \right]
\]

where the first two lines reflect the private bank trade-off, and the last line reflects the social trade-off.

Liquidations are given by

\[
\Omega = \int_{R}^{R_l} R f_H(R|\theta) dR
\]

so that we have

\[
\frac{\partial \Omega}{\partial \theta_n} = \int_{R}^{R_l} R \frac{f_H(R|\theta)}{\partial \theta_n} dR.
\]

Substituting in above, we obtain

\[
0 = E \left[ \left( \lambda c_2(R) + c_2(R) \left( 1 + \mu \left( 1 - \frac{\partial f_L(R|\theta)}{f_H(R|\theta)} \right) \right) \right) \frac{\partial f_H(R|\theta)}{f_H(R|\theta)} \right] \\
+ \lambda E \left[ \frac{\partial \gamma}{\partial \Omega} \Omega Y_0 \cdot \left[ \int_{R}^{R_l} R \frac{f_H(R|\theta)}{\partial \theta_n} dR \right] \right]
\]

giving the result.
B.5 Heterogeneous Investors and the Allocation of Securities

In the baseline model, investors are homogeneous and risk neutral, so that the distribution of standard and bail-in debt among investors is irrelevant. A key practical concern is what investors should hold what form of debt, since bail-in debt holders will experience losses when it is written down. Particular concern has been expressed about protecting retail investors from losses that are large relative to their wealth\(^{43}\), and to preventing institutional investors who are potentially exposed to fire sales from bearing losses from bail-ins.\(^{44}\)

To capture these elements, we extend the model to include two classes of bank investors, “institutional” and “retail.” To make the problem interesting, we include aggregate risk. Institutional investors are able to invest across all banks, but still retain exposure to the aggregate state and have preferences that may depend on bank liquidation discounts. Retail investors are only able to invest in a single bank and retain exposure to the idiosyncratic return of that bank. For simplicity, we abstract away from other potential components of these investors’ portfolio choice problems, instead allowing for state dependent preferences. All investors are price takers, and purchase state-contingent payoffs from the banks they invest in. Nevertheless, we show that in equilibrium all investors purchase a combination of the standard and bail-in debt contracts issued by banks.

Denote \(q(R,s)\) the (endogenous) probability-normalized price of a unit of payoff from a bank that realizes state \((R,s)\).\(^{45}\) Institutional investors are indexed by \(i \in I\), have initial wealth \(w_i^0\), and preferences \(u_i^0(c_i^0) + E [u_i^1(c_i^1 | s, \gamma(s))]\). Retail investors are indexed by \(j \in J\), have initial wealth \(w_j^0\), and preferences \(u_j^0(c_j^1) + E [u_j^1(c_j^1 | s)]\). Both \(I\) and \(J\) are finite sets, and we interpret each investor type as corresponding to a continuum of (atomistic) agents of that type. Both types of agents have

\(^{43}\)The resolution of four Italian banks in 2015 sparked a political backlash due to losses to retail investors. Financial Times, “Italy bank rescues spark bail-in debate as anger at Renzi grows,” December 22, 2015.

\(^{44}\)Article 44 of BRRD states that “[m]ember states shall ensure that in order to provide for the resolvability of institutions and groups, resolution authorities limit...the extent to which other institutions hold liabilities eligible for a bail-in tool.”

\(^{45}\)Note that the bank will go bankrupt in some states, implying not all liabilities are repaid at full face value. For simplicity, we price units of payout directly, rather than face value.
period-0 budget constraints given by

$$c_k^0 + \sum_s \pi(s) \int_R q(R,s)x^k(R,s)f_H(R|s)dR = w_k^0, \quad k \in I \cup J.$$ 

However, they differ in their choice of $c_1$. Institutional investors are able to diversify across banks, so that $c_1^i(s) = \int_R x^i(R,s)f_H(R|s)dR$. Retail investors are not able to diversify across banks, and so have $c_1^j(R,s) = x^j(R,s)$. Given the contract payoff $x(R,s)$ from the bank, market clearing for liabilities is given by

$$\sum_{k \in I \cup J} \mu^k x_1^k(R,s) = x(R,s)$$

where $\mu^k$ is the mass of investors of type $k \in I \cup J$.

We focus on the case where the mass of retail investors is sufficiently small that it does not exhaust the returns of the bank in any state $(R,s)$. That is, $\sum_j \mu^j x_1^j(R,s) < x(R,s)$. As a result, both retail and institutional investors price bank liabilities on the margin. We now characterize the equilibrium of the private economy without government intervention.

**Proposition 15.** Suppose that in equilibrium $\sum_j \mu^j x_1^j(R,s) < x(R,s)$. In the private equilibrium:

1. The price $q(R,s) = q(s)$ depends only on the aggregate state $s$.

2. Optimal bank contracts combine standard and bail-in debt.

3. Retail investors only purchase standard debt, and their consumption profile $c_1^j(R,s) = c_1^j(s)$ only depends on the aggregate state $s$. Consumption profiles of retail investors are given by

$$\frac{\partial u_1^j \left(c_1^j(s)|s \right)}{\partial c_1^j(s)} = q(s) \frac{\partial u_0^j \left(c_0^j \right)}{\partial c_0^j}$$

4. Institutional investors purchase both standard and bail-in debt. Consumption profiles of institutional investors are given by

$$\frac{\partial u_1^i \left(c_1^i(s), \gamma(s) \right)}{\partial c_1^i(s)} = q(s) \frac{\partial u_0^i \left(c_0^i \right)}{\partial c_0^i}$$
Even though retail investors are tied to a specific bank, their equilibrium consumption profile does not depend on the idiosyncratic state. This implies not only that retail investors exclusively purchase standard debt, but also that retail investors are first in line for repayment in the event of bank liquidation. In other words, in equilibrium they purchase claims that have the highest priority for repayment. Since retail investors are often depositors, one natural interpretation of this result is that of deposit priority.\footnote{These deposits are not insured in this section, but are repaid due to their priority. In Appendix B.8, we consider deposit insurance.} However, it extends beyond deposits, and furthermore suggests that retail bondholders may also benefit from priority. This suggests a role for non-bail-in able long-term debt, as a way to codify protection for retail investors.

Institutional investors are not exposed to the idiosyncratic state due to their ability to diversify, but are exposed to the aggregate state. Institutional investors face greater losses on the aggregate state when either they are more risk tolerant, or less exposed to bank fire sales. This suggests that the ideal holders of bail-in debt will be institutional investors with limited risk aversion (or ability to diversify using other securities) and limited commonality with the banking sector, so that they are not affected by fire sales.

Finally, consider what would happen if we relaxed the assumption $\sum_j \mu_j x_j^1(R, s) < x(R, s)$. Consider an aggregate state $s$ where $\sum_j \mu_j x_j^1(R, s) = x(R, s)$ for a range of returns $R \leq R^*$. For $R > R^*$, institutional investors are the marginal pricing agent, and $q(R, s) = q(s)$ is a constant. For $R < R^*$, retail investors are the marginal pricing agents, and $q(R, s) \geq q(s)$. Given monotone liabilities contracts, $q(R, s)$ will be falling in $R$. Contracts will still be debt, but the optimal thresholds are affected by the fact that retail investors suffer larger losses in liquidation, pushing $q(R, s)$ higher above $q(s)$. This generates an additional trade-off for the bank in deciding the optimal composition of standard and bail-in debt.

### B.5.1 Proof of Proposition 15

Suppose that there is a state-contingent Arrow price $q(R, s) = q(s)$ that depends only on the aggregate state. Contracts still take the form of standard and bail-in debt, following the same steps as in the
proof of Proposition 2.

Now, consider the investor side. Begin first with institutional investors, whose Lagrangian is given by

\[ L_i = u_i^0 (c_i^0) + \sum_s \pi(s) u_i^1 (c_i^1 | s, \gamma(s)) + \lambda_i \left[ w_0^i - c_0^i - \sum_s \pi(s) \int_R q(R,s)c_i^1(R,s)f_H(R|s)dR \right] \]

\[ + \sum_s \pi(s) \mu_i(s) \left[ \int_R x_i^j(R,s)f_H(R|s)dR - c_i^0(s) \right]. \]

Given the non-negativity constraint \( x_i^j(R,s) \geq 0 \), we have

\[ \frac{\partial L_i}{\partial x_i^j(R,s)} = - \left[ \lambda_i q(R,s) - \mu_i(s) \right] \pi(s)f_H(R|s) \leq 0. \]

This equation holds with equality only at the lowest value of \( q(R,s) \) in state \( s \). In other words, investors only purchase \( x_i^j(R,s) > 0 \) if \( q(R,s) = q(s) \), where \( q(s) \) is defined to be the lowest price of a state-contingent security for some return state \( R \) in state \( s \).

Suppose then that in equilibrium \( \sum_j \mu_j^i x_j^j(R,s) < x_i^j(R,s) \). Then, at least one institutional investor \( i \) is purchasing \( x_i^j(R,s) > 0 \). As a result, we have \( q(R,s) = q(s) \) for all \( R \) in state \( s \), that is the price is constant in aggregate state \( s \). Moreover, \( q(s)\lambda_i^i = \mu_i^i(s) \).

From here, we can obtain \( \lambda_i^j \) from the FOC for \( c_i^0 \) and \( \mu_i^j(s) \) from the FOC for \( c_i^1 \). Substituting in, we obtain

\[ \frac{\partial u_i^1 (c_i^1(s)|s, \gamma(s))}{\partial c_i^1(s)} = \frac{\partial u_i^0 (c_i^0)}{\partial c_i^0}q(s). \]

giving us the characterization of the consumption rules of institutional investors.

Finally, consider type-\( j \) (retail) investors. Given the constant price \( q(s) \), their Lagrangian is

\[ L^j = u_0^j (c_0^j) + E \left[ u_1^j (c_1^j(R,s)|s) \right] + \lambda^j \left[ w_0^j - c_0^j - \sum_s \pi(s) \int_R q(s)c_1^j(R,s)f_H(R|s)dR \right], \]
so that we have optimality condition for $c_1^i(R,s)$

$$\frac{\partial u^i_1 \left( c_1^i(R,s)|s \right)}{\partial c_1^i(R,s)} = \lambda^i q(s).$$

As a result, $c_1^i(R,s) = c_1^i(s)$ is constant within state $s$. The indifference condition follows immediately by combining with the FOC for $c_0^i$. This concludes the proof.

**B.6 Outside Equity**

The baseline model featured no role for (outside) equity-like instruments in the bank’s capital structure. We extend the model to incorporate risk aversion and risk shifting, ingredients known to generate a role for equity-like claims. Optimal contracts still feature a region of liquidations and a region of “bail-ins,” where the bank is held to its continuation agency rent. Above the bail-in region, the contract involves equity-like claims.\footnote{See e.g. Hilscher and Raviv (2014) for analysis of CoCo design on risk shifting.}

Banks are risk averse and have utility $u(c_1 + c_2)$ from consumption, while investors are risk averse and have utility $v(x_1 + x_2)$. Bank utility and marginal utility are finite at 0, and we normalize $u(0) = 0$. We incorporate risk shifting by extending the bank’s monitoring decision to $e \in \{L, H, RS\}$, where $e = RS$ is “risk shifting” and $e \in \{L, H\}$ are the high and low monitoring choices from before. Risk shifting does not generate a private benefit but affects the return density, $f_{RS}(R)$.\footnote{We could incorporate a private benefit or cost of risk shifting without qualitatively changing results.}

Define the likelihood ratios $\lambda_{L,H}(R) = \frac{f_{L}(R)}{f_{H}(R)}$ and $\lambda_{RS,H}(R) = \frac{f_{RS}(R)}{f_{H}(R)}$. Risk shifting inefficiently pushes mass towards the extremes of the distribution, which we formalize by defining a point $R_{RS} \in [R, \overline{R}]$ such that $\frac{\partial \lambda_{RS,H}(R)}{\partial R} < 0$ for $R < R_{RS}$ and $\frac{\partial \lambda_{RS,H}(R)}{\partial R} \geq 0$ for $R \geq R_{RS}$.

As before, we assume optimal contracts enforce $e = H$. The no-risk-shifting constraint is

$$\int_{R} u(c(R)) \left( f_{H}(R) - f_{RS}(R) \right) dR \geq 0 \tag{18}$$

while the incentive constraint is the same as before, except with $u(c(R))$. Investor participation
is given by
\[ Y_0 - A = \int_R v(x(R)) f_H(R) dR. \]

Define \( \lambda_H(R) = \frac{\mu_L}{\mu} \lambda_{L,H}(R) + \frac{\mu_{RS}}{\mu} \lambda_{RS,H}(R) \) and \( \mu = \mu_L + \mu_{RS} \).

To simplify exposition, we will assume that the characterization that follows satisfies both consumption monotonicity for the bank and liability monotonicity for investors.\(^{49}\) Characterization of contracts in settings that do not satisfy monotonicity is beyond the scope of this paper. Moreover, we assume that the region \( 1 + \mu (1 - \lambda_H(R)) < 0 \) is a connected set. This simplifies exposition.

**Proposition 16.** Let \( |S| = 1 \). Suppose that the region \( 1 + \mu (1 - \lambda_H(R)) < 0 \) is a connected set. The privately optimal contract is as follows.

1. In the region where \( 1 + \mu (1 - \lambda_H(R)) < 0 \), there are liquidations and bail-ins.
2. In the region where \( 1 + \mu (1 - \lambda_H(R)) \geq 0 \), there are bail-ins and “equity.” The equity sharing rule is
   \[ u'(c(R)) \left( 1 + \mu (1 - \lambda_H(R)) \right) = \lambda v'(RY_0 - c(R)) \]

The motivations behind the liquidation region and the bail-in region are as in the baseline model (we interpret bail-in region as automatically wiping out outside equity, but again there is an equivalence between principal write down and debt-equity conversion for bail-in debt). In the liquidation region, all other liabilities are wiped out. In the bail-in region, only “debt” holders are repaid. Consider next the “outside equity” region. First, bank risk aversion moderates payouts to the bank, smoothing the bank consumption profile on the upside and so giving away some of the equity value to investors. Second, bank consumption decreases with the average likelihood \( \lambda_H(R) \).

In the region \( R \leq R_{RS}, \lambda_H(R) \) is decreasing in \( R \) and so banker consumption is increasing. However, when \( R \geq R_{RS}, \lambda_{L,H} \) is falling while \( \lambda_{RS,H} \) is rising. This second effect, which comes from the risk shifting motivation, moderates payoffs to banks in high return states, which signal a higher likelihood that the bank engaged in risk shifting.

\(^{49}\)Note that because both agents are risk averse, there is less scope for live-or-die contracts.
We could also derive the socially optimal contract, which would internalize the fire sale spillover cost of liquidations. However, conditional on not liquidating, bank and planner incentives are aligned, suggesting that the planner needs only to control the trade-off between liquidations and non-liquidations, and not the trade-off between bail-ins and “equity.”

B.6.1 Proof of Proposition 16

Given the assumption of consumption monotonicity, if there is a liquidation region, it satisfies a threshold rule \( R \leq R_l \). We define the optimal contract in terms of this threshold rule and in terms of liabilities \( x(R) \) above this threshold. The bank’s Lagrangian is given by

\[
\mathcal{L} = \int_{R \geq R_l} u(c(R)) f_H(R) dR \\
+ \mu_L \left[ \int_{R \geq R_l} u(c(R)) \left( f_H(R) - f_L(R) \right) dR - BY_0 \right] + \mu_{RS} \left[ \int_R u(c(R)) \left( f_H(R) - f_{RS}(R) \right) dR \right]
\]

\[
+ \lambda \left[ A + \int_{R \leq R_l} v(\gamma RY_0) f_H(R) dR + \int_{R \geq R_l} v(RY_0 - c(R)) f_H(R) dR - Y_0 \right]
\]

\[
+ \int_{R \geq R_l} \chi(R) [c(R) - bRY_0] f_H(R) dR
\]

Define \( \tilde{\lambda}_H(R) = \frac{\mu_L}{\mu} \lambda_{L,H}(R) + \frac{\mu_{RS}}{\mu} \lambda_{RS,H}(R) \) and \( \mu = \mu_L + \mu_{RS} \). We can combine the second line and obtain

\[
\mathcal{L} = \int_{R \geq R_l} u(c(R)) f_H(R) dR \\
+ \mu \left[ \int_{R \geq R_l} u(c(R)) \left( 1 - \tilde{\lambda}_H(R) \right) f_H(R) dR - \frac{\mu_L}{\mu} BY_0 \right]
\]

\[
+ \lambda \left[ A + \int_{R \leq R_l} v(\gamma RY_0) f_H(R) dR + \int_{R \geq R_l} v(RY_0 - c(R)) f_H(R) dR - Y_0 \right]
\]

\[
+ \int_{R \geq R_l} \chi(R) [c(R) - bRY_0] f_H(R) dR
\]

\[
^{50}\text{If effort were a continuous choice variable that affected bank returns, there would be an incentive to govern this margin. See Mendicino et al. (2018) for a numerical study of this problem.}
\]
The derivative in $R_l$ is given by

$$\frac{1}{f_H(R_l)} \frac{\partial \mathcal{L}}{\partial R_l} = -u(c(R_l)) \left[ 1 + \mu \left( 1 - \lambda_H(R_l) \right) \right] + \lambda \left[ v(\gamma RY_0) - v(RY_0 - c(R_l)) \right]$$

so that liquidations may be optimal when $1 + \mu \left( 1 - \lambda_H(R_l) \right) < 0$, that is when the average likelihood ratio is high. At low values of $R_l$, both the risk shifting and shirking problems have high likelihoods, so that $\lambda_H$ is large. As a result, bank consumption contributes negatively to welfare. Provided that this negative contribution outweighs the resource cost to investors, we have $R_l > R$.

Next, consider the region above $R_l$. The FOC for consumption $c(R)$ is

$$0 = u'(c(R)) \left( 1 + \mu (1 - \lambda_H(R)) \right) - \lambda v'(RY_0 - c(R)) + \chi(R)$$

so that we have $\chi(R) > 0$ when $1 + \mu (1 - \lambda_H(R)) < 0$. As a result, for all values $1 + \mu (1 - \lambda_H(R)) < 0$, we either have liquidation or bail-in.

Finally, for $1 + \mu (1 - \lambda_H(R)) > 0$, we either have bail-in or an interior consumption value. When consumption is interior, it satisfies a risk sharing rule

$$u'(c(R)) \left( 1 + \mu (1 - \lambda_H(R)) \right) = \lambda v'(RY_0 - c(R))$$

giving us an “equity” sharing rule.

Finally, the only role of assuming $1 + \mu (1 - \lambda_H(R)) < 0$ is a connected set in the proof is to ensure that it there are no points with $1 + \mu (1 - \lambda_H(R)) \geq 0$ below $R_l$.

### B.7 Premium for standard debt

In the baseline model, the incentive problem is the only motivation for issuance of standard debt. In practice, standard debt can enjoy a premium relative to all other instruments, meaning it can pay a lower rate of return to investors. There are two natural stories for such a premium. The first is that standard debt takes the form of demand deposits, which enjoy a liquidity premium and require a
lower rate of return. The second is that standard debt enjoys preferential tax treatment. We show that contracts still feature standard and bail-in debt, and that the trade-off is largely the same up to the consideration of the return premium. We then discuss potential issues with a pure premium story for standard debt.

Suppose that standard debt has required return $\frac{1}{1+r}$, where $r > 0$. We obtain the following result.

**Proposition 17.** Suppose the model is extended to include a premium for standard debt. Optimal contracts combine standard and bail-in debt. The private optimality condition for standard debt is

$$\mu b \left( \frac{f_L(R)}{f_H(R)} - 1 \right) = b + \lambda [(1 - b) - \gamma] + r \left[ \lambda [(1 - b) - \gamma] - \lambda \frac{1 - F_H(R)}{R_l f_H(R)} \right].$$

while the optimality condition for bail-in debt is the same as in Proposition 2. The tax on $R_l$ that decentralizes the socially optimal contract is

$$\tau_l = -(1 + r) R_l f_H(R) \frac{\partial \gamma(\Omega)}{\partial \Omega} \int_R^{R_l} R Y_0 f_H(R) dR$$

while the tax on bail-in debt is $\tau_u = 0$.

Relative to the baseline case where $r = 0$, when $r > 0$ we have the term

$$r \left[ \lambda [(1 - b) - \gamma] - \lambda \frac{1 - F_H(R)}{R_l f_H(R)} \right]$$

in the private optimality condition, reflecting an additional cost/benefit trade-off of increasing use of standard debt. This term contains two additional effects of the presence of the liquidity premium. On the one hand, the higher liquidity premium implies that the costs of liquidation go up, because the resources lost would have been repaid to investors who have a high willingness to pay. On the other hand, replacing bail-in debt with standard debt increases payoff to investors with high willingness to pay in non-liquidation states. The bank privately trades off these two forces in
choosing the optimal standard debt level, in addition to the incentive forces.

**Premium versus Incentive Problems.** If \( r > 0 \), then the bank is willing to issue standard debt even in the absence of an incentive problem, that is if \( B = b = 0 \) and hence \( \mu = 0 \). The premium story alone can generate use of standard debt in the bank’s capital structure. However, in the absence of the incentive problem the logic of Corollary 4 applies. The bank (without loss of generality) uses equity as its other instrument. The planner can implement optimal regulation with an equity requirement. By including the incentive problem, our model provides a role for bail-in debt in optimal contracts.

What if instead \( B > 0, b = 0, \) and \( r > 0 \), so that standard debt has value from a premium perspective, but not from an incentive perspective (relative to bail-in debt). In this case, the optimal contract would combine standard and bail-in debt. However, this story on its own is problematic for two reasons.

The first is that because bail-ins typically apply to long-term debt, which were also non-contingent prior to the crisis, the premium story revolves around premiums on long-term debt, which is likely due to tax incentives. But if the government is subsidizing (non-contingent) long-term debt, this suggests it must provide some fundamental economic benefit. Our model provides a fundamental economic benefit of non-contingent long-term debt.

A second and closely related way to understand this issue is that in the event that \( b = 0 \), banks have strong incentives to protect themselves against liquidations by backing their non-contingent claims with liquid assets such as treasuries. This relates to a fundamental question in the banking literature: why are illiquid assets paired with fragile (often deposit) financing? Our model endogenously pairs illiquid assets with fragile (non-contingent) financing, rather than exogenously imposing it. Optimal regulation in our model respects the fundamental activity of banks: backing illiquid assets with fragile funding. A model that relies exclusively on a standard debt premium

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51 As a technical aside, of course a bank with no incentive problem and an expected return greater than 1 would, given linear technology, scale up to infinity. This issue is fixed simply by assuming that banks operate a concave technology \( Y_0 = f(I_0) \) to produce projects.
naturally lends itself to a “narrow banking” result, where not only the planner but also banks prefer to use safe treasuries to keep the bank from ever failing.

We could nevertheless adopt this view. The main result that would change is the non-optimality of bailouts (Proposition 7), which would no longer generically hold. We would be back into an incomplete markets world, in which bailouts may be desirable to mitigate fire sales, in a standard way. Moreover in the case of deposit insurance, the planner would always prefer to bail out the bank, rather than liquidating and repaying depositors. Bailing out the bank would save resources without distorting bank incentives, and so would be strictly preferred to liquidation.

B.7.1 Proof of Proposition 17

Relative to the baseline model, the only change is that the participation constraint becomes

\[ Y_0 - A = \int_R^{\bar R_l} (1 + r) \gamma R Y_0 f_H(R) dR + \int_{R \geq R_l}^{\bar R_u} [(1 + r)(1 - b) R_l Y_0 + x_1(R)] Y_0 f_H(R) dR \]

where \( x_1(R) \) is repayment pledged to other investors. Note that it is immediate that standard debt enjoys priority over other liabilities, since it has the lower required rate of return. The proof that optimal contracts combine standard and bail-in debt follows as in the proof of Proposition 2. As a result, the optimization problem that determines \( R_l \) and \( R_u \) is the same as before, except that the participation constraint is now

\[ Y_0 - A = \int_R^{\bar R_l} (1 + r) \gamma R Y_0 f_H(R) dR + \int_{R \geq R_l}^{\bar R_u} [(1 + r)(1 - b) R_l + (1 - b)(R - R_l)] Y_0 f_H(R) dR \]

\[ + \int_{R_u}^{\bar R_u} [(1 + r)(1 - b) R_l + (1 - b)(R_u - R_l)] Y_0 f_H(R) dR \]

This yields the private optimality condition for \( R_l \)
$$0 = -bR_l Y_0 f_H(R_l) - \mu b R_l Y_0 \left( 1 - \frac{f_L(R_l)}{f_H(R_l)} \right) f_H(R_l)$$

$$+ \lambda \left[ (1 + r) \gamma R_l Y_0 f_H(R_l) - (1 + r)(1 - b) R_l Y_0 f_H(R_l) \right] + \lambda \int_{R_l}^R r(1 - b) Y_0 f_H(R) dR$$

which rearranges to

$$\mu b \left( \frac{f_L(R_l)}{f_H(R_l)} - 1 \right) = b + \lambda [(1 - b) - \gamma] + r \left[ \lambda [(1 - b) - \gamma] - \lambda \frac{1 - F_H(R_l)}{R_l f_H(R_l)} \right]$$

Because $R_u$ is not directly impacted by the liquidity premium, the optimality condition for $R_u$ is as before, assuming that $R_u > R_l$.

The planning problem features a wedge of the same form as before. The only difference is that the wedge is now weighted by $1 + r$, reflecting the higher liquidation losses. In other words, the planning problem is decentralized by the tax

$$\tau_l = -(1 + r) R_l f_H(R_l) \frac{\partial \gamma(\Omega)}{\partial \Omega} \int_{R_l}^R R Y_0 f_H(R) dR.$$  

As before, $R_u$ does not contribute to liquidations, and therefore $\tau_u = 0$.

### B.8 Insured Deposits and Bailouts

In addition to fire sale spillovers and moral hazard, another goal of bail-ins is to reduce the costs of protecting insured deposits. We consider the addition of a group of insured deposits, and explore how the planner chooses to protect depositors.

For simplicity, we assume that $\gamma < 1 - b$ does not depend on liquidations (no fire sale spillover). We further allow for the planner to commit ex ante to the desired combination of bailouts and insurance, so that the planner can always tie their hands and commit to no bailouts if desired. As a result, bailouts in this section will only occur if they are ex ante optimal.
The bank is constrained to issue insured deposits as a fixed fraction of its total assets, that is it issues \((1 - b)R_d Y_0\) in insured deposits for some fixed threshold \(R_d > R\). We abstract away from the socially optimal determination of \(R_d\), instead focusing on how the planner chooses to protect a given set of depositors.\(^{52}\) The bank is always insolvent if \(R < R_d\), absent intervention, regardless of its other liabilities. Because deposits are insured, the planner is liable for any shortfall relative to the face value \((1 - b)R_d Y_0\). Insured deposits are always at the top of the creditor hierarchy in liquidation.\(^{53}\)

Because the bank chooses bailouts with commitment, we set the political cost \(\kappa = 0\). When the planner bails out the bank, the bailout cost is

\[
\text{Cost}_{\text{No Liquidation}} = \tau ((1 - b)R_d Y_0 + x(R) - (1 - b)RY_0)
\]

where \(x(R)\) is any liabilities in excess of \((1 - b)R_d Y_0\) that the planner does not write down. When the planner instead allows the bank to fail, the creditor hierarchy implies the cost to deposit insurance is

\[
\text{Cost}_{\text{Liquidation}} = \min \{\tau ((1 - b)R_d Y_0 - \gamma RY_0), 0\}
\]

When \(x(R) = 0\), the cost of rescuing the bank with a bailout is lower than the cost of rescuing the bank under liquidation, due to the loss of pledgeable income in liquidation.

The planner solves for the optimal contract, which includes the rescue decision (either via bailout or via liquidation and repayment by insurance).\(^{54}\) We constrain bank consumption to be monotonic, that is \(c(R)\) must be nondecreasing in \(R\).\(^{55}\) which was satisfied by optimal contracts in the baseline model. This implies that bailouts must be monotonic: if a an insolvent bank \(R\) is bailed out, then all insolvent banks \(R' \geq R\) must also be bailed out. This rules out the possibility that the

\(^{52}\)For example, the planner may use deposit insurance to backstop risk averse depositors.

\(^{53}\)In practice, banks may issue wholesale funding which is not insured but runs prior to resolution.

\(^{54}\)A technical aside is that it is possible that the planner does not find it optimal to allow the bank to scale up as much as possible due to the cost of insuring deposits. We assume this is not the case, for example if \(R_d\) is close to \(R\).

\(^{55}\)If \(c(R) > c(R')\) but \(R < R'\), the bank could increase its payoff ex post by destroying assets to bring its return down to \(R\). We look for contracts where value destruction is not ex post optimal.
planner bails out a bank with $R < R_d$ to protect depositors but liquidates a bank with $R > \frac{1-b}{\gamma} R_d$ for incentive reasons.

**Proposition 18.** Suppose that $c(R)$ must be monotonic, there are no fire sales, and there are insured deposits. The socially optimal contract consists of insured deposits $R_d$, standard debt $R_l \geq R_d$, and bail-in debt $R_u \geq R_l$. The following are true regarding the use of deposit insurance and bailouts.

1. If $R_l > R_d$, there is deposit insurance but no bailouts. The bank is liquidated when $R \leq R_l$.

2. If $R_l = R_d$, there is a threshold $R_L \leq R_d$ such that the bank is liquidated when $R \leq R_L$ and bailed out when $R_L \leq R \leq R_d$. The indifference condition is for bailouts (when interior) is

$$b + \tau (1 - b - \gamma) = \mu b \left( \frac{f_L(R_L)}{f_H(R_L)} - 1 \right).$$

Proposition 18 illustrates the trade-off between two mechanisms for protecting insured deposits. Bailing out the bank reduces the taxpayer cost of deposit insurance, but provides worse incentives for the bank. Whenever the planner allows use of standard debt in excess of insured deposits, that is $R_l > R_d$, then necessarily the planner will commit to rescue depositors but not the bank. In this case, there is deposit insurance but no bailouts.  

If $R_l = R_d$ and $R_L < R_d$, the planner uses bailouts ex post in order to reduce the cost of protecting depositors. This may or may not imply that the planner wishes to restrict use of insured deposits ex ante to avoid bailouts, depending on the motivation for deposit insurance. If deposit insurance is a way to provide a backstop to risk-averse depositors that the bank cannot provide itself, or if it is a way to stop sunspot runs, the planner may wish to allow enough insured deposits that it sometimes engages in bailouts.

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56 If bailouts are chosen in a time-inconsistent manner and if $R_d > R^{BO}$, there will have a mixture of bailouts and insurance independent of whether or not it is desirable. The planner will optimally set $R_l = R_d$.  

76
B.8.1 Proof of Proposition 18

Due to consumption monotonicity, there is a threshold $R_L \geq R$ for bank liquidation, with $R_L = R$ corresponding to no liquidations. As in the proof of Proposition 7, there are no bailouts above $R_d$, due to the taxpayer burden. We can thus split the problem into two parts.

First, suppose that the liquidation threshold satisfies $R_L > R_d$, and suppose that the planner finds it optimal to engage in bailouts in a states $R < R_d$. By consumption monotonicity, there are also bailouts for $R_d \leq R \leq R_L$. But then because transfers to regular investors are wasteful, it is optimal to set $R_L = R_d$, as in the proof of Proposition 7. The optimal contract does not feature both $R_L > R_d$ and bailouts.

Consider then the form of the optimal contract when $R_L > R_d$. Because there are no bailouts, the social objective function is

$$\int c_2(R)f_H(R)dR - \int_{R \leq R_L} \tau \max \{(1 - b)R_d - \gamma R, 0\} Y_0 f_H(R) dR$$

while the corresponding investor participation constraint is

$$Y_0 - A = \int_{R_L}^R \max \{(1 - b)R_d, \gamma R\} Y_0 f_H(R) dR + \int_{R \geq R_L} \left((1 - b)R_d + x(R)\right) f_H(R) dR$$

and where incentive compatibility is the same as in the baseline model. From here, note that the trade-off above $R_L$ is the same as in the baseline model. The model again combines standard and bail-in debt, as in the baseline model.

Consider next the optimal contract when $R_L < R_d$. $R_L$ then also corresponds to the bailout threshold, such that there are bailouts when $R_L \leq R \leq R_d$, and where $R_L = R_d$ corresponds to no bailouts. The resulting social objective function is

$$\int c_2(R)f_H(R)dR - \int_{R_L}^{R_d} \tau \left[(1 - b)R_d - \gamma R\right] Y_0 f_H(R) dR - \int_{R_L}^{R_d} \tau (1 - b)\left(R_d - R\right) Y_0 f_H(R) dR$$
while investor repayment is given by

\[ Y_0 - A = (1 - b)R_d Y_0 + \int_{R_d}^{R} x(R) f_H(R) dR \]

reflecting that depositors are always repaid. Finally, incentive compatibility is as in the baseline model. Optimal contracts again combine standard and bail-in debt.

Consider the choice of the liquidation threshold \( R_L \). The trade-off is the same as in the baseline model, expect that an increase in the liquidation threshold leads to a tax burden on taxpayers rather than a cost to investors. That is, the FOC for the liquidation threshold is

\[ 0 = -bR_L - \mu b R_L \left( 1 - \frac{f_L(R_L)}{f_H(R_L)} \right) - \tau \left[ (1 - b)R_d - \gamma R_L - (1 - b)(R_d - R_L) \right] \]

which simplifies to

\[ b + \tau (1 - b - \gamma) = \mu b \left( \frac{f_L(R_L)}{f_H(R_L)} - 1 \right). \]

The only change is that the effective costs of liquidations has risen, due to the greater burden on taxpayers (\( \tau > \lambda \)). If the solution to this equation features \( R_L < R_d \), then there are bailouts in states \( R_L \leq R \leq R_d \).