

# Favoring the Winner or Loser in Repeated Contests

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Should a firm favor a weaker or stronger employee in a contest? Despite a widespread emphasis on rewarding the best employees, managers continue to tolerate and even favor poor performers. Contest theory reveals that evenly matched contests are the most intense, which implies that a contest designer can maximize each player's effort by artificially boosting the underdog's chances. We apply this type of "handicapping" to a two-period repeated contest between employees, in which the only information available about their abilities is their performance in the first period. In this setting, employees are strategic and forward looking, such that they fully anticipate the potential impact of the first-period contest result on the second-period contest and thus adjust their behaviors accordingly. The manager also incorporates these strategic behaviors of employees when determining an optimal handicapping policy. If employees' abilities are sufficiently different, favoring the first-period loser in the second period increases the total effort over both periods. However, if abilities are sufficiently similar, we find the opposite result occurs: total effort increases the most in response to a handicapping strategy of favoring the first-period winner.

*Key words:* game theory; contests; handicap; incentives; ratcheting; moral hazard

*History:* Received: February 10, 2012; accepted: May 14, 2013; Preyas Desai served as the editor-in-chief and Ganesh Iyer served as associate editor for this article. Published online in *Articles in Advance*.

## 1. Introduction

Given their limited resources, firms often face a tough dilemma on how best to motivate their employees: do they invest in laggards or reward their top performers? Still, this fundamental management question has not gained a consensus answer. Many experts recommend that managers should reward their top employees and avoid coddling weaker employees with lower standards. For example, top sales agents often receive more training and obtain more back-office resources (Farrell and Hakstian 2001). Indeed, \$1.3 billion in training expenses are "devoted to grooming leaders" (Kranz 2007, p. 22) and managers are often told to look for top performers to receive specialized learning opportunities (Kranz 2007). However, more than 60% of employees surveyed indicated that their managers tolerate poor performers implying that top performers were not being recognized (Marchetti 2007). Furthermore, nearly two-thirds of managers claim they spend a majority of their time dealing with and helping poor performers (Chang 2003, 2004). Such results may be strategic in the sense that favoring a weak performer might improve his chances of success, encouraging him to work harder. Not only would this improve performance by raising the skill level of employees, but it would also increase the general competition among employees by improving the performance of weak employees (Farrell and

Hakstien 2001, Chang 2004). It is therefore unclear what the best strategy is for the manager.

To address the issue of whether a firm should favor weaker or stronger employees, we adopt a contest theory approach. Managers often reward employees based on overall evaluations over a certain period of time, rather than on a narrowly defined sales contest (which is a short-term temporary monetary incentive program for salespeople). In this study, we refer to this entire evaluation time period as the contest. As such, we broadly define a *contest* as any competition between employees, including competition for limited support, resources, or promotion to higher ranks. We refer to several reward systems firms use to motivate and encourage employees to expend their efforts, such as monetary bonuses, promotions, or more subtle forms of various privileges, such as the contest prize.

Contests typically involve two or more employees competing for a single prize, and the employee who performs best usually wins (i.e., winner-take-all contest). By adding this winner-take-all component, firms can induce a significant increase in effort. However, the effect is unclear when a manager faces heterogeneous employees who differ in their abilities. Weaker employees often recognize their small chance of winning and thus have little motivation to increase their effort (Hart et al. 1989, Murphy et al. 2004, Corsun

et al. 2006). Consequently, stronger employees, anticipating less competition, also may respond with limited changes to their behavior or even lower effort. In this case, the contest fails to properly motivate the employees or meet the goals of the manager.

Previous research in economics and marketing suggests guidelines for contests. For example, contest theory suggests leveling the playing field by granting an advantage (which is also called “handicapping” in the contest literature) to a weaker employee, which may increase overall effort (Lazear and Rosen 1981, Baye et al. 1993, Baik 1994, Liu et al. 2007). The advantage increases the weaker employee’s chances of success, similar to giving a weaker golfer a “handicap,” which should make the contest more intense.<sup>1</sup> Therefore, helping the weaker employee can make all employees compete harder to win the contest.

However, in practice, some contests actually favor the stronger employees. For example, successful sales agents often receive more lucrative territories or product lines (Skiera and Albers 1998), are assigned less administrative responsibilities, obtain more back-office resources, or have more training (Farrell and Hakstian 2001, Krishnamoorthy et al. 2005). Similar examples exist in other fields. Successful researchers tend to have more grant opportunities (Che and Gale 2003), winners of the regular season receive home field advantage in a sport’s postseason, and winners of early speed trials gain the most favorable position in car races (Mastromarco and Runkel 2009). Similarly, by winning previous contests, well-established firms and incumbent politicians often enjoy easier runs in subsequent contests. In most cases, such favoring of the winner over the loser seems to contradict the principle of maximizing effort by leveling the playing field.

We note that these existing models (for example, Lazear and Rosen 1981, Baye et al. 1993, Baik 1994) assume that the contest designer can perfectly distinguish between high- and low-ability employees. However, most managers in reality face uncertainty about their employees’ abilities in dynamic settings, whereby a prior round of the contest is both a source of profit for the firm and the source of information about the relative strength of the employees. Therefore, managers can base their assessment, in part, on employees’ past performances in previous periods. Past performance clearly offers a strong, albeit noisy, signal of ability.

In this article, we consider a two-period repeated contest with uncertainty where the contest designer (i.e., manager) faces heterogeneous employees but

does not know the exact abilities of each individual employee. The manager only receives a noisy signal of employees’ abilities through first-period contest results. In the second-period contest, the manager can assign a handicap to favor the winner or loser of the previous contest. However, in a dynamic setting where the two contests are linked through the handicapping policy, such a policy might create a new incentive problem. For example, if employees anticipate that winning a current contest will hurt them in the future, they have less incentive to win the first contest.<sup>2</sup> This “ratcheting,” by which employees modify their efforts in the current period to alter incentives in future periods, is a common problem for workforce management (Weitzman 1980, Freixas et al. 1985, Chung et al. 2011, Misra and Nair 2011).

Favoring the loser of the first contest thus creates an incentive problem of ratcheting in which both players reduce their attempts to win in the first period so they can take advantage of second-period handicapping. On the other hand, favoring the winner of the first contest would also create another type of incentive problem of “moral hazard,” because the winner no longer has to work as hard in the second contest. The manager should weigh the trade-offs of these two different policies (favoring the winner versus favoring the loser) to maximize the employees’ total effort.

Here, we adopt a Tullock (1980) contest model, which uses the standard ratio-form contest success function. The model clearly demonstrates that employees’ incentives change over time with different handicapping policies (i.e., favoring weaker versus stronger employees) and fully considers the strategic effects of the employees (i.e., ratcheting versus moral hazard). In particular, if abilities are sufficiently similar, surprisingly, favoring the first-period winner in the second period increases the total effort over both periods—rewarding the top performer is optimal. However, if abilities are sufficiently different, the opposite result holds, and total effort is maximized by adopting a handicapping policy that favors the first-period loser—investing in laggards is optimal. As such, the model suggests a clear handicapping policy guideline for the manager when faced with a heterogeneous workforce with uncertain abilities in a dynamic contest environment. We use this contest model and its handicapping policy as an analogy to understand the firm’s fundamental dilemma of when a firm should favor weaker or stronger employees to motivate all employees.

More specifically, we first find that when employees are sufficiently different in their abilities, the standard

<sup>1</sup> We use the paradoxical terminology of handicapping, as popularized by its usage in association with golf, which implies that a higher handicap helps a player.

<sup>2</sup> For example, Taylor and Trogon (2002) show that low-ranking National Basketball Association teams lose more games to ensure they get a better draft pick for the next season. See also Stein and Rapoport (2005) and Yildirim (2005).

handicapping policy of favoring the loser of the first period is optimal. Even though rewarding the loser in the second period decreases the total first-period effort because of the ratcheting effect, the employees' strategic considerations for the second-period handicap overcome this effort loss in the first period. This is because rewarding the loser in the second period reduces the stronger employee's first-period incentives more than it reduces the weak employees's first-period incentives. In this way, the first-period contest becomes more equitable through the handicapping policy of favoring the loser. This mitigates the negative impact on total first-period effort for very large ability differences.

On the other hand, when employees' abilities are sufficiently similar, we find that favoring the winner of the first-period contest is optimal. With little difference in their abilities, the extent of advantage the winner of the first contest receives is also small. Accordingly, the loser of the first-period contest still has a fair chance to win the second-period contest even with a small disadvantage and thus experiences sufficient incentive to exert effort. Hence, the increase in effort in the first period as a result of the future rewards for winning outweighs the effort loss in the second period when employees' abilities are sufficiently similar.

The rest of this article is organized as follows. Section 2 describes the related literature. Section 3 presents our main model of dynamic contest and its analysis. We provide extensions to our model in §4, and we conclude in §5.

## 2. Literature Review

The theoretical foundations of contests stem from the economics literature. Since Tullock (1980) published his seminal contest model in which players compete for a single prize through the expenditure of their resources (sometimes called a Tullock or ratio-form contest), several models have been proposed for different types of contests (Lazear and Rosen 1981, Moldovanu and Sela 2001). In particular, we follow Meyer (1991, 1992) to determine how the uncertainty about employee types affects a manager's handicapping policy. Meyer (1991, 1992) analyzes a multiperiod contest using a Lazear–Rosen (Lazear and Rosen 1981) difference-form contest, in which success is a function of the difference in (ability-adjusted) effort levels. The Lazear–Rosen model differs from most models in the contest literature in that (1) it does not follow the standard Tullock contest form (which uses the ratio-form success function) and (2) it does not use a linear cost function for effort. In our model, we instead adopt the standard Tullock contest model and attain pure-strategy equilibria where both sides

exert effort. This is not possible in difference-form asymmetric contests with nonlinear costs (Hirshleifer 1989, Baik 2004).

In this sense, our study extends the robustness of Meyer's (1992) results in the symmetric case to a Tullock (1980) ratio-form contest and uncovers new results in the asymmetric case. Meyer (1992) shows that in a promotion setting, giving an advantage to the winner increases overall effort (or, equivalently, minimizes the cost of prizes to induce the same effort). This result is limited, however, to only the case of symmetric players, and we extend it to investigate the incentive problem in a two-period repeated contest between two asymmetric employees (i.e., individuals with different abilities) who strategically choose their efforts in response to the handicapping policy. Meyer (1991) analyzes the asymmetric players under uncertainty, but the incentive effect of handicapping, which is the main focus of this paper, is ignored because employees do not make any strategic decisions in Meyer's (1991) model. In contrast, we focus on the incentive's costs and benefits from the different handicapping policy under uncertainty with asymmetric players.

There is another stream of literature in contest theory that considers a repeated contest with asymmetric information between players about their abilities or valuations. The players update their beliefs about the abilities or valuations of their rivals based on performance in earlier periods. Münster (2009) allows for two types of valuations, and players can strategically choose efforts in the first-period contest to affect beliefs in the second-period contest. Hörner and Sahuguet (2007) analyze a similar setup in a repeated auction setting. Similarly, Krämer (2007) models an infinitely repeated contest where players are unsure about their relative abilities but learn about the state over time. He finds that in some cases, the least able agent persistently wins. However, this literature differs from our paper as we consider the information asymmetry between the firm and employees, and we focus on the firm's optimal handicapping policy (whether to reward the loser or winner). These papers do not consider this issue.

The model of contests (particularly, the use of handicapping) has been extensively applied in various contexts outside of sales contests. For example, Tsoulouhas et al. (2007) use a contest model to study the issue of employee promotion selection and show when it is optimal to handicap insiders or outsiders for chief executive officer selection. Horsky et al. (2012) investigate the advertising agency selection problem using a contest model, and Harbaugh and Ridlon (2010) apply the contest model to the all-pay auction setting. In particular, Harbaugh and Ridlon (2010) have a theme and structure similar to the current model in that both investigate the issue

of optimal handicapping policy between asymmetric players in a two-period contest setting. However, unlike Harbaugh and Ridlon (2010), who only find support for favoring the loser for all ability differences (i.e., the handicapping policy of rewarding the loser always maximizes total bids in the all-pay auction setting), our model clearly identifies the conditions in which favoring the winner could be optimal. Furthermore, the bid equilibrium in Harbaugh and Ridlon (2010) can only be found in mixed strategies, which is not realistic in most contest settings. Our model overcomes this inconsistency of a mixed-strategy equilibrium and provides richer results as they relate to competition between asymmetric employees.<sup>3</sup>

Also, the use of handicapping, or bias, broadly applies to procurement auction settings (Laffont and Tirole 1988, 1991; Celentani and Ganuza 2002; Burguet and Che 2004). In multiattribute auctions where contracts are awarded based on multiple attributes such as price or quality, the auction designer uses a scoring function to compare bids and the bid with the highest score wins (Engelbrecht-Wiggans et al. 2007). In this setting, the auction designer may bias her subjective evaluation of attributes or distort the relative weights of the various attributes to favor a specific bidder. Laffont and Tirole (1991) and Celentani and Ganuza (2002) investigate the issue of favoritism in this multiattribute auction settings. Also, Laffont and Tirole (1988) study whether to favor the incumbent in a regulated market and find that the auction should be biased in favor of the new entrant. On the other hand, Burguet and Che (2004) examine the optimal scoring system in multiattribute procurement auctions and find that in the presence of corruption, handicapping the efficient firm is not optimal and exacerbates the inefficient allocation as a result of bribery. The optimal scoring rule for the buyer may even favor the efficient firm. Our paper can contribute to this scoring auction literature by identifying the conditions when it is optimal to favor the weaker or stronger bidder in a dynamic setting.

Several contests have also been examined with company objectives other than effort maximization, such as to boost employee morale (Murphy et al. 2004), increase sales (Brown and Peterson 1994), improve customer satisfaction metrics (Hauser

et al. 1994), increase accuracy in employee promotion selection (Rosen 1986, Meyer 1991, Tsoulouhas et al. 2007, Ryvkin and Ortmann 2008), or identify the most qualified bidder in the auction setting (Burguet and Che 2004, Hubbard and Paarsch 2009).

In marketing, most theoretical research focuses on the issue of optimal contest design. Kalra and Shi (2001) is among the first papers to have examined sales contests from a game-theoretical perspective. They identify specific conditions in which the optimal contest design structure should include multiple prizes at varying levels to induce greater effort by all salespeople. Krishna and Morgan (1998) further show that winner-take-all contests are optimal when contestants are risk neutral. Lim (2010) uses a behavioral economics model to demonstrate that a contest with a higher proportion of winners than losers can yield greater effort than one with fewer winners under certain conditions. Finally, using laboratory and field experiments, Chen et al. (2011) and Lim et al. (2009) show empirically that the prize structure of a sales contest indeed affects the effort of contestants.

In contrast, we do not address the issue of optimal structure of contests or prize amount. Instead, our focus is on finding the optimal handicapping policy (whether to reward the loser or winner) taking the common winner-take-all structures (Krishna and Morgan 1998) and prizes as given. This will give us insight for answering management's fundamental dilemma as to whether to favor the stronger or weaker employees.<sup>4</sup>

### 3. Model

We consider a model where two employees,  $A$  and  $B$ , are competing in a contest setting. These employees are heterogeneous in their abilities, denoted by the parameters  $a_A > 0$  and  $a_B > 0$ . Without loss of generality, we simply denote one with relatively low ability as  $A$  and the other with relatively high ability as  $B$ . Therefore, employee  $B$  is stronger and more effective than the other player, employee  $A$ , in ability;  $a_A \leq a_B$ . They compete to win a single prize in a contest by exerting effort that increases their probability of winning. Note that this classification is a *relative* term for comparison between two employees. Both of them could have very low or very high abilities. We also allow the possibility that two employees have the same ability level, which is captured by  $a_A = a_B$ .

Let  $e_A$  and  $e_B$  represent the efforts exerted by employee  $A$  and employee  $B$ , respectively. We assume that efforts are unobservable by the manager, who only identifies a winner of the contest. The ability parameters directly influence the effectiveness of

<sup>3</sup>Harbaugh and Ridlon (2010) examine the auction setting where the expected payoff to the weaker player is zero, and therefore, there is no strategic effect of the handicapping policy on the weaker player in the first period (since his payoff cannot be lower than zero). Hence, their model does not fully incorporate the tension between two different incentives problems (ratcheting versus moral hazard) created by different handicapping policies, which is the focus this current paper. Thus, their model cannot find the pure-strategy equilibrium or the conditions under which favoring the winner can be optimal.

<sup>4</sup>Shin and Sudhir (2010) study another management dilemma of whether a firm should reward its own customers or new customers and, thus, has similar flavor with the current study.

converting effort into performance. Thus, higher ability leads to higher performance, all else being equal. The probability of  $A$  winning the contest is his effort relative to the total effort of both  $A$  and  $B$ . We specify employee  $A$ 's contest success probability ( $s_A$ ) by following the standard Tullock (1980) ratio-form contest success function:<sup>5</sup>

$$s_A = \frac{a_A e_A}{a_A e_A + a_B e_B}, \quad (1)$$

where  $\partial s_A / \partial e_A > 0$  and  $\partial^2 s_A / \partial e_A^2 < 0$  for all  $e_A$ , such that the extra effort exerted by player  $A$  increases his probability of winning at a decreasing rate. Furthermore, the probability of winning decreases with his rival's effort whereas the ability parameter increases the probability of success as it becomes greater. In other words, the stronger employee  $A$  becomes, the more likely he is to win the prize.

We define the relative ability as  $\sigma = a_A / a_B$ ; here,  $0 \leq \sigma \leq 1$  since  $a_A \leq a_B$ . Although employees know each other's ability, the manager only receives information on relative ability and does not know the exact ability of each employee. This is because employees are spending more time with each other than they are with the manager. For example, coworkers are typically engaged in the same or similar tasks, and thus, they have naturally more interactions and know more about each other.<sup>6</sup> Therefore, we assume that employees have a greater knowledge about their coworkers than the manager does through their personal interactions (Marchetti 2004).

Given this basic setup, we first look at the case of a one-period static contest before we analyze our main model of two-period repeated contests. This benchmark model serves as a baseline for our main model analysis.

### 3.1. Benchmark: One-Period Static Case

In this benchmark case, we consider a simple one-period contest where we summarize the standard results in the contest literature.

<sup>5</sup> A number of studies have provided a microfoundation for the Tullock ratio-form specification from an axiomatic approach (Skaperdas 1996, Clark and Riis 1998), which is the main reason why Tullock's ratio-form model has become a standard in contest theory literature.

<sup>6</sup> This is consistent with sociology literature on social network and community ties (Wellman et al. 1988, Podolny and Baron 1997) that supports that similarity breeds more connections (McPherson et al. 2001). Because people generally only have significant contacts with others similar to themselves in sociodemographics such as social status or rank (Campbell et al. 1986, McPherson et al. 2001), coworkers in a similar level in the workplace hierarchy are more likely to interact with each other and thus "know more about" each other (Hampton and Wellman 2000).

Two employees,  $A$  and  $B$ , are competing for a privately valued prize, denoted as  $v_A$  and  $v_B$ ,<sup>7</sup> respectively, and have unit costs in effort. Following the standard Tullock (1980) ratio-form contest success function as stated above, employee  $A$ 's utility function is written as

$$\begin{aligned} U_A &= s_A \cdot v_A - e_A = \frac{a_A e_A}{a_A e_A + a_B e_B} v_A - e_A \\ &= \frac{\sigma e_A}{\sigma e_A + e_B} v_A - e_A. \end{aligned} \quad (2)$$

In a similar way, employee  $B$ 's utility function is  $U_B = (1 - s_A) \cdot v_B - e_B$ . From the first-order condition,<sup>8</sup> it is standard to find that the equilibrium levels of efforts for employees  $A$  and  $B$  are, respectively,

$$e_A^* = \frac{\sigma v_B}{(v_B / v_A + \sigma)^2} \quad \text{and} \quad e_B^* = \frac{\sigma v_A}{(\sigma v_A / v_B + 1)^2}, \quad (3)$$

where  $\partial e_i / \partial v_i > 0$  and  $\partial e_i / \partial \sigma > 0$  for all  $i \in \{A, B\}$ , such that equilibrium efforts increase in their own valuation of the prize ( $v_i$ ) and the relative ability  $\sigma$ .

We can immediately see that the ratio of efforts  $e_A / e_B$  is equal to the ratio of valuations  $v_A / v_B$ . In particular, when valuations are identical ( $v_A = v_B$ ), the equilibrium levels of efforts are identical, regardless of ability differences (i.e.,  $e_A^* = e_B^*$ ). At the same time, equilibrium effort levels depend on ability differences and increase as ability differences become smaller (i.e.,  $\sigma$  becomes greater). This captures the intuition that an evenly matched contest is most intense. These are standard Tullock contest results found in the literature (e.g., Baik 1994, Nti 1999, Rosen 1986).

We assume that the firm (manager) can treat employees differently by favoring or granting an advantage to a particular employee, which affects the outcome of the contest. We call this artificial bias a "handicapping policy." This handicapping policy can take several forms in practice: for example, employees can be assigned to different environments or product lines (Skiera and Albers 1998) or given different amounts of training or back-office resources (Meyer 1992, Krishnamoorthy et al. 2005).

Let  $h$  be the handicapping policy. The handicap,  $h \geq 0$ , has a multiplicative effect on the ability parameter  $\sigma = a_A / a_B$  such that the relative ability now becomes  $h\sigma$ . When  $h > 1$ , the firm favors the weaker employee, and handicapping reduces asymmetry.

<sup>7</sup> Throughout the paper, we maintain the assumption that  $v_A = v_B = v$ . However, as we show in our main model, the handicapping causes the implicit valuations in the first period to vary for different employees. Therefore, the results from this asymmetric valuation case are key to understanding the mechanism underlying our main results.

<sup>8</sup> The first-order conditions for employees  $A$  and  $B$  are  $\sigma e_B v_A / (\sigma e_A + e_B)^2 = 1$  and  $\sigma e_A v_B / (\sigma e_A + e_B)^2 = 1$ , respectively. The second-order sufficiency condition for  $A$  (and  $B$ ),  $\partial^2 U_A / \partial e_A^2 = -(2e_B \sigma^2 / (e_B + \sigma e_A)^3) v_A < 0$ , also holds.

When  $h < 1$ , the firm favors the stronger employee, and thus, handicapping amplifies the asymmetry. For simplicity, we assume that heterogeneity only appears in ability, and valuations are set to  $v_A = v_B = v$ .<sup>9</sup>

Recall that equilibrium effort levels are the same when valuations are identical regardless of ability asymmetry. From (3), we express total effort as

$$e^{\text{Total}} = e_A + e_B = 2 \frac{\sigma h}{(\sigma h + 1)^2} v. \quad (4)$$

The objective of the manager in our setting is to maximize the total employee efforts  $e^{\text{Total}}$ .<sup>10</sup> Because any asymmetry between players reduces total contest effort, the manager has an incentive to make the contest more equitable by giving an advantage to the weaker player. When the manager knows the abilities of the employees (perfect information case), the optimal policy is to favor the weaker employee by the exact reciprocal amount of the ability difference,  $h^* = 1/\sigma$ , equalizing the two employees' abilities. Not surprisingly, this maximizes total effort such that it is equal to total effort when employees are symmetric. This is a well-known result from Tullock (1980).

However, in practice, the manager does not necessarily have perfect information about the identities of the stronger and weaker employees. The manager may have a good sense of his employee pool (hence, knows the distribution of abilities) but does not know the exact location of each individual on the distribution.<sup>11</sup> For example, in hiring new employees, the firm has expectations of performance from stronger and weaker performers given its employee pool (thus, their ability difference  $\sigma$ ) but cannot determine the identity of the weaker or stronger employee. In this case, closing the ability gap by favoring one of the players might reduce total effort if it is erroneously applied to the stronger player, further increasing the ability disparity.

In practice though, managers receive multiple noisy signals about employees' abilities, such as their absenteeism, project completion, and prior performance evaluations or previous contest results. The manager

<sup>9</sup> Employee heterogeneity can be expressed in terms of cost of effort, prize valuation, and ability. We assume these costs and valuations are constant between players and allow only for the ability to differ. See Baye et al. (1996) for a discussion of the equivalence of contests with asymmetric costs, valuations, and abilities.

<sup>10</sup> Although we leave the firm's objective in this reduced form for now, we later formally construct a micromodel of this reduced form in which we show that the manager only needs to find the optimal  $h$  that maximizes the total employee efforts.

<sup>11</sup> For example, a company with very lucrative job positions such as consulting or investment banking usually attracts many good talents and thus has a low variance and high  $\sigma$ , whereas many start-up companies or low-profile manufacturing companies have a pool of employees with higher variance and lower  $\sigma$ .

receives such a noisy signal  $\gamma$  about employees' relative abilities. This signal is incorrect with probability  $p$  and correct with probability  $(1 - p)$ . In other words, with probability  $p$ , employee  $A$  ( $B$ ) is incorrectly identified as stronger (weaker), and with probability  $(1 - p)$ , employee  $A$  ( $B$ ) is correctly identified as the weaker (stronger) employee.

**PROPOSITION 1.** *When a signal  $\gamma$  is sufficiently informative (i.e.,  $p$  is sufficiently small), a handicapping policy that favors the perceived weaker employee ( $h^* > 1$ ) is optimal for all ability differences ( $\sigma < 1$ ). Moreover, this optimal handicapping policy is strictly smaller than  $1/\sigma$  ( $h^* < 1/\sigma$ ).*

**PROOF.** See the appendix.  $\square$

In a static contest, the optimal handicapping policy under uncertainty is to favor the perceived weaker employee. However, this handicapping policy does not fully compensate the ability difference resulting from the uncertainty. This is different from the perfect information case, where the handicapping policy precisely equalizes the two employees' abilities by fully compensating the exact amount of ability difference ( $1/\sigma$ ).<sup>12</sup>

### 3.2. Dynamic Model

In this section,<sup>13</sup> we extend the static benchmark model to the case of a two-period repeated game and explicitly model the source of noisy information  $\gamma$  as the result of a first-period contest. That is, in this two-period model,  $\gamma$  is realized as a win or lose (i.e., the endogenous outcome) of the first-period contest between strategic, forward-looking employees.

Suppose now that the manager observes a two-period repeated contest of prize  $v$  for each period, such that the value of the prize does not vary between periods;  $v_1 = v_2 = v$ . By keeping the values of each contest the same, we can focus on the direct effect of  $h$  on the equilibrium outcome. Prior to the first-period contest, the manager commits to a handicapping policy  $h$ , which is observable to the employees.

<sup>12</sup> This result is akin to that of Lazear and Rosen (1981) in a difference-form contest. As they also show that the optimal handicapping does not necessarily result in a fair game (i.e., equalizing the ability difference), and the actual handicap can be less than  $1/\sigma$ . Our result replicates their finding under uncertainty in a Tullock contest. Also, the closest model to our single-period setting is that of Dahm and Porteiro (2008), who consider the effect of a noisy signal on the efforts of agents in a Tullock contest model. However, they do not consider the optimal handicapping policy, which is the focus of this paper. In their setting, the firm does not strategically set the optimal handicapping to induce higher effort; it just incorporates this additional noisy information when it selects a better agent.

<sup>13</sup> It is important to note that the dynamic model in this research studies a two-period repeated contest, not a contest that lasts over two periods. In other words, we study the dynamic relation of two contests, but not a dynamic contest.

In practice, firms explicitly proclaim the way a winner is selected with well-defined criteria as well as a publicized benefit for the winner or the loser in the future. In the first period, the employees compete in a contest, and the manager gains information about the employees' relative abilities by observing who wins or loses. This signal is still *noisy* because the success function, or winning probability, of the first-period contest is stochastic (see Equation (1)); more importantly, employees may strategically adjust their behaviors. Using the information about who wins the contest in the first period, the manager assesses, with some probability, who is weaker or stronger and assigns the predetermined  $h$  for the second-period contest. Hence, the manager treats the result of the first-period contest as the signal about employees' relative abilities.

Here, employees are strategic and forward looking, such that they fully anticipate the potential impact of the first-period contest result on the second-period contest, and thus they adjust their behaviors accordingly. The manager also considers the employees' strategic behaviors when determining an optimal handicapping policy.

**3.2.1. Second Period.** We start by considering the second period. Instead of receiving an exogenous signal  $\gamma$  with uncertainty  $p$ , the manager receives the endogenous signal from the first-period contest as a win or loss, with probability equal to the first-period success function. As we see from the benchmark, the second-period equilibrium efforts of both employees are identical since  $v_A = v_B = v$ . However, given the handicapping policy  $h$ , the exact level of effort depends on which employee won the first-period contest.

The impact of handicapping policy  $h$  on the ability gap is either  $\sigma h$  if the handicapping policy of favoring the loser is correctly applied to the weaker employee or  $\sigma/h$  if it is incorrectly applied to the stronger employee. For example, when  $\sigma = a_A/a_B = \frac{1}{2}$ , employee  $A$  must exert twice as much effort as employee  $B$  to equal his chance of winning the contest without any handicapping policy. If a handicapping policy of  $h = 1/\sigma = 2$  is correctly applied to the weaker employee  $A$  (i.e.,  $h\sigma = ha_A/a_B = 1$ ),  $A$  only has to equal  $B$ 's effort to achieve an equal chance of winning. However,  $A$  would have to exert four times as much effort as employee  $B$  if the handicap were incorrectly applied to  $B$  (i.e.,  $\sigma/h = a_A/ha_B = \frac{1}{4}$ ).

Hence, if the weaker employee  $A$  wins the contest in the first period, he is believed (incorrectly) to be the stronger employee, and the handicapping policy of favoring the loser is erroneously applied to the truly

stronger employee. In this case, we can find the equilibrium second-period efforts from Equation (3). The equilibrium effort in the second period will be

$$e^f = e_A^f = e_B^f = \frac{\sigma h}{(\sigma + h)^2} v. \quad (5)$$

Again, both employees exert the same amount of effort  $e^f$  in equilibrium since the valuation for the contest is the same (i.e.,  $v_A = v_B = v$ ). The superscript  $f$  represents their "false" identity.

But if employee  $A$  loses the contest in the first period, he will be correctly identified as the weaker employee and receive a handicap of  $h$ . Equilibrium levels of effort for both employees in the second period are hence

$$e^t = e_A^t = e_B^t = \frac{\sigma h}{(1 + \sigma h)^2} v, \quad (6)$$

where the superscript  $t$  represents their "true" identity.

To calculate the second-period success function, we insert these equilibrium efforts into Equation (1). Thus, the second-period success functions for employee  $A$  are conditional on the outcome of the first-period contest. If  $A$  loses ( $L$ ) in the first period (and thus, equilibrium effort is  $e^t = (\sigma h/(1 + \sigma h)^2)v$ ), the contest success function for employee  $A$  is

$$\begin{aligned} s_{A|L} &= \Pr(A \text{ wins} \mid A \text{ loses first-period contest}) \\ &= \frac{\sigma h}{1 + \sigma h}. \end{aligned} \quad (7)$$

If he wins ( $W$ ) in the first period (and thus, equilibrium effort is  $e^f = (\sigma h/(\sigma + h)^2)v$ ), the contest success function for employee  $A$  is

$$\begin{aligned} s_{A|W} &= \Pr(A \text{ wins} \mid A \text{ wins first-period contest}) \\ &= \frac{\sigma}{\sigma + h}. \end{aligned} \quad (8)$$

Employee  $A$ 's and employee  $B$ 's second-period expected utilities are, respectively,

$$\begin{aligned} E(U_{A,2}) &= s_A \cdot [(s_{A|W})v - e_2^f] + (1 - s_A) \\ &\quad \cdot [(s_{A|L})v - e_2^t] \\ \text{and } E(U_{B,2}) &= (1 - s_A) \cdot [(1 - s_{A|W})v - e_2^f] \\ &\quad + s_A \cdot [(1 - s_{A|L})v - e_2^t], \end{aligned} \quad (9)$$

where  $s_A$  is the probability that employee  $A$  wins in the first-period contest ( $s_A = \sigma e_A/(\sigma e_A + e_B)$  from Equation (1)).

**3.2.2. First Period.** Now, we turn to the first-period utility to understand the employees' optimal effort decisions. Since employees are strategic and forward looking, they incorporate the fact that their

first-period efforts affect not only their current first-period utility but also their future second-period utility. Thus, employee  $A$  chooses an effort level in the first period that maximizes his total expected utility:

$$E(U_A) = s_A v - e_A + \delta \{ s_A \cdot [(s_{A|W})v - e_2^f] + (1 - s_A)[(s_{A|L})v - e_2^t] \}, \quad (10)$$

where  $\delta$  is the discount factor, which we normalize to 1 for simplicity. The first-order condition is<sup>14</sup>

$$\begin{aligned} \frac{\partial s_A}{\partial e_A} v + \frac{\partial s_A}{\partial e_A} [(s_{A|W})v - e_2^f] - \frac{\partial s_A}{\partial e_A} [(s_{A|L})v - e_2^t] &= 1 \\ \Leftrightarrow \frac{\partial s_A}{\partial e_A} \omega_A = 1 &\Leftrightarrow \frac{\sigma e_B}{(\sigma e_A + e_B)^2} \omega_A = 1, \end{aligned} \quad (11)$$

where

$$\begin{aligned} \omega_A &= v + \{ [(s_{A|W})v - e_2^f] - [(s_{A|L})v - e_2^t] \} \\ &= v + v \left[ \left( \frac{\sigma}{\sigma + h} \right)^2 - \left( \frac{\sigma h}{1 + \sigma h} \right)^2 \right]. \end{aligned}$$

Here,  $\omega_A$  captures employee  $A$ 's implicit value of winning the first-period contest. The first term ( $v$ ) represents the utility from the first-period contest, and the second  $\{ [(s_{A|W})v - e_2^f] \}$  and third  $\{ [(s_{A|L})v - e_2^t] \}$  terms represent the extra future value (or cost) of winning the first-period contest, depending on  $\sigma$  and  $h$ .

Similarly, the first-order condition for maximizing employee  $B$ 's total utility function is

$$\frac{\partial(1 - s_A)}{\partial e_B} \omega_B = 1 \Leftrightarrow \frac{\sigma e_A}{(\sigma e_A + e_B)^2} \omega_B = 1, \quad (12)$$

where

$$\begin{aligned} \omega_B &= v + \{ [(1 - s_{A|L})v - e_2^t] - [(1 - s_{A|W})v - e_2^f] \} \\ &= v + v \left[ \left( \frac{1}{1 + \sigma h} \right)^2 - \left( \frac{h}{\sigma + h} \right)^2 \right]. \end{aligned}$$

Here,  $\omega_B$  represents employee  $B$ 's implicit value of winning the first-period contest.

From Equations (11) and (12), we find that first-period equilibrium efforts are

$$\begin{aligned} e_A^* &= \frac{\sigma \omega_B}{(\omega_B / \omega_A + \sigma)^2} \quad \text{and} \\ e_B^* &= \frac{\sigma \omega_A}{(\sigma \omega_A / \omega_B + 1)^2}. \end{aligned} \quad (13)$$

As a result, the probability of employee  $A$  winning the first-period contest is simply

$$s_A = \frac{\sigma \omega_A}{\sigma \omega_A + \omega_B}. \quad (14)$$

<sup>14</sup> It is easy to check that the second-order condition,  $\partial^2 U_A / \partial e_A^2 < 0$ , is still satisfied.

Note that the implicit valuations in the first period,  $\omega_A$  and  $\omega_B$ , are independent of first-period efforts ( $e_A, e_B$ ). That is, they are treated as exogenous prizes just as in the static, single-period contest case. However, a change in handicapping policy  $h$  (whether to favor the winner or loser of the first-period contest as well as how much benefit or favor to give) affects the effort levels of both employees in the first period by either increasing or decreasing the implicit value of winning in the first period ( $\omega_A, \omega_B$ ). This is the strategic effect of  $h$  on the first-period efforts.

The following proposition shows how differently a handicapping policy affects the implicit value of winning in the first period for different employees, which ultimately affects their effort levels.

**PROPOSITION 2.** *The value of winning the first-period contest decreases in handicapping policy  $h$  ( $\partial \omega_A / \partial h < 0$ ,  $\partial \omega_B / \partial h < 0$ ). Moreover,*

(a) *under a handicapping policy of favoring the loser (i.e., when  $h > 1$ ), the value of winning the first-period contest is such that  $\omega_B < \omega_A < v$ ;*

(b) *under a handicapping policy of favoring the winner (i.e., when  $h < 1$ ), the value of winning the first-period contest is such that  $\omega_B > \omega_A > v$ ; and*

(c) *without a handicapping policy (i.e.,  $h = 1$ ), the value of winning the first-period contest converges to the static single-period case for both employees:  $\omega_A = \omega_B = v$ .*

**PROOF.** See the appendix.  $\square$

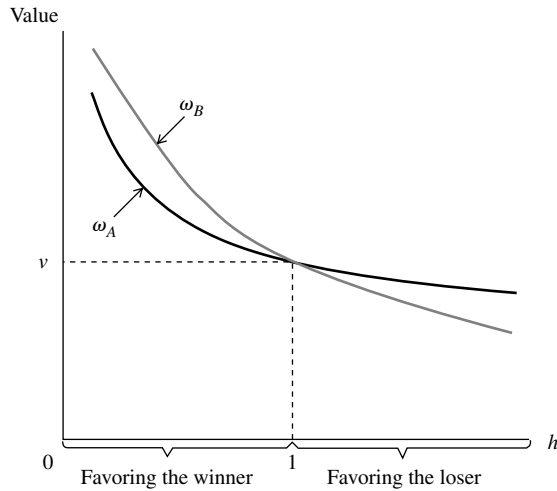
Proposition 2 suggests that when  $h$  becomes larger (i.e., the manager favors the loser of the first period more), the value of winning in the first period decreases for both employees ( $\partial \omega_A / \partial h < 0$  and  $\partial \omega_B / \partial h < 0$ ) since the loser can benefit in the second period from the handicapping policy  $h$ . This is the strategic effect of  $h$  due to the dynamic relationship between two-period contests. What is more interesting and surprising is that the value for the stronger employee is greater or smaller than that of the weaker employee depending on the handicapping policy (i.e., whether  $h < 1$  or  $h > 1$ ). Figure 1 illustrates this relationship between the handicapping policy ( $h$ ) and the value of winning the first-period contest (when  $\sigma = 0.5$ ).

When the firm favors the loser (i.e.,  $h > 1$ ), it reduces the value of winning the first-period contest because of the future punishment for current success:  $\omega_A < v$  and  $\omega_B < v$ . As a result, both employees may modify their efforts by holding back in the first period. This dynamic arises from the ratchet effect identified in previous literature (Weitzman 1980, Freixas et al. 1985). Moreover, this decrease in value is greater for the stronger employee who is more likely to win the current contest, and thus,  $\omega_B < \omega_A < v$  for  $h > 1$ .

On the other hand, when the firm favors the winner ( $h < 1$ ), the value of winning the first-period contest is higher than the static single-period case as a



Figure 1 The Value of Winning the First-Period Contest



result of the future rewards for the current success:  $\omega_A > v$ ,  $\omega_B > v$  for  $h < 1$ . More importantly, the value of winning for the stronger employee is greater than that of the weaker employee when  $h < 1$ . This is because the stronger employee  $B$  benefits more from the increased asymmetry in the second-period contest, whereas the weaker employee  $A$  is still more likely to lose even with an advantage since the handicapping does not fully compensate the ability difference as a result of the uncertainty (see Proposition 1). Hence, employee  $B$  would value winning the first-period contest more than employee  $A$  would ( $\omega_A < \omega_B$ ).

The implicit value of winning varies for each employee under different handicapping policies and thus affects their effort levels differently. From Equation (13), it is clear that the direct effect of its own implicit value of winning is to increase the first-period effort:  $\partial e_A / \partial \omega_A > 0$  and  $\partial e_B / \partial \omega_B > 0$ . The higher the stake, the more effort employees exert.

However, changes in rival's efforts are more ambiguous. The indirect effect of the implicit value of winning for employee  $A$  ( $B$ ) on the effort of employee  $B$  ( $A$ ) is not necessarily monotonic:

$$\begin{aligned} \frac{\partial e_A}{\partial \omega_B} &= \frac{\sigma \omega_A^2 (\sigma \omega_A - \omega_B)}{(\sigma \omega_A + \omega_B)^3} \geq 0 \Leftrightarrow \sigma \geq \frac{\omega_B}{\omega_A}, \\ \frac{\partial e_B}{\partial \omega_A} &= \frac{\sigma \omega_B^2 (\omega_B - \sigma \omega_A)}{(\sigma \omega_A + \omega_B)^3} \leq 0 \Leftrightarrow \sigma \geq \frac{\omega_B}{\omega_A}. \end{aligned} \quad (15)$$

The following proposition summarizes these indirect effects of the implicit value of winning on the effort of the other competitor with different handicapping policies, which is the key factor that drives our main results of optimal handicapping choice in the next section.

PROPOSITION 3. (a) Under a handicapping policy of favoring the loser ( $h > 1$ ), where  $\omega_B < \omega_A < v$ ,

$$\frac{\partial e_A}{\partial \omega_B} < 0 \quad \text{and} \quad \frac{\partial e_B}{\partial \omega_A} > 0$$

if employee abilities are very different ( $\sigma < \omega_B / \omega_A$ ),

$$\frac{\partial e_A}{\partial \omega_B} > 0 \quad \text{and} \quad \frac{\partial e_B}{\partial \omega_A} < 0$$

otherwise ( $\sigma \geq \omega_B / \omega_A$ ).

(b) Under a handicapping policy of favoring the winner ( $h \leq 1$ ), where  $\omega_B > \omega_A > v$ ,

$$\frac{\partial e_A}{\partial \omega_B} \leq 0 \quad \text{and} \quad \frac{\partial e_B}{\partial \omega_A} \geq 0.$$

PROOF. See the appendix.  $\square$

A handicapping policy of favoring the loser obviously reduces both players' valuations but in different amounts. When abilities are sufficiently different ( $\sigma < \omega_B / \omega_A$ ), a decline in  $\omega_A$  causes employee  $B$  to lower his effort ( $\partial e_B / \partial \omega_A > 0$ ) since he is playing an even weaker opponent. However, employee  $A$  lowers his effort when the value to employee  $B$  increases ( $\partial e_A / \partial \omega_B < 0$ ). This result stems from the fact that the increased effort from the stronger player as a result of the increase in the valuation of winning  $\omega_B$  makes the first-period contest even more asymmetric such that weaker player (employee  $A$ ) has very little chance of winning. In this case, the weaker player simply reduces his effort cost in the first period.

On the other hand, when employees' abilities are similar ( $\sigma \geq \omega_B / \omega_A$ ), a change in  $h$  has the opposite effect. Employee  $A$  surprisingly lowers his effort further when the value to employee  $B$  declines ( $\partial e_A / \partial \omega_B > 0$ ). Likewise, when  $\omega_A$  increases, employee  $B$  increases his effort. In other words, the weaker employee has a fair chance of winning the contest when employees' abilities are similar and thus tries to match his rival in the contest. However, the stronger employee,  $B$ , now finds it optimal to reduce his effort cost in the first period and tries to benefit from the handicapping policy of favoring the loser in the second period, which makes the contest even more asymmetric.

In addition, a handicapping policy of favoring the winner ( $h < 1$ ) increases both players' valuations for winning the first-period contest. An increase in  $\omega_A$  causes employee  $B$  to raise his effort ( $\partial e_B / \partial \omega_A > 0$ ) to match the competitor in the contest. However, employee  $A$  lowers his effort when the value to employee  $B$  increases ( $\partial e_A / \partial \omega_B < 0$ ). This is very similar to the case of when employees are very different under a handicapping policy of favoring the loser: the increased asymmetry in the first period as a result of

the increase in  $\omega_B$  causes employee  $A$  to have very little chance of winning. Hence, employee  $A$  finds it optimal to reduce his effort cost, even for all ability differences ( $\partial e_A/\partial \omega_B \leq 0$  and  $\partial e_B/\partial \omega_A \geq 0$ ).

This indirect effect of handicapping policy on the rival's effort is the key factor that mitigates the first-period effort losses from favoring the loser and explains how a handicapping policy of favoring the loser can increase total effort if the contest is asymmetric enough. To what degree these effects impact overall effort will be shown in the next section.

### 3.3. Optimal Handicapping Policy: Favoring the Loser or Winner

In the previous section, we showed that favoring the winner ( $h < 1$ ) has the direct effect of increasing the valuation of winning the first period ( $\omega_A, \omega_B$ ), leading to higher total efforts in the first period. But it also has a separate and unique indirect effect on each employee's effort ( $\partial e_A/\partial \omega_B < 0, \partial e_B/\partial \omega_A > 0$ ). Furthermore, favoring the winner may increase the ability gap in the second period, lowering total efforts in the second period. The manager should consider all three effects when she chooses the optimal handicapping policy.

Let  $e_1^{\text{Total}}$  and  $e_2^{\text{Total}}$  be the total efforts of the first period and second period, respectively. Given a handicapping policy  $h$  and ability difference  $\sigma$ , the total expected equilibrium effort for both periods is

$$\begin{aligned} E[e^{\text{Total}}(h, \sigma)] &= E[e_1^{\text{Total}}(h, \sigma) + e_2^{\text{Total}}(h, \sigma)] \\ &= e_{A1} + e_{B1} + [s_A \cdot 2 \cdot e_2^f + (1 - s_A) \cdot 2 \cdot e_2^t], \end{aligned}$$

where

$$\begin{aligned} e_{A1}(\omega_A, \omega_B, \sigma) &= \frac{\sigma \omega_B}{(\omega_B/\omega_A + \sigma)^2}, \\ e_{B1}(\omega_A, \omega_B, \sigma) &= \frac{\sigma \omega_A}{(\sigma \omega_A/\omega_B + 1)^2}, \\ \omega_A(h, \sigma) &= v \left[ 1 + \left( \frac{\sigma}{\sigma + h} \right)^2 - \left( \frac{\sigma h}{1 + \sigma h} \right)^2 \right] \\ &= v \cdot \hat{\omega}_A(h, \sigma | v = 1), \\ \omega_B(h, \sigma) &= v \left[ 1 + \left( \frac{1}{1 + \sigma h} \right)^2 - \left( \frac{h}{\sigma + h} \right)^2 \right] \\ &= v \cdot \hat{\omega}_B(h, \sigma | v = 1), \\ e_2^f(h, \sigma) &= \frac{v(\sigma h)}{(\sigma + h)^2}, \quad e_2^t(h, \sigma) = \frac{v(\sigma h)}{(1 + \sigma h)^2}, \\ \text{and } s_A &= \frac{\sigma \omega_A}{\sigma \omega_A + \omega_B}. \end{aligned} \quad (16)$$

Recall that since employees' valuations are identical in the second period, their equilibrium efforts in that period are also the same; if employee  $A$  wins in

the first period (with probability  $s_A$ ), the equilibrium effort is  $e_2^f$  for both employees, whereas if he loses in the first period (with probability  $1 - s_A$ ), it is  $e_2^t$ .

The objective of the manager is to maximize the firm's profit, which is a function of total employee effort  $\pi_M = \psi \cdot e^{\text{Total}} - C(v)$ , where  $\psi$  is the parameter that captures the firm's product efficiency and  $C(v)$  is the cost for prize  $v$ .

Here, the contest prize clearly affects the equilibrium effort levels; i.e., more precisely,  $e^{\text{Total}} = e^{\text{Total}}(h, v, \sigma)$ . Thus, when the manager designs the contest in practice, she should optimize not only the handicapping policy  $h$  but also the contest prize amount  $v$  for a given  $\sigma$ . However, all  $\omega_A$  and  $\omega_B$  are linear functions of  $v$  (i.e.,  $\omega_A(h, v, \sigma) = v \cdot \hat{\omega}_A(h, \sigma | v = 1)$  and  $\omega_B(h, v, \sigma) = v \cdot \hat{\omega}_B(h, \sigma | v = 1)$ , where  $\hat{\omega}_A(h, \sigma | v = 1)$  and  $\hat{\omega}_B(h, \sigma | v = 1)$  are  $\omega_A, \omega_B$  when  $v = 1$ ). Thus, the first-period efforts  $e_{A1}$  and  $e_{B1}$  are also linear functions of  $v$  (i.e.,  $e_{A1}(\omega_A, \omega_B, \sigma) = (\sigma \omega_B)/((\omega_B/\omega_A) + \sigma)^2 = (v\sigma \hat{\omega}_B)/((\hat{\omega}_B/\hat{\omega}_A) + \sigma)^2$  and  $e_{B1}(\omega_A, \omega_B, \sigma) = (\sigma \omega_A)/(\sigma(\omega_A/\omega_B) + 1)^2 = (v\sigma \hat{\omega}_A)/(\sigma(\hat{\omega}_A/\hat{\omega}_B) + 1)^2$ ). Moreover,  $s_A$  (which is equal to  $(\sigma \omega_A)/(\sigma \omega_A + \omega_B) = (\sigma \hat{\omega}_A)/(\sigma \hat{\omega}_A + \hat{\omega}_B)$ ) is independent of  $v$ , and therefore, the second-period expected efforts ( $s_A \cdot 2 \cdot e_2^f + (1 - s_A) \cdot 2 \cdot e_2^t$ ) are again a linear function of  $v$ . Therefore,  $e^{\text{Total}}(h, v, \sigma) = v \cdot e^{\text{Total}}(h, \sigma)$ .

Now, the firm's profit function can be rewritten as  $\pi_M = v \cdot \psi \cdot e^{\text{Total}}(h, \sigma) - C(v)$ . This implies that the choice of optimal handicapping policy  $h$  is independent of the contest prize  $v$ . Hence, to maximize profit, the manager only needs to find the optimal  $h$  that maximizes the total employee efforts for any given  $v$ . In other words, for any given contest prize, our result of a handicapping policy is always optimal, and thus the objective of the manager reverts to finding the optimal  $h$  irrespective of the value of  $v$ .<sup>15</sup> However, it is important to note that, in general, the optimal contracting will be a function of both the handicapping  $h$  and the prize value  $v$ . Only when the prize value  $v$  is symmetric does the problem degenerate to one with optimizing only the handicapping  $h$ . Otherwise, the firm should optimally choose both the handicapping  $h$  and the optimal size of the prize  $v$  simultaneously.

Also, we note that  $h$  affects employees' implicit valuations in the first period through  $\omega_A$  and  $\omega_B$ . Given a manager's handicapping policy in the second period, employees choose their efforts in the first period, fully anticipating the consequence of their choices in the second period.

<sup>15</sup> The optimal prize  $v$  clearly affects the incentives of the agents by steepening or moderating the efforts. The optimal  $v$  can be found as  $C'(v^*) = \hat{e}^T$ , and thus, it is independent of  $h$  and depends only on the specification of cost function  $C(v)$ . For example, if we impose a convex cost of  $C(v) = v^2/2$ , then  $v^* = \hat{e}^T$ .

**PROPOSITION 4.** For any given  $v > 0$ , when employee abilities are very different ( $\sigma$  is sufficiently small), a handicapping policy of favoring the loser ( $h > 1$ ) maximizes the expected total effort. Otherwise, when employee abilities are similar ( $\sigma$  is sufficiently large), a handicapping policy of favoring the winner ( $h < 1$ ) maximizes total effort.

**PROOF.** See the appendix.  $\square$

There is a fundamental trade-off between the effort levels across two periods when the manager employs a handicapping policy. On the one hand, a handicapping policy of favoring the loser ( $h > 1$ ) always increases effort in the *second* period because it levels the playing field and encourages the weaker player. Yet favoring the loser reduces the incentives of players to win the first-period contest because of the future punishment for the winner, tempering the gains from the second period.

On the other hand, a handicapping policy of favoring the winner ( $h < 1$ ) always increases effort in the *first* period because of the additional future reward for the winner in the second period. However, this reduces effort in the second period because the winner of the first contest would no longer need to work as hard in the second period, tempering the gains from the first period.

Overall, the manager must balance these trade-offs when choosing a handicapping strategy. When employees are very different in their abilities (i.e., small  $\sigma$ ), the handicapping policy of favoring the loser ( $h > 1$ ) can intensify competition between employees in the second period, and this benefit of increased effort outweighs the loss of effort in the first period. Both employees reduce effort in the first period to take advantage of future benefit, but employees do not race to the bottom as a result of the strategic indirect effect of the weaker employee  $A$  ( $\partial e_A / \partial \omega_B < 0$  from Proposition 3). The value to stronger employee  $B$  decreases significantly (i.e.,  $\omega_B < \omega_A < v$  from Proposition 2), thus lowering his first-period effort. This has an indirect effect on the weaker employee's effort because he now has a higher chance to win the first-period contest against a more restrained stronger employee and, thus, increases his effort. This mitigates the loss in total first-period effort for the manager. This only occurs when employees are very different in their abilities (i.e., small  $\sigma$ ), and in this case, favoring the loser ( $h > 1$ ) maximizes expected total effort.

In contrast, when employee abilities are very similar (i.e., large  $\sigma$ ), the cost in lost effort associated with the handicapping policy of favoring the loser ( $h > 1$ ) is greater. Again, the value for winning the first-period contest decreases for both employees  $A$  and  $B$ , reducing their efforts. Unlike the case when  $\sigma$  is small (or employees are very different), the indirect strategic

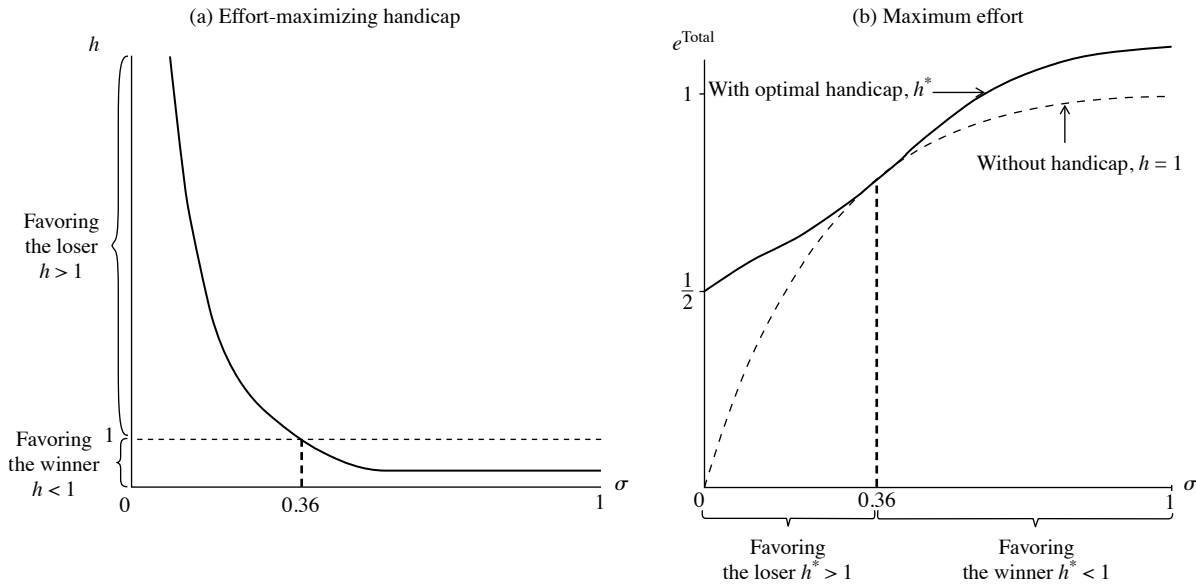
effect of the reduced implicit value causes the weaker employee to further reduce his effort ( $\partial e_A / \partial \omega_B > 0$  from Proposition 3), instigating a true "race to the bottom." Note that this indirect effect is greater than the indirect strategic effect on employee  $B$  since  $\omega_B < \omega_A < v$  (i.e.,  $|\partial e_A / \partial \omega_B| > |\partial e_B / \partial \omega_A|$ ). Furthermore, because the abilities are similar, any advantage gained from the handicapping policy would only marginally improve effort in the second period. Therefore, when employee abilities are very similar, a handicapping policy in favor of the loser ( $h > 1$ ) cannot be advantageous for the manager.

A handicapping policy in favor of the winner ( $h < 1$ ), on the other hand, motivates employees to compete more intensely in the first period, outweighing the costs of lackluster performance in the second period. In other words, the potential incentive problem of moral hazard from favoring the winner is not severe when players are very similar in their abilities. Even with a small advantage to the winner, the loser still has sufficient incentive to exert effort in the second-period contest. In anticipation of such effort, the stronger player still responds to the contest with sufficiently high effort level in the second period. Hence, when the ability difference is small (i.e.,  $\sigma$  is sufficiently large), the cost of effort loss in the second period is more than compensated by the increased effort in the first period—favoring the winner ( $h < 1$ ) maximizes the expected total effort. For example, when employees are identical in ability,  $\sigma = 1$ , the total expected effort is  $(3h + 1)v / (h + 1)^2$ , which is maximized by a handicapping policy of favoring the winner  $h = \frac{1}{3}$ .

This is in stark contrast to the static case, where favoring the perceived weaker player ( $h > 1$ ) is optimal for all ability differences. In contrast, in a dynamic setting, the manager sometimes maximizes the total effort by favoring the perceived stronger player who wins the first-period contest. By increasing the incentives in the first-period contest through the future rewards for the first-period contest success, the manager can maximize the total efforts from both employees. This only arises from the dynamic incentives created by the handicapping policy, and hence, in the static case, we could not find the situation where favoring the perceived stronger player optimal.

We illustrate the relationship between the relative ability  $\sigma$  and the optimal handicapping policy  $h$  in Figure 2(a), when  $v = 1$ . This clearly demonstrates that favoring the winner ( $h < 1$ ) is beneficial to the manager's attempt to raise effort when employees are similar in their abilities (in this particular example, when  $\sigma > 0.36$ ). Favoring the loser ( $h > 1$ ) is beneficial only when employees are sufficiently different in their abilities (when  $\sigma < 0.36$ ).

Figure 2 Optimal Handicapping Policy and Expected Total Effort for  $v = 1$



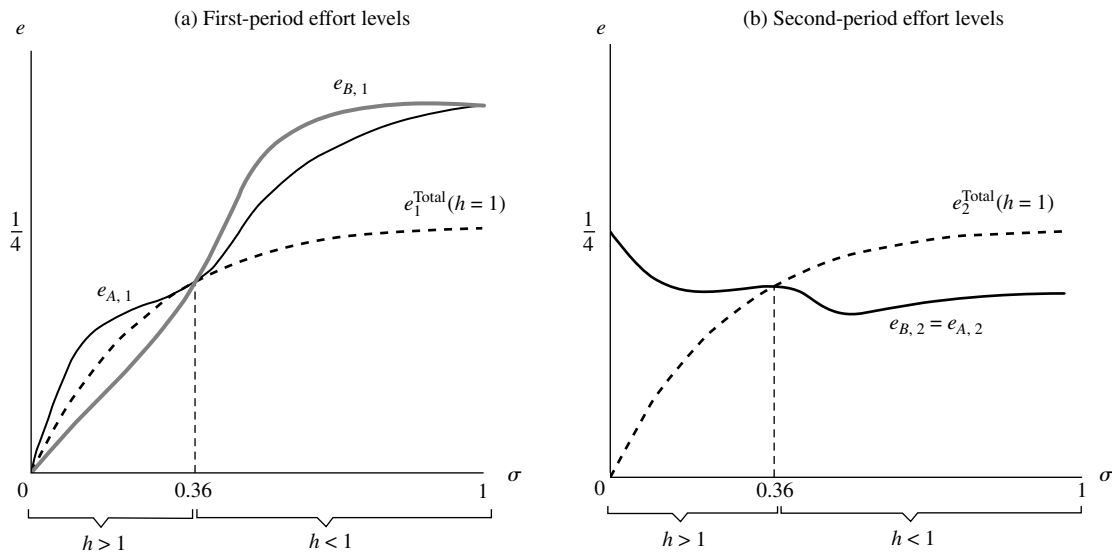
Next, we investigate the relationship between the relative ability  $\sigma$  and the expected total effort from both periods  $E(e^{\text{Total}})$  under the optimal handicapping policy  $h^*$ . As Figure 2(b) shows, the expected total effort  $E(e^{\text{Total}})$  under the optimal policy  $h^*$  is always greater than the expected total effort without any handicapping policy (i.e.,  $h = 1$  for all  $\sigma$ ).

To better understand the underlying forces behind this result, we decompose the total effort into individual effort by period, as shown in Figure 3. First, we note that there is a single crossover between the first- and second-period effort at the point where the optimal handicapping policy is  $h = 1$  (in this particular case,  $\sigma = 0.36$ ). A handicapping policy of  $h = 1$  is equivalent to the case of a no handicapping policy

(or static contest case). Hence, effort levels from both the first and the second periods are identical. Second, Figure 3(b) also illustrates that the efforts of both employees are equal in the second period because the exogenous value for the contest prize ( $v$ ) is the same for both.

When employees' abilities are sufficiently different ( $\sigma < 0.36$ ), a handicapping policy of favoring the loser  $h > 1$  clearly raises the second-period effort by both employees (see Figure 3(b)). Moreover, the first-period effort by the weaker employee  $A$  is greater than that of the stronger employee  $B$  (see Figure 3(a)). This is because the potential effort loss in the first period is mitigated by the indirect effect identified in Proposition 3. The decreased value of winning

Figure 3 Effort by Period and Employee Type as a Function of Ability Differences for  $h = h^*$



the first period, or “loser’s bonus,” has a strategic effect on employee *B* to reduce his effort. This reduction in effort from a restrained employee *B* causes employee *A* to increase his effort in the first period. In other words, the change in the total first-period effort is ambiguous and small. This effect is offset by the overwhelming increase in effort in the second period.

On the other hand, when employees’ abilities are quite similar ( $\sigma > 0.36$ ), favoring the winner increases the first-period effort by raising the value of winning in the first period for both employees (see Figure 3(a)). Although the strategic effect from an increased valuation of winning in the first period is greater for stronger employee *B* (see Proposition 2), the indirect effect from the weaker player is also increased effort. These effects are higher than the effort reduction from increased asymmetry in the second-period contest.

#### 4. Extension: Commitment vs. Flexibility

In this extension, we relax two key assumptions in our main model: (1) the manager credibly commits to a handicapping policy prior to the first-period contest,<sup>16</sup> and (2) the employees have a greater knowledge about their coworkers than the manager. Though these assumptions seem reasonable (since firms tend to operate in an environment with enforceable contracts with its employees, clients, and suppliers, and employees have naturally more interactions with each other), an interesting issue about whether the firm prefers to commit to a handicapping policy arises if the manager expects to receive more relevant information regarding the identities of employees (who is the weaker employee, who is the stronger one) during the first period.<sup>17</sup> In these circumstances, eliminating all ex post possibilities through commitment

<sup>16</sup> This is a critical assumption because our main result of the “favoring the winner” case in Proposition 4 only arises with commitment. Without commitment, the manager only maximizes the second-period efforts and thus always favors the loser since favoring the weaker is optimal under uncertainty for a static case (see Proposition 1).

<sup>17</sup> In this case, one may consider expanding the contract space and condition the handicapping policy on the total effort. There are several papers in contest theory literature that allow agents to update their beliefs about the abilities of their rivals based on performance in earlier periods (Hörner and Sahuguet 2007, Krämer 2007, Münster 2009). However, these papers only consider strategic interactions between employees and ignore the issue of firm’s strategic choice of handicapping policy. In contrast, we consider not only strategic interactions between employees but also strategic interactions between the firm and employees (i.e., optimal handicapping policy). This extra layer of complexity makes it intractable to allow conditioning the handicapping policy on the first-period effort.

seems unlikely to be a good idea, and the manager may value the flexibility to act on this information.

To address this issue, we compare our main model of commitment with the following benchmark scenario of no commitment. In this benchmark case of no commitment, we also relax the assumption that employees have superior information to the manager—the manager perfectly learns the abilities of both employees at the end of the first period. This setup serves to “stack the deck” against finding commitment-related benefit because it assumes the best possible situation for the no-commitment (flexibility) case; that is, the information that the manager can act on at the beginning of the second period is now perfect information. In reality, the information is far from perfect, which should lower the potential benefit from the flexibility. Hence, with this “perfect information” under the no-commitment case, one may wonder whether the manager still prefers to commit to a handicapping policy.

We first note that under the no-commitment situation, the manager chooses a policy *h* only after the first period. If the manager knows employees’ identities perfectly, she can achieve the first-best outcome in the second period by favoring the weaker employee by the exact amount of the ability difference,  $h = 1/\sigma$ , yielding  $v/2$  in total second-period effort (Tullock 1980). Since employees fully anticipate that the manager will perfectly know their identities in the second period, there is no strategic component of effort in the first period.<sup>18</sup> Thus, the first period in the repeated contest reverts to the static contest result where total effort is equal to  $2\sigma v/(\sigma + 1)^2$ . With perfect information, the total effort from both players in both periods is

$$\bar{e}^{\text{Total}} = \frac{2\sigma v}{(\sigma + 1)^2} + \frac{v}{2} = \frac{v(1 + 6\sigma + \sigma^2)}{2(1 + \sigma)^2}, \quad (17)$$

where  $\bar{e}^{\text{Total}}$  denotes the effort level under no commitment, which simplifies to  $v$  when  $\sigma = 1$ .

We can easily see that the total effort under commitment is lower than that of no commitment ( $e^{\text{Total}} < \bar{e}^{\text{Total}}$ ) over the range of asymmetry where favoring the loser is optimal (i.e., when employees are very different or  $\sigma$  is sufficiently low), since the

<sup>18</sup> Here, we are simply imposing the best possible scenario for the no-commitment case by assuming that the manager receives perfect information. However, in practice, the issue of whether or how the manager can acquire the perfect information is important. For example, employees may strategically conceal their abilities to avoid ratcheting. Once one has a micromodel about the interactions between the manager and employees during the contest, then one can analyze the dynamics of information revelation. But this is beyond the scope of the current research. We thank an anonymous reviewer for highlighting this important issue.

second-period effort is always higher under no commitment ( $e_2^{\text{Total}} < \bar{e}_2^{\text{Total}}$ ), and the first-period effort is lower than in the static case under this handicapping policy ( $e_1^{\text{Total}} < \bar{e}_1^{\text{Total}}$  since  $\omega_i \leq v$  for all  $i = \{A, B\}$ ; see Proposition 2). Hence, the manager values flexibility more than commitment when employees are very different.

However, we find that even if the manager can benefit from flexibility by obtaining the perfect information at the end of first period, she prefers to commit to a handicapping policy of favoring the winner based on the first-period contest result when  $\sigma$  is sufficiently large or employees are quite similar.<sup>19</sup> In spite of having perfect information about employees, the manager can be better off by forgoing this information and committing to the handicapping policy of favoring the winner when employees are quite similar. In this case, the employees' future consideration raises the first-period effort because of the handicapping policy of favoring the winner. This increase in the first-period effort can more than compensate the effort loss from the inefficiency caused by using noisy information based on the first-period contest result (instead of using perfect information about employee types). For example, in the symmetric case of  $\sigma = 1$ , the optimal handicapping is  $h^* = \frac{1}{3}$ , and the total effort is  $e^{\text{Total}} = \frac{9}{8}v > v = \bar{e}^{\text{Total}}$ .

This result highlights the value of a dynamic nature of handicapping policy; by dividing the contest into separate but mutually dependent contests, a manager can increase total effort above that achieved from perfect information by committing to a handicapping policy. Therefore, the effect of a handicapping policy is not merely shifting employee's efforts between two periods, but in fact, it increases the total effort level above the perfect information case as a result of employees' strategic behaviors arising from the dynamic relationship between two contests through a handicapping policy.

## 5. Conclusion

Managers constantly face the problem of motivating a heterogeneous workforce. In particular, the issue of whether managers should invest in laggards or reward their top performers to motivate heterogeneous employees still does not have a consensus answer. In this research, we use a Tullock contest model and handicapping policy to address this issue. We show that by dividing the contest into separate but mutually dependent contests, a manager can

increase total effort by committing to a certain handicapping policy.

A conflict arises, however, between favoring a poor performer and rewarding a top performer in a dynamic setting. Favoring the loser increases effort in the second period at the expense of reducing each employee's incentive to win the first period because of the future punishment for the winner; this can be seen as the "ratchet effect." Also, favoring the winner increases effort in the first period because of the future reward for the winner but would also create another type of incentive problem—the "moral hazard," where the winner no longer has to work as hard. The manager should weigh the trade-offs of these two different policies (favoring the winner versus favoring the loser) to maximize the employees' total effort.

We find that if abilities are sufficiently similar, favoring the winner in the second period increases the total effort over both periods—rewarding the top performer is optimal. However, if abilities are sufficiently different, the opposite result holds, and the total effort is maximized by adopting a handicapping policy that favors the loser—investing in laggards is optimal. As such, the model suggests a clear handicapping policy guideline for the manager when faced with a heterogeneous workforce with uncertain abilities in a dynamic contest environment. Moreover, handicapping is common practice in various settings: from sports events (for example, golf and sailing) to social systems (such as affirmative actions).<sup>20</sup> This study can broadly apply to those various settings and provide useful insight about what types of systems can help participants in those settings to put in their best efforts and increase the efficiency of the system.

Also, we show that a manager can increase total effort above the level achieved from perfect information by committing to a handicapping policy. This arises from employees' strategic behaviors because of the dynamic relationship between two contests through handicapping policy. Furthermore, this implies that even if the manager knew the ability of each employee, a handicapping policy of favoring the winner has a greater incentive effect than evening the contest by favoring the loser. This result uniquely contributes to the existing contest literature concerning maximizing effort and handicapping.

There are many other factors that affect the effectiveness of a handicapping policy such as fairness or prize structure. For instance, handicapping the weaker employee may cause the stronger employee to feel that the contest has been unfairly altered such that the latter's work appears underappreciated,

<sup>19</sup> The issue of commitment and flexibility is investigated in many different settings, and similar results are reported (for example, Amador et al. 2006, Zeithammer 2007). It has been omitted here for brevity, but the detailed proof of this result is available from the authors upon request.

<sup>20</sup> We thank an anonymous reviewer for encouraging us to think about the broader implications of handicapping and suggesting these examples.

which may eventually lead to disenchantment for the stronger player. This point is worth examining from a goal theory perspective, as Murphy et al. (2004) do. Our paper complements this goal theory perspective and contributes to understanding why favoring the loser could lower the effort from a stronger employee. Yet the proposed model sometimes seems to favor the Matthew effect, by which winners only win because they have won in the past, not because of their superior ability (Merton 1968). We believe that our model can offer another explanation for the Matthew effect.

Also, we have only examined a two-period repeated contest. A natural extension would involve a multiperiod contest; it would be interesting to examine whether our main results can hold beyond two periods. Unlike the two period case where the handicapping policy is employed only once at the beginning of the second period, the manager can employ different handicapping policies for each period, and thus, the optimal handicapping profile would involve a series of different handicapping policies over entire periods, which is beyond the scope of our current work. We leave it for future research to explore this issue.

Finally, we only examine the two-employee case; other research has examined a much larger pool of participants, in which case the optimal proportion of winners in a contest can be an important issue (Lim 2010). However, if managers only have to distinguish between a pair of leaders, it might not be practical to include more than two agents in a competition, say, for the same client account. Nevertheless, an application to the multiemployee case would broaden the implications of our study.

### Acknowledgments

The authors are grateful to the editor, the associate editor, and two anonymous reviewers for detailed feedback that greatly improved the paper. They thank Kyung Hwan Baik, Michael Baye, Timothy Derdenger, Eunyu Lee, Noah Lim, John Maxwell, Subrata Sen, K. Sudhir, Birger Wernerfelt, and especially Rick Harbaugh for their helpful comments. Also, they thank seminar participants at the Quantitative Marketing and Economics Conference at Chicago, the North East Marketing Consortium Conference at Harvard, Carnegie Mellon University, Indiana University, Sungkyunkwan University, the University of Rochester, the University of Texas at Austin, Washington University in St. Louis, and Yale University for helpful feedback.

### Appendix

**PROOF OF PROPOSITION 1.** Given a handicapping policy  $h$  and uncertainty  $p$ , we can calculate the equilibrium efforts under handicapping from Equation (3). With probability  $1 - p$ , employee  $A$  is (correctly) identified as the weaker employee and receives a handicap of  $h$ . Equilibrium levels of effort for both employees then are  $e^t = e_A^t$

$= e_B^t = (\sigma h / (1 + \sigma h)^2)v$ . Since  $v_A = v_B = v$ , both employees exert the same amount of effort  $e^t$  in equilibrium, where the superscript  $t$  represents their true identity. But with probability  $p$ , employee  $A$  is identified (incorrectly) as the stronger employee, and the handicap is erroneously applied to the truly stronger employee  $B$ . The equilibrium levels of effort for both employees are  $e^f = e_A^f = e_B^f = (\sigma h / (\sigma + h)^2)v$ , where the superscript  $f$  represents their false identity.

For a given  $\sigma \in [0, 1]$  and  $p \in [0, \frac{1}{2}]$ , the manager solves the following maximization problem:

$$\max_{h>0} E(e^{\text{Total}}) = p \left( 2v \left( \frac{\sigma/h}{(1 + \sigma/h)^2} \right) \right) + (1 - p) \left( 2v \left( \frac{\sigma h}{(1 + \sigma h)^2} \right) \right).$$

The first-order condition with respect to  $h$  yields

$$\frac{\partial E(e^{\text{Total}})}{\partial h} = 2\sigma v \left( \frac{(1 - p)(1 - h\sigma)}{(1 + \sigma h)^3} - \frac{p(h - \sigma)}{(h + \sigma)^3} \right).$$

It can be easily seen that  $[\partial E(e^{\text{Total}})/\partial h]_{h=1} = 2\sigma v(((1 - 2p) \cdot (1 - \sigma))/(1 + \sigma)^3) \geq 0$  (equality holds when  $\sigma = 1$ ), and  $\partial E(e^{\text{Total}})/\partial h < 0$  for all  $h \geq (1/\sigma) (> 1)$ . By continuity, there must be at least one point  $h^* \in [1, 1/\sigma]$  at which  $[\partial E(e^{\text{Total}})/\partial h]_{h=h^*} = 0$ .

Moreover, we can show that  $\partial E(e^{\text{Total}})/\partial h > 0$  for all  $h < 1$ .

1. When  $h \leq \sigma (< 1)$ , it is trivially satisfied.

2. When  $\sigma < h < 1$ , we show that  $(1 - p)(1 - h\sigma)/(1 + \sigma h)^3 > p(h - \sigma)/(h + \sigma)^3$  by rearranging the inequality

$$\begin{aligned} \frac{(1 - p)(1 - h\sigma)}{(1 + \sigma h)^3} &> \frac{p(h - \sigma)}{(h + \sigma)^3} \\ \Leftrightarrow \left( \frac{1 - p}{p} \right) \left( \frac{h + \sigma}{1 + \sigma h} \right)^3 \left( \frac{1 - h\sigma}{h - \sigma} \right) &> 1 \\ \Leftrightarrow p < \frac{(h + \sigma)^3(1 - h\sigma)}{(1 - \sigma^2)[h(1 + h^2) + h(1 + h^2)\sigma^2 - (1 - 6h^2 + h^4)\sigma]} &= \bar{p}. \end{aligned}$$

In the second inequality, as  $p \downarrow 0$ , the (left-hand side) goes to infinity, and thus, the inequality is always satisfied. More precisely, we can find the condition such that when  $p < \bar{p}$ ,  $\partial E(e^{\text{Total}})/\partial h > 0$  for all  $h < 1$ .

In summary, when  $p < \bar{p}$ ,  $\partial E(e^{\text{Total}})/\partial h > 0$  for all  $h \leq 1$ , and  $\partial E(e^{\text{Total}})/\partial h < 0$  for all  $h \geq (1/\sigma) (> 1)$ . Therefore,  $E(e^{\text{Total}})$  attains its maximum at some point  $h^* \in [1, 1/\sigma]$ , where  $[\partial E(e^{\text{Total}})/\partial h]_{h=h^*} = 0$  by the mean value theorem. Hence, the optimal handicapping policy is strictly smaller than  $1/\sigma$ .

**PROOF OF PROPOSITION 2.** First, it is obvious that  $\omega_A = \omega_B = v$  when  $h = 1$ . Next, note that  $\omega_A = v + \sigma^2 v / (\sigma + h)^2 - (\sigma h)^2 v / (1 + \sigma h)^2$  and  $\omega_B = v + (1 / (1 + \sigma h))^2 v - (h / (\sigma + h))^2 v$ . Hence,

$$\begin{aligned} \omega_B - \omega_A &= \left[ \left( \frac{1}{1 + \sigma h} \right)^2 - \left( \frac{h}{\sigma + h} \right)^2 - \frac{\sigma^2}{(\sigma + h)^2} + \frac{(\sigma h)^2}{(1 + \sigma h)^2} \right] v \\ &= \left[ \frac{1 + (\sigma h)^2}{(1 + \sigma h)^2} - \frac{\sigma^2 + h^2}{(\sigma + h)^2} \right] v \\ &= \frac{2v\sigma h(1 - h^2)(1 - \sigma^2)}{(1 + \sigma h)^2(\sigma + h)^2}. \end{aligned} \tag{18}$$

Therefore,  $\omega_B - \omega_A > 0$  if and only if  $h < 1$  (note that  $\sigma < 1$ ).

Moreover,  $\omega_A = v + \sigma^2 v / (\sigma + h)^2 - (\sigma h)^2 v / (1 + \sigma h)^2 > v$  if and only if  $h < 1$ , because  $\sigma^2 v / (\sigma + h)^2 < (\sigma h)^2 v / (1 + \sigma h)^2 \iff 1 < h$ . The results in the proposition follow.

**PROOF OF PROPOSITION 3.** From the first-order condition,  $\partial e_A / \partial \omega_B = \sigma \omega_A^2 (\sigma \omega_A - \omega_B) / (\sigma \omega_A + \omega_B)^3$  and  $\partial e_B / \partial \omega_A = -\sigma \omega_B^2 (\sigma \omega_A - \omega_B) / (\sigma \omega_A + \omega_B)^3$ .

1. Under a handicapping policy of favoring the loser,  $\omega_B < \omega_A < v$ . Hence, if  $\sigma < \omega_B / \omega_A$ , then  $\partial e_A / \partial \omega_B = \sigma \omega_A^2 (\sigma \omega_A - \omega_B) / (\sigma \omega_A + \omega_B)^3 < 0$  and  $\partial e_B / \partial \omega_A = -\sigma \omega_B^2 (\sigma \omega_A - \omega_B) / (\sigma \omega_A + \omega_B)^3 > 0$ . Otherwise (i.e.,  $\sigma \geq \omega_B / \omega_A$ ),  $\partial e_A / \partial \omega_B > 0$  and  $\partial e_B / \partial \omega_A < 0$ .

2. Under a handicapping policy of favoring the winner,  $\omega_B > \omega_A > v$ . Thus,  $\partial e_A / \partial \omega_B = \sigma \omega_A^2 (\sigma \omega_A - \omega_B) / (\sigma \omega_A + \omega_B)^3 < 0$  and  $\partial e_B / \partial \omega_A = \sigma \omega_B^2 (\omega_B - \sigma \omega_A) / (\sigma \omega_A + \omega_B)^3 > 0$ .

3. Without a handicapping policy,  $\omega_A = \omega_B = v$ . Thus,  $\partial e_A / \partial \omega_B = 0$  and  $\partial e_B / \partial \omega_A = 0$ .

**PROOF OF PROPOSITION 4.** First, we show that there exists a unique equilibrium of efforts by both employees for all  $\sigma$  and  $h$ . We have already established that the first- and second-order conditions are satisfied, yielding the existence of an equilibrium. Therefore, to prove uniqueness, we examine the Hessian of  $U_A$  and  $U_B$ :

$$H = \begin{bmatrix} \frac{\partial^2 U_A}{\partial e_A^2} & \frac{\partial^2 U_B}{\partial e_A \partial e_B} \\ \frac{\partial^2 U_A}{\partial e_A \partial e_B} & \frac{\partial^2 U_B}{\partial e_B^2} \end{bmatrix}, \quad (19)$$

which simplifies to

$$H = \begin{bmatrix} \frac{\partial^2 s_A}{\partial e_A^2} v_A & -\frac{\partial^2 s_A}{\partial e_A \partial e_B} v_B \\ \frac{\partial^2 s_A}{\partial e_B \partial e_A} v_A & -\frac{\partial^2 s_A}{\partial e_B^2} v_B \end{bmatrix}, \quad (20)$$

since valuations are exogenous to efforts. If  $H$  is negative definite for all  $e_A$  and  $e_B$ , then the equilibrium is unique (Rosen 1965). This claim is easy to verify since the first component is negative by assumption and the determinant,

$$|H| = -\frac{\partial^2 s_A}{\partial e_A^2} \frac{\partial^2 s_A}{\partial e_B^2} v_A v_B - \frac{\partial^2 s_A}{\partial e_A \partial e_B} \frac{\partial^2 s_A}{\partial e_A \partial e_B} v_A v_B > 0, \quad (21)$$

is positive definite, provided that

$$-\frac{\partial^2 s_A}{\partial e_A^2} \frac{\partial^2 s_A}{\partial e_B^2} > \frac{\partial^2 s_A}{\partial e_A \partial e_B} \frac{\partial^2 s_A}{\partial e_A \partial e_B}. \quad (22)$$

We have already established existence, so we can evaluate Equation (22) for each success function:

• From Equation (7), when  $\Pr(A \text{ wins second period} | A \text{ loses first period}) = s_{A|L} = \sigma h e_A / (\sigma h e_A + e_B)$ ,

$$|H| > 0, \quad \text{because } \frac{h^2 \sigma^2}{(e_B + e_A h \sigma)^4} > 0.$$

• From Equation (8), when  $\Pr(A \text{ wins second period} | A \text{ wins first period}) = s_{A|W} = (e_A (\sigma / h)) / (e_A (\sigma / h) + e_B)$ ,

$$|H| > 0, \quad \text{because } \frac{h^2 \sigma^2}{(e_A \sigma + e_B h)^4} > 0.$$

• From Equation (1), when  $\Pr(A \text{ wins first period}) = s_A = \sigma e_A / (\sigma e_A + e_B)$ ,

$$|H| > 0, \quad \text{because } \frac{\sigma^2}{(e_B + e_A \sigma)^4} > 0.$$

Next, we show that the optimal handicapping policy is decreasing in  $\sigma$ . From Equation (16), we know that for a given  $\sigma$ , total effort is

$$E[e^{\text{Total}}(h, \sigma)] = e_{A1} + e_{B1} + [s_A \cdot 2 \cdot e_2^f + (1 - s_A) \cdot 2 \cdot e_2^t],$$

where

$$\begin{aligned} e_{B1}(\omega_A, \omega_B, \sigma) &= \frac{\sigma \omega_B}{((\omega_B / \omega_A) + \sigma)^2}, \\ e_{B1}(\omega_A, \omega_B, \sigma) &= \frac{\sigma \omega_A}{(\sigma(\omega_A / \omega_B) + 1)^2}, \quad s_A = \frac{\sigma \omega_A}{\sigma \omega_A + \omega_B}, \\ \omega_A(h, \sigma) &= v + v \left[ \left( \frac{\sigma}{\sigma + h} \right)^2 - \left( \frac{\sigma h}{1 + \sigma h} \right)^2 \right], \\ \omega_B(h, \sigma) &= v + v \left[ \left( \frac{1}{1 + \sigma h} \right)^2 - \left( \frac{h}{\sigma + h} \right)^2 \right], \\ e_2^f(h, \sigma) &= \frac{v(\sigma h)}{(\sigma + h)^2}, \quad \text{and} \quad e_2^t(h, \sigma) = \frac{v(\sigma h)}{(1 + \sigma h)^2}. \end{aligned} \quad (23)$$

At equilibrium  $h = h^*$ , the first-order condition with respect to  $h$  satisfies

$$\begin{aligned} \frac{\partial E[e^{\text{Total}}]}{\partial h} \Big|_{h=h^*} &= \frac{\partial E[e^{\text{Total}}]}{\partial \omega_{AB}} \frac{d\omega_A}{dh} + \frac{\partial E[e^{\text{Total}}]}{\partial \omega_B} \frac{d\omega_B}{dh} + \frac{dE[e^{\text{Total}}]}{dh} \\ &= \left[ \frac{\partial e_{A1}}{\partial \omega_A} + \frac{\partial e_{B1}}{\partial \omega_A} + 2(e_2^f - e_2^t) \frac{\partial s_A}{\partial \omega_A} \right] \frac{d\omega_A}{dh} \\ &\quad + \left[ \frac{\partial e_{A1}}{\partial \omega_B} + \frac{\partial e_{B1}}{\partial \omega_B} + 2(e_2^f - e_2^t) \frac{\partial s_A}{\partial \omega_B} \right] \frac{d\omega_B}{dh} \\ &\quad + 2s_A \frac{de_2^f}{dh} + 2(1 - s_A) \frac{de_2^t}{dh} = 0, \end{aligned}$$

where

$$\frac{\partial e_{A1}}{\partial \omega_A} = \frac{2\sigma \omega_A \omega_B^2}{(\sigma \omega_A + \omega_B)^3} > 0, \quad \frac{\partial e_{B1}}{\partial \omega_A} = \frac{\sigma \omega_B^2 (\omega_B - \sigma \omega_A)}{(\sigma \omega_A + \omega_B)^3} > 0,$$

$$e_2^f - e_2^t \geq 0 \quad (\text{when } (1 - h^2)(1 - \sigma^2) \geq 0),$$

$$\frac{\partial s_A}{\partial \omega_B} = \frac{\sigma \omega_B}{(\sigma \omega_A + \omega_B)^2} > 0,$$

$$\frac{\partial \omega_A}{\partial h} = -2\sigma^2 \left( \frac{1}{(h + \sigma)^3} + \frac{h}{(1 + \sigma h)^3} \right) < 0,$$

$$\frac{\partial \omega_B}{\partial h} = -2\sigma \left( \frac{h}{(h + \sigma)^3} + \frac{1}{(1 + \sigma h)^3} \right) < 0,$$

$$\frac{\partial e_{A1}}{\partial \omega_B} = \frac{-\sigma \omega_A^2 (\omega_B - \sigma \omega_A)}{(\sigma \omega_A + \omega_B)^3}, \quad \frac{\partial e_{B1}}{\partial \omega_B} = \frac{2\sigma^2 \omega_A^2 \omega_B}{(\sigma \omega_A + \omega_B)^3} > 0,$$

$$\frac{\partial s_A}{\partial \omega_A} = \frac{-\sigma \omega_A}{(\sigma \omega_A + \omega_B)^2} < 0, \quad \frac{de_2^f}{dh} = \frac{\sigma(\sigma - h)}{(h + \sigma)^3}, \quad \text{and}$$

$$\frac{de_2^t}{dh} = \frac{\sigma(1 - h\sigma)}{(1 + h\sigma)^3}. \quad (24)$$



By substituting these results, the first-order condition of Equation (24) simplifies to

$$\begin{aligned} & \frac{\partial E[e^{\text{Total}}]}{\partial h} \\ &= -2\sigma^2 \left[ \frac{2\sigma\omega_A\omega_B^2 + \sigma\omega_B^2(\omega_B - \sigma\omega_A)}{(\sigma\omega_A + \omega_B)^3} + \frac{(1-\sigma)^2(1-h)^2\sigma\omega_B}{(\sigma\omega_A + \omega_B)^2} \right] \\ & \quad \cdot \left( \frac{(1+\sigma h)^3 + h(h+\sigma)^3}{(h+\sigma)^3(1+\sigma h)^3} \right) \\ & \quad - 2\sigma \left[ \frac{-\sigma\omega_A^2(\omega_B - \sigma\omega_A) + 2\sigma^2\omega_A^2\omega_B}{(\sigma\omega_A + \omega_B)^3} - \frac{\sigma\omega_A(1-\sigma)^2(1-h)^2}{(\sigma\omega_A + \omega_B)^2} \right] \\ & \quad \cdot \left( \frac{h(1+\sigma h)^3 + (h+\sigma)^3}{(h+\sigma)^3(1+\sigma h)^3} \right) \\ & \quad + \frac{2\sigma}{\sigma\omega_A + \omega_B} \left( \frac{(1+\sigma h)^3 + (h+\sigma)^3}{(h+\sigma)^3(1+\sigma h)^3} \right) = 0 \\ \Leftrightarrow & -\sigma[2\sigma\omega_A\omega_B^2 + \sigma\omega_B^2(\omega_B - \sigma\omega_A) \\ & \quad + (1-\sigma)^2(1-h)^2\sigma\omega_B(\sigma\omega_A + \omega_B)]((1+\sigma h)^3 + h(h+\sigma)^3) \\ & \quad + [\sigma\omega_A^2(\omega_B - \sigma\omega_A) - 2\sigma^2\omega_A^2\omega_B \\ & \quad + (1-\sigma)^2(1-h)^2\sigma\omega_A(\sigma\omega_A + \omega_B)](h(1+\sigma h)^3 + (h+\sigma)^3) \\ & \quad + (1+\sigma h)^3 + (h+\sigma)^3(\sigma\omega_A + \omega_B)^2 = 0, \end{aligned}$$

where  $\omega_A(h, \sigma) = v + v[(\sigma/(\sigma+h))^2 - (\sigma h)/(1+\sigma h)^2]$  and  $\omega_B(h, \sigma) = v + v[(1/(1+\sigma h))^2 - (h/(\sigma+h))^2]$ .

Using this, we can further simplify the first-order condition and define the implicit function  $F(h, \sigma)$ :

$$\begin{aligned} & \frac{\partial E[e^{\text{Total}}]}{\partial h} = 0 \\ \Leftrightarrow & F(h, \sigma) = [\sigma(X^2Y^2 - h^2Y^2 + X^2)(3X^2Y^2 - h^2Y^2 + X^2 \\ & \quad - \sigma X^2Y^2 - \sigma(1-h^2)(2-\sigma)(1+2h\sigma+h^2)) \\ & \quad + \sigma(1-h^2)(1-\sigma)^2(X^2Y^2 - h^2Y^2 + X^2) \\ & \quad \cdot ((1+\sigma)X^2Y^2 - h^2Y^2 + X^2 - \sigma^3(Y^2 - X^2))] \\ & \quad \times (Y^3 + hX^3) + [(X^2Y^2 - h^2Y^2 + Y^2)(2\sigma X^2Y^2 \\ & \quad - h^2Y^2 + X^2 + (X^2Y^2 - \sigma^2(1-h^2)(2-\sigma) \\ & \quad \cdot (1+2h\sigma+h^2))) + (X^2Y^2 - \sigma^2Y^2 + \sigma^2X^2) \\ & \quad \cdot ((1-\sigma)X^2Y^2 - h^2Y^2 + X^2 - \sigma^3(Y^2 - X^2))] \\ & \quad \times (hY^3 + X^3) + X^3Y^2 - Y^2 = 0, \end{aligned}$$

where  $X = h + \sigma$  and  $Y = 1 + \sigma h$ .

We differentiate  $F(h, \sigma)$  with respect to  $h, \sigma$ . After a few algebraic steps, we can see that  $dF/dh < 0$  for all  $h > 0$  and  $\sigma \in [0, 1]$ , and  $dF/d\sigma > 0$  if  $h > 0$  and  $\sigma \leq \hat{\sigma}$ , where  $\hat{\sigma}$  solves the equation  $(dF/d\sigma)(h, \hat{\sigma}) = 0$ . By the implicit function theorem,  $dh/d\sigma = -(\partial F/\partial \sigma)/(\partial F/\partial h) < 0$ . All we need to show is that when  $\sigma$  is large such that  $\sigma > \hat{\sigma}$ , it is the case that  $h^* < 1$ . We prove it by contradiction. Suppose that  $\sigma$  is sufficiently large and the optimal handicapping policy is  $h^* \geq 1$ . Then, it must be the case that  $(\partial E[e^{\text{Total}}]/\partial h)(h=1) > 0$ . However, we can easily see that

$$\begin{aligned} \frac{\partial E[e^{\text{Total}}(h=1)]}{\partial h} &= 2(1+5\sigma-5\sigma^3-33\sigma^5-37\sigma^4-9\sigma^5 \\ & \quad + 5\sigma^6 + \sigma^7) < 0, \quad \text{if } \sigma > \hat{\sigma} \approx 0.38. \end{aligned}$$

This contradicts the assumption. Thus, when  $\sigma > \hat{\sigma}$ , it is the case that  $h^* < 1$ .

In particular, when  $\sigma = 1$ ,  $E[e^{\text{Total}}(h, \sigma = 1)] = v(1+3h)/(1+h)^2 > 1$  from Equation (16). Total effort is therefore maximized at  $h = 1/3$ .

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