

Bail-Ins, Optimal Regulation, and Crisis Resolution

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Abstract

We provide a framework to study bail-in regimes for banks. In the presence of a monitoring problem, the optimal bank capital structure combines standard debt, which liquidates the bank and provides strong monitoring incentives, and bail-in debt, which recapitalizes the bank but provides weaker incentives. A bail-in regime, which increases use of bail-in debt, is the optimal regulatory policy when liquidation is socially costly due to fire sales or bailouts. Bail-ins fully replace bailouts. Bail-ins can generate self-fulfilling crises in long-term debt markets, leading to bank runs. Debt guarantees and an expanded notion of lender of last resort prevent these crises, and should complement bail-ins in the crisis resolution toolkit.

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1 Introduction

In the aftermath of the 2008 financial crisis, the question of orderly bank resolution has received significant attention on both sides of the Atlantic. In many advanced economies, governments employed bailouts to stem financial turbulence in late 2008 and early 2009.¹ Bailouts were arguably very effective at stabilizing financial markets, but have been criticized for leading to moral hazard and perverse redistribution.² As a result, the US (Title II of the Dodd-Frank Act) and the EU (Bank Recovery and Resolution Directive) have introduced “bail-ins,” which allow the government to impose haircuts on (long-term) debt holders. The Dodd-Frank Act lists ensuring that “creditors and shareholders will bear the losses of the financial company” as one of the primary purposes of bail-ins, and requires that “[n]o taxpayer funds shall be used to prevent the liquidation of any financial company under [Title II].”³ Nevertheless, there is limited formal economic analysis on bail-ins.

Two important concerns arise from the introduction of bail-ins. The first is that if bank solvency can be improved by replacing standard debt with bail-in debt, then what prevents banks from efficiently doing so using private contracts?⁴ Moreover, why are bail-ins a preferable instrument to other liability instruments, such as equity, or to other recapitalization methods such as bailouts.

The second is that during a crisis, the possibility of haircuts may destabilize financial markets. In the words of former Treasury Secretary Timothy Geithner, “*the consequence of the haircuts imposed on creditors in the case of Lehman and Washington Mutual was a dramatic escalation in the scope and intensity of the run*” (Geithner (2016)). More recently in the resolutions of Veneto Banca and Banco Popolare di Vicenza in 2017, the Italian government spared senior debt holders from a bail-in in part due to concerns about investor confidence in other fragile Italian banks.⁵

Understanding these issues requires a framework in which debt is part of an optimal

¹Two examples in the US are the Troubled Asset Relief Program (TARP), which authorized the government to buy toxic bank assets, and the Temporary Liquidity Guarantee Program (TLGP), which provided for guarantees of bank debt.

²The Dodd-Frank Wall Street Reform and Consumer Act (Dodd-Frank Act) lists “protect[ing] the American taxpayer by ending bailouts” as one of its main objectives, and lists “minimiz[ing] moral hazard” (Section 204) as one of the purposes of bail-ins.

³Dodd-Frank Act Sections 204 and 214.

⁴For example, banks could use contingent convertible (CoCo) securities that have gained traction in Europe, which are an internal recapitalization instrument with a trigger event (for example, the bank’s capital ratio falling below some threshold) for either a principal write-down or a conversion into equity.

⁵For example, then Bank of Italy Deputy Governor Fabio Panetta stated “I think resolution would have been very costly not just in monetary terms but also in terms of confidence.” Political concerns and guarantee obligations were also important factors in the decision. Financial Times, “Why Italy’s €17bn bank rescue deal is making waves across Europe,” June 26, 2017.

liability structure, so as to understand the impacts of a regulator changing that liability structure. We provide such a framework in an optimal contracting model based on an incentive problem. Banks must monitor the quality of their loans both at the onset of the lending relationship and in its continuation. Because monitoring is not contractible at either stage, the optimal contract must incentivize monitoring, which involves the bank keeping a sufficient stake (“agency rent”) in its loan performance not only ex ante but also in continuation. Banks write optimal liability contracts in a complete markets setting, but must respect the underlying incentive problem.

In this framework, we show that the privately optimal bank contract can be implemented with a combination of two debt instruments: *non-bail-inable*, or standard, debt and *bail-in* debt. Standard debt has a face value that does not depend on the bank’s return, and leads to insolvency and liquidation when bank returns are low. This provides strong monitoring incentives to the bank for initial monitoring, by eliminating the bank’s continuation agency rent. Bail-in debt provides weaker incentives, by holding the bank to the continuation agency rent, but does not require a costly liquidation of the bank. Bail-in debt is useful because although it cushions against liquidation, it still provides maximal cash flow transfer on the downside. Both instruments retain the upside for the bank. Banks privately find it optimal to issue only standard debt when liquidation values are not too low.

We then study the design of socially optimal bank regulation in the presence of a fire sale externality – more bank liquidations reduce the recovery value to any individual bank in liquidation. The social planner writing the bank’s contract internalizes the fire sale, but must account for the bank’s incentive problem. The socially optimal contract can also be implemented with a combination of standard and bail-in debt. However, the social planner reduces the use of standard debt to mitigate the fire sale. A bail-in regime implements the socially optimal contract by changing the contingency properties of bank debt ex post, and so is the optimal regulatory regime.

The model helps to understand the role of bail-ins in the regulatory regime, as opposed to other forms of liability regulation such as equity requirements. Bail-in debt is less efficient at addressing the bank’s incentive problem than standard debt, but avoids social costs of bank failures. By contrast, equity worsens the bank incentive problem even further by giving away the upside of the bank to investors. This provides the role for bail-in debt in the regulatory regime.

The model further sheds light on why bail-ins are particularly appropriate regulatory mechanism for banks, as opposed to non-financial corporates. Non-financials may have higher average liquidation discounts than banks, but are less exposed to fire sales. Our

model predicts that non-financials adopt capital structures that are easier to resolve and make socially efficient use existing debt restructuring mechanisms such as Chapter 11. By contrast, banks take on capital structures that are more difficult to resolve, making socially inefficient use of private restructuring mechanisms due to the fire sale externality.

We next introduce the possibility of time-inconsistent bailouts, and study the trade-off between bail-ins and bailouts for recapitalizing insolvent banks. The privately optimal bank contract makes no use of bail-in debt whenever banks expect to receive bailouts. By contrast, the socially optimal contract uses sufficient bail-in debt to recapitalize the banks without ever engaging in bailouts. Bail-ins fully replace bailouts. The intuition is that whereas bailouts are socially costly due to distortionary taxation, the costs of bail-ins are efficiently priced into bank contracts *ex ante*. As a result, the planner prefers recapitalization via bail-ins over bailouts. This coincides with a core principle of post-crisis resolution, that the costs of bank resolution should be borne by bank investors and not by taxpayers.⁶

The model sheds light on the difference between the pre- and post-crisis worlds, and the role of bail-ins. Prior to the crisis, private use of bail-in debt was limited because the private cost of bankruptcy was low relative to the social cost, due to fire sale externalities and moral hazard from bailouts. Bail-ins reduce the social cost of bank failures while respecting the underlying private incentive problem that gave rise to debt contracts in the first place, and so constitute optimal regulation in the model.

We then turn to the second key question of the paper, of whether bail-ins can destabilize bank refinancing efforts. In order to do so, we show that privately and socially optimal contracts in our model can be implemented using standard (short-term) debt and bail-inable long-term debt, and model one of the important institutional features of bail-ins: they subordinate long-term debt to short-term debt in the event of bank failure. We argue that this implementation of the optimal contract in our model in fact naturally generates this property.

We demonstrate the existence of rollover crises for fundamentally solvent banks. A rollover crisis is a self-fulfilling prophecy in which long-term debt purchasers believe they are about to be bailed in and become unwilling to purchase newly issued long-term debt, so that the bank cannot refinance itself and is forced into bankruptcy and liquidation. Since the outstanding stock of short-term debt is senior to long-term debt in resolution, long-term debt receives no payoff, justifying the equilibrium beliefs. Bail-ins generate

⁶See Sections 204 and 214 of the Dodd-Frank Act as cited above. The Dodd-Frank Act further states that “[t]axpayers shall bear no losses from the exercise of any authority under this title” (Section 214). See also e.g. [French et al. \(2010\)](#).

refinancing instability in a bank that was fundamentally solvent, and can lead to inefficient bank failures.

Finally, we study policy options to bolster market stability and prevent rollover crises, as a complement to an effective bail-in regime. First, we show that an expanded *lender of last resort facility* (LOLR), which promises to lend *long-term* debt to banks, can eliminate rollover crises. Second, we show that an extension of *temporary* guarantees to new long-term debt during crisis times can help stabilize the market and prevent rollover crises, even though these guarantees are not fulfilled in equilibrium. These proposals have precedent in programs used by the US government during the 2008 financial crisis; for example, the *Temporary Liquidity Guarantee Program* (TLGP), which extended debt guarantees to new issuances of long-term debt.

These results imply that even while bail-ins may be an optimal regulatory regime, they can result in instabilities for banks during times of market stress. As a result, “bailout” policies that protect new debt issuances are a desirable complement to a well-functioning bail-in regime, even while bailout policies that protect existing debt are not.

Related Literature. First, we relate to a growing literature on bail-ins.⁷ Dewatripont and Tirole (2018) explore how bail-ins can complement liquidity regulation. Keister and Mitkov (2017) show that banks may not write down their (deposit) creditors if they anticipate government bailouts. Chari and Kehoe (2016) use a costly state verification model and show that bail-ins are not required in the optimal regulatory regime. In their model, costly state verification implies that standard debt contracts are the only renegotiation-proof contracts, so that the possibility of bail-ins leads to a reduction in standard debt issuance but not the use of bail-in debt. Mendicino et al. (2018) numerically explore the optimal composition of bail-in debt and equity in the presence of both private benefit taking and risk shifting, taking contracts as given and with a regulatory objective of protecting insured deposits. Pandolfi (2018) studies a related incentive problem to ours, but takes standard debt contracts as given. The paper argues that bailouts may be desirable in conjunction with bail-ins when bail-ins limit investment scale by weakening bank incentives. Walther and White (2020) show that precautionary bail-ins of long-term debt can signal adverse information about a bank’s balance sheet and cause a bank run, leading to an overly weak bail-in regime. Our contribution is to derive an optimal bank contract that combines standard and bail-in debt, to rationalize bail-ins as optimal regulation, and to study rollover

⁷In addition to the papers described below, see also e.g. Berger et al. (2020), Bolton and Oehmke (2019), and Colliard and Gromb (2018). See also a related literature on optimal derivatives protection, e.g. Biais et al. (2016) and Biais et al. (2019).

crises and their implications for the crisis resolution toolkit.

Second, we relate to a literature on contingent convertible debt securities as a recapitalization instrument, including studying the possibility of multiple equilibria arising from the conversion trigger.⁸ Multiplicity in our model arises from the relationship between short-term and long-term debt, and results in a bank run. We further study the role of lender of last resort and debt guarantees in preventing multiplicity.

Third, we relate to the literature on theories of debt in both the banking and corporate finance contexts.⁹ Our model incorporates an incentive theory of debt and combines two views of the role of debt: that debt is valuable both for cash flow transfer and for the explicit liquidation (or control rights transfer). In our model, standard debt generates liquidations and bail-in debt generates cash flow transfers.

Fourth, we relate to the literature on macroprudential regulation.¹⁰ This literature features two common motivations for macroprudential regulation – pecuniary externalities (e.g. fire sales) and fiscal externalities (bailouts) – and studies optimal ex ante regulation, possibly in conjunction with ex post bailouts. Our focus is on ex post bail-ins as an optimal policy, and whether it is a complement or substitute to macroprudential regulation and bailouts.

Finally, we connect to the literature on debt dilution. A large finance literature has focused on different methods of debt dilution, including issuance of new senior debt and maturity shortening. [Brunnermeier and Oehmke \(2013\)](#) show that maturity shortening can result from a deliberate desire to dilute longer-maturity creditors. [He and Milbradt \(2016\)](#) show that multiple equilibria can arise if long-term creditors expect to be diluted by future short-term creditors, resulting in a gradual maturity shortening. In our model, multiplicity arises because bail-ins imply existing short-term debt dilutes new long-term debt and results in an immediate (fundamental) bank run.

⁸See e.g. [Flannery \(2002\)](#), [Raviv \(2004\)](#), and [Sundareshan and Wang \(2015\)](#). See also [Flannery \(2014\)](#) for a broader overview of the literature.

⁹For example, [Calomiris and Kahn \(1991\)](#), [Dewatripont and Tirole \(1994\)](#), [Diamond and Dybvig \(1983\)](#), [Diamond and Rajan \(2001\)](#), [Hébert \(2018\)](#), [Innes \(1990\)](#), [Jensen and Meckling \(1976\)](#), [Nachman and Noe \(1994\)](#), and [Townsend \(1979\)](#). We also connect in particular to the related literature that emphasizes the monitoring role of banks, e.g. [Diamond \(1984\)](#) and [Holmstrom and Tirole \(1997\)](#).

¹⁰For example, [Bianchi \(2011\)](#), [Bianchi \(2016\)](#), [Bianchi and Mendoza \(2010\)](#), [Caballero and Krishnamurthy \(2001\)](#), [Dávila and Korinek \(2018\)](#), [Farhi et al. \(2009\)](#), [Farhi and Tirole \(2012\)](#), [Farhi and Werning \(2016\)](#), [Korinek and Simsek \(2016\)](#), [Lorenzoni \(2008\)](#), [Schmitt-Grohé and Uribe \(2016\)](#), and [Stein \(2012\)](#).

2 Model

We develop a three-period model with three economic agents: banks, investors, and arbitrageurs. Banks sign contracts with investors to raise investment funds, while arbitrageurs purchase bank projects that are liquidated prior to maturity. We tailor the model to address the core trade-off of bail-ins: between standard debt and bail-in debt. Our baseline model will have no role for instruments such as equity, or for other trade-offs that affect the use of debt (e.g. tax benefits). We consider such extensions in the appendix.

The three-period economy, $t = 0, 1, 2$, has a unit continuum of banks, investors, and arbitrageurs. Banks invest in a project of variable scale $Y_0 = A_0 + I_0 > 0$ by using their own funds, $A_0 > 0$, and funds $I_0 \geq 0$ from (date 0) investors. Investors are deep-pocketed at date 0 and can finance any investment scale.

Banks and investors are risk-neutral and do not discount the future. We denote bank consumption by (c_0, c_1, c_2) , so that bank expected utility is given by $E_0 [c_0 + c_1 + c_2]$. We denote the payments to investors by (x_1, x_2) . x_t is the actual amount received by investors, and is distinct from the face value of liabilities. Investor expected utility from the bank contract is $E_0 [-I_0 + x_1 + x_2]$. Contracts are subject to limited liability constraints, given by¹¹

$$c_0, c_1, c_2, x_1, x_2 \geq 0 \tag{1}$$

Banks need to refinance any liabilities that mature at date 1. They raise these funds from a set of (date 1) risk-neutral, no-discounting, deep-pocketed investors. Any projects they are forced to liquidate at this point are purchased by arbitrageurs, who generate an equilibrium liquidation price.

The economy features idiosyncratic uncertainty, but no aggregate uncertainty.¹²

2.1 Bank Projects

Banks extend financing to firms, thereby establishing a lending and monitoring relationship with those firms. When first extending funds to firms, banks monitor their borrowers, ensuring that the projects undertaken are of good quality. In doing so, banks develop specialized knowledge of that firm, and are uniquely able to monitor and collect from the firm in continuation. This relationship is the foundation of banking in our model. We refer to these relationships as bank projects.¹³

¹¹In our model, these are equivalent to the limited liability constraints $c_1 + c_2 \geq 0$ and $x_1 + x_2 \geq 0$.

¹²See Appendix B.2 for aggregate uncertainty.

¹³For simplicity, firms in our model earn zero surplus from the lending relationship.

Our model proceeds similarly to a multi-period version of [Innes \(1990\)](#). The bank project experiences a stochastic quality shock R at date 1, adjusting its scale to $Y_1 = RY_0$, at which point uncertainty is resolved. The shock R is idiosyncratic with a density $f_e(R)$ that has support over $[\underline{R}, \bar{R}]$. The state R is contractible, but the distribution of R depends on the bank's non-contractible monitoring effort $e \in \{H, L\}$, where $e = H$ is high monitoring effort and $e = L$ is low monitoring effort. $f_e(R)$ satisfies the *monotone likelihood ratio property* (MLRP), that is $\frac{\partial}{\partial R} \left[\frac{f_L(R)}{f_H(R)} \right] < 0$. MLRP is a standard assumption in generating debt contracts, and implies that high (low) returns are a signal that the bank exerted high (low) monitoring effort. We assume throughout the paper that optimal contracts induce high monitoring, so that $e = H$.¹⁴

Because monitoring effort is non-contractible, the bank chooses e to maximize bank utility after contracts have been signed. Given the consumption profile under the contract signed, the bank exerts high monitoring effort if

$$E [c_1(R) + c_2(R)|e = H] \geq E [c_1(R) + c_2(R)|e = L] + BY_0$$

where $B > 0$ is a private benefit of exerting low monitoring effort (“shirking”). We rearrange this incentive compatibility constraint to obtain the representation

$$E \left[(c_1(R) + c_2(R)) \left(1 - \frac{f_L(R)}{f_H(R)} \right) \middle| e = H \right] \geq BY_0. \quad (2)$$

Higher payoffs $c_1(R) + c_2(R, s)$ in states where the likelihood ratio $\frac{f_L(R)}{f_H(R)}$ is low relax incentive compatibility because these states signal that monitoring effort was high.

Although the quality shock R is realized at date 1, the project does not mature until date 2 and yields no dividend at date 1. If the project survives to date 2, it generates 1 unit of the consumption good per unit of final scale, $Y_2 = Y_1$. Only a portion $(1 - b)Y_2$ is pledgeable to investors, while the remaining portion bY_2 is retained by the bank. This non-pledgeable portion bY_2 is an agency rent in the continuation monitoring or collection problem of the bank.¹⁵ Holding projects to maturity implies a maximum pledgeability constraint $c_2 \geq bY_2$.

Projects can be liquidated prematurely at date 1, in which case they yield $\gamma Y_1 < Y_1$

¹⁴This can be guaranteed by assuming that $E[R|e = L] + B < 1$, so that the project is negative NPV under low effort. If optimal contracts induced low effort, incentive compatibility would either not bind or would lead the bank to adopt a capital structure that rewarded the bank for low returns and punished it for high returns.

¹⁵See e.g. [Holmstrom and Tirole \(1997\)](#). We assume that the continuation payoff under shirking is sufficiently low that shirking in continuation is never optimal.

units of the consumption good at date 1 and nothing at date 2, with the proceeds accruing entirely to investors.¹⁶ We assume that $\gamma < 1 - b$, so that liquidating the project is not desirable from an investor repayment perspective and so are not desirable ex post. Liquidations may be ex ante efficient for incentive reasons, since the bank can be paid 0 when liquidated but must be paid the agency rent bY_2 when not liquidated.

Finally, because banks are risk-neutral and do not discount the future, banks are indifferent to whether they consume at date 1 or date 2. We set $c_1(R) = 0$ to ease the exposition going forward.

2.2 Bank Liabilities

In order to raise investment $I_0 \geq 0$, banks pledge state-contingent liabilities to investors at date 0, which promise a face value of $L_t(R) \geq 0$ to be paid in period t . Banks may pledge a face value of liabilities in excess of pledgeable income. In such states, the bank is then unable to repay its liabilities in full. It enters bankruptcy and liquidates its assets.¹⁷

Without loss of generality, we assume that banks pledge one-period liabilities contracts, so that $L_2(R) = 0$. This is without loss of generality in our model since investors are indifferent to the period in which repayment occurs. Given a liability structure $L_1(R)$, the resulting payoff profiles of banks and investors are

$$(c_2(R), x_1(R)) = \begin{cases} (RY_0 - L_1(R), L_1(R)), & L_1(R) \leq (1 - b)RY_0 \\ (0, \gamma RY_0), & L_1(R) > (1 - b)RY_0 \end{cases} \quad (3)$$

where $c_1(R) = x_2(R) = L_2(R) = 0$. To understand this payoff profile, if $L_1(R) \leq (1 - b)RY_0$, the bank can roll over its face value of liabilities by raising money from date 1 investors, who break even at the same face value $L_1(R)$. Date 0 investors receive $x_1(R) = L_1(R)$, while the bank receives $c_2(R) = Y_2 - L_1(R)$. If instead $L_1(R) > (1 - b)RY_0$, the face value of liabilities exceeds pledgeable income and the bank is liquidated, yielding payoffs $x_1(R) = \gamma(s)RY_0$ and $c_2(R) = 0$.

The voluntary investor participation constraint states that investors must at least break even in expectation on the contract they signed, and it is given by

$$Y_0 - A_0 \leq E[x_1(R) | e = H]. \quad (4)$$

¹⁶We think of the liquidation discount as arising from selling projects to second-best users, who have not developed the knowledge of the firm lending relationship that the bank has. In this sense, we also assume the banker is not severable from the bank.

¹⁷We could obtain the same results by assuming the bank can both pledge liabilities and make a commitment to liquidation. In this sense, excess liabilities serve as the method of committing to liquidation.

where $I_0 = Y_0 - A_0$ is the amount financed by investors.

Finally, we assume that liabilities $L_1(R)$ must be monotone, that is

$$R \geq R' \Rightarrow L_1(R) \geq L_1(R'). \quad (5)$$

Monotonicity is a common assumption in many settings of optimal contracts or security design. It generates the flat face value of liabilities in high-return states.¹⁸

2.3 Arbitrageurs and Liquidation Values

At date 1, arbitrageurs can purchase bank projects and convert them into the consumption good using a production technology $\mathcal{F}(\Omega)Y_0$, where Ω is the economy-wide fraction of bank projects purchased relative to the initial scale. Arbitrageur surplus at date 1 from purchasing projects is $\mathcal{F}(\Omega)Y_0 - \gamma\Omega Y_0$, yielding an equilibrium liquidation price

$$\gamma = \gamma(\Omega) = \frac{\partial \mathcal{F}}{\partial \Omega}, \quad \Omega = \int_R \alpha(R) R f_H(R) dR \quad (6)$$

where $\alpha(R) \in \{0, 1\}$ indicates whether or not a bank liquidates in state R , with $\alpha(R) = 1$ denoting liquidation. When $\frac{\partial \gamma}{\partial \Omega} = \frac{\partial^2 \mathcal{F}}{\partial \Omega^2} < 0$, there is a *fire sale spillover* from bank liquidations: more liquidations reduce the liquidation value.

Arbitrageurs have initial wealth $\bar{A} - A_0$, but cannot borrow against future income. Their total date 0 welfare is $u(\bar{A} - A_0) + (\mathcal{F}(\Omega)Y_0 - \gamma\Omega)Y_0$, with $u'(\bar{A}) > 1$ so that the borrowing constraint binds. The intertemporal borrowing constraint ensures that fire sales result in Pareto inefficiencies by creating a wedge between the date 0 and date 1 marginal values of wealth. It operates similarly to the case where there are multiple date 1 aggregate states, and incomplete markets prevent arbitrageurs from equating the marginal value of wealth across date 1 states.

2.4 Bank Optimal Contracting and Equilibrium

Every bank takes the equilibrium liquidation values γ as given, and signs a contract (L_1, Y_0) with investors. Banks maximize their own expected utility

$$\max_{L_1, Y_0} E [c_2(R) | e = H],$$

¹⁸For example, one justification offered is that banks would be incentivized to pad their returns, for example by secretly borrowing from a third party.

subject to incentive compatibility (2), investor participation (4), monotonicity (5), and limited liability (1), where c_2 and x_1 are given by equation (3). Figure 1 presents a simple timeline underlying this contracting problem.

Since all banks are identical ex ante, all banks sign the same equilibrium contract. Therefore, $\alpha(R) = \mathbf{1}_{L_1(R) > (1-b)RY_0}$, where $\mathbf{1}_{\{\cdot\}}$ is the indicator function. An equilibrium of the economy is a set of liquidation values γ such that the contract (L_1, Y_0) is optimal, given γ , and such that liquidation values are determined by equation (6), given (L_1, Y_0) .

3 Privately and Socially Optimal Contracts

In this section, we characterize the privately optimal contract written by banks that take as given the liquidation value γ . We compare the privately optimal contract to the contract written by the social planner who internalizes the fire sale spillover. In both cases, the optimal contract can be implemented by a combination of two debt instruments. The first, which we call standard debt, has a fixed face value that does not depend on R , and liquidates the bank in low-return states. The second, which we call bail-in debt, has a face value that can be written down based on R , and restores bank solvency when total debt exceeds the pledgeable income. Although the bank and planner both agree that the optimal liability structure combines standard and bail-in debt, they disagree on the relative use of the two instruments. The planner wishes to use less standard debt than the bank, internalizing the fire sale spillover.

3.1 Privately Optimal Contract

We begin by characterizing the privately optimal bank contract in terms of two thresholds, R_l and R_u . We then associate these two thresholds with the two debt instruments. These thresholds are sufficient statistics for the privately optimal liability structure of the bank.

Proposition 1. *A privately optimal bank contract has a liability structure*

$$L_1(R, s) = \begin{cases} (1-b)R_l Y_0, & R \leq R_l \\ (1-b)RY_0, & R_l \leq R \leq R_u \\ (1-b)R_u Y_0, & R_u \leq R \end{cases}$$

where $0 \leq R_l \leq R_u \leq \bar{R}$. The bank is liquidated if and only if $R \leq R_l$. These thresholds, when

interior and not equal,¹⁹ are given by

$$\underbrace{\mu b \left(\frac{f_L(R_l)}{f_H(R_l)} - 1 \right)}_{\text{Incentive Provision}} = \underbrace{b + \lambda (1 - b - \gamma)}_{\text{Liquidation Costs}} \quad (7)$$

$$0 = E \left[\underbrace{\lambda - 1}_{\text{Investor Repayment}} - \underbrace{\mu \left(1 - \frac{f_L(R)}{f_H(R)} \right)}_{\text{Incentive Provision}} \mid R \geq R_u, e = H \right] \quad (8)$$

where $\mu > 0$ is the Lagrange multiplier on incentive compatibility (2) and $\lambda > 1$ is the Lagrange multiplier on investor participation (4).

Proof. All proofs are contained in the appendix.²⁰ ■

Before discussing the properties of the privately optimal contract, we associate these two thresholds R_l and R_u with the two debt instruments that we discussed before. We associate R_l with standard debt and R_u with bail-in debt.

To understand this terminology consider Figure 2, which illustrates the optimal liability contract. There are three regions of the liability structure. The first region, where $R \leq R_l$, is one where the total face value of liabilities is constant, but exceeds the pledgeable income of the bank. In this region, the bank is liquidated, and investors only receive partial repayment on the face value of their debt contracts. This is a standard debt contract.

In the third region, above R_u , investors receive a constant payoff equal to the total face value of liabilities, so that $(R_u - R_l)Y_0$ corresponds to the total level of bail-in debt. What distinguishes bail-in debt from standard debt is that in the second region, where $R_l \leq R \leq R_u$, the face value of bail-in debt is written down to $((1 - b)R - R_l)Y_0$. This recapitalizes the bank and allows it to continue operating, rather than being liquidated. We refer to this as bail-in debt because its face value can be written down (“bailed in”) based on the idiosyncratic state.

¹⁹For the remainder of the paper, we assume that the thresholds are interior and not equal, except when explicitly stated otherwise. Generally speaking, R_l will be interior when the likelihood ratio $\frac{f_L(R)}{f_H(R)}$ is sufficiently large, that is when \underline{R} is a sufficiently good signal of low effort. R_u will be interior when $\frac{f_L(\bar{R})}{f_H(\bar{R})}$ is sufficiently small and $\mu > \lambda - 1$, that is when \bar{R} is a sufficiently good signal of high effort.

²⁰In the proof of this proposition, see Appendix A.1.1 for a comment on non-uniqueness of face value of liabilities $L_1(R)$ below the threshold R_l . Non-uniqueness arises in this region because any face value of liabilities above $(1 - b)RY_0$ results in bank liquidation. We have chosen the face value of liabilities that correspond to standard debt, which seems most natural in the context of banks and bail-ins. Moreover, uniqueness is restored if there is an $\epsilon \rightarrow 0$ premium for standard debt, for example due to tax benefits of debt. The face value of liabilities is unique above R_l .

Corollary 2. *The privately optimal contract can be implemented with a combination of standard debt with face value $(1 - b)R_l Y_0$, which cannot be written down contingent on the idiosyncratic state R , and bail-in debt with face value $(1 - b)(R_u - R_l)$, which can be written down contingent on the idiosyncratic state.*

For the remainder of the paper, we associate standard and bail-in debt with the thresholds R_l and R_u , respectively, rather than writing out their associated (face value) liabilities.

The core property of standard debt is that it forces liquidations in low-return states. Equation (7) describes the marginal trade-off the banker faces in replacing a unit of bail-in debt with a unit of standard debt. On the one hand, liquidating the bank results in a total resource loss $b + \lambda(1 - b - \gamma)$ to the bank and investors. On the other hand, pledging to liquidate the bank provides higher-powered monitoring incentives at date 0, reflected in the term $\mu b \left(\frac{f_L(R_l)}{f_H(R_l)} - 1 \right)$, by depriving the bank of the non-pledgeable income bRY_0 . In particular, the liquidation threshold features $\frac{f_L(R_l)}{f_H(R_l)} > 1$. That is, at R_l the likelihood ratio is greater than 1, implying that the state provides a stronger signal that low effort may have been exerted.

By contrast, for a given level of standard debt, an additional unit of bail-in debt does not change liquidations but does transfer value from banks to investors. The marginal trade-off is summarized in equation (8). On the one hand, the binding investor participation constraint implies this transfer is valuable ($\lambda - 1 > 0$), as it allows the bank to increase project scale. On the other hand, increasing the total debt level reduces bank consumption in high-return states, where the likelihood ratio $\frac{f_L(R)}{f_H(R)}$ is low and the signal of high effort is stronger. This weakens bank monitoring incentives and tightens the incentive compatibility constraint (2). The optimal level of bail-in debt equalizes these two effects on the margin.

The Role of Agency Problems. Our model features three ingredients that are jointly necessary to generate contracts that consist of combinations of standard and bail-in debt: the ex ante incentive problem ($B > 0$), limited pledgeability ($b > 0$), and costly liquidations ($\gamma < 1$). In the absence of any one of these elements, contracts in our model would not combine standard and bail-in debt.

Corollary 3. *The privately optimal contract can be implemented with a single liability instrument if $B = 0$, $b = 0$, or $\gamma = 1$.*

1. *If $B = 0$, then the privately optimal contract can be implemented with equity.*

2. If $b = 0$, then the privately optimal contract can be implemented with bail-in debt.
3. If $\gamma = 1$, then the privately optimal contract can be implemented with standard debt.

When $B = 0$, a standard Modigliani-Miller logic applies. The bank can ensure incentive compatibility with any monotone consumption policy $c_2(R)$, and does not need to use liquidations as a disciplining device. As a result, without loss of generality the bank uses entirely equity financing. The second and third cases show that $B > 0$ is not sufficient to generate a privately optimal contract that combines standard and bail-in debt. When $B > 0$, the privately optimal contract employs some debt instrument for ex ante incentive reasons. If $b = 0$, then all income is pledgeable to investors, and the bank can set $c_2(R) = 0$ without liquidating. The bank only uses only bail-in debt. If $\gamma = 1$ but $b > 0$, there is a limit to pledgeable income, but no bankruptcy costs from liquidation. Banks can repay any amount $x_1(R) \leq RY_0$ by liquidating bank projects, and the pledgeability constraint ceases to be relevant. Banks use only standard debt.

In both the second and third cases, the key property of debt is the full cash flow transfer from the bank to investors in low-return states. This corresponds to a common understanding of debt in the optimal contracting literature: the core property of debt is its payoff profile $x_1(R) = \min\{RY_0, R_u Y_0\}$.²¹ In the absence of pledgeability limitations, this value transfer is achieved with bail-in debt. In the absence of bankruptcy costs, this value transfer is achieved with standard debt.

However, if there are both limited pledgeability and bankruptcy costs, then bail-in debt cannot enact a full cash flow transfer, while standard debt enacts a full cash flow transfer at a resource cost. A role emerges for both forms of debt in the optimal contract.

Why Didn't Banks Issue Bail-in Debt before 2008? Although Proposition 1 states that privately optimal bank contracts combine standard and bail-in debt, bail-in debt is largely a post-crisis innovation that was "introduced" by bail-in regulation: it places contingencies into debt contracts where few (if any) had previously existed. This leads to the question: Under what conditions would the privately optimal contract feature no bail-in debt?²²

The case where banks do not issue bail-in debt, that is $R_u = R_l$, is a corner solution of the model. This case is summarized in the following corollary to Proposition 1.

²¹For example, see Hébert (2018).

²²Unfavorable tax or regulatory treatment may also have contributed to a lack of bail-in debt issuance. Prior to the crisis, regulatory requirements were generally equity requirements, which bail-in debt would not count towards. We see some support for this force mattering in the fact that CoCos have grown in use post-crisis in the EU, where they often count towards regulatory capital requirements, but not in the US, where they do not count towards regulatory capital requirements.

Corollary 4. *Suppose that the solution R_l to equation (7) satisfies*

$$E \left[\lambda - 1 - \mu \left(1 - \frac{f_L(R)}{f_H(R)} \right) \mid R \geq R_l, e = H \right] \leq 0 \quad (9)$$

Then, banks do not issue bail-in debt, that is $R_l = R_u$.

To understand Corollary 4, the left-hand side of equation (9) is the marginal value of increasing R_u above R_l , as in equation (8) from Proposition 1. When this marginal value is negative, the incentive costs of increasing the total debt level outweigh the investor repayment benefits, and the bank chooses $R_l = R_u$.

A key determinant of the trade-off between standard and bail-in debt is the cost of liquidations, γ . When liquidation values are not too low, the resource loss from liquidations is not too large and R_l increases,²³ pushing the bank towards $R_l = R_u$. Banks are less likely to issue bail-in debt when liquidation values are high.

3.2 Socially Optimal Contract

The social planner internalizes the fire sale spillover and writes bank contracts to maximize social welfare. For expositional simplicity, we assign a welfare weight of zero to arbitrageurs. Appendix B.1 illustrates generic Pareto inefficiency of the privately optimal contract, and shows that a similar characterization applies with positive welfare weights.

As a result, social welfare is bank welfare, with the only difference between the private bank contracting problem and the social planning problem is that the planner internalizes the fire sale spillover. The social planner writes contracts subject to the same conditions as banks, namely incentive compatibility (2), investor participation (4), monotonicity (5), and limited liability (1), with c_2 and x_1 given by equation (3). In other words, any socially optimal contract must also be privately feasible.

Proposition 5. *The socially optimal contract can be implemented with a combination of standard and bail-in debt. These debt levels are associated with endogenous thresholds R_l and R_u . Whenever $R_l < R_u$ is interior, R_l is given by*

$$\mu b \left(\frac{f_L(R_l)}{f_H(R_l)} - 1 \right) = b + \lambda ((1 - b) - \gamma) + \underbrace{\lambda \left| \frac{\partial \gamma(\Omega)}{\partial \Omega} \right| \int_{R_l}^{R_u} R f_H(R) dR}_{\text{Fire Sale Spillover. } \geq 0} \quad (10)$$

²³To see this, combine equation 7 with MLRP.

while R_u is given as before by equation (8).

Even though the planner uses the same debt instruments as the bank, the term $\frac{\partial \gamma}{\partial \Omega} < 0$ generates an additional *social* cost of liquidation in the planner's optimality condition for R_l . This liquidation cost term represents the only difference between the private and social optimality conditions in equations (7) and (10), respectively. Fire sale spillovers generate an additional social cost: the project liquidations of one bank increase the resource loss to all other banks that liquidate projects at the same depressed prices. By contrast, there is no additional wedge in the determination of R_u , since a greater total debt level arising from more bail-in debt does not change total liquidations. Relative to private banks, the planner prefers relatively less use of standard debt in favor of more use of bail-in debt. However, because the planner respects the underlying incentive problem, the planner prefers to replace standard debt with bail-in debt, which generates the cash flow transfer without costly liquidations.

In the case of Corollary 4, where banks had not found it optimal to issue bail-in debt, the additional social cost of bankruptcy implies optimal regulation can introduce bail-in debt into the bank's capital structure. The planner's motivation to do so does not result from an incentive to *complete markets* by writing contracts that banks could not write on their own. Rather, private contracts feature too little or no bail-in debt because banks do not internalize the fire sale spillover.

Bail-ins as Optimal Regulation. Proposition 5 provides the contract that the planner writes for banks, under ex-ante regulation which contractually specifies bail-ins. Bail-ins are often associated with an ex post resolution authority: the planner takes a debt contract with a fixed face value, and writes down that face value ex post. Both forms of authority are used in practice, with the US emphasizing ex post resolution and the EU being more accommodating of contractual recapitalization.²⁴

In our model, there is a straightforward equivalence between the two regulatory methods. Under the ex ante (contractual) implementation, the planner caps bank issuance of standard debt at R_l and requires all remaining debt $R_u - R_l$ to have contractual write-down provisions that restore solvency to the bank. Under the ex post (resolution authority)

²⁴In the US, banks are required to maintain a certain level of *total loss-absorbing capital* (TLAC), principally long-term debt and equity, to safeguard the bank against poor returns. Debt used to satisfy TLAC requirements must be plain-vanilla, implying a fixed face value, while debt with contractual contingencies cannot generally be used to satisfy TLAC requirements. In particular, "eligible external LTD [is] prohibited from including contractual triggers for conversion into or exchange for equity." 82 FR 8266. See Avdjiev et al. (2017) for background on the European case.

implementation, the planner caps the amount of *non-bail-inable* debt at R_l and designates the remaining $R_u - R_l$ to be *bail-inable*. The resolution authority can write down bail-inable debt ex post to recapitalize the bank, resulting in the same outcome. Since the model predicts that these two methods are equivalent, it is not necessarily surprising that we see both methods used in practice.²⁵

3.3 Bail-in Debt or Equity?

Proposition 1 highlights why standard debt can be a valuable loss-absorbing instrument for banks, relative to equity. Bail-in debt combines the incentive properties of standard debt with the loss-absorbing properties of equity. It generates a cash flow transfer below $R_u(s)$ and a flat investor payoff above $R_u(s)$, similar to standard debt, but does so without liquidating the bank. By contrast, equity transfers the upside of the bank to investors, which worsens incentives. Bail-in debt therefore achieves a capital structure that standard debt and equity combined cannot. Under the incentive problem of the baseline model, banks prefer bail-in debt to equity as a loss-absorbing instrument.

As in the privately optimal contract, bail-in debt, rather than equity, is used as the loss absorbing instrument. Although the social planner and the bank disagree about the costs of bankruptcy, they agree about the underlying incentive problem. As a result, the planner replaces standard debt with bail-in debt, which addresses the underlying incentive problem that standard debt was designed to solve. This provides a role for bail-in debt rather than equity.²⁶

Although the baseline model does not feature equity, in Appendix B.5 we add a role for equity in the model by incorporating risk aversion and risk shifting. We show that the core trade-off between standard debt and bail-in debt exists as in the baseline model.

3.4 Relationship to Debt Renegotiation and Restructuring

In practice, bail-in debt is generally associated with banks. However, the core (private) optimal contracting model of the paper could also be applied to non-financial corporates, some of whom may have high liquidation discounts even in the absence of fire sales. In principle, this suggests that non-financial corporates might also wish to use bail-in debt.

²⁵It is not necessarily surprising to see some preference for ex post implementation, given that there would be an incentive for regulatory arbitrage around contractual triggers to the extent possible.

²⁶In Appendix B.5, we add risk shifting and risk aversion, which generates a role for both bail-in debt and equity. The intuition for the role of bail-in debt is unchanged.

One interpretation in this spirit can be provided in the context of debt renegotiation and restructuring. Chapter 11 of the US Bankruptcy Code provides a reorganization and debt restructuring process for non-financials, allowing them to avoid liquidation under Chapter 7.²⁷ Our model predicts that if failures of non-financial firms are not associated with externalities such as fire sales or bailouts, they will use Chapter 11 efficiently.

In comparison to non-financials, banks take on sufficiently more short-term debt and do not make use of a Chapter 11 type process. In our model, one way to understand this difference is as follows. Banks and other financials may be relatively easier to liquidate during normal times, but are also associated with externalities and bailouts in crises. Our model suggests that both of these forces encourage banks to adopt capital structures that leave them with larger quantities of short-term, or non-resolvable, debt, and therefore do not make use of Chapter 11. By contrast, non-financials may have greater liquidation discounts on average, but may not be as vulnerable to fire sales or as likely to receive bailouts.²⁸ Our model suggests that they adopt capital structures that make them easier to resolve ex post, and make use of Chapter 11.²⁹

One important consideration is that the design of Chapter 11 may simply not be appropriate for banks due to the financial nature of their activities.³⁰ Reflecting this, the US Treasury Department has adopted a proposal for a Chapter 14 bankruptcy process, with the aim of creating a process in the spirit of Chapter 11 that is tailored to banks.³¹ Our model predicts that banks would privately under-utilize Chapter 14 relative to the social optimum by over-issuing standard (short-term) debt, leaving a role for a bail-in regime.

3.5 Discussion

We now provide additional discussion, including connecting our results to real-world regimes.

²⁷To the extent non-financials can control bankruptcy decisions ex ante, for example with their capital structure choices, a process like Chapter 11 provides an ex post alternative to ex ante contractual write-down arrangements. This is in the spirit of the equivalence between ex ante contractual provisions and ex post bail-ins in our model.

²⁸In this sense, we might think of financial companies as having a high average liquidation value γ , but also having a high sensitivity $\frac{\partial \gamma}{\partial \Omega}$. By contrast, non-financials may have a lower average liquidation value, but also a much lower sensitivity $\frac{\partial \gamma}{\partial \Omega}$.

²⁹For example, the percent of nonfinancial corporate debt that is short-term is approximately 32%, while the ratio of nonfinancial corporate debt to the market value of equities is approximately 34%. (US Flow of Funds)

³⁰See French et al. (2010).

³¹See US Department of Treasury (2018). See Scott and Taylor (2012) for the original proposal.

Short- and Long-Term Debt. In practice, bail-in regimes focus on write-downs of long-term debt, with short-term debt given priority over long-term debt in both resolution and normal insolvency.³² Both the privately and socially optimal contracts can be implemented by issuing R_l in standard short-term debt and $R_u - R_l$ in bail-inable long-term debt. Moreover, our model predicts that short-term debt should enjoy absolute priority over long-term debt in bankruptcy and liquidation, given this implementation.³³ To understand why, in the threshold state R_l , long-term debt is fully bailed in while short-term debt is fully repaid. If on the other hand the bank underwent normal bankruptcy and short- and long-term debt were equal claimants, long-term debt would receive positive repayment. This would be inconsistent with a No Creditor Worse Off principle of bank resolution, which requires that resolution be ex post *Pareto efficient* relative to insolvency.³⁴

CoCos. Bail-in debt in our model can be interpreted as a form of *contingent convertible* (CoCo) debt, a form of contractual bail-in instrument that has gained prominence in Europe.³⁵ The most natural interpretation in this context is that bail-in debt in our model is a principal write-down CoCo debt security that applies at the point of non-viability.³⁶

Firing the Banker. We assumed that the banker was not severable from the bank, and so could not be fired. In this case, standard debt can be viewed as enforcing a transfer of control rights to investors, whose best option is to liquidate the bank. The model could be extended to include costly firings of the banker, where a control rights transfer might happen with a change of management but without liquidation. If there were a trade-off between liquidation and firing the banker, banks would still over-use costly liquidations, not internalizing the fire sale.³⁷ The planner would prefer to reduce liquidations, and might choose instead to force a change of management.³⁸

³²For example, in the US the top-tier bank holding company is subject to a “clean holding company” requirement, which bars it from issuing short-term debt to external investors. See 12 CFR §252.64.

³³In practice, short-term debt priority has three implementations. The first is contractual: bail-in debt is junior to short-term debt. The second is organizational: short-term debt is issued at the operating subsidiary, whereas long-term debt is issued at the top-tier holding company. The third is legal: national bankruptcy law confers priority to short-term debt in the case of banks. The three are equivalent in our model, which may help understand why the method of guaranteeing short-term debt priority varies across countries.

³⁴See e.g. Article 73 of BRRD.

³⁵See Avdjiev et al. (2017) and Flannery (2014) for more background on CoCos.

³⁶Bail-in debt can also be expressed as a debt-equity conversion in our model, so that our model does not speak to the optimal form of CoCos (principal write-down and debt-equity conversion).

³⁷For example, if the cost of reorganizing under new management is greater than the loss from liquidation, the planner nevertheless may prefer firing the banker to avoid the fire sale spillover.

³⁸Of course, this assumes no diminishing marginal value in changing management. For example, a planner might face an issue similar to a fire sale if forced to change the management at many banks

Dynamic Agency Problems. We simplified the continuation (date 1) agency problem to a required agency rent. Although a formal dynamic model is beyond the scope of this paper, it is worth discussing briefly the forces that would arise with multiple periods of effort choice. In this setting, liquidations (“standard debt”) and maximal cash flow transfers (“bail-ins”) would still be valuable tools for incentive provision after low returns, while greater payoffs to the bank after high returns would likewise be valuable. This suggests that these instruments would still arise in the optimal contract along some histories.³⁹ However, if future shirking (low effort) reduced agency rents, then an incentive scheme that induced future shirking might constitute an optimal form of money burning, similar to liquidation. This might be seen as a form of optimal debt overhang. This might lead to an additional conflict between a social planner, who wishes to reduce overhang to increase effort and reduce the probability of failures, and the bank, who wants to use overhang as an incentive device.

4 Bailouts and Time Consistency

We now introduce bailouts to the model and study their role in the optimal regime. Bailouts may be welfare-enhancing in our model, even if they are chosen ex post without commitment, because they can mitigate the fire sale spillover.⁴⁰ However, a core principle of post-crisis regulation is that banks and bank investors, not taxpayers, should bear the costs of bank resolution.

We first study the private bank contracting problem and show that bailouts can completely eliminate banks’ incentives to issue bail-in debt. This offers another explanation for why banks wrote few contingencies into their debt contracts prior to the crisis. We then show that optimal regulation limits the use of standard debt so that no ex post bailouts occur: bail-ins fully replace bailouts. A planner who could tie her hands and never engage in bailouts would always prefer to do so.

The contracting problem is the same as before, except that banks understand they may be bailed out when insolvent.

simultaneously, where the marginal ability of the new banker hired declines as the planner is forced to search for a greater number of new bankers.

³⁹However, incentive provision would be linked over periods, likely leading to history dependence - future punishments enhance current incentives when they follow after current low returns, but exacerbate current incentives when they follow after current high returns.

⁴⁰Another view (Chari and Kehoe (2016)) is that a planner may be tempted ex post to bail out banks to prevent resource losses from liquidation, even in the absence of fire sale spillovers. The results of this section also hold in this case.

4.1 Ex Post Bailout Authority

The planner can bail out insolvent banks in order to prevent liquidations and fire sales. Bailouts are chosen ex post according to a utilitarian welfare function, and are financed by taxpayers.

The bailout required to recapitalize an insolvent bank is $T_1(R) = L_1(R) - (1 - b)RY_0$. Bailouts are associated with two costs. First, bailouts have an ex post (date 1) political or administrative cost κY_1 ,⁴¹ which we think of as corresponding to a political backlash against bailouts. Second, bailouts have an ex ante (date 0) cost $(\tau - 1)T_1(R)$ to taxpayers, which we think of as corresponding to distortionary costs of taxation. The distortionary cost $\tau - 1$ is sufficiently large that transfers from taxpayers to banks are not welfare-enhancing for redistributive reasons alone.⁴² The timing of the two costs in our model generates simple bailout rules.

We conjecture a threshold rule for bailouts, so that insolvent banks with $R \geq R^{BO}$ are bailed out, and then verify the rule is optimal. The optimal threshold R^{BO} is the solution to the ex post bailout problem

$$\max_{R^{BO} \leq R_l} \underbrace{\int_R^{R^{BO}} (\gamma(\Omega) - 1) R Y_0 f_H(R) dR}_{\text{Resource Losses from Liquidations}} - \underbrace{\int_{R^{BO}}^{R_l} \kappa R Y_0 f_H(R) dR}_{\text{Political Costs of Bailouts}}$$

where the transfer $T_1(R)$ does not appear in the objective function because the planner is utilitarian and the distortionary cost arises ex ante. The ex post bailout decision trades off losses from bank liquidation against the political cost of bailing out banks. The optimal bailout rule, when interior, is given by

$$\underbrace{1 - \gamma(\Omega)}_{\text{Resource Loss}} - \underbrace{\frac{\partial \gamma(\Omega)}{\partial \Omega} \int_R^{R^{BO}} R f_H(R) dR}_{\text{Fire Sale Spillover}} = \underbrace{\kappa}_{\text{Political Cost}} \quad (11)$$

Equation (11) implies a threshold bailout rule, as conjectured, with a unique solution R^{BO} .⁴³ If the planner continued bailing out banks beyond R^{BO} , concavity of γ implies that total loss falls below the political cost of bailouts. As a result, we have a threshold rule.

Even though R^{BO} is unique, equation (11) implies strategic complementarities in

⁴¹The political cost scales with bank size in order to prevent banks from “outgrowing” the cost.

⁴²See the proof of Proposition 7 for a formal condition. Even if redistribution were desirable, they could be done with ex ante lump-sum rather than ex post bailouts. We can interpret bank resources A as including any desirable redistribution.

⁴³To ensure strict (rather than weak) optimality of the threshold rule, we can add a distortionary cost $\tau_1 \rightarrow 0$ of transfers at date 1.

bank risk taking: the planner only has an incentive to engage in bailouts ex post if the equilibrium contract features $R_l > R^{BO}$.⁴⁴ If the equilibrium contract sets $R_l < R^{BO}$, but a single bank instead writes an alternative contract $R'_l > R^{BO}$, that bank will not be bailed out. As a result, there may be multiple equilibria of the date 0 private bank contracting problem. Our main result on the private contracting problem provides a generic result, and is agnostic to the equilibrium that is actually selected.

4.2 Privately Optimal Contract

As before, the privately optimal bank contract combines standard and bail-in debt. The only difference is that insolvent banks are now bailed out when the equilibrium contract sets $R_l \geq R^{BO}$.⁴⁵ We show that whenever the equilibrium contract sets $R_l > R^{BO}$, then $R_u = R_l$ and banks issue no bail-in debt. Bailouts crowd out bail-ins.

Proposition 6. *Suppose there are time-inconsistent bailouts. If the equilibrium private contract sets $R_l \geq R^{BO}$, then it also sets $R_u = R_l$. There is bail-in debt only if there are no bailouts.*

To understand Proposition 6, suppose that the equilibrium contract features $R_u > R_l \geq R^{BO}$. In states $R_l \leq R \leq R_u$, bail-in debt is written down and the bank consumes $c_2 = bRY_0$. Suppose that a single bank deviated by writing a contract that set $R'_l = R_u$, and otherwise left contract terms unchanged. The bank now receives bailouts over the range $R_l \leq R \leq R_u = R'_l$, meaning that standard debt holders are fully repaid, while the bank consumes $c_2 = bRY_0$. The investor participation constraint is relaxed, so that this contract strictly dominates the equilibrium contract, contradicting that $R_u > R_l \geq R^{BO}$ was an equilibrium optimal contract.⁴⁶

Proposition 6 provides a moral hazard view of limits to the private use of bail-in debt: banks do not use bail-in debt when they expect to be bailed out. The moral hazard view is particularly strong in the presence of fire sale spillovers, where resource losses are larger and bailout incentives stronger.

⁴⁴See e.g. Farhi and Tirole (2012). To see the complementarity in our model, the left hand side of equation 11 is increasing in the marginal bank that is liquidated in equilibrium. When there are bailouts, the marginal bank that is liquidated has return $R = R^{BO}$. When $R_l^* < R^{BO}$, the marginal bank that is liquidated has $R = R_l^*$, and so the value of rescuing any failing bank is below the cost κ .

⁴⁵Bank contract incentives do not change in the region of the contract above R_l . Below R_l , banks want to maximize on the bailout subsidy whenever possible. Due to liability monotonicity (5), this implies standard debt.

⁴⁶This result does not rely on fire sale spillovers. If $1 - \gamma > \kappa$ but γ does not depend on Ω , then $R^{BO} = \underline{R}$ and every insolvent bank is bailed out. Proposition 6 implies that no bail-in debt is issued.

4.3 Socially Optimal Contract With Bailouts

The moral hazard problem of Proposition 6 provides an additional role for bank regulation. We study the socially optimal contract in the presence of ex post bailouts, and show that it sets $R_l \leq R^{BO}$ so that no bailouts occur. This is true whether or not there are fire sale spillovers.

Proposition 7. *Suppose there are time-inconsistent bailouts, and suppose that τ is sufficiently large that ex ante transfers from taxpayers to banks are not welfare enhancing. The socially optimal contract sets $R_l \leq R^{BO}$, and there are no bailouts. Bank welfare is non-decreasing in R^{BO} .*

When banks fail ex post, the planner is tempted to bail them out to mitigate fire sales and prevent resource losses. However, bailouts are costly to taxpayers in a socially undesirable way. Although bail-ins are costly to investors ex post, this cost is priced into bank contracts ex ante. The planner prefers to recapitalize failing banks using bail-ins rather than bailouts, avoiding perverse redistribution from taxpayers to banks and bank investors. A planner would therefore prefer to commit never to bail out failing banks and instead use bail-ins to mitigate fire sales.

In the absence of commitment power, bailouts are chosen in a time-inconsistent manner. A planner that prefers ex ante to liquidate a bank for incentive reasons may prefer ex post to bail out the bank to mitigate fire sales. Since liquidation is not time consistent, the planner must choose between bail-in debt and bailouts to recapitalize the bank. Since bailouts are socially costly, the planner chooses bail-ins. Bail-ins are an imperfect but time-consistent substitute for a commitment against bailouts. Nevertheless, since the planner is forced to increase the use of bail-in debt to prevent bailouts, the optimal bank contract is distorted relative to the optimal commitment contract. Bank welfare increases in the strength of the commitment against bailouts.

Proposition 7 reflects the principle that banks and bank investors, not taxpayers, should bear the cost of bank recapitalization. Optimal regulation fully replaces bailouts with bail-ins.

5 Rollover Crises

We now turn to the second question of the paper, which is whether the prospect of bail-ins is a source of bank instability. We characterize rollover crisis equilibria, which are self-fulfilling prophecies where bail-in debt holders believe they are about to be bailed in, and

refuse to refinance an otherwise healthy bank at date 1. The bank becomes too fragile to recapitalize itself and suffers a bank run, justifying the equilibrium beliefs.

To generate rollover crises, we introduce a notion of fragility in the date 1 economy. We extend the model to four periods, and incorporate uncertainty at date 2 via a second quality shock. The extended model features the same form of optimal contracts as in Section 3. We adopt the implementation of this optimal contract that utilizes standard short-term debt and bail-inable long-term debt. Given the uncertainty in continuation, banks will try to refinance the maturing short-term debt by replacing it with long-term debt in order to avoid future liquidations. In the best equilibrium of the model, refinancing is successful, whereas in the rollover crisis equilibrium the bank will fail to refinance itself.

The results of this section apply to both the privately optimal contract and the socially optimal contract with fire sales.

5.1 Extended Model

There are now four periods, $t = 0, 1, 2, 3$. Figure 3 provides an illustration of the extended timeline. Banks sign complete markets contracts at date 0 and experience a quality shock R at date 1, with the same incentive problem at date 0. Banks also experience a quality shock R_2 at date 2, with $R_2 \sim F_2$ on $[\underline{R}, \bar{R}]$ and $E[R_2] = 1$, after which uncertainty resolves. Projects mature at date 3 and pay off $Y_3 = Y_2 = R_2 Y_1$ units of the consumption good, but may be liquidated prior to maturity. The date 1 limited pledgeability constraint takes the form of a maximum date 1 debt level $R_b Y_1$, where $R_b \in [\underline{R}, \bar{R}]$, so that we have

$$\int_{R \leq R_b} R f_2(R_2) dR_2 + \int_{R \geq R_b} R_b f_2(R_2) dR_2 \equiv 1 - b. \quad (12)$$

As before, $(1 - b)RY_0$ is the maximum pledgeable (expected) repayment to investors. Liquidations are not necessary for incentive provision in the continuation problem, meaning that all liquidations on the equilibrium path will occur at date 1. However, the liquidation value $\gamma(\Omega)$ at date 1 is persistent and applies also at date 2. We think of dates 1 and 2 as subsequent stages of a crisis, where fire sale prices remain depressed throughout.

The extended model is identical to the baseline model, up to the additional period, if we assume the rollover problem at date 1 (replacing short-term debt with long-term debt) always results in successful refinancing when the bank is solvent. The optimal contract can therefore be implemented with R_l in short-term debt, used to liquidate the bank at date 1, and $R_u - R_l$ of long-term debt that can be written down to restore solvency.⁴⁷

⁴⁷The contract may also specify covenants to restrict dilution incentives at date 1. At the end of this

Debt Seniority. An important issue for rollover crises is the seniority structure. As in the baseline model, standard (short-term) debt is naturally senior to bail-in (long-term) debt under a No Creditor Worse Off condition for insolvency and resolution. This holds because in the threshold solvency state R_l , bail-in debt is fully written down and receives no payoff. If bail-in debt received positive payoffs in insolvency, it would be better off under liquidation than resolution (bail-in), giving natural seniority to standard debt. This corresponds to the institutional design of bail-in regimes, which subordinate bail-in debt to (standard) short-term debt.

5.2 Refinancing Problem

We now study the refinancing problem at date 1, where a solvent bank refinances its short-term debt. To simplify exposition, we focus on the limiting case where $R_l = R_u$, so that there is no initial long-term debt.⁴⁸ Let $D_1 = (1 - b)R_l Y_0$ denote the initial level of standard (short-term) debt and Y_0 the project scale. The bank at date 1 refinances itself using a combination of (standard) short-term debt D_2 and (bail-inable) long-term debt L_3 .⁴⁹

Throughout this section, we study the refinancing problem of a fundamentally solvent bank, with $R_l < R$. Fundamentally insolvent banks are always liquidated at date 1. We will also rule out all equilibria except two: the best equilibrium with successful refinancing (corresponding to the baseline model), and the rollover crisis equilibrium.

At date 1, the bank operates in a Walrasian market, where q_1^D is the price of a new unit of short-term debt, and q_1^L is the price of a new unit of long-term (bail-in) debt. We assume contracts are fully visible to all creditors to rule out conventional rat race dynamics.⁵⁰ This assumption is reflected in the following pair of constraints

$$q_1^D D_2 \leq \int_{D_2 \geq R_2 Y_1} \gamma R_2 Y_1 f_2(R_2) dR_2 + \int_{D_2 \leq R_2 Y_1} D_2 f_2(R_2) dR_2 \quad (13)$$

$$q_1^L L_3 \leq \int_{D_2 \leq R_2 Y_1} \min\{R_2 Y_1 - D_2, L_3\} f_2(R_2) dR_2 \quad (14)$$

which state that the expected payoff to debt holders must be at least as high as the amount

section, we show that covenants do not affect existence of rollover crises.

⁴⁸This assumption is not required to generate rollover crises. Outstanding long-term debt would only increase the probability of rollover crises.

⁴⁹Our results generalize immediately to the case where banks can also issue other liabilities that are junior to short-term debt under the same logic as will follow.

⁵⁰We could also interpret this as arising from explicit bond covenants.

they pay, given market prices.⁵¹ They imply the bank cannot deliberately dilute new bank creditors.

Best Equilibrium. Under these constraints, there is a unique best equilibrium under which the bank successfully refinances itself and never liquidates after date 1.

Lemma 8. *There is a unique best equilibrium with no liquidations after date 1. It has prices $\bar{q}_1^D = 1$ and $\bar{q}_1^L < 1$. It has short-term debt issuance $\bar{D}_2 = \underline{R}Y_0$ and long-term debt issuance $\bar{L}_3 = \frac{D_1 - \bar{D}_2}{\bar{q}_1^L}$.*

The best equilibrium of Lemma 8 is the outcome of the refinancing problem associated with the optimal contract considered in the first part of the paper. Under this equilibrium, the problem behaves as-if periods 2 and 3 were combined into a single period. Our baseline model can equivalently be viewed as internalizing this best equilibrium.

5.3 Rollover Crisis Equilibrium

We now show that in addition to the best equilibrium, there also exists a rollover crisis equilibrium. In the rollover crisis equilibrium, long-term debt markets fear a bail-in is imminent and quote a price of 0 for new long-term debt. The bank fails to refinance itself as a result, resulting in the bank failing and being liquidated. As a result, long-term debt, which is junior in liquidation, indeed receives no payoff, justifying the equilibrium beliefs and price.⁵² Unlike conventional bank runs which result from sunspots in short-term debt markets, rollover crisis equilibria result from sunspots in *long-term debt markets*.

We show that a rollover crisis equilibrium generically exists in a region above the short-term debt level R_l .⁵³

⁵¹We use stronger conditions than necessary in order to guarantee the debt issuances of the best equilibrium are unique. Rollover crises are not affected by using these stronger conditions. See Appendix B.8.

⁵²To be clear, we are not looking for traditional bank run equilibria, and we assume that a costless lender of last resort stands ready to stop a sunspot bank run. Rollover crises will result in a bank run, however, because short-term debt alone cannot refinance the bank.

⁵³We assume that the maximum amount pledgeable to investors is higher in continuation than in liquidation even with short-term debt,

$$\sup_{d_2 \leq R_b} \int_{\underline{R}}^{d_2} \gamma R_2 f_2(R_2) dR_2 + \int_{d_2}^{\bar{R}} d_2 f_2(R_2) dR_2 > \gamma \quad (15)$$

where we define $d_2 = D_2/Y_1$ to be the continuation debt-to-asset ratio. Repayment to short-term debt holders therefore cannot be increased by liquidating the bank. As a result, if the bank cannot refinance itself, there is sufficient short-term debt to liquidate the bank.

Proposition 9. Let $D_1 = (1 - b)R_l Y_0$ be the short-term debt level, with $R_l > \underline{R}$. A rollover crisis equilibrium exists for all date-1 returns $R \in [R_l, R^*)$, where R^* is given by

$$\frac{D_1}{R^* Y_0} = \underbrace{\sup_{d_2 \leq R_b} \int_{\underline{R}}^{d_2} \gamma R_2 f_2(R_2) dR_2 + \int_{d_2}^{\bar{R}} d_2 f_2(R_2) dR_2}_{\text{Maximum Funds Raisable With Only Short-Term Debt}} \quad (16)$$

In the rollover crisis equilibrium, the bank is unable to refinance itself at date 1 and is liquidated. The equilibrium price of new long-term debt is $q_1^L = 0$.

To understand the intuition behind the rollover crisis equilibrium, consider the threshold bank for which $D_1 = (1 - b)Y_1$. This threshold bank is just able to recapitalize itself in the best equilibrium, and survives only by issuing the securities packages with $D_2 \leq \underline{R}Y_1$ and $L_3 = R_b Y_1 - D_2$. In particular, there is no securities package $(D_2, 0)$ and price q_1 that would refinance the bank in a Walrasian equilibrium. As a result, this threshold bank, if faced with the only option of refinancing itself with short-term debt, would suffer a run by its short-term creditors.⁵⁴ This generates the rollover crisis equilibrium: holders of long-term debt expect to be bailed in and quote a price of 0, leading to a bank run and liquidation. Since new long-term debt is subordinated to short-term debt, it would receive no recovery value in the liquidation, justifying the equilibrium price of 0 and completing the equilibrium.

Fundamentally, the rollover crisis arises because the bank finds itself needing to lengthen its maturity structure at date 1. When it finds itself solvent but with low returns at date 1, its short-term-debt-to-asset ratio is higher than necessary for incentive provision in continuation. As a result, the bank seeks to keep itself viable by replacing some of its short-term debt with long-term debt, in order to prevent inefficiently costly future liquidations which reduce income pledgeable to investors. The rollover crisis prevents the bank from accessing long-term debt markets, leaving it with no choice but to attempt to roll over its short-term debt. However, short-term debt leaves the bank exposed to future liquidations, leaving the bank unable to raise sufficient new funds to fully roll over its short-term debt. As a result, the bank fails to refinance itself and suffers liquidation at date 1.

Rollover crises exist because the bail-in regime confers explicit priority to short-term debt over long-term debt. The conferred priority applies not only under bail-in resolution, but also under liquidation, leading to the instability. This further implies that a rollover

⁵⁴A lender of last resort also cannot break even on any securities package $(D_2, 0)$ below R^* .

crisis exists for any other liabilities that a bank could issue, such as equity, provided those instruments are subordinated to short-term debt. Although “deposit priority” may help alleviate traditional bank runs, it has the potential to generate instability in long-term debt markets.

Propagation of Rollover Crises. The existence of rollover crises does not rely on fire sales, only on liquidation discounts. However, fire sales contribute to the propagation of rollover crises, leading to more frequent crises and larger fire sales. The rollover crisis problem is particularly severe in times of market stress.

Because we have multiple equilibria, we must adopt an equilibrium selection rule. We adopt a simple selection rule: banks experience a rollover crisis with probability $0 < p < 1$ and experience the best equilibrium with probability $1 - p$ when in the rollover crisis region. By the law of large numbers, equilibrium liquidations and the liquidation value are a solution to

$$\gamma^* = \gamma(\Omega^*), \quad \Omega^* = \epsilon + \int_{\underline{R}}^{R_l} R f_H(R) dR + p \int_{R_l}^{R^*(\gamma^*)} R f_H(R) dR \quad (17)$$

where ϵ is an exogenous liquidation shock that we use to illustrate the feedback loop. The rollover crisis threshold R^* depends on the equilibrium liquidation value γ^* , as illustrated in Proposition 9, so that we have a fixed point problem. We characterize the feedback loop by starting from $\epsilon = 0$, and then study the equilibrium response of total liquidations Ω^* with respect to ϵ . If there is no feedback loop, then $\frac{\partial \Omega^*}{\partial \epsilon} = 1$.

Proposition 10. *Starting from an equilibrium of the date-1 economy with $\epsilon = 0$, an exogenous increase in liquidations ϵ generates a total increase in equilibrium liquidations*

$$\left. \frac{\partial \Omega^*}{\partial \epsilon} \right|_{\epsilon=0} = 1 + \frac{p \left| \frac{\partial \gamma(\Omega^*)}{\partial \Omega^*} \right| \left| \frac{\partial R^*(\gamma^*)}{\partial \gamma^*} \right| R^* f_H(R^*)}{1 - p \left| \frac{\partial \gamma(\Omega^*)}{\partial \Omega^*} \right| \left| \frac{\partial R^*(\gamma^*)}{\partial \gamma^*} \right| R^* f_H(R^*)} \Bigg|_{\epsilon=0} \quad (18)$$

The core of the feedback loop in Proposition 10 is the sensitivity of the liquidation discount to liquidations, $\frac{\partial \gamma}{\partial \Omega}$. When this sensitivity is higher, $\left. \frac{\partial \Omega^*}{\partial \epsilon} \right|_{\epsilon=0}$ increases, and the exogenous shock to liquidations is amplified by the propagation of rollover crises. The increase in liquidations lowers the liquidation value, expanding the region of crises. More banks become subject to rollover crises, pushing down liquidation values further. By contrast, if liquidation values are not sensitive to liquidations ($\frac{\partial \gamma}{\partial \Omega} = 0$) then there is no feedback loop and no propagation.

In normal times, when fire sale spillovers are limited, rollover crises may therefore be relatively contained. By contrast, during financial crises, when spillovers may be more severe, they can propagate, increasing bank failures and exacerbating fire sales. This propagation effect reflects the concern that even though bail-ins may have beneficial properties for bank resolution during relatively normal times or for the resolution of individual banks, they may generate adverse effects during times of systemic crises.

5.4 Private Solutions to Rollover Crises

We now ask whether there are private solutions to the rollover crises that do not require changes to optimal payoff structure of contracts in Section 3. We discuss covenants, alternative contract implementations, lines of credit, early triggers, and seniority.

Covenants. A natural conjecture is that bond covenants, which are a common tool for addressing debt dilution or rat race incentives, would be effective in preventing rollover crises. We show that bond covenants do not prevent rollover crises.

A bond covenant is an arbitrary set of restrictions on the refinancing structures available to the bank at date 1,

$$C_1(D_2, L_3 | D_1, Y_1, q_1) \leq \mathbf{0},$$

where C_1 is some vector-valued function. Define the *vacuous covenant* by $C_1(D_2, L_3 | D_1, Y_1, q_1) = \mathbf{0}$, which places no restrictions on the bank. All previous results have assumed the vacuous covenant.

Proposition 11. *If a rollover crisis equilibrium exists under the vacuous covenant $C_1(D_2, L_3 | D_1, Y_1, q_1) = \mathbf{0}$, then it also exists under any other covenant $C_1(D_2, L_3 | D_1, Y_1, q_1)$.*

Proposition 11 shows that covenants are not a solution to rollover crises. To understand why, rollover crises are justified in equilibrium by the *outstanding* stock of short-term debt, not by new short-term debt. As a result, covenants cannot rule out rollover crises.

Control Rights. Rollover crises arise from the need to refinance maturing short-term debt, that is from the use of short-term debt as the means of enforcing liquidations. This implies that using an alternative method of enforcing liquidation (or control rights transfer) would help to rule out the rollover crisis problem. For example, the optimal contract

could in theory be implemented using entirely long-term debt, along with a contractual specification of when liquidation would occur.

Line of Credit. Rollover crises arise because the prices at which banks borrow at date 1 are not contracted upon. This implies that if banks can contract at date 0 for its refinancing at date 1, the bank can rule out a rollover crisis. This resembles a line of credit, which would specify that in states $R \leq R_l$, the bank can borrow the best equilibrium quantities at the best equilibrium prices. Importantly although the line of credit can rule out rollover crises, the availability and terms of the line of credit are state contingent.

Early Triggers. A natural conjecture is that early triggers (or a precautionary bail-in) – writing down existing long-term debt prior to resolution – can help recapitalize the bank and prevent rollover crises.⁵⁵ However, the existence of rollover crises was proven in the case where there was *no* existing long-term debt, so that early triggers have in effect already been applied. Although early triggers may help prevent the bank from drifting into the rollover crisis region, once in that region the bank is susceptible to the rollover crisis even if the early trigger is applied.

Seniority. Rollover crises arise because bail-in debt is subordinated to standard short-term debt. One possibility is to allow the bank at date 1 to issue standard long-term debt, which would not be bail-inable but would rank *pari passu* short-term debt in insolvency proceedings.⁵⁶

There are two issues with this solution. The first is that although the bank's viability increases with standard long-term debt, it may nevertheless be more fragile than with bail-in long-term debt, so that the value of the bank would fall. This would therefore be a second-best solution to the problem. The second is that although new long-term debt might be *de jure pari passu* to short-term debt, it would nevertheless be *de facto* junior due to its longer maturity.⁵⁷ This implies that although its recovery value might not be 0 in

⁵⁵Early triggers are a property of Contingent Convertible (CoCo) instruments, where the trigger (e.g. based on the stock price or capital ratio) results in an automatic write-down or conversion from debt to equity with the goal of alleviating debt overhang in a distressed bank. A second possible implementation leverages the institutional structure of bail-in resolution as currently practiced in the U.S. under Title II, where bail-ins are applied at the level of the top-tier bank holding company (BHC) prior to operating subsidiaries and intermediate BHCs. A bank could therefore mimic an early trigger by issuing new long-term debt out of operating subsidiaries rather than the top-tier holding company.

⁵⁶Non-bail-inability could be contractually designated, or could be implemented (e.g. in the US) by differentiating between top-tier BHC debt that can be bailed in (and is subordinated), and operating subsidiary debt which cannot be bailed in and is *de jure pari passu* with subsidiary short-term debt.

⁵⁷See e.g. Brunnermeier and Oehmke (2013) and He and Milbradt (2016).

liquidation, it would be lower than its legal *pari passu* status indicates. Provided that the *de facto* seniority of short-term debt sufficiently reduces the recovery value of long-term debt, rollover crises will continue to exist.

6 Policy Responses to Rollover Crises

Section 5 showed that the optimal contracts of Section 3 were susceptible to rollover crises at date 1 when using a natural contract implementation with short- and long-term debt. We now consider *ex post* policies the planner could adopt to prevent rollover crises, given this implementation. We consider two policies that have precedent in policies employed during the 2008 financial crisis. The first is an expanded lender of last resort that extends both short- and long-term loans to banks. The second is temporary guarantees of new issuances of long-term debt. We show that both policies can rule out the possibility of rollover crises.

6.1 Expanded Lender of Last Resort

A common solution to sunspot bank runs by short-term debt holders (i.e. depositors) is a lender of last resort (LOLR), which extends short-term loans to fundamentally solvent banks faced with a sunspot bank run.⁵⁸ Although rollover crisis equilibria in our model are a form of coordination failure, the coordination failure arises from long-term debt, not short-term debt. A conventional LOLR extending short-term loans is therefore not a sufficient policy in this case,⁵⁹ but a LOLR facility might be successful if it provided long-term debt loans to distressed banks.⁶⁰ A facility of this form is not without precedent. For example, the Capital Purchase Program (CPP) implemented under TARP in 2008, made \$250 billion available for the purchase of preferred stock in banks, with the goal of making funding available in a fragile market. The nature and goals of CPP were similar to those of our proposal.⁶¹

⁵⁸For example, the Federal Reserve set up the Term Securities Lending Facility (TSLF) in March 2008 to provide liquidity to financial institutions and restore credit market functioning, allowing them to swap illiquid collateral for liquid Treasury securities, which could then act as collateral in new funding agreements in credit markets.

⁵⁹A conventional LOLR facility could provide a temporary stopgap while the government attempted to orchestrate a public or private sector rescue program.

⁶⁰These loans would be subordinated to short-term debt, and would be potentially bail-inable and uncollateralized. The LOLR would break even in expectation but not on every realized path. This LOLR facility differs starkly from the usual facility and violates Bagehot best practice principles.

⁶¹The US Treasury Department, in describing CPP, states that it “helped bolster the capital position of viable institutions of all sizes and built confidence in these institutions and the financial system as

We model the extended LOLR facility as follows. At the same time as the bank accesses private markets, it can also access the LOLR which is willing to make available to the bank any package (D_2^{LOLR}, L_3^{LOLR}) at prices q_1^{LOLR} so that it breaks even in expectation. LOLR claims rank *pari passu* with private sector claims of the same instrument.⁶² Provided that the bank successfully refinances with issuances $(D_2, L_3, D_2^{LOLR}, L_3^{LOLR})$, the break-even prices of the LOLR are given by

$$q_1^{D,LOLR} = 1 - \int_{D_2 + D_2^{LOLR} \geq R_2 Y_1} \left[1 - \frac{\gamma R_2 Y_1}{D_2 + D_2^{LOLR}} \right] f_2(R_2) dR_2 \quad (19)$$

$$q_1^{L,LOLR} = \int_{D_2 + D_2^{LOLR} \leq R_2 Y_1} \min \left\{ \left(R_2 Y_1 - D_2 - D_2^{LOLR} \right) \frac{1}{L_3 + L_3^{LOLR}}, 1 \right\} f_2(R_2) dR_2 \quad (20)$$

Given market prices q_1 for private sector debt, the bank can refinance itself if there are a securities package $(D_2, L_3, D_2^{LOLR}, L_3^{LOLR})$ and corresponding LOLR prices q_1^{LOLR} that allow the bank to raise D_1 in revenue, while satisfying the private sector no-rat-race conditions.

Because the bank can always refinance itself successfully by borrowing best equilibrium quantities from the LOLR at best equilibrium prices, rollover crises are eliminated. Since LOLR debt and private sector debt rank *pari passu*, private sector prices must be the best equilibrium prices. The bank can then refinance itself entirely from the private sector.

Proposition 12. *Suppose there is an extended LOLR facility. Suppose private sector and LOLR claims rank pari passu. Rollover crises are eliminated. The bank can refinance itself entirely in private markets at best equilibrium prices.*

Proposition 12 is similar in spirit to the standard LOLR solution to sunspot bank runs: by making loans available to the bank, the LOLR prevents the onset of the crisis. Although these loans only happen off equilibrium in the model, in practice such a facility would likely be used to some degree by banks during crises. This leads to two natural practical concerns. The first is that because the LOLR extends uncollateralized, long-term loans and would lose money on some realized paths (but not in expectation), the operating principles are different from those of a standard LOLR. The second is that the facility would be subject to familiar moral hazard concerns: it may unintentionally lend to insolvent banks,

a whole." <https://www.treasury.gov/initiatives/financial-stability/TARP-Programs/bank-investment-programs/cap/Pages/default.aspx>.

⁶²In practice, the government tends to give itself high priority on claims in resolution. See 12 CFR §380.21.

or lend at prices lower than break-even prices.⁶³ This would lead the LOLR to subsidize distressed banks, amounting to a partial bailout.

6.2 Debt Guarantees

Deposit insurance is a second common solution to sunspot bank runs. In its idealized form, deposit insurance prevents the run and is not filled on the equilibrium path. In a rollover crisis, although insurance would allow the bank to roll over its short-term debt and avoid liquidation, insurance would be filled on the equilibrium path.

We consider instead an extension of temporary guarantees to new issuances of long-term debt, and show that these guarantees can successfully rule out rollover crises without being filled on the equilibrium path. This has precedent in the Temporary Liquidity Guarantee Program (TLGP) instituted in 2008, under which the US government provided guarantees to new issuances of senior unsecured debt with the goal of “preserving confidence in the banking system and encouraging liquidity.”⁶⁴ These guarantees were “temporary” in that they expired no later than June 2012.

We model guarantees as follows: at the beginning of date 1, the government extends a temporary guarantee to all new issuances of long-term debt. Guarantees oblige the government to cover any losses relative to face value during the guarantee period.⁶⁵ Guarantees expire at the end of date 1, after which the debt is once again subject to the bail-in regime. In other words, if at date 1 the bank enters bankruptcy and liquidates, the government is obligated to pay $1 - q_1^{L,B}$ to a holder of new long-term debt, where $q_1^{L,B} \leq 1$ is the recovery value of new long-term debt in liquidation. This guarantee extension eliminates rollover crises.

Proposition 13. *A temporary guarantee of new long-term debt that expires at the end of period 1 eliminates rollover crises. Guarantees are not filled on the equilibrium path.*

Proposition 13 is closely related to the rationale for deposit insurance. Since debt is guaranteed, investors cannot expect a price of 0, ruling out the rollover crisis and allowing the bank to refinance itself. Guarantees are never filled on the equilibrium path because

⁶³One particular manifestation of this is that the break-even prices of long-term debt are bank-dependent, because it is not guaranteed to be repaid in full. A LOLR operating during stress times may not wish to price discriminate between banks to avoid reputational damage to distressed banks.

⁶⁴12 CFR Part 370. <https://www.fdic.gov/news/board/08bodtlgp.pdf>

⁶⁵We have assumed away existing long-term debt. Guarantees would not need to be extended to existing long-term debt. This is consistent with TLGP.

they expire the end of date 1. This “temporary” aspect is consistent with the principle of TLGP, where the guarantees expired after a certain time frame.

It is too strong in practice to assume that guarantees can be timed perfectly to never be filled. Some guarantees would be filled on the equilibrium path, leading to moral hazard concerns.⁶⁶ Proposition 13 is an idealized result that helps explain why debt guarantee programs such as TLGP may be a valuable part of a crisis resolution toolkit.

6.3 Bailouts versus Debt Guarantees

Sections 6.1 and 6.2 show that expanded LOLR facilities and long-term debt guarantees can be valuable components of a planner’s crisis resolution toolkit. However, post-crisis regulation has limited the ability of the US government to engage in such policies as part of a commitment against future bailouts.⁶⁷ Our model suggests an important distinction between bailouts of existing debt and protection of new debt issued during stress times, even when such protection is “bailout-like.” Optimal regulation in Section 4 replaces bailouts of existing debt with bail-ins. Guarantees serve a different function and can be a valuable stabilization tool during a crisis. A commitment against bailouts of existing debt need not preclude protection of new debt during a crisis.

7 Extensions

In Appendix B, we provide a set of extensions.

In Appendix B.1, we study the case with positive arbitrageur welfare weights and show generic inefficiency of the privately optimal contract.

In Appendix B.2, we allow for the possibility of aggregate risk. Optimal contracts take the same form, but both instruments are contingent on the aggregate state, implying the addition of a dual price trigger or aggregate risk hedge in the optimal contract. Banks inefficiently limit contingencies on aggregate risk when there are fire sales or bailouts.

In Appendix B.3, we consider the interaction between macroprudential (asset-side) and liability-side regulation by introducing multiple investment projects. While bail-ins still constitute optimal liability regulation, asset-side regulation is also required to

⁶⁶The Debt Guarantee Program (DGP) portion of TLGP, which involved guarantees of new loans, at its peak guaranteed approximately \$345 billion in debt. Approximately \$153 million in guarantees were filled. A second component of TLGP, the Transaction Account Guarantee Program, provided guarantees for noninterest-bearing transaction accounts and resulted in estimated losses of \$2.5 billion. These numbers are reported by the FDIC as of February 2019 (<https://www.fdic.gov/regulations/resources/tlgp/index.html>).

⁶⁷See Geithner (2016).

implement the social optimum because the asset composition affects the probability of bank failure and liquidation.

In Appendix B.4, we study the allocation of bail-in securities among heterogeneous investors with different risk tolerances and different exposures to the banking sector. We show that retail investors, who maintain greater exposures to individual banks, and institutional investors who experience spillovers from fire sales should hold safer (non-bail-inable) claims.

In Appendix B.5, we incorporate a role for equity-like claims into the bank's capital structure by incorporating bank risk aversion and risk shifting. There is still a role for bail-in debt in addressing the incentive problem of the baseline model. We show that although the bank and the planner disagree over the quantity of standard debt versus total loss-absorbing capital (bail-in debt + equity), they do not disagree over the composition of total loss-absorbing capital.

In Appendix B.6, we allow for standard debt to command a premium over other instruments, including bail-in debt. This increases use of standard debt and helps to explain why, in practice, the level of standard debt banks employ is so high. The marginal trade-off for banks is still influenced by the incentive problem and leads to use of bail-in debt. In absence of the incentive problem, there would be no reason to use bail-in debt over equity.

In Appendix B.7, study the trade-off between bailouts and bail-ins in protecting insured deposits when banks are allowed to issue insured deposits as part of their standard debt. The planner faces a trade-off between greater deposit insurance (i.e. taxpayer) losses when liquidating the bank, and worse bank incentives when bailing out the bank. Bailouts may be desirable to lessen the taxpayer burden of deposit insurance. All non-deposit investors are fully bailed in whenever the planner bails out the bank. This motivates the possibility of having a deposit guarantee scheme, even in the absence of bailouts of other forms of debt.

In Appendix B.8, we characterize the weaker form no-rat-race conditions used to rule out deliberate dilution motives.

In Appendix B.9, we characterize conditions under which rollover crises can generate multiple equilibria of the date 1 aggregate economy.

8 Conclusion

We characterize optimal bank contracts under a monitoring incentive problem. Privately optimal bank contracts combine standard and bail-in debt. Banks' private use of bail-in debt is limited when there are fire sales or bailouts. A bail-in regime, which increases use of bail-in debt relative to standard regime, is the optimal regulatory policy. Optimal regulation replaces bailouts with bail-ins. This helps to understand the introduction and design of bail-in regimes in the US and EU.

We also show that the prospect of bail-ins can have destabilizing effects on bank refinancing during times of market stress, leading to rollover crises and bank failures. Rollover crises can be addressed using policies such as extended lender of last resort facilities and debt guarantees. These crisis resolution tools complement an effective bail-in regime, and should be retained in the crisis resolution toolkit.

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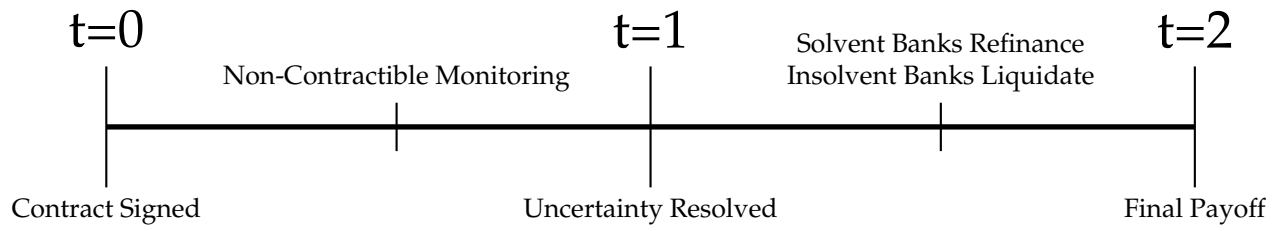


Figure 1: This figure presents a simple timeline for the model.

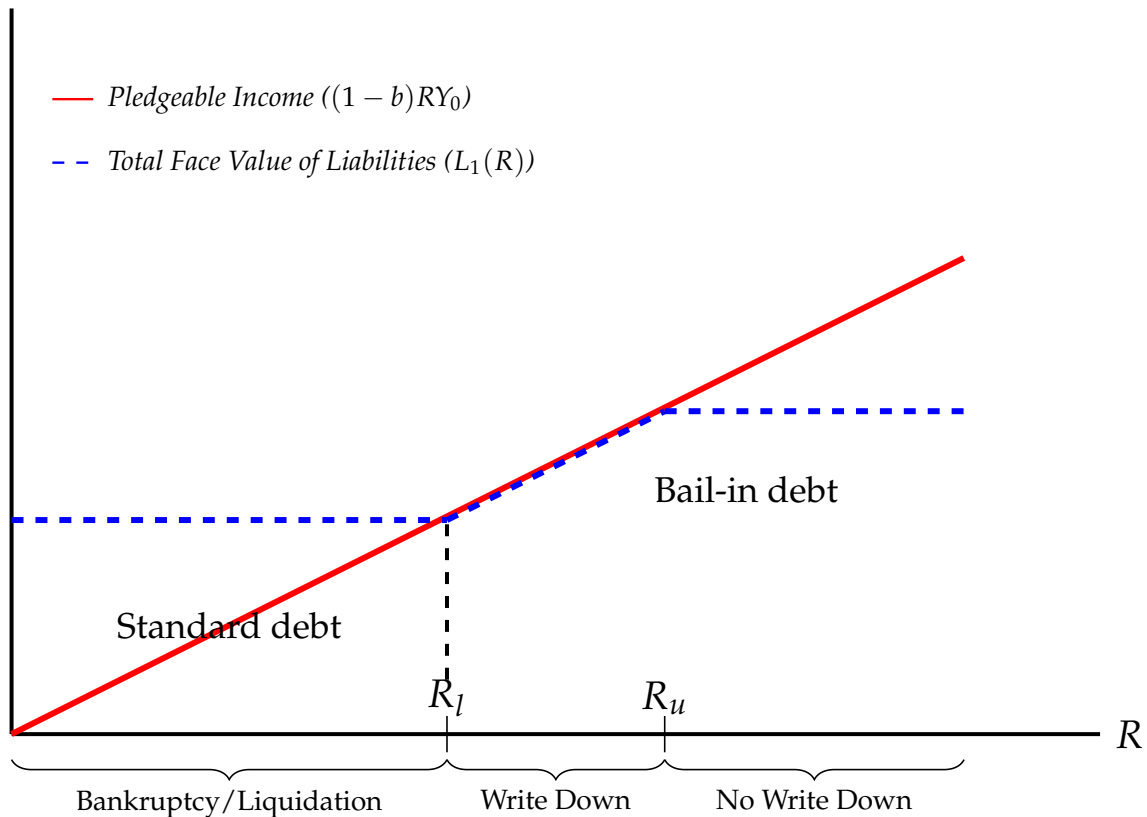


Figure 2: This figure provides an illustration for the privately optimal contract. Up to a threshold R_l , bank liabilities are constant and exceed pledgeable income, leading to liquidations (“standard debt”). Between R_l and R_u , the face value of liabilities is written down to coincide with pledgeable income (“bail-in” or “write-down”). Above R_u , the face value of liabilities is constant (“bail-in debt”).

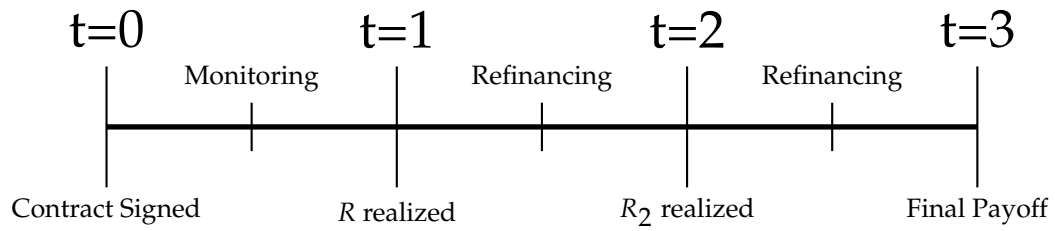


Figure 3: This figure presents a simple timeline for the extended model (Section 5).

Online Appendix

A Proofs

A.1 Proof of Proposition 1

Consider the program

$$\max_{L_1, Y_0} E [c_2(R) | e = H]$$

Subject to

$$E \left[c_2(R) \left(1 - \frac{f_L(R)}{f_H(R)} \right) \middle| e = H \right] \geq BY_0$$

$$Y_0 - A = \int_{R|\alpha=1} \gamma(s)RY_0 f_H(R) dR + \int_{R|\alpha=0} L_1(R) f_H(R) dR$$

$$R \geq R' \Rightarrow L_1(R) \geq L_1(R')$$

$$c_2(R) \geq (1 - \alpha(R))bRY_0$$

$$L_1(R) \geq 0$$

and recall the second to last constraint is limited pledgeability. It is helpful to redefine the problem in the investor payoff space, and then to define the implementing liability structure $L_1(R)$. Total investor payoff $x(R)$ is given by

$$x(R) = \alpha(R)\gamma RY_0 + (1 - \alpha(R))RY_0 - c_2(R)$$

where $\alpha(R) \in \{0, 1\}$ is the liquidation rule. We treat $\alpha(R)$ as a choice variable, and then back out the liability structure that implements it. Note that because banks are repaid 0 when $\alpha(R) = 1$, it is irrelevant whether we multiply $c_2(R)$ by $1 - \alpha(R)$. Given this characterization, investor voluntary participation can be rewritten as

$$Y_0 - A = E [\alpha\gamma RY_0 + (1 - \alpha)RY_0 - c_2 | e = H].$$

We begin by studying the optimization problem not subject to liability monotonicity, and show that it generates a non-monotone contract. The Lagrangian of this relaxed

problem is

$$\begin{aligned}
\mathcal{L} = & E [c_2 | e = H] + \mu \left[E \left[c_2 \left(1 - \frac{f_L(R)}{f_H(R)} \right) | e = H \right] - BY_0 \right] \\
& + \lambda [E [\alpha(R)\gamma(s)RY_0 + (1 - \alpha)RY_0 - c_2 | e = H] + A - Y_0] \\
& + E [\chi (c_2 - (1 - \alpha) bRY_0) | e = H] \\
& + E [\zeta ((\alpha\gamma RY_0 + (1 - \alpha)RY_0 - c_2)) | e = H]
\end{aligned}$$

From here, first order condition for bank consumption as

$$\begin{aligned}
0 = & f_H(R) + \mu \left(1 - \frac{f_L(R)}{f_H(R)} \right) f_H(R) - \lambda f_H(R) + \chi(R) f_H(R) - \zeta(R) f_H(R) \\
= & \left[1 - \lambda + \mu \left(1 - \frac{f_L(R)}{f_H(R)} \right) \right] f_H(R) + \chi(R) - \zeta(R)
\end{aligned}$$

By MLRP, there is a threshold R^* such that $\chi(R) > 0$ for $R \leq R^*$ and $\zeta(R) > 0$ for $R \geq R^*$, implying that $x(R) = L_1(R) = 0$ for all $R \geq R^*$. This threshold is given by

$$1 - \lambda + \mu \left(1 - \frac{f_L(R^*)}{f_H(R^*)} \right) = 0. \quad (21)$$

However, this contract violates liability monotonicity unless $L_1(R) = 0$ for all R . Therefore, we have an upper pooling region in the optimal contract, where liabilities and investor repayment are constant.⁶⁸

It is worth remarking that the contract not subject to monotonicity is of the live-or-die form.⁶⁹ It implies that banks will be either liquidated or held to the agency rent when $R < R^*$, with all remaining repayment going to investors. By contrast, the bank receives the full resources of the bank when $R > R^*$. This contract is optimal because it provides strong incentives to the bank. Because all agents are risk-neutral, they are willing to accept this extreme payoff structure. However, this payoff structure violates liability monotonicity, and so is not implementable.

We now characterize the optimal contract using the following strategy. First, we conjecture pooling thresholds R_u with corresponding liabilities $x_u \equiv x(R_u) = L_1(R_u)$, so that $x(R) = x_u$ for all $R \geq R_u$. The live-or-die result of the contract not subject to

⁶⁸If $L_1(R) = 0$, then the entire contract is pooled. If $R^* = \bar{R}$, then the results that follow apply setting $R_u = \bar{R}$ to be the pooling threshold.

⁶⁹See Innes (1990).

monotonicity implies such a pooling threshold exists.⁷⁰ We then solve for the optimal contract below R_u , taking as given R_u and x_u , subject to a relaxed monotonicity constraint $x(R) \leq x_u \forall R \leq R_u$, and verify that the resulting contracting is monotone. In doing so, we characterize the space of implementable contracts (that satisfy monotonicity). Finally, we optimize over the choice of R_u and x_u .

Conjecture pooling thresholds R_u with liabilities x_u . The associated Lagrangian is given by

$$\begin{aligned} \mathcal{L} = & E [c_2 | e = H] + \mu \left[E \left[c_2 \left(1 - \frac{f_L(R)}{f_H(R)} \right) | e = H \right] - BY_0 \right] \\ & + \lambda [E [\alpha \gamma RY_0 + (1 - \alpha)RY_0 - c_2(R) | e = H] + A - Y_0] \\ & + E [\chi (c_2 - (1 - \alpha) bRY_0) | e = H] \\ & + E [v (x_u - (\alpha \gamma RY_0 + (1 - \alpha)RY_0 - c_2)) | e = H] \end{aligned}$$

where the final line is the relaxed monotonicity constraint, and where we have anticipated that limited liability $x(R) \geq 0$ does not bind below R_u for feasible contracts. Taking the derivative in consumption $c_2(R)$ for $R \leq R_u$, we obtain

$$0 = 1 + \mu \left(1 - \frac{f_L(R)}{f_H(R)} \right) - \lambda + \chi(R) + v(R).$$

Observe that the resulting contract is non-monotone if $R_u > R^*$ (we would have $\zeta(R) > 0$ so that $x(R) = 0$), by the same logic as above. Therefore, we can discard candidate contracts with $R_u > R^*$. This implies that $1 + \mu \left(1 - \frac{f_L(R_u)}{f_H(R_u)} \right) - \lambda < 0$ among the set of viable contracts.

Now, consider the derivative in liquidations $\alpha(R)$, given by⁷¹

$$\frac{\partial \mathcal{L}}{\partial \alpha(R)} \propto \lambda (\gamma - 1) + \chi(R)b + v(R) (1 - \gamma)$$

When $\alpha(R) = 1$, $v(R) = 1$ is possible at at most a single point, in particular at $\gamma RY_0 = x_u$. $\alpha(R) = 1$ therefore generically implies $\chi(R) > 0$ and $v(R) = 0$. From the FOC for $c_2(R)$, we have (almost everywhere) that when $\alpha(R) = 1$

$$\chi(R) = \lambda - 1 - \mu \left(1 - \frac{f_L(R)}{f_H(R)} \right)$$

⁷⁰Note that this is without loss, since the pooling threshold could be $R_u = \bar{R}$ if $R^* = \bar{R}$.

⁷¹Implicitly, we are treating $\alpha(R)$ as a continuous variable in performing the differentiation. To do so, we implicitly incorporate the constraint $\alpha(R)(1 - \alpha(R)) = 0$, which ensures that implementable contracts must set $\alpha(R) \in \{0, 1\}$. The logic below is unaffected.

which, combined with the liquidation rule, yields

$$\frac{\partial \mathcal{L}}{\partial \alpha(R)} \propto \lambda (\gamma - 1) + \left(\lambda - 1 - \mu \left(1 - \frac{f_L(R)}{f_H(R)} \right) \right) b.$$

By MLRP, there is a threshold rule $R \leq R_l$ for liquidations.

Finally, in the region (if non-empty) between R_l and R_u , by MLRP we have

$$1 + \mu \left(1 - \frac{f_L(R)}{f_H(R)} \right) - \lambda < 1 + \mu \left(1 - \frac{f_L(R_u)}{f_H(R_u)} \right) - \lambda < 0$$

so that we have either $\chi(R) > 0$ or $\nu(R) > 0$. This implies that $x(R) = \min\{(1 - b)RY_0, x_u\}$ for all $R_l \leq R \leq R_u$.

As a result, the optimal contract is a three-part liability structure. First, there is a threshold R_l such that $\alpha(R) = 1$ and $x(R) = \gamma RY_0$ for $R \leq R_l$, and $\alpha(R) = 0$ for $R \geq R_l$. Second, there is a threshold $R_u \geq R_l$ such that $x(R) = \min\{(1 - b)RY_0, x_u\}$ for $R \leq R_u$ and $x(R) = x_u$ for $R \geq R_u$. Note finally that there cannot be a discontinuity in liabilities at R_u . If there were a discontinuity, we would have

$$x_u > \lim_{R \uparrow R_u} x(R) = (1 - b)R_u Y_0$$

and liabilities would exceed pledgeable income at R_u . The capital structure is therefore continuous at R_u .

Finally, the above capital structure can be implemented by a liabilities contract $L_1(R) = (1 - b)R_l Y_0$ for $R \leq R_l$ and $L_1(R) = x(R)$ for $R > R_l$. This liability structure is monotone, and so we have implementable contracts.

In sum, the optimal contract lies within a class of contracts characterized by thresholds R_l and R_u and corresponding liability structure above. This proves the first part of the proposition.

Now, we characterize the optimal thresholds R_l and R_u . Considering the case where these thresholds are interior, $\underline{R} < R_l \leq R_u \leq \bar{R}$ we have the optimization problem

$$\max_{R_l, R_u, Y_0} \int_{R_l}^{R_u} bRY_0 f_H(R) dR + \int_{R_u}^{\bar{R}} [R - (1 - b)R_u] Y_0 f_H(R|s) dR$$

subject to

$$\int_{R_l}^{R_u} bRY_0 (f_H(R) - f_L(R)) dR + \int_{R_u}^{\bar{R}} [R - (1 - b)R_u] Y_0 (f_H(R) - f_L(R)) dR \geq BY_0$$

$$Y_0 - A = \int_{\underline{R}}^{R_l} \gamma R Y_0 f_H(R) dR + \int_{R_l}^{R_u} (1 - b) R Y_0 f_H(R) dR + \int_{R_u}^{\bar{R}} (1 - b) R_u Y_0 f_H(R) dR$$

Under the same multiplier convention, the optimality condition for R_l is

$$0 = -b R_l Y_0 - \mu b R_l Y_0 \left(1 - \frac{f_L(R_l)}{f_H(R_l)} \right) + \lambda (\gamma - (1 - b)) R_l Y_0,$$

which reduces to

$$\mu b \left(\frac{f_L(R_l)}{f_H(R_l)} - 1 \right) = b + \lambda (1 - b - \gamma).$$

Similarly, the optimality condition for R_u is

$$0 = \int_{R_u}^{\bar{R}} [-(1 - b) Y_0 f_H(R) - \mu (1 - b) Y_0 (f_H(R) - f_L(R)) + \lambda (1 - b) Y_0 f_H(R)] dR,$$

which reduces to

$$0 = E \left[\lambda - 1 - \mu \left(1 - \frac{f_L(R)}{f_H(R)} \right) \middle| R \geq R_u, e = H \right].$$

This completes the proof.

A.1.1 A Remark on Contract Uniqueness

The optimal contract is not generally unique in the following sense. In the region $R \leq R_l$, the bank only needs a liability face value that is sufficient to liquidate the bank, and so any contract with monotone face value $L_1(R) > (1 - b) Y_0$ in this region is optimal. We selected the contract with a flat face value below R_l due to its correspondence to standard debt. The face value of liabilities above R_l is uniquely determined. Moreover, in the presence of an $\epsilon \rightarrow 0$ premium for standard debt (e.g. as in Appendix B.6), the implementation using standard debt becomes uniquely optimal.

A.2 Proof of Corollary 2

Consider the proposed liability structure. The amount $(1 - b) R_l Y_0$ of standard debt liquidates the bank when $R \leq R_l$, generating the lower region. $(1 - b)(R_u - R_l)$ is written down in the region $R_l \leq R \leq R_u$, so that the bank is always held to the agency rent over this region. The full debt level $(1 - b) R_u Y_0$ is repaid above R_u . Therefore, we replicate the contract in Proposition 1.

A.3 Proof of Corollary 3

We split the proof into the different cases.

Case 1: Suppose first that $B = 0$, but $b > 0$ and $\gamma(s) < 1 - b$. We impose $(1 - b)E[R] < 1$ to obtain a finite solution.

The result is a Modigliani-Miller type result. Incentive compatibility is now

$$E \left[c_2(R) \left(1 - \frac{f_L(R)}{f_H(R)} \right) \mid e = H \right] \geq 0.$$

Let $c_2(R)$ be some monotone consumption rule. We have

$$E \left[c_2(R) \left(1 - \frac{f_L(R)}{f_H(R)} \right) \mid e = H \right] = \text{cov} \left(c_2(R), 1 - \frac{f_L(R)}{f_H(R)} \right) \geq 0$$

where the inequality follows from MLRP. As a result, any monotone consumption rule is implementable. We can span the frontier of expected repayment splits between banks and investors, with $\Pi \in [0, (1 - b)E[R]]$ to investors and $(1 - b)E[R] - \Pi$ to bankers, with monotone consumption rules (e.g. equity). Because all agents are risk-neutral, all that matters is the expected revenue division, and there is no need to liquidate the bank. Equity allocation rules $E \in [0, 1 - b]$, with investors receiving shares E and banks retaining equity $1 - E$, generate monotone consumption profiles and so are incentive compatible. They also span the range of possible surplus divisions. As a result, pure equity constitutes an optimal contract.

Case 2: Consider next $b = 0$. The RHS of (7) then collapses to $\lambda(1 - \gamma)$ while the LHS collapses to 0, and so banks never choose to liquidate. Optimal contracts use only bail-in debt.

Case 3: Consider finally $\gamma = 1$. Any face value $L_1(R) \leq RY_0$ can then be repaid by liquidating assets, so that bank consumption is $c_2(R) = RY_0 - L_1(R)$ for any $L_1(R) \leq RY_0$. Therefore for any liability structure $L_1(R)$, we can define

$$(c_2(R), x(R)) = \begin{cases} (RY_0 - L_1(R), L_1(R)), & L_1(R) \leq RY_0 \\ (0, RY_0), & L_1(R) \geq RY_0 \end{cases}$$

where the relevant liquidation function $\alpha(R) \in [0, 1]$ is defined from the liability structure. For example, without loss of generality we could define $\alpha(R) = \frac{x(R)}{RY_0}$. As a result, minimum

pledgeability never binds.

Defining the problem in the repayment space, we then have

$$\max \int_R [RY_0 - x(R)] f_H(R) dR,$$

subject to

$$\int_R [RY_0 - x(R)] (f_H(R) - f_L(R)) dR \geq BY_0$$

$$Y_0 - A = \int_R x(R) f_H(R) dR$$

$$R \geq R' \Rightarrow x(R) \geq x(R')$$

with $0 \leq x(R) \leq RY_0$. Relaxing monotonicity, the FOC for $x(R)$ is given by

$$\frac{\partial \mathcal{L}}{\partial x(R)} = \left[-1 - \mu \left(1 - \frac{f_L(R)}{f_H(R)} \right) + \lambda \right] f_H(R)$$

yielding a threshold rule R^* such that $x(R) = RY_0$ for $R \leq R^*$ and $x(R) = 0$ for $R \geq R^*$. This results in an upper pooling region R_u with liabilities x_u . Because $R_u < R^*$ as in the proof of Proposition 1, we have $x(R) = RY_0$ for all $R \leq R_u$. Continuity implies $L_1(R) = R_u Y_0$ for all R , and so the contract is standard debt.

A.4 Proof of Corollary 4

Characterizing the value of R_l from equation (7), we then take its solution and plug it into the RHS of equation (8). The RHS of equation (8) corresponds to the value of increasing R_u on the margin. Supposing that it is ≤ 0 at R_l , then the value of increasing R_u above R_l on the margin is non-positive. Moreover, by MLRP the RHS of equation (8) is non-positive for all $R > R_l$. As a result, we move to a corner solution where $R_u = R_l$. Note that R_l may fall below the value implied by equation (7), but over the region below this value, the bank prefers to issue standard debt, per the optimality condition of equation (7).

A.5 Proof of Proposition 5

Consider the program of the social planner

$$\max_{L_1, Y_0} E [c_2 | e = H]$$

subject to

$$\begin{aligned}
E \left[c_2 \left(1 - \frac{f_L(R)}{f_H(R)} \right) \middle| e = H \right] &\geq BY_0 \\
Y_0 - A &= \int_{R|\alpha=1} \gamma(\Omega)RY_0f_H(R)dR + \int_{R|\alpha=0} L_1(R)f_H(R)dR \\
R \geq R' &\Rightarrow L_1(R) \geq L_1(R') \\
c_2 &\geq (1 - \alpha(R))bRY_0 \\
\Omega &= \int \alpha(R)Rf_H(R)dR
\end{aligned}$$

The proof follows as in the proof of Proposition 1. Redefine the payoff space over $x(R)$ and solve for the optimal contract without imposing monotonicity. The first order condition for $c_2(R)$ is the same as in the proof of Proposition 1, since $c_2(R)$ does not directly affect Ω . This implies as before that we obtain a pooling region at the top.

As before, take R_u and x_u as given, and solve for the optimal contract for $R \leq R_u$. The same steps imply that implementable contracts must satisfy $R_u < R^*$. The FOC for optimal liquidations $\alpha(R)$ is now

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial \alpha(R)} &\propto \lambda (\gamma(\Omega) - 1) RY_0f_H(R) + \chi(R)bRY_0f_H(R) + \nu(R) (1 - \gamma) RY_0f_H(R) \\
&+ \frac{\partial \gamma(\Omega)}{\partial \Omega} \frac{\partial \Omega}{\partial \alpha(R)} \lambda \int_{R'} \alpha(R')R'Y_0f_H(R')dR'
\end{aligned}$$

Substituting in the derivative $\frac{\partial \Omega}{\partial \alpha(R)} = Rf_H(R)$, we obtain

$$\frac{\partial \mathcal{L}}{\partial \alpha(R)} \propto \lambda (\gamma - 1) + \chi(R)b + \nu(R) (1 - \gamma) + \frac{\partial \gamma(\Omega)}{\partial \Omega} \lambda \int_{R'} \alpha(R')R'f_H(R')dR'$$

The additional wedge $\frac{\partial \gamma(\Omega)}{\partial \Omega} \lambda \int_{R'} \alpha(R')R'f_H(R')dR'$ is negative and independent of R . The same steps apply as in the proof of Proposition 1, yielding a liquidation threshold rule R_l . Because as before $R_u < R^*$, we have $x(R) = \min\{(1 - b)RY_0, x_u\}$ in the region $R_l \leq R \leq R_u$. Thus, the set of candidate optimal contracts is the same as in the private equilibrium, and the implementation of Corollary 2 holds.

Lastly, we characterize the optimal choices of R_l and R_u for interior solutions. The optimality condition for R_u is identical to the private optimality condition, since it does

not affect the liquidation value. By contrast, the social optimality condition for R_l satisfies

$$b + \lambda((1 - b) - \gamma) = \mu b \left(\frac{f_L(R_l)}{f_H(R_l)} - 1 \right) + \frac{\lambda \frac{\partial \Omega}{\partial R_l}}{R_l \gamma_0 f_H(R_l)} \frac{\partial \gamma(\Omega)}{\partial \Omega} \int_{\underline{R}}^{R_l} R \gamma_0 f_H(R) dR.$$

Substituting in $\frac{\partial \Omega}{\partial R_l} = R_l f_H(R_l)$ and rearranging, we obtain

$$\mu b \left(\frac{f_L(R_l)}{f_H(R_l)} - 1 \right) = b + \lambda((1 - b) - \gamma) - \lambda \frac{\partial \gamma(\Omega)}{\partial \Omega} \int_{\underline{R}}^{R_l} R f_H(R) dR.$$

This completes the proof.

A.6 Proof of Proposition 6

The proof proceeds by contradiction. Suppose that the equilibrium optimal contract sets $R_u > R_l \geq R^{BO}$. Because $R_l \geq R^{BO}$, banks are bailed out when insolvent in any state $R \geq R^{BO}$. Consider any single bank writing a contract, and suppose that bank offers an alternate contract that sets $R'_l = R_u$, but otherwise leaves the liability contract otherwise unchanged. Since the bank is now bailed out in states $R_l \leq R \leq R_u$ rather than bailed in, investor repayment increases, the participation constraint is relaxed, and investors increase date 0 payment to the bank. Suppose the bank immediately consumes that additional payment, so that project scale is unaffected. The consumption profile of the bank after date 0 under the revised contract is identical to the original, so that the contract remains incentive compatible. Moreover, bank welfare is strictly higher under the revised contract than the original, because the bank consumes the additional date 0 payment from investors. But then this contract yields higher utility than the equilibrium contract, contradicting that it the equilibrium contract was optimal. As a result, if in equilibrium $R_l \geq R^{BO}$, then $R_u = R_l$.⁷²

A.7 Proof of Proposition 7

We adopt the following proof strategy. We will consider a contract that results in bailouts, and show that it is equivalent to a contract that: (1) features bail-ins (rather than bailouts) ex post; and, (2) implements an ex ante lump sum transfer from taxpayers to the bank. Because ex ante transfers from taxpayers to banks are assumed undesirable, it follows immediately

⁷²Note that it is possible that the bank may, in the presence of bailouts, no longer find it optimal to offer a contract that enforces high effort, due to the bailout guarantee. The above argument is unaffected. We can generally rule out the possibility that bailouts induce low effort by introducing aggregate risk and assuming that the probability of bailout states is not too high.

that the same (bail in) contract without the ex ante lump sum transfer dominates the bailout contract. Finally, we will derive the required condition on τ so that ex ante transfers from taxpayers to banks are indeed not optimal.

Define contracts over a space $\{R_l, R_u, T\}$, where T is an ex ante lump-sum transfer from the government (via taxpayers) to banks, above and beyond any bailouts. We use these transfers as an accounting tool to compare contracts.

Consider an implementable contract $\Gamma = \{R_l, R_u, 0\}$ with $R_l > R^{BO}$. Let $T(R_l, R_u)$ be the (ex ante) value of the bailout transfer under this contract, and define an alternative contract Γ' by $R'_l = \min\{R_l, R^{BO}\}$, $R'_u = R_u$, and $T' = T(R_l, R_u)$. The contract Γ' is implementable: bank consumption is identical, and investor repayment is satisfied by ex ante transfers rather than ex post (bailout) transfers. As a result, welfare under Γ' is at least as high as under the contract Γ .⁷³

Now, consider an alternate contract $\Gamma'' = \{R'_l, R'_u, 0\}$. This contract is implementable because it generates the same consumption-to-asset ratio $c_2(R)/Y_0$ as contract Γ' , with only the project scale Y_0 being different. In order to compare the welfare of these contracts, note that for any incentive compatible bank contract with thresholds (R_l, R_u) and transfers T , equilibrium bank welfare is given by

$$\underline{V}(R_l, R_u)Y_0(R_l, R_u, T | R_l^*, R_u^*)$$

where we have defined

$$\underline{V}(R_l, R_u) = \int_{R_l}^{R_u} bRf_H(R)dR + \int_{R_u}^{\bar{R}} (R - (1-b)R_u)f_H(R)dR.$$

Total investor repayment under the contract Π per unit of scale is given by

$$\Pi = \int_{\underline{R}}^{R_l} \gamma Rf_H(R)dR + \int_{R_l}^{R_u} (1-b)Rf_H(R)dR + \int_{R_u}^{\bar{R}} (1-b)R_u f_H(R)dR$$

so that the project scale is given by

$$Y_0(R_l, R_u | R_l^*, R_u^*) = \frac{A + T}{1 - \Pi(R_l, R_u | R_l^*, R_u^*)}.$$

⁷³We will not have to consider political costs of bailouts for the proof, but we state “at least as high” for formality.

Substituting in, we obtain equilibrium bank welfare under the contract as

$$\frac{\underline{V}(R_l, R_u)}{1 - \Pi(R_l, R_u | R_l^*, R_u^*)} (A + T).$$

Social welfare at date 0, given a lump sum tax cost $\tau > 1$, is given by⁷⁴

$$\frac{\underline{V}(R_l, R_u)}{1 - \Pi(R_l, R_u | R_l^*, R_u^*)} (A + T) - \tau T$$

Now, we can compare contracts Γ' and Γ'' . Because these two contracts differ only in the date 0 lump sum transfer but feature the same thresholds, a sufficient condition for the welfare gain from switching to contract Γ'' from contract Γ' is

$$\tau T' - \frac{\underline{V}(R'_l, R'_u)}{1 - \Pi(R'_l, R'_u | R'_l, R'_u)} T' \geq 0$$

or in other words

$$\tau \geq \max_{R_l, R_u} \frac{\underline{V}(R_l, R_u)}{1 - \Pi(R_l, R_u | R_l, R_u)}.$$

Under this condition, contract Γ'' is preferable to the contract Γ' . Because contract Γ' yields at least as high welfare as contract Γ , then contract Γ'' is preferable to contract Γ . In other words, there are no bailouts.

Note that because $R_l \leq R^{BO}$ acts as a constraint on the socially optimal contract, it immediately follows that bank welfare is non-decreasing in R^{BO} . In particular, if any contract $\Gamma = \{R_l, R_u, 0\}$ that is implementable given R^{BO} is also implementable for any $R^{BO'} \geq R^{BO}$, implying that social welfare cannot be lower. Lastly, recall that social welfare and bank welfare are the same in the absence of bailouts.

A.8 Proof of Lemma 8

If the best equilibrium is to have no liquidations, it must set $D_2 \leq \underline{R}Y_1$. Short-term debt is then always repaid in full, and $\bar{q}_1^D = 1$, since it is senior. Because $\underline{R}Y_1 < D_1$, the bank cannot refinance itself with short-term debt $D_2 \leq \underline{R}Y_1$ alone, and therefore must raise some long-term debt. Long-term debt will not be fully repaid, and has price $\bar{q}_1^L < 1$. Given the lower long-term debt price, the “no rat race” condition of equation 13 binds and we have $D_2 = \underline{R}Y_1$. The total funds raised from long-term debt must satisfy the repayment

⁷⁴We can ignore the political costs of bailouts.

condition

$$D_1 - \underline{R}Y_1 = \int_{\underline{R}}^{\bar{R}} \min\{L_3, R_2 Y_1 - \underline{R}Y_1\} f_2(R_2) dR_2.$$

Noting that the RHS is increasing in L_3 , there is a unique solution \bar{L}_3 to this equation. From here, \bar{q}_1^L is given by the definition $\bar{q}_1^L = \frac{D_1 - \underline{R}Y_1}{\bar{L}_3}$. The prices and quantities satisfy the no rat race conditions by construction. As a result, the best equilibrium has unique prices and quantities, and does not feature liquidations.

A.9 Proof of Proposition 9

Suppose that markets quote $q_1^L = 0$. The bank can only raise funds by issuing short-term debt, with maximum raisable funds

$$\sup_{D_2 \leq R_b Y_1} \int_{\underline{R}}^{D_2/Y_1} \gamma R_2 Y_1 f_2(R_2) dR_2 + \int_{D_2/Y_1}^{\bar{R}} D_2 f_2(R_2) dR_2$$

Define $d_2 = D_2/Y_1$, then the bank cannot refinance itself if

$$\frac{D_1}{Y_1} > \sup_{d_2 \leq R_b} \int_{\underline{R}}^{d_2} \gamma R_2 f_2(R_2) dR_2 + \int_{d_2}^{\bar{R}} d_2 f_2(R_2) dR_2.$$

The right-hand side is a constant that does not depend on D_1 , Y_1 , or R . As a result, we obtain a threshold rule in Y_1 (given D_1), which implies a threshold rule in R^* , so that rollover crises may exist when $R \leq R^*$. Lastly, note that equation (15) guarantees that the existing stock D_1 is sufficient to liquidate the bank whenever there is a rollover crisis. As a result, a hypothetical unit of new long-term debt receives no recovery value in liquidation, completing the equilibrium.

Finally, at $R = R_l$ we have

$$\frac{D_1}{R_l Y_0} = (1 - b) > \sup_{d_2 \leq R_b} \int_{\underline{R}}^{d_2} \gamma R_2 f_2(R_2) dR_2 + \int_{d_2}^{\bar{R}} d_2 f_2(R_2) dR_2$$

so that a rollover crisis equilibrium always exists at $R = R_l$. Hence, there is always an interval of existence $R \in [R_l, R^*)$.

A.10 Proof of Proposition 10

Consider the fixed point problem

$$\gamma^* = \gamma \left(\epsilon + \int_{\underline{R}}^{R_l} R f_H(R) dR + p \int_{R_l}^{R^*(\gamma^*)} R f_H(R) dR \right).$$

Totally differentiating in ϵ and evaluating at $\epsilon = 0$, we obtain

$$\frac{\partial \gamma^*}{\partial \epsilon} = \frac{\partial \gamma(\Omega^*)}{\partial \Omega^*} \left(1 + p \frac{\partial R^*(\gamma^*)}{\partial \gamma^*} \frac{\partial \gamma^*}{\partial \epsilon} R^* f_H(R^*) \right)$$

which rearranges to

$$\frac{\partial \gamma^*}{\partial \epsilon} = \frac{\frac{\partial \gamma(\Omega^*)}{\partial \Omega^*}}{1 - \frac{\partial \gamma(\Omega^*)}{\partial \Omega^*} p \frac{\partial R^*(\gamma^*)}{\partial \gamma^*} R^* f_H(R^*)}.$$

Next, evaluating the derivative in $\frac{\partial \Omega^*}{\partial \epsilon}$ and substituting in, we obtain

$$\begin{aligned} \frac{\partial \Omega^*}{\partial \epsilon} &= 1 + p \frac{\partial R^*(\gamma^*)}{\partial \gamma^*} \frac{\partial \gamma^*}{\partial \epsilon} R^* f_H(R^*) \\ &= 1 + \frac{p \frac{\partial \gamma(\Omega^*)}{\partial \Omega^*} \frac{\partial R^*(\gamma^*)}{\partial \gamma^*} R^* f_H(R^*)}{1 - p \frac{\partial \gamma(\Omega^*)}{\partial \Omega^*} \frac{\partial R^*(\gamma^*)}{\partial \gamma^*} R^* f_H(R^*)}. \end{aligned}$$

Finally, differentiating equation (16) in γ^*

$$\frac{\partial R^*}{\partial \gamma^*} = -\frac{(R^*)^2 \Upsilon_0}{D_1} \int_{\underline{R}}^{d_2^*} R_2 f_2(R_2) dR_2$$

and substituting in, we obtain the final result

$$\frac{\partial \Omega^*}{\partial \epsilon} = 1 + \frac{p \left| \frac{\partial \gamma(\Omega^*)}{\partial \Omega^*} \right| \left| \frac{\partial R^*(\gamma^*)}{\partial \gamma^*} \right| R^* f_H(R^*)}{1 - p \left| \frac{\partial \gamma(\Omega^*)}{\partial \Omega^*} \right| \left| \frac{\partial R^*(\gamma^*)}{\partial \gamma^*} \right| R^* f_H(R^*)}.$$

Noting that $\frac{\partial R^*}{\partial \gamma^*}$ and $\frac{\partial \gamma}{\partial \Omega^*}$ are negative, we have adopted the absolute value notation for clarity.

A.11 Proof of Proposition 11

Suppose that markets quote $q_1^L = 0$. Let \mathcal{C}^0 be the space of tuples $C = (q_1, D_2, L_3)$ with $q_1^L = 0$. A rollover crisis equilibrium exists under the vacuous covenant if there does not

exist any tuple $C \in \mathcal{C}^0$ such that $q_1^D D_2 \geq D_1$ and that is consistent with the no rat race conditions (13) and (14).

Now, consider an alternate covenant C_1 . When markets quote $q_1^L = 0$, the set of tuples that satisfy covenant C_1 is $\mathcal{C}^1 \subset \mathcal{C}^0$. But since no tuple $C \in \mathcal{C}^0$ refinances the bank, no tuple $C \in \mathcal{C}^1$ refinances the bank either. As a result, existence of rollover crises under the vacuous covenant implies existence of rollover crises under any other covenant.

A.12 Proof of Proposition 12

The modified private sector no rat race conditions, given *pari passu* claims, are

$$q_1^D D_2 \leq \int_{D_2 + D_2^{LOLR} \geq R_2 Y_1} \gamma R_2 Y_1 \frac{D_2}{D_2 + D_2^{LOLR}} f_2(R_2) dR_2 + \int_{D_2 + D_2^{LOLR} \leq R_2 Y_1} D_2 f_2(R_2) dR_2 \quad (22)$$

$$q_1^L L_3 \leq \int_{D_2 + D_2^{LOLR} \leq R_2 Y_1} \min \left\{ \left(R_2 Y_1 - D_2 - D_2^{LOLR} \right) \frac{L_3}{L_3 + L_3^{LOLR}}, L_3 \right\} f_2(R_2) dR_2 \quad (23)$$

Conjecture a rollover crisis equilibrium with $q_1^L = 0$. Suppose the bank borrows best equilibrium quantities from the LOLR, and does not borrow from the private sector. The break-even LOLR debt prices are given by

$$q_1^{D,LOLR} = \frac{1}{D_2} \int_{\bar{D}_2 \geq R_2 Y_1} \gamma R_2 Y_1 f_2(R_2) dR_2 + \int_{\bar{D}_2 \leq R_2 Y_1} \bar{D}_2 f_2(R_2) dR_2 = \bar{q}_1^D$$

$$q_1^{L,LOLR} = \frac{1}{L_3} \int_{\bar{D}_2 \leq R_2 Y_1} \min \{ (R_2 Y_1 - \bar{D}_2), \bar{L}_3 \} f_2(R_2) dR_2 = \bar{q}_1^L$$

which are the best equilibrium prices, given that $\bar{D}_2 = \underline{R} Y_1$. Therefore, the bank raises total funds $\bar{q}_1^D \bar{D}_2 + \bar{q}_1^L \bar{L}_3 = D_1$ from the LOLR, and successfully refinances itself. The rollover crisis equilibrium is eliminated.

Finally, because the LOLR and private sector rank *pari passu*, in equilibrium $q_1^{D,LOLR} = q_1^D$ and $q_1^{L,LOLR} = q_1^L$. Equilibrium market prices are therefore best equilibrium prices, and the bank can refinance itself entirely from the private sector using best equilibrium issuances. The best equilibrium is restored.

A.13 Proof of Proposition 13

Suppose that a rollover crisis equilibrium occurs, and consider the value of a hypothetical unit of new private sector long-term debt. The government guarantee implies the unit is repaid its full face value of 1, so that $q_1^L = 1$. But then the bank can refinance itself with long-term debt, and a rollover crisis equilibrium does not exist.⁷⁵

Consider now instead the best equilibrium prices \bar{q}_1 . The bank can always refinance itself with the best equilibrium quantities (\bar{D}_2, \bar{L}_3) , and therefore survives until period 2. Guarantees expire at the end of period 1, and so are never filled, regardless of the refinancing package chosen by the bank. As a result, the bank indeed chooses the best equilibrium quantities (D_2, L_3) , and the best equilibrium is restored.

B Extensions

B.1 Pareto Efficiency

We now study whether the socially optimal contract in Section 3 is indeed Pareto efficient relative to the privately optimal contract. Recall that we have assumed that $u'(\bar{A}) > 1$. We obtain the following result.

Proposition 14. *Let $\frac{\partial \gamma}{\partial \Omega} < 0$. The socially optimal contract features R_L given by*

$$\mu b \left(\frac{f_L(R_L)}{f_H(R_L)} - 1 \right) = b + \lambda ((1 - b) - \gamma) + \lambda \left(1 - \frac{1}{u'(\bar{A} - A_0)} \right) \left| \frac{\partial \gamma}{\partial \Omega} \right| \Omega.$$

As a result, the privately optimal contract is not Pareto efficient given $u'(\bar{A}) > 1$.

Pareto efficient improvements arise because arbitrageurs are borrowing constrained, so that their marginal utility at date 0 exceeds that at date 1. Efficiency is achieved by transferring resources to arbitrageurs at date 0 in order to compensate them for resource losses from lower surplus from bank liquidations.

When we take $\omega^A \rightarrow 0$, we have the optimal allocation has $A_0 \rightarrow \bar{A}$ and $u'(\bar{A} - A_0) \rightarrow +\infty$, and we obtain the first order condition of Proposition 5.

⁷⁵If $q_1^L = 1$, then the stronger no rat race conditions we have applied in the main paper will be violated. Instead, we need to use the weaker form no rat race conditions in Appendix B.8. Under these conditions, we have the quoted price $\bar{q}_1^L = 1$. The bank chooses issuance $(D_2, L_3) = (0, \bar{D}_2 + \bar{L}_3)$, which lowers the price to the level in equation (29) and gives us an equilibrium.

B.1.1 Proof of Proposition 14

We can characterize a Pareto efficient contract by adopting the welfare function

$$E [c_2 | e = H] + \omega^A [u(\bar{A} - A_0) + (\mathcal{F}(\Omega) - \gamma(\Omega)\Omega) Y_0],$$

where ω^A is the welfare weight on arbitrageurs. The optimality of standard and bail-in debt follows the same steps as in the proofs of Propositions 1 and 5. However, in writing the optimal choice of threshold R_l , we now account for arbitrageur surplus and obtain

$$b + \mu b \left(\frac{f_L(R_l)}{f_H(R_l)} - 1 \right) = b + \lambda ((1 - b) - \gamma) + (\lambda - \omega^A) \left| \frac{\partial \gamma}{\partial \Omega} \right| \Omega.$$

Finally, the optimality condition for A_0 is given by $\lambda = \omega^A u'(\bar{A} - A_0)$, so that we obtain

$$b + \mu b \left(\frac{f_L(R_l)}{f_H(R_l)} - 1 \right) = b + \lambda ((1 - b) - \gamma) + \lambda \left(1 - \frac{1}{u'(\bar{A} - A_0)} \right) \left| \frac{\partial \gamma}{\partial \Omega} \right| \Omega$$

which completes the proof when $u'(\bar{A}) > 1$.

B.2 Aggregate Risk

To incorporate aggregate risk into the model, we add an aggregate state $s \in S$ of the economy at date 1. For expositional simplicity, we assume that S is a finite set, with probability measure $\pi(s)$.

The aggregate state s affects the return distribution, so that we have $f_e(R|s)$. All contracts can be written on the aggregate state. MLRP now applies contingent on the aggregate state, and liability monotonicity is also contingent on the aggregate state.

From here, the characterization of privately optimal contracts follows almost identically to before.

Proposition 15. *A privately optimal bank contract has a liability structure*

$$L_1(R, s) = \begin{cases} (1 - b)R_l(s)Y_0, & R \leq R_l(s) \\ (1 - b)RY_0, & R_l \leq R \leq R_u(s) \\ (1 - b)R_u(s)Y_0, & R_u(s) \leq R \end{cases}$$

where $0 \leq R_l(s) \leq R_u(s) \leq \bar{R}$ are aggregate-state-contingent thresholds. The bank is liquidated if

and only if $R \leq R_l(s)$. These thresholds, when interior and not equal, are given by

$$\underbrace{\mu b \left(\frac{f_L(R_l(s)|s)}{f_H(R_l(s)|s)} - 1 \right)}_{\text{Incentive Provision}} = \underbrace{b + \lambda (1 - b - \gamma(s))}_{\text{Liquidation Costs}} \quad (24)$$

$$0 = E \left[\underbrace{\lambda - 1}_{\text{Investor Repayment}} - \underbrace{\mu \left(1 - \frac{f_L(R|s)}{f_H(R|s)} \right)}_{\text{Incentive Provision}} \mid R \geq R_u(s), s, e = H \right] \quad (25)$$

where $\mu > 0$ is the Lagrange multiplier on incentive compatibility (2) and $\lambda > 1$ is the Lagrange multiplier on investor participation (4).

Proof. The proof follows the same steps as the proof of Proposition 1. ■

In contrast to the baseline model, both instruments are contingent on the aggregate state, reflecting that the terms of bank contracts adjust to verifiable events that are beyond a bank's control. For example, if all else equal a state s has lower returns due to an aggregate (TFP) shock, equation (7) implies it should have a lower liquidation threshold.⁷⁶

In the context of CoCos, conditioning the level of bail-in debt on both the idiosyncratic state (i.e. individual bank health) and aggregate state (i.e. banking sector health) can be thought of as a dual price trigger.⁷⁷ In this context, there is $R_l(s_{\min})$ of fully non-contingent debt, and $R_u(s_{\max})$ of bail-in debt with a dual-price trigger. The dual price trigger writes down bail-in debt automatically to $R_u(s)$ based on the aggregate state s , and allows for it to be additionally written down to $R_l(s) - R_l(s_{\min})$ to restore bank solvency. The dual price trigger thus conditions recapitalization of banks on the aggregate state as well as the idiosyncratic state.⁷⁸

From here, the results on the socially optimal contract proceed identically, with the state contingency. Similarly, the bailout results can also be derived, where the result is that no bail-in debt is issued for state s whenever there are bailouts in state s .

This helps to understand the limits of bank contingencies on verifiable aggregate risk. Although aggregate risk is verifiable and not a result of bank shirking, banks neglect fire sales and expect to receive bailouts in bad aggregate states. This limits the extent to which they write contingencies on aggregate risk.

⁷⁶See Dewatripont and Tirole (2012) for a related argument.

⁷⁷See e.g. Allen and Tang (2015) and McDonald (2013).

⁷⁸Alternatively, we could consider it a combination of $R_l(s_{\min})$ of fully non-contingent debt, $R_u(s_{\max})$ of debt with a dual price trigger but no automatic write-down, and an aggregate risk hedge that mimicked the automatic write-down.

B.2.1 Bail-in Equivalence with Aggregate Risk

As highlighted above, the degree of bail-inability of debt depends on the aggregate state. Such rules either must be contractually pre-written into debt contracts, or must be written into the rules governing the operations of the bail-in authority. Provided that such rules are specified to govern ex-post resolution, the equivalence between ex-ante contractual provisions and ex-post resolution follows as in the baseline model.

In the US, such rules could be implemented using the organizational structure of the bank. Bank holding companies are required to maintain an amount of loss-absorbing debt at the level of the top-level holding company. The goal is to resolve the top-level holding company while allowing operating subsidiaries to continue operations without being affected by the resolution of the holding company. In principle, however, if a full write-down of the liabilities at the holding company level is not sufficient to recapitalize the bank, recapitalization would require bail-ins of debt at the operating subsidiaries. One could structure the governing rules of the bail-in authority to condition the ability of that authority to resolve operating subsidiaries based on the state of the economy. Operating subsidiaries could be resolved by the bail-in authority in crises, but not in normal times.

It is not clear whether aggregate state contingent rules governing the bail-in authority could credibly be implemented and followed. A bail-in authority is likely to be tempted to recapitalize a bank if there is enough long-term debt available to do so, suggesting the potential for time inconsistency in bail-ins.

B.3 Macprudential Regulation and Bail-ins

In the baseline model, the fact that banks have a single investment project means that liability-side regulation is sufficient. In practice, banks asset allocations also affect their risk profiles. We now show that macroprudential (asset-side) regulation is a necessary complement to bail-ins when banks can affect risks using both sides of their balance sheet.

We augment the model as follows. Banks choose a contractible vector $\theta = (\theta_1, \dots, \theta_N)$ of asset allocations. The total return R on bank scale Y_0 follows a density $f_e(R|\theta)$, which depends on the allocation θ . $f_e(R|\theta)$ satisfies MLRP (conditional on (θ)) over the relevant range of allocations θ . To simplify exposition, the support of R is an interval $[\underline{R}, \bar{R}]$ that does not depend on θ . Otherwise, the setup is the same as before.⁷⁹

⁷⁹In Appendix B.3.1, we show how a standard asset allocation problem generates a density function of this form. If the shirking benefit $B(\theta)$ depended on the allocation, e.g. because riskier assets are more difficult to monitor, the planner and banker would agree on how θ affects B . Assets in our model all sell at the same discount and generate the same fire sale spillover. If they differed in terms of liquidation discounts and fire sale spillovers, there would be an additional regulatory incentive on this margin.

As before, optimal liability contracts combine contingent and standard debt, and the trade-off between standard and bail-in debt reflects the same forces as before.⁸⁰ We now characterize the optimal asset allocation rule under the socially optimal contract.

Proposition 16. *The socially optimal contract has FOC for θ_n*

$$0 = E \left[\underbrace{\left(\lambda x(R) + c_2(R) \left(1 + \mu \left(1 - \frac{\partial f_L(R|\theta)/\partial \theta_n}{\partial f_H(R|\theta)/\partial \theta_n} \right) \right) \right)}_{\text{Private Bank Benefit}} \frac{\partial f_H(R|\theta)/\partial \theta_n}{f_H(R|\theta)} \right] \\ + \lambda E \left[\underbrace{\frac{\partial \gamma}{\partial \Omega} \Omega Y_0 \cdot \left[\int_{\underline{R}}^{R_l} R \frac{\partial f_H(R|\theta)}{\partial \theta_n} dR \right]}_{\text{Fire Sale Cost}} \right]$$

The first line of Proposition 16 reflects the private trade-off to banks of a change in asset composition, corresponding to changes in the return distribution. These changes are weighted by the (weighted) sum of payoff to investors in those states, and to banks in those states, where the weighting reflects both the direct value of payoffs, and the incentive value of payoffs. The second line of Proposition 16 reflects the *social* cost of changes in asset composition. The social cost arises when changes in the return distribution affect the magnitude of the fire sale spillover, by altering the measure Ω of bank liquidations. When an asset increases the probability that the banks' total return is lower than R_l , larger allocations to that asset result in more severe fire sale spillovers. The social cost term penalizes investment in such assets. The social cost term exists whenever $R_l > \underline{R}$, that is whenever liability-side regulation has not completely eliminated bank failures.

Proposition 16 illustrates that macroprudential (asset) regulation is a necessary complement to bail-ins (liability regulation). Macroprudential regulation and bail-ins co-exist in the regulatory regime because they control fire sales in different manners. For a given level of asset risk, bail-ins mitigate fire sales by reducing the liquidation threshold. For a given liquidation threshold, macroprudential regulation mitigates fire sales by reducing the probability that a bank will fall below that threshold.⁸¹ These two aspects of regulation are not generally perfect substitutes, so they co-exist under the optimal regulatory regime.

Even though macroprudential regulation and bail-ins are not perfect substitutes, Proposition 16 suggests that bail-ins are a partial substitute for macroprudential regulation. Stronger liability regulation pushes the magnitude of the additional wedge in the asset allocation decision towards zero, by reducing the size of the liquidation region.

⁸⁰Given that θ is contractible, the proof follows the same steps as Proposition 1.

⁸¹Macroprudential regulation in our model closely risk weights on loss-absorbing capital.

B.3.1 Multiple Assets Density Function

Suppose that there are $N + 1$ assets between which the bank allocates its funds. Denote $\omega \in [\underline{\omega}, \bar{\omega}]$ to be the underlying idiosyncratic state of the bank, with associated density $f_e^\omega(\omega)$, where $e \in \{H, L\}$. Suppose that $f_e^\omega(\omega)$ satisfies MLRP, so that $\frac{\partial}{\partial \omega} \left(\frac{f_H^\omega(\omega)}{f_L^\omega(\omega)} \right) > 0$.

Asset $n \in \{1, \dots, N + 1\}$ generates a return $R_n(\omega)$ per unit. Let $\theta = (\theta_1, \dots, \theta_{N+1})$ be a vector that determines the asset allocations $\theta_1 Y_0, \dots, \theta_{N+1} Y_0$. Allocations θ satisfy a technological restriction $\mathcal{F}(\theta) = 0$, for example there may be a concave technology. Note that to coincide with the previous parts, we assume the technology is linear in the scale Y_0 , and only (potentially) concave in the asset weights. If $\mathcal{F}(\theta) = \sum_{n=1}^{N+1} \theta_n - 1$, we have a simple linear technology with equal cost of investment across assets.

We invert θ_{N+1} from $(\theta_1, \dots, \theta_N)$ via \mathcal{F} , so that we can internalize the constraint. We denote the total return to the bank, given an asset allocation vector θ , by

$$R(\omega\theta) = \sum_{n=1}^{N+1} \theta_n R_n(\omega) Y_0$$

where θ_{N+1} is derived from the technology $\mathcal{F}(\theta) = 0$, given $\theta_1, \dots, \theta_N$.

Suppose that conditional on θ , there is an injective mapping between ω and R . In this case, R identifies ω , given θ , and we can write contracts on R . We assume that the mapping is injective over the relevant range of asset allocations θ . For example, this will be the case if asset allocations are non-negative ($\theta_n \geq 0$) and individual asset returns are monotone in ω . Without loss of generality, we assume the injective mapping is monotone increasing: high states ω identify high returns R , consistent with the interpretation of $e = H$ as “high effort.”

Denote $R^{-1}(R|\theta)$ to be the inverse function mapping the total return R into the idiosyncratic state ω . The inverse function does not depend directly on e , but rather the density will depend on e . We now derive the density of R , conditional on θ . We have

$$F_e(R|\theta) = \Pr(R(\omega|\theta) \leq R|e) = \Pr(\omega \leq R^{-1}(R|\theta) | e) = F_e^\omega(R^{-1}(R|\theta)).$$

Differentiating in R , we obtain the density function:

$$f_e(R|\theta) = f_e^\omega(R^{-1}(R|\theta)) \frac{\partial R^{-1}(R|\theta)}{\partial R}$$

We impose the simplifying assumption that the support $[\underline{R}, \bar{R}]$ of the density is invariant to the allocation θ . If the support depended on the portfolio allocation, we would

have boundary terms in derivatives. The principal term of relevance would be how the lower boundary of the support moves in the asset allocation, which reflects changes in the measure of the liquidation region. These effects are qualitatively the same as the direct effects of changing the measure from changes in the density. For simplicity, we keep the support fixed.

Finally, we can show that this function satisfies monotone likelihood. Differentiating the likelihood ratio in R , we obtain

$$\begin{aligned}
\frac{d}{dR} \left(\frac{f_H(R|\theta)}{f_L(R|\theta)} \right) &= \frac{d}{dR} \left(\frac{f_H^\omega(R^{-1}(R|\theta))}{f_L^\omega(R^{-1}(R|\theta))} \right) \\
&= \frac{\frac{\partial f_H^\omega}{\partial \omega} \frac{\partial R^{-1}(R|\theta)}{\partial R} f_L^\omega - \frac{\partial f_L^\omega}{\partial \omega} \frac{\partial R^{-1}(R|\theta)}{\partial R} f_H^\omega}{(f_L^\omega)^2} \\
&= \frac{\partial}{\partial \omega} \left(\frac{f_H^\omega}{f_L^\omega} \right) \frac{\partial R^{-1}(R|\theta)}{\partial R} \\
&> 0
\end{aligned}$$

where in the last line, we have used MLRP on f_e^ω combined with monotonicity of R^{-1} .

As a result, we obtain a representation of the problem as a density $f_e(R|\theta)$. Implicitly, we differentiate in $(\theta_1, \dots, \theta_N)$, where we have internalized θ_{N+1} as arising from the technology.

B.3.2 Proof of Proposition 16

Consider the optimal contract of the social planner. Holding fixed the debt levels R_l and R_u , the derivative of the planner's Lagrangian in θ_n is given by

$$\begin{aligned}
0 &= E \left[c_2 \frac{\partial f_H(R|\theta) / \partial \theta_n}{f_H(R|\theta)} \right] + \mu E \left[c_2 \left(\frac{\partial f_H(R|\theta) / \partial \theta_n}{f_H(R|\theta)} - \frac{\partial f_L(R|\theta) / \partial \theta_n}{f_H(R|\theta)} \right) \right] \\
&\quad + \lambda E \left[x \frac{\partial f_H(R|\theta) / \partial \theta_n}{f_H(R|\theta)} \right] \\
&+ \lambda E \left[\int_{\underline{R}}^{R_l} \frac{\partial \gamma(\Omega)}{\partial \Omega} \frac{\partial \Omega}{\partial \theta_n} R Y_0 f_H(R|\theta) dR \right]
\end{aligned}$$

where the first two lines reflect the private bank trade-off, and the last line reflects the social trade-off. Liquidations are given by

$$\Omega = \int_{\underline{R}}^{R_l} R f_H(R|\theta) dR$$

so that we have

$$\frac{\partial \Omega}{\partial \theta_n} = \int_{\underline{R}}^{R_l} R \frac{\partial f_H(R|\theta)}{\partial \theta_n} dR.$$

Substituting in above, we obtain

$$0 = E \left[\underbrace{\left(\lambda x(R) + c_2(R) \left(1 + \mu \left(1 - \frac{\partial f_L(R|\theta)/\partial \theta_n}{\partial f_H(R|\theta)/\partial \theta_n} \right) \right) \right) \frac{\partial f_H(R|\theta)/\partial \theta_n}{f_H(R|\theta)}}_{\text{Private Bank Benefit}} \right] \\ + \lambda E \left[\underbrace{\frac{\partial \gamma}{\partial \Omega} \Omega Y_0 \cdot \left[\int_{\underline{R}}^{R_l} R \frac{\partial f_H(R|\theta)}{\partial \theta_n} dR \right]}_{\text{Fire Sale Cost}} \right]$$

giving the result.

B.4 Heterogeneous Investors and the Allocation of Securities

In the baseline model, investors are homogeneous and risk neutral, so that the distribution of standard and bail-in debt among investors is irrelevant. A key practical concern is what investors should hold what form of debt, since bail-in debt holders will experience losses when it is written down. Particular concern has been expressed about protecting retail investors from losses that are large relative to their wealth⁸², and to preventing institutional investors who are potentially exposed to fire sales from bearing losses from bail-ins.⁸³

To capture these elements, we extend the model to include two classes of bank investors, “institutional” and “retail.” To make the problem interesting, we include aggregate risk. Institutional investors are able to invest across all banks, but still retain exposure to the aggregate state and have preferences that may depend on bank liquidation discounts. Retail investors are only able to invest in a single bank and retain exposure

⁸²The resolution of four Italian banks in 2015 sparked a political backlash due to losses to retail investors. Financial Times, “Italy bank rescues spark bail-in debate as anger at Renzi grows,” December 22, 2015.

⁸³Article 44 of BRRD states that “[m]ember states shall ensure that in order to provide for the resolvability of institutions and groups, resolution authorities limit...the extent to which other institutions hold liabilities eligible for a bail-in tool.”

to the idiosyncratic return of that bank. For simplicity, we abstract away from other potential components of these investors' portfolio choice problems, instead allowing for state dependent preferences. All investors are price takers, and purchase state-contingent payoffs from the banks they invest in. Nevertheless, we show that in equilibrium all investors purchase a combination of the standard and bail-in debt contracts issued by banks.

Denote $q(R, s)$ the (endogenous) probability-normalized price of a unit of payoff from a bank that realizes state (R, s) .⁸⁴ Institutional investors are indexed by $i \in I$, have initial wealth w_0^i , and preferences $u_0^i(c_0^i) + E [u_1^i(c_1^i | s, \gamma(s))]$. Retail investors are indexed by $j \in J$, have initial wealth w_0^j , and preferences $u_0^j(c_1^j) + E [u_1^j(c_1^j | s)]$. Both I and J are finite sets, and we interpret each investor type as corresponding to a continuum of (atomistic) agents of that type. Both types of agents have period-0 budget constraints given by

$$c_0^k + \sum_s \pi(s) \int_R q(R, s) x^k(R, s) f_H(R|s) dR = w_0^k, \quad k \in I \cup J.$$

However, they differ in their choice of c_1 . Institutional investors are able to diversify across banks, so that $c_1^i(s) = \int_R x^i(R, s) f_H(R|s) dR$. Retail investors are not able to diversify across banks, and so have $c_1^j(R, s) = x^j(R, s)$. Given the contract payoff $x(R, s)$ from the bank, market clearing for liabilities is given by

$$\sum_{k \in I \cup J} \mu^k x_1^k(R, s) = x(R, s)$$

where μ^k is the mass of investors of type $k \in I \cup J$.

We focus on the case where the mass of retail investors is sufficiently small that it does not exhaust the returns of the bank in any state (R, s) . That is, $\sum_j \mu^j x_1^j(R, s) < x(R, s)$. As a result, both retail and institutional investors price bank liabilities on the margin. We now characterize the equilibrium of the private economy without government intervention.

Proposition 17. *Suppose that in equilibrium $\sum_j \mu^j x_1^j(R, s) < x(R, s)$. In the private equilibrium:*

1. *The price $q(R, s) = q(s)$ depends only on the aggregate state s .*
2. *Optimal bank contracts combine standard and bail-in debt.*
3. *Retail investors only purchase standard debt, and their consumption profile $c_1^j(R, s) = c_1^j(s)$*

⁸⁴Note that the bank will go bankrupt in some states, implying not all liabilities are repaid at full face value. For simplicity, we price units of payout directly, rather than face value.

only depends on the aggregate state s . Consumption profiles of retail investors are given by

$$\frac{\partial u_1^j(c_1^j(s)|s)}{\partial c_1^j(s)} = q(s) \frac{\partial u_0^j(c_0^j)}{\partial c_0^j}$$

4. Institutional investors purchase both standard and bail-in debt. Consumption profiles of institutional investors are given by

$$\frac{\partial u_1^i(c_1^i(s)|s, \gamma(s))}{\partial c_1^i(s)} = q(s) \frac{\partial u_0^i(c_0^i)}{\partial c_0^i}$$

Even though retail investors are tied to a specific bank, their equilibrium consumption profile does not depend on the idiosyncratic state. This implies not only that retail investors exclusively purchase standard debt, but also that retail investors are first in line for repayment in the event of bank liquidation. In other words, in equilibrium they purchase claims that have the highest priority for repayment. Since retail investors are often depositors, one natural interpretation of this result is that of deposit priority.⁸⁵ However, it extends beyond deposits, and furthermore suggests that retail bondholders may also benefit from priority. This suggests a role for non-bail-inable long-term debt, as a way to codify protection for retail investors.

Institutional investors are not exposed to the idiosyncratic state due to their ability to diversify, but are exposed to the aggregate state. Institutional investors face greater losses on the aggregate state when either they are more risk tolerant, or less exposed to bank fire sales. This suggests that the ideal holders of bail-in debt will be institutional investors with limited risk aversion (or ability to diversify using other securities) and limited commonality with the banking sector, so that they are not affected by fire sales.

Finally, consider what would happen if we relaxed the assumption $\sum_j \mu^j x_1^j(R, s) < x(R, s)$. Consider an aggregate state s where $\sum_j \mu^j x_1^j(R, s) = x(R, s)$ for a range of returns $R \leq R^*$. For $R > R^*$, institutional investors are the marginal pricing agent, and $q(R, s) = q(s)$ is a constant. For $R < R^*$, retail investors are the marginal pricing agents, and $q(R, s) \geq q(s)$. Given monotone liabilities contracts, $q(R, s)$ will be falling in R . Contracts will still be debt, but the optimal thresholds are affected by the fact that retail investors suffer larger losses in liquidation, pushing $q(R, s)$ higher above $q(s)$. This generates an additional trade-off for the bank in deciding the optimal composition of standard and

⁸⁵These deposits are not insured in this section, but are repaid due to their priority. In Appendix B.7, we consider deposit insurance.

bail-in debt.

B.4.1 Proof of Proposition 17

Suppose that there is a state-contingent Arrow price $q(R, s) = q(s)$ that depends only on the aggregate state. Contracts still take the form of standard and bail-in debt, following the same steps as in the proof of Proposition 1.

Now, consider the investor side. Begin first with institutional investors, whose Lagrangian is given by

$$\begin{aligned} \mathcal{L}^i = & u_0^i(c_0^i) + \sum_s \pi(s) u_1^i(c_1^i|s, \gamma(s)) + \lambda^i \left[w_0^i - c_0^i - \sum_s \pi(s) \int_R q(R, s) x^i(R, s) f_H(R|s) dR \right] \\ & + \sum_s \pi(s) \mu^i(s) \left[\int_R x^i(R, s) f_H(R|s) dR - c_1^i(s) \right]. \end{aligned}$$

Given the non-negativity constraint $x^i(R, s) \geq 0$, we have

$$\frac{\partial \mathcal{L}^i}{\partial x^i(R, s)} = - \left[\lambda^i q(R, s) - \mu^i(s) \right] \pi(s) f_H(R|s) \leq 0.$$

This equation holds with equality only at the lowest value of $q(R, s)$ in state s . In other words, investors only purchase $x^i(R, s) > 0$ if $q(R, s) = q(s)$, where $q(s)$ is defined to be the lowest price of a state-contingent security for some return state R in state s .

Suppose then that in equilibrium $\sum_j \mu^j x_1^j(R, s) < x(R, s)$. Then, at least one institutional investor i is purchasing $x^i(R, s) > 0$. As a result, we have $q(R, s) = q(s)$ for all R in state s , that is the price is constant in aggregate state s . Moreover, $q(s) \lambda^i = \mu^i(s)$.

From here, we can obtain λ^i from the FOC for c_0^i and $\mu^i(s)$ from the FOC for c_1^i . Substituting in, we obtain

$$\frac{\partial u_1^i(c_1^i(s)|s, \gamma(s))}{\partial c_1^i(s)} = \frac{\partial u_0^i(c_0^i)}{\partial c_0^i} q(s).$$

giving us the characterization of the consumption rules of institutional investors.

Finally, consider type- j (retail) investors. Given the constant price $q(s)$, their Lagrangian is

$$\mathcal{L}^j = u_0^j(c_0^j) + E \left[u_1^j(c_1^j(R, s)|s) \right] + \lambda^j \left(w_0^j - c_0^j - \sum_s \pi(s) \int_R q(s) c_1^j(R, s) f_H(R|s) dR \right),$$

so that we have optimality condition for $c_1^i(R, s)$

$$\frac{\partial u_1^j \left(c_1^j(R, s) | s \right)}{\partial c_1^j(R, s)} = \lambda^j q(s).$$

As a result, $c_1^j(R, s) = c_1^j(s)$ is constant within state s . The indifference condition follows immediately by combining with the FOC for c_1^j . This concludes the proof.

B.5 Risk Aversion and Risk Shifting

The baseline model featured no role for equity-like instruments in the bank's capital structure. We extend the model to incorporate risk aversion and risk shifting, ingredients known to generate a role for equity-like claims. Optimal contracts still feature a region of liquidations and a region of "bail-ins," where the bank is held to its continuation agency rent. Above the bail-in region, the contract involves equity-like claims.⁸⁶

Banks are risk averse and have utility $u(c_1 + c_2)$ from consumption, while investors are risk averse and have utility $v(x_1 + x_2)$. Bank utility and marginal utility are finite at 0, and we normalize $u(0) = 0$. We incorporate risk shifting by extending the bank's monitoring decision to $e \in \{L, H, RS\}$, where $e = RS$ is "risk shifting" and $e \in \{L, H\}$ are the high and low monitoring choices from before. Risk shifting does not generate a private benefit but affects the return density, $f_{RS}(R)$.⁸⁷ Define the likelihood ratios $\lambda_{L,H}(R) = \frac{f_L(R)}{f_H(R)}$ and $\lambda_{RS,H}(R) = \frac{f_{RS}(R)}{f_H(R)}$. Risk shifting inefficiently pushes mass towards the extremes of the distribution, which we formalize by defining a point $R_{RS} \in [\underline{R}, \bar{R}]$ such that $\frac{\partial \lambda_{RS,H}(R)}{\partial R} < 0$ for $R < R_{RS}$ and $\frac{\partial \lambda_{RS,H}(R)}{\partial R} \geq 0$ for $R \geq R_{RS}$.

As before, we assume optimal contracts enforce $e = H$. The no-risk-shifting constraint is

$$\int_{\underline{R}}^{\bar{R}} u(c(R)) (f_H(R) - f_{RS}(R)) dR \geq 0 \quad (26)$$

while the incentive constraint is the same as before, except with $u(c(R))$. Investor participation is given by

$$Y_0 - A = \int_{\underline{R}}^{\bar{R}} v(x(R)) f_H(R) dR.$$

Define $\bar{\lambda}_H(R) = \frac{\mu_L}{\mu} \lambda_{L,H}(R) + \frac{\mu_{RS}}{\mu} \lambda_{RS,H}(R)$ and $\mu = \mu_L + \mu_{RS}$.

To simplify exposition, we will assume that the characterization that follows satisfies

⁸⁶See e.g. [Hilscher and Raviv \(2014\)](#) for analysis of CoCo design on risk shifting.

⁸⁷We could incorporate a private benefit or cost of risk shifting without qualitatively changing results.

both consumption monotonicity for the bank and liability monotonicity for investors.⁸⁸ Characterization of contracts in settings that do not satisfy monotonicity is beyond the scope of this paper. Moreover, we assume that the region $1 + \mu(1 - \bar{\lambda}_H(R)) < 0$ is a connected set. This simplifies exposition.

Proposition 18. *Let $|S| = 1$. Suppose that the region $1 + \mu(1 - \bar{\lambda}_H(R)) < 0$ is a connected set. The privately optimal contract is as follows.*

1. *In the region where $1 + \mu(1 - \bar{\lambda}_H(R)) < 0$, there are liquidations and bail-ins.*
2. *In the region where $1 + \mu(1 - \bar{\lambda}_H(R)) \geq 0$, there are bail-ins and “equity.” The equity sharing rule is*

$$u'(c(R)) (1 + \mu(1 - \bar{\lambda}_H(R))) = \lambda v'(RY_0 - c(R))$$

The motivations behind the liquidation region and the bail-in region are as in the baseline model. Consider next the “equity” region. First, bank risk aversion moderates payouts to the bank, smoothing the bank consumption profile on the upside and so giving away some of the equity value to investors. Second, bank consumption decreases with the average likelihood $\bar{\lambda}_H(R)$. In the region $R \leq R_{RS}$, $\bar{\lambda}_H(R)$ is decreasing in R and so banker consumption is increasing. However, when $R \geq R_{RS}$, $\lambda_{L,H}$ is falling while $\lambda_{RS,H}$ is rising. This second effect, which comes from the risk shifting motivation, moderates payoffs to banks in high return states, which signal a higher likelihood that the bank engaged in risk shifting.

We could also derive the socially optimal contract, which would internalize the fire sale spillover cost of liquidations. However, conditional on not liquidating, bank and planner incentives are aligned, suggesting that the planner needs only to control the trade-off between liquidations and non-liquidations, and not the trade-off between bail-ins and “equity.”⁸⁹

B.5.1 Proof of Proposition 18

Given the assumption of consumption monotonicity, if there is a liquidation region, it satisfies a threshold rule $R \leq R_l$. We define the optimal contract in terms of this threshold rule and in terms of liabilities $x(R)$ above this threshold. The bank’s Lagrangian is given

⁸⁸Note that because both agents are risk averse, there is less scope for live-or-die contracts.

⁸⁹If effort were a continuous choice variable that affected bank returns, there would be an incentive to govern this margin. See [Mendicino et al. \(2018\)](#) for a numerical study of this problem.

by

$$\begin{aligned}
\mathcal{L} &= \int_{R \geq R_l} u(c(R)) f_H(R) dR \\
&+ \mu_L \left[\int_{R \geq R_l} u(c(R)) (f_H(R) - f_L(R)) dR - B Y_0 \right] + \mu_{RS} \left[\int_R u(c(R)) (f_H(R) - f_{RS}(R)) dR \right] \\
&+ \lambda \left[A + \int_{R \leq R_l} v(\gamma R Y_0) f_H(R) dR + \int_{R \geq R_l} v(R Y_0 - c(R)) f_H(R) dR - Y_0 \right] \\
&+ \int_{R \geq R_l} \chi(R) [c(R) - b R Y_0] f_H(R) dR
\end{aligned}$$

Define $\bar{\lambda}_H(R) = \frac{\mu_L}{\mu} \lambda_{L,H}(R) + \frac{\mu_{RS}}{\mu} \lambda_{RS,H}(R)$ and $\mu = \mu_L + \mu_{RS}$. We can combine the second line and obtain

$$\begin{aligned}
\mathcal{L} &= \int_{R \geq R_l} u(c(R)) f_H(R) dR \\
&+ \mu \left[\int_{R \geq R_l} u(c(R)) [1 - \bar{\lambda}_H(R)] f_H(R) dR - \frac{\mu_L}{\mu} B Y_0 \right] \\
&+ \lambda \left[A + \int_{R \leq R_l} v(\gamma R Y_0) f_H(R) dR + \int_{R \geq R_l} v(R Y_0 - c(R)) f_H(R) dR - Y_0 \right] \\
&+ \int_{R \geq R_l} \chi(R) [c(R) - b R Y_0] f_H(R) dR
\end{aligned}$$

The derivative in R_l is given by

$$\frac{1}{f_H(R_l)} \frac{\partial \mathcal{L}}{\partial R_l} = -u(c(R_l)) \left[1 + \mu (1 - \bar{\lambda}_H(R_l)) \right] + \lambda \left[v(\gamma R Y_0) - v(R Y_0 - c(R_l)) \right]$$

so that liquidations may be optimal when $1 + \mu (1 - \bar{\lambda}_H(R_l)) < 0$, that is when the average likelihood ratio is high. At low values of R_l , both the risk shifting and shirking problems have high likelihoods, so that $\bar{\lambda}_H$ is large. As a result, bank consumption contributes negatively to welfare. Provided that this negative contribution outweighs the resource cost to investors, we have $R_l > \underline{R}$.

Next, consider the region above R_l . The FOC for consumption $c(R)$ is

$$0 = u'(c(R)) (1 + \mu(1 - \bar{\lambda}_H(R))) - \lambda v'(R Y_0 - c(R)) + \chi(R)$$

so that we have $\chi(R) > 0$ when $1 + \mu(1 - \bar{\lambda}_H(R)) < 0$. As a result, for all values $1 + \mu(1 - \bar{\lambda}_H(R)) < 0$, we either have liquidation or bail-in.

Finally, for $1 + \mu(1 - \bar{\lambda}_H(R)) > 0$, we either have bail-in or an interior consumption value. When consumption is interior, it satisfies a risk sharing rule

$$u'(c(R)) (1 + \mu(1 - \bar{\lambda}_H(R))) = \lambda v'(RY_0 - c(R))$$

giving us an “equity” sharing rule.

Finally, the only role of assuming $1 + \mu(1 - \bar{\lambda}_H(R)) < 0$ is a connected set in the proof is to ensure that there are no points with $1 + \mu(1 - \bar{\lambda}_H(R)) \geq 0$ below R_l .

B.6 Premium for standard debt

In the baseline model, the incentive problem is the only motivation for issuance of standard debt. In practice, standard debt can enjoy a premium relative to all other instruments, meaning it can pay a lower rate of return to investors. There are two natural stories for such a premium. The first is that standard debt takes the form of demand deposits, which enjoy a liquidity premium and require a lower rate of return. The second is that standard debt enjoys preferential tax treatment. We show that contracts still feature standard and bail-in debt, and that the trade-off is largely the same up to the consideration of the return premium. We then discuss potential issues with a pure premium story for standard debt.

Suppose that standard debt has required return $\frac{1}{1+r}$, where $r > 0$. We obtain the following result.

Proposition 19. *Suppose the model is extended to include a premium for standard debt. Optimal contracts combine standard and bail-in debt. The private optimality condition for standard debt is*

$$\mu b \left(\frac{f_L(R_l)}{f_H(R_l)} - 1 \right) = b + \lambda [(1 - b) - \gamma] + r \left[\lambda [(1 - b) - \gamma] - \lambda \frac{1 - F_H(R_l)}{R_l f_H(R_l)} \right].$$

while the optimality condition for bail-in debt is the same as in Proposition 1. The tax on R_l that decentralizes the socially optimal contract is

$$\tau_l = -(1 + r) R_l f_H(R_l) \frac{\partial \gamma(\Omega)}{\partial \Omega} \int_{\underline{R}}^{R_l} R Y_0 f_H(R) dR$$

while the tax on bail-in debt is $\tau_u = 0$.

Relative to the baseline case where $r = 0$, when $r > 0$ we have the term

$$r \left[\lambda [(1 - b) - \gamma] - \lambda \frac{1 - F_H(R_l)}{R_l f_H(R_l)} \right]$$

in the private optimality condition, reflecting an additional cost/benefit trade-off of increasing use of standard debt. This term contains two additional effects of the presence of the liquidity premium. On the one hand, the higher liquidity premium implies that the costs of liquidation go up, because the resources lost would have been repaid to investors who have a high willingness to pay. On the other hand, replacing bail-in debt with standard debt increases payoff to investors with high willingness to pay in non-liquidation states. The bank privately trades off these two forces in choosing the optimal standard debt level, in addition to the incentive forces.

Premium versus Incentive Problems. If $r > 0$, then the bank is willing to issue standard debt even in the absence of an incentive problem, that is if $B = b = 0$ and hence $\mu = 0$. The premium story alone can generate use of standard debt in the bank's capital structure. However, in the absence of the incentive problem the logic of Corollary 3 applies. The bank (without loss of generality) uses equity as its other instrument.⁹⁰ The planner can implement optimal regulation with an equity requirement. By including the incentive problem, our model provides a role for bail-in debt in optimal contracts.

What if instead $B > 0$, $b = 0$, and $r > 0$, so that standard debt has value from a premium perspective, but not from an incentive perspective (relative to bail-in debt). In this case, the optimal contract would combine standard and bail-in debt. However, this story on its own is problematic for two reasons.

The first is that because bail-ins typically apply to long-term debt, which were also non-contingent prior to the crisis, the premium story revolves around premiums on long-term debt, which is likely due to tax incentives. But if the government is subsidizing (non-contingent) long-term debt, this suggests it must provide some fundamental economic benefit. Our model provides a fundamental economic benefit of non-contingent long-term debt.

A second and closely related way to understand this issue is that in the event that $b = 0$, banks have strong incentives to protect themselves against liquidations by backing their non-contingent claims with liquid assets such as treasuries. This relates to a fundamental question in the banking literature: why are illiquid assets paired with fragile (often deposit) financing? Our model endogenously pairs illiquid assets with fragile (non-contingent) financing, rather than exogenously imposing it. Optimal regulation in our model respects the fundamental activity of banks: backing illiquid assets with fragile funding. A model

⁹⁰As a technical aside, of course a bank with no incentive problem and an expected return greater than 1 would, given linear technology, scale up to infinity. This issue is fixed simply by assuming that banks operate a concave technology $Y_0 = f(I_0)$ to produce projects.

that relies exclusively on a standard debt premium naturally lends itself to a “narrow banking” result, where not only the planner but also banks prefer to use safe treasuries to keep the bank from ever failing.

We could nevertheless adopt this view. The main result that would change is the non-optimality of bailouts (Proposition 7), which would no longer generically hold. We would be back into an incomplete markets world, in which bailouts may be desirable to mitigate fire sales, in a standard way. Moreover in the case of deposit insurance, the planner would always prefer to bail out the bank, rather than liquidating and repaying depositors. Bailing out the bank would save resources without distorting bank incentives, and so would be strictly preferred to liquidation.

B.6.1 Proof of Proposition 19

Relative to the baseline model, the only change is that the participation constraint becomes

$$Y_0 - A = \int_{\underline{R}}^{R_l} (1+r)\gamma R Y_0 f_H(R) dR + \int_{R \geq R_l}^{R_u} [(1+r)(1-b)R_l Y_0 + x_1(R)] Y_0 f_H(R) dR$$

where $x_1(R)$ is repayment pledged to other investors. Note that it is immediate that standard debt enjoys priority over other liabilities, since it has the lower required rate of return. The proof that optimal contracts combine standard and bail-in debt follows as in the proof of Proposition 1. As a result, the optimization problem that determines R_l and R_u is the same as before, except that the participation constraint is now

$$Y_0 - A = \int_{\underline{R}}^{R_l} (1+r)\gamma R Y_0 f_H(R) dR + \int_{R_l}^{R_u} [(1+r)(1-b)R_l + (1-b)(R - R_l)] Y_0 f_H(R) dR \\ + \int_{R_u}^{\bar{R}} [(1+r)(1-b)R_l + (1-b)(R_u - R_l)] Y_0 f_H(R) dR$$

This yields the private optimality condition for R_l

$$0 = -bR_l Y_0 f_H(R_l) - \mu b R_l Y_0 \left(1 - \frac{f_L(R_l)}{f_H(R_l)}\right) f_H(R_l) \\ + \lambda [(1+r)\gamma R_l Y_0 f_H(R_l) - (1+r)(1-b)R_l Y_0 f_H(R_l)] + \lambda \int_{R_l}^{\bar{R}} r(1-b) Y_0 f_H(R) dR$$

which rearranges to

$$\mu b \left(\frac{f_L(R_l)}{f_H(R_l)} - 1 \right) = b + \lambda [(1 - b) - \gamma] + r \left[\lambda [(1 - b) - \gamma] - \lambda \frac{1 - F_H(R_l)}{R_l f_H(R_l)} \right]$$

Because R_u is not directly impacted by the liquidity premium, the optimality condition for R_u is as before, assuming that $R_u > R_l$.

The planning problem features a wedge of the same form as before. The only difference is that the wedge is now weighted by $1 + r$, reflecting the higher liquidation losses. In other words, the planning problem is decentralized by the tax

$$\tau_l = -(1 + r)R_l f_H(R_l) \frac{\partial \gamma(\Omega)}{\partial \Omega} \int_{\underline{R}}^{R_l} R Y_0 f_H(R) dR.$$

As before, R_u does not contribute to liquidations, and therefore $\tau_u = 0$.

B.7 Insured Deposits and Bailouts

In addition to fire sale spillovers and moral hazard, another goal of bail-ins is to reduce the costs of protecting insured deposits. We consider the addition of a group of insured deposits, and explore how the planner chooses to protect depositors.

For simplicity, we assume that $\gamma < 1 - b$ does not depend on liquidations (no fire sale spillover). We further allow for the planner to commit ex ante to the desired combination of bailouts and insurance, so that the planner can always tie their hands and commit to no bailouts if desired. As a result, bailouts in this section will only occur if they are ex ante optimal.

The bank is constrained to issue insured deposits as a fixed fraction of its total assets, that is it issues $(1 - b)R_d Y_0$ in insured deposits for some fixed threshold $R_d > \underline{R}$. We abstract away from the socially optimal determination of R_d , instead focusing on how the planner chooses to protect a given set of depositors.⁹¹ The bank is always insolvent if $R < R_d$, absent intervention, regardless of its other liabilities. Because deposits are insured, the planner is liable for any shortfall relative to the face value $(1 - b)R_d Y_0$. Insured deposits are always at the top of the creditor hierarchy in liquidation.⁹²

Because the bank chooses bailouts with commitment, we set the political cost $\kappa = 0$.

⁹¹For example, the planner may use deposit insurance to backstop risk averse depositors.

⁹²In practice, banks may issue wholesale funding which is not insured but runs prior to resolution.

When the planner bails out the bank, the bailout cost is

$$\text{Cost}_{\text{No Liquidation}} = \tau ((1 - b)R_d Y_0 + x(R) - (1 - b)RY_0)$$

where $x(R)$ is any liabilities in excess of $(1 - b)R_d Y_0$ that the planner does not write down. When the planner instead allows the bank to fail, the creditor hierarchy implies the cost to deposit insurance is

$$\text{Cost}_{\text{Liquidation}} = \min\{\tau ((1 - b)R_d Y_0 - \gamma R Y_0), 0\}$$

When $x(R) = 0$, the cost of rescuing the bank with a bailout is lower than the cost of rescuing the bank under liquidation, due to the loss of pledgeable income in liquidation.

The planner solves for the optimal contract, which includes the rescue decision (either via bailout or via liquidation and repayment by insurance).⁹³ We constrain bank consumption to be *monotonic*, that is $c(R)$ must be nondecreasing in R ,⁹⁴ which was satisfied by optimal contracts in the baseline model. This implies that bailouts must be monotonic: if a an insolvent bank R is bailed out, then all insolvent banks $R' \geq R$ must also be bailed out. This rules out the possibility that the planner bails out a bank with $R < R_d$ to protect depositors but liquidates a bank with $R > \frac{1-b}{\gamma} R_d$ for incentive reasons.

Proposition 20. *Suppose that $c(R)$ must be monotonic, there are no fire sales, and there are insured deposits. The socially optimal contract consists of insured deposits R_d , standard debt $R_l \geq R_d$, and bail-in debt $R_u \geq R_l$. The following are true regarding the use of deposit insurance and bailouts.*

1. *If $R_l > R_d$, there is deposit insurance but no bailouts. The bank is liquidated when $R \leq R_l$.*
2. *If $R_l = R_d$, there is a threshold $R_L \leq R_d$ such that the bank is liquidated when $R \leq R_L$ and bailed out when $R_L \leq R \leq R_d$. The indifference condition is for bailouts (when interior) is*

$$b + \tau (1 - b - \gamma) = \mu b \left(\frac{f_L(R_L)}{f_H(R_L)} - 1 \right).$$

Proposition 20 illustrates the trade-off between two mechanisms for protecting insured deposits. Bailing out the bank reduces the taxpayer cost of deposit insurance, but provides

⁹³A technical aside is that it is possible that the planner does not find it optimal to allow the bank to scale up as much as possible due to the cost of insuring deposits. We assume this is not the case, for example if R_d is close to \underline{R} .

⁹⁴If $c(R) > c(R')$ but $R < R'$, the bank could increase its payoff ex post by destroying assets to bring its return down to R . We look for contracts where value destruction is not ex post optimal.

worse incentives for the bank. Whenever the planner allows use of standard debt in excess of insured deposits, that is $R_l > R_d$, then necessarily the planner will commit to rescue depositors but not the bank. In this case, there is deposit insurance but no bailouts.⁹⁵

If $R_l = R_d$ and $R_L < R_d$, the planner uses bailouts ex post in order to reduce the cost of protecting depositors. This may or may not imply that the planner wishes to restrict use of insured deposits ex ante to avoid bailouts, depending on the motivation for deposit insurance. If deposit insurance is a way to provide a backstop to risk-averse depositors that the bank cannot provide itself, or if it is a way to stop sunspot runs, the planner may wish to allow enough insured deposits that it sometimes engages in bailouts.

B.7.1 Proof of Proposition 20

Due to consumption monotonicity, there is a threshold $R_L \geq \underline{R}$ for bank liquidation, with $R_L = \underline{R}$ corresponding to no liquidations. As in the proof of Proposition 7, there are no bailouts above R_d , due to the taxpayer burden. We can thus split the problem into two parts.

First, suppose that the liquidation threshold satisfies $R_L > R_d$, and suppose that the planner finds it optimal to engage in bailouts in a states $R < R_d$. By consumption monotonicity, there are also bailouts for $R_d \leq R \leq R_L$. But then because transfers to regular investors are wasteful, it is optimal to set $R_L = R_d$, as in the proof of Proposition 7. The optimal contract does not feature both $R_L > R_d$ and bailouts.

Consider then the form of the optimal contract when $R_L > R_d$. Because there are no bailouts, the social objective function is

$$\int c_2(R) f_H(R) dR - \int_{R \leq R_L} \tau \max\{(1-b)R_d - \gamma R, 0\} Y_0 f_H(R) dR$$

while the corresponding investor participation constraint is

$$Y_0 - A = \int_{\underline{R}}^{R_L} \max\{(1-b)R_d, \gamma R\} Y_0 f_H(R) dR + \int_{R \geq R_L} ((1-b)R_d + x(R)) f_H(R) dR$$

and where incentive compatibility is the same as in the baseline model. From here, note that the trade-off above R_L is the same as in the baseline model. The model again combines standard and bail-in debt, as in the baseline model.

Consider next the optimal contract when $R_L < R_d$. R_L then also corresponds to the

⁹⁵If bailouts are chosen in a time-inconsistent manner and if $R_d > R^{BO}$, there will have a mixture of bailouts and insurance independent of whether or not it is desirable. The planner will optimally set $R_l = R_d$.

bailout threshold, such that there are bailouts when $R_L \leq R \leq R_d$, and where $R_L = R_d$ corresponds to no bailouts. The resulting social objective function is

$$\int c_2(R) f_H(R) dR - \int_{\underline{R}}^{R_L} \tau [(1-b)R_d - \gamma R] Y_0 f_H(R) dR - \int_{R_L}^{R_d} \tau (1-b) (R_d - R) Y_0 f_H(R) dR$$

while investor repayment is given by

$$Y_0 - A = (1-b)R_d Y_0 + \int_{R_d}^{\bar{R}} x(R) f_H(R) dR$$

reflecting that depositors are always repaid. Finally, incentive compatibility is as in the baseline model. Optimal contracts again combine standard and bail-in debt.

Consider the choice of the liquidation threshold R_L . The trade-off is the same as in the baseline model, expect that an increase in the liquidation threshold leads to a tax burden on taxpayers rather than a cost to investors. That is, the FOC for the liquidation threshold is

$$0 = -bR_L - \mu bR_L \left(1 - \frac{f_L(R_L)}{f_H(R_L)} \right) - \tau \left[(1-b)R_d - \gamma R_L - (1-b)(R_d - R_L) \right]$$

which simplifies to

$$b + \tau (1 - b - \gamma) = \mu b \left(\frac{f_L(R_L)}{f_H(R_L)} - 1 \right).$$

The only change is that the effective costs of liquidations has risen, due to the greater burden on taxpayers ($\tau > \lambda$). If the solution to this equation features $R_L < R_d$, then there are bailouts in states $R_L \leq R \leq R_d$.

B.8 No Rat Race Conditions

In the main paper, we have used equations (13) and (14) to rule out rat race dynamics. These conditions are stronger than is necessary as we discuss in this section. Suppose instead that the market quotes debt prices \bar{q}_1^L and \bar{q}_1^D . We define the equilibrium prices associated with an issuance (D_2, L_3) by

$$q_1^D = \min\{\bar{q}_1^D, \hat{q}_1^D\} \tag{27}$$

$$q_1^L = \min\{\bar{q}_1^L, \hat{q}_1^L\} \tag{28}$$

where \hat{q}_1^D and \hat{q}_1^L are the payoff-neutral prices, given by

$$\hat{q}_1^D = \frac{1}{D_2} \left[\int_{D_2 \geq R_2 Y_1} \gamma R_2 Y_1 f_2(R_2) dR_2 + \int_{D_2 \leq R_2 Y_1} D_2 f_2(R_2) dR_2 \right]$$

$$\hat{q}_1^L = \frac{1}{L_3} \left[\int_{D_2 \leq R_2 Y_1} \min\{R_2 Y_1 - D_2, L_3\} f_2(R_2) dR_2 \right]$$

Under these conditions, equilibrium prices are the maximum between the payoff-neutral prices, and the prices quoted by the market. In other words, the bank internalizes that certain issuances may lower market prices for its debt, but can never increase them.

This distinction is not relevant for the majority of the analysis, and so we present the simpler conditions of equations (13) and (14). The simpler conditions have the benefit that the best equilibrium is unique, whereas it is not unique under the extended conditions. To illustrate why it is not unique, suppose that (\bar{D}_2, \bar{L}_3) are the best equilibrium quantities. Then, there is also a best equilibrium associated with quantities $(0, \bar{D}_2 + \bar{L}_3)$. The prices are $q_1^D = 1$ and

$$q_1^L = \frac{1}{\bar{D}_2 + \bar{L}_3} \left[\int_R \min\{R_2 Y_1, \bar{D}_2 + \bar{L}_3\} f_2(R_2) dR_2 \right] \quad (29)$$

However, this is not an equilibrium under the no rat race conditions (13) and (14), since the bank always prefers to issue $D_2 = \underline{R}Y_1$ to capitalize on the higher price of short-term debt.

Rollover crisis equilibria are not affected by this definition, because given a market price $\bar{q}_1^L = 0$, we have $q_1^L = 0$ regardless of the payoff-neutral price \hat{q}_1^L .

B.9 Rollover Crises and Aggregate Multiplicity

Rollover crises can generate multiple *aggregate* equilibria of the economy for a fixed equilibrium selection rule p for any individual bank, provided that γ is sufficiently sensitive to additional liquidations. This reflects another form of fragility during crisis times, similar to the feedback loop.

Proposition 21. Fix $p \in (0, 1)$ and suppose that $\frac{\partial \gamma}{\partial \Omega} < 0 \forall \Omega \geq 0$. For any aggregate equilibrium γ^* , $\exists c > 0$ such that if at γ^* we have

$$\left| \frac{\partial \gamma}{\partial \Omega} \right| > c,$$

then there also exist at least two additional equilibria: one with lower γ and one with higher γ .

When liquidation values fall rapidly in response to sell-offs, our economy becomes more fragile and subject to multiple equilibrium. Crisis times (where γ may be highly

responsive) are likely characterized by such heightened sensitivity. A bail-in regulatory regime is therefore more likely to contribute to fragility in long-term debt markets during crises. Finally, we note that Proposition 21 is generic: there is *always* a sufficiently high sensitivity that produces multiple aggregate equilibria. The only question is how high that elasticity is.

B.9.1 Proof of Proposition 21

Consider equation (17)

$$\gamma^* = \gamma \left(\int_{\underline{R}}^{R_l} R f_H(R) + p \int_{R_l}^{R^*(\gamma^*)} R f_H(R) dR \right)$$

and define the gap

$$\Delta(\gamma^*) = \gamma^* - \gamma \left(\int_{\underline{R}}^{R_l} R f_H(R) + p \int_{R_l}^{R^*(\gamma^*)} R f_H(R) dR \right)$$

which is the gap between the equilibrium price, and the implied equilibrium price from the liquidation function. Given an inherited contract with $R_l > \underline{R}$, take the range of values $\gamma \in [\underline{\gamma}, \bar{\gamma}]$ that can be obtained (where $\underline{\gamma} = \gamma(E[R])$ and $\bar{\gamma} = \gamma(0)$). Since $R_l > \underline{R}$, we have $\Delta(\bar{\gamma}) > 0$. Since $p < 1$, we have $\Delta(\underline{\gamma}) < 0$.

Every zero of Δ is an equilibrium of the economy at date 1. Suppose that we have an equilibrium at γ^* , that is $\Delta(\gamma^*) = 0$, and suppose that $\Delta'(\gamma^*) < 0$. Then, for sufficiently small $\epsilon > 0$ we have $\Delta(\gamma^* + \epsilon) < 0 < \Delta(\gamma^* - \epsilon)$. Given that $\Delta(\bar{\gamma}) < 0$ and $\Delta(\underline{\gamma}) > 0$, then by continuity we must have two additional zeros, one at a point above γ^* , and one at a point below γ^* . Given these points are zeros of Δ , they are also equilibria.

Finally, we characterize when we have $\Delta'(\gamma^*) < 0$. Differentiating, we obtain the required condition

$$1 < \frac{\partial \gamma}{\partial \Omega} \frac{\partial \Omega}{\partial R^*} \frac{\partial R^*}{\partial \gamma}.$$

We have $\frac{\partial \Omega}{\partial R^*} = p R^* f_H(R^*)$. Differentiating equation (16), we obtain

$$-\frac{D_1}{(R^*)^2 Y_0} \frac{\partial R^*}{\partial \gamma^*} = \int_{\underline{R}}^{d_2^*} R_2 f_2(R_2) dR_2$$

where d_2^* is the solution to the supremum problem in equation (16). Observe that both $\frac{\partial \Omega}{\partial R^*}$ and $\frac{\partial R^*}{\partial \gamma^*}$ depend on γ^* , but not on $\frac{\partial \gamma}{\partial \Omega}$. Note that both of these depend on γ , but not on $\frac{\partial \gamma}{\partial \Omega}$.

As a result, we have multiple aggregate equilibria if at γ^* we have

$$\left| \frac{\partial \gamma}{\partial \Omega} \right| > c$$

where c is given by

$$\frac{1}{c} = \left| \frac{\partial \Omega}{\partial R^*} \frac{\partial R^*}{\partial \gamma^*} \right| = pR^* f_H(R^*) \frac{(R^*)^2 \gamma_0}{D_1} \int_{\underline{R}}^{d_2^*} R_2 f_2(R_2) dR_2$$

concluding the proof.