MODELING TERM STRUCTURES OF SWAP SPREADS*

Hua He

Yale School of Management
135 Prospect Street Box 208200
New Haven, CT 06552

December 1999
Last Revised: March 2001

Abstract

Swap spreads, the interest rate differentials between the fixed rates on fixed-for-floating swap contracts and the yields-to-maturity on maturity-matched government bonds, define a market for one of the most actively transacted securities in the global fixed-income arena. A large universe of fixed-income securities including corporate bonds and mortgaged-back securities use interest rate swap spreads as a key benchmark for pricing and hedging. Swap spreads have received renewed attention since the Fall of 1998 when their volatile movements contributed in a significant way to the financial turmoil that led the US Fed to cut short-term interest rates by 75 basis points. In this paper we present new insights on how to analyze term structure of interest swap spreads. Specifically, we focus on the determinants of swap spreads and show how quantities such as the spread of short-term LIBOR over GC-repo rates, the liquidity premium commended by government bonds, and the risk premium required for holding long-term bonds/swaps jointly determine term structures of swap spreads.

*The author thanks seminar participants at UT Austin, Wharton, Yale, Federal Reserve Board, and the Financial Engineering Workshop at U. of Chicago for helpful comments, and thanks Lehman Brothers for providing the data. Support from the International Center for Finance at Yale School of Management is gratefully acknowledged.

1 Introduction

Swap spreads, the interest rate differentials between the fixed rates on fixed-for-floating swap contracts and the yields-to-maturity on maturity-matched government bonds (priced at par), define a market for one of the most actively transacted securities in the global fixed-income arena. A large universe of fixed-income securities including corporate bonds and mortgaged-back securities use interest rate swap spreads as a key benchmark for pricing and hedging. Swap spreads have received renewed attention since the Fall of 1998 when their volatile movements contributed in a significant way to the financial turmoil that led the US Fed to cut the Federal Fund Rate by 75 basis points. While the financial crisis of 1998 is well behind us, to date swap spreads in US continue to be near their historical highs and volatile.

In this paper we present new insights on how to analyze term structures of swap spreads. Our objective is to set up a simple analytical framework so as to explain some of the extraordinary movements in interest rate swaps or swap spreads occurred in recent years. We contribute to the literature of interest rate swaps in three aspects. First, we derive a new formula for swap spreads based on the idea that swaps are financed by LIBOR rates which are generally higher than GC-repo rates used for financing government bonds. This contrasts sharply to the existing literature which attributes swap spreads to the default risk of the swap counterparty. Second, we provide an analytical framework which enables us to study the term structure of swap spreads and the determinants of swap spreads. In particular, we examine show how economic factors such as *short-term financing spreads* (i.e., LIBOR over GC-repo rates), the *liquidity premium* commended by government bonds, and the *risk premium* required for holding long-term government bonds or swaps jointly determine term structures of swap spreads. To our knowledge, this represents one of the few papers in the literature that look at the term structure swap spreads from a no-default perspective. Third, we apply our analytical framework to the US swap and bond data and obtain empirical evidence which supports our analytical claims with regard to the term structure of swap spreads and its relationship with the short-term financing spreads, liquidity premium and risk premium.

When an investor enters a swap agreement as a fixed receiver in a plain-vanilla fixed-for-floating swap, the investor is promised to receive from the counterparty a series of semi-annual fixed payments in exchange for paying the counterparty a series of semi-annual floating payments. While the fixed payments are determined at the outset of the swap agreement, the floating payments are to be determined at later dates, based on the six-month LIBOR rates prevailing at the beginning of each payment period. This simple structure of swaps suggests that swap spreads can be originated from the following three sources: 1)the credit worthiness of the counter-parties involved; 2) the floating leg (LIBOR) and its spread over the GC-repo; and 3) other economic forces that have the potential to affect the term structure of interest rates.

The credit worthiness of counterparties has traditionally been taken as the primary factor affecting the fair market swap rates or swap spreads. Indeed, an overwhelming number of academic studies have assumed that the main driving force behind the interest rate differentials between swaps and government bonds is the risk of counterparty default on its swap obligation, see Cooper and Mello (1991), Litzenberger (1992), Sorensen and Bollier (1994), Duffie and Huang (1996), etc. While this assumption was reasonable when the swap market was at its early stage of development, the current industry practice of swap agreements assumes that both counterparties enter a netting and collateral arrangement, known as the *Master Swap Agreement*. Under this agreement, both parties net out all of the exiting swap positions and impose collateral against each other based on daily net mark to market values of all open positions. Firms with worse credit ratings do not have to pay up to enter swap transactions with firms with better credit ratings, i.e., the mid-market swap curve is universal to every counterparty. The current industry practice has essentially removed (in a significant way if not completely) the risk of default by either counterparty so that, for all practical purposes, swaps shall be valued without the consideration of counterparty risk. In a world in which counterparty default risk can be totally ignored, an investor receiving fixed in a swap agreement is equivalent to holding a long position in a default-free government bond while at the same time financing the long position by paying a six-month LIBOR interest rate.

To understand the fair market swap rates from the perspective that swaps are financed by LIBOR rates, we consider a competing investment strategy – buy a government bond with the same maturity as the swap while financing the purchase of the government bond via a term repo at a standard rate for general collateral (GC). Since the credit worthiness of the banks involved in resetting LIBOR rates ranges from AA to A (or worse), the six-month LIBOR rates have traditionally been quoted at a positive spread over the six-month GC term repo rates. Consequently, it is natural to expect that investors who receive fixed in a fixed-for-floating swap be awarded with a fixed rate that is slightly higher than the yield-to-maturity on an otherwise comparable government bond (priced at par). In this manner, investors are compensated for the extra financing cost they have to bear due to the credit premium inherited in the six-month LIBOR rates. In other words, the fair market swap spreads are existed as a compensation for the short-term financing spreads between the six-month LIBOR rates and the six-month term GC-repo rates.

Notwithstanding our fair value theory based on short-term financing spreads, casual empirical observations suggest that short-term financing spreads have traditionally been well-behaved, fluctuating narrowly around a mean of 15-25 basis points, even during the period of financial crisis in 1998. In other words, the volatility we observed in the swap market recently cannot possibly be triggered by the volatility of short-term financing spreads. This leads us to pursue other economic factors affecting the term structure of swap spreads. Specifically, we note that the term structure of interest rates is generally determined by expectation of interest rates in the near and distant future and by the risk premium required for investors to holding longer term bonds or swaps. While the short-end of the curve is driven largely by expectations, the long-end of the curve is driven primarily by the risk premium. In addition, the liquidity advantage commended by government bonds, e.g., the on-the-runs, may cause bonds to enjoy a liquidity premium vs their swap counterparts. As both risk premium and liquidity premium are key determinants of the term structure of default-free interest rates, the risk premium differential and the liquidity premium differential between bonds and swaps are likely to be the key determinants of the term structure of swap spreads.

In this paper we set up a multi-factor term structure framework to analyze the determinants of swap spreads and the shape of swap spread curve. Our model assumes that all swaps are traded *default-free*. We first show in a formal setting that swap rates and their spreads over the default-free government bond yields are fully determined by the dynamics of the short-term financing spreads. The fair market swap rates are set in such a way that the present value of the (default-free) fixed payments equalizes the present value of the (default-free) floating payments. Under the assumption that the short-term financing spreads are independent of the current term structure of default-free interest rates, the fair market swap spreads can be shown to equal the present value of the short-term financing spreads properly amortized over the swap maturity. Next, we take a closer look at the relationship between the term structure of swap rates and the term structure of government bond yields. Using a 3-factor term structure model, we are able to dissect the term structure of interest rates by examining factors such as level, slope and curvature, see discussed in Litterman and Scheinkman (1991). In the same way that the level, slope and curvature of the government bond curve are driven by the market rate expectation, risk premium and liquidity premium, we demonstrate using our 3-factor model that the level, slope and curvature of the swap spread curve can be driven by the market expectation of future short-term financing spreads, the risk premium and liquidity premium differentials between bonds and swaps.

The basic intuition behind our analysis is as follows. Market expectations of interest rates in the future affect long-term swap rates as well as long-term bond yields. Expectations of higher (or lower) rates in the future generally result in higher (or lower) long-term swap rates as well as higher long-term bond yields. However, the net effect on the difference between the level of swap rates and the level of bond yields shall be negligible. However, expectations of future short-term financing spreads affect only the swap rates. Expectations of higher short-term financing spreads in the future can lead to higher long-term swap rates relative to bonds, thereby increasing the spread differential between the two level factors. Similarly, market expectations of interest rate movements in the near and distant future also affect the slope of the yield curve in the short and intermediate sectors. But, such effect is applicable equally to bonds and swaps. The net effect on the slope differential between the swap curve and the government curve may be trivial. In other words, the slope of swap spreads shouldn't be materially affected by the expectations of future interest rates.

Risk premium, also known as term premium, refers to the additional expected return fixed-income investors demand in order to compensate for the risk of holding longer term bonds (or swaps). Bond (swap) risk premium makes long-term interest rates higher than it would otherwise be. When fixed income investors demand more risk premium for holding longer term swaps than bonds of equal maturity, the long-term swap rates may be higher than it would other wise be. Bond (swap) risk premium also makes the yield curve steeper than it may otherwise be. When fixed income investors demand more risk premium for holding longer term swaps than for holding bonds of equal maturity, the term structure of swap rates becomes steeper than that of government bond yields, making the term structure of swap spreads upward sloping. Note the risk premium differential between swaps and bonds is driven by the supply and demand of government bonds verses the supply and demand of swaps.

Liquidity premium refers to the additional premium investors are willing to pay for bonds with liquidity. In general, on-the-run bonds are far more liquid than swaps with equivalent maturity. The liquidity premium can affect the pricing of all government bonds relative swaps, widening or narrowing the overall spread between swaps and government bonds. It can also affect a specific sector of the government bonds so that it may steepen or flatten the term structure of swap spreads. For example, if the 10-year on-the-runs commend more liquidity premium than the 2-year on-the-runs, then it may cause the bond yield curve to be flatter than it would otherwise be. The liquidity deferential in bonds and swaps may cause the swap curve to be steeper than the bond curve.

The empirical results we obtained are very encouraging. First, as expected, we find that the short-term financing spreads are positively correlated with the long-term swap spreads. However, short-term financing spreads can only explain a small fraction of the recent movements in long-term swap spreads. Long-term swap spreads peaked over 100 basis points in the aftermath of LTCM crisis in the Fall 1998, the Y2K liquidity crisis in the Summer 1999, and the Treasury buyback in the Spring 2000. In all these three events, we find that the short-term financing spreads only moved up moderately (from a mean of 20 basis points to a high of 40-50 basis points). Second, the risk premium played a significant role in determining the term structure of swap spreads. On average, the risk premium required by long-dated swaps was about 2.38% higher than that of long-term government bonds, making the overall swap curve steeper than the bond curve, especially in the long-end. During the 1998 financial crisis, the risk premium required by long-term bonds exceeded the risk premium required by long-term swaps by more than 3%, making the term structure of swap spreads humped in the long-end. In the Spring 2000 after US Treasury Department announced it buy-back program, the risk premium required by long-term bonds dropped significantly, and the risk premium required by long-dated swaps exceeded that of bonds by as much as 8%, making the term structure of swap spreads strictly upward sloping. Movements in the relative risk premiums between swaps and bonds contributed significantly to movements in the term structure of swap spreads in recent years. Finally, the liquidity premium also had a big impact on the term structure of swap spreads. While liquidity premium is difficult to quantify in general, we take a look at the yield spreads between the on-the-run and off-the-run bonds. Specialness commended by the on-the-run bonds serves a good indicator for assessing liquidity premium. For the sample period covered in our study (from 1995-2000), the time series of yield spreads between the on/off-the-runs have shown a clear sign of gradual widening in spreads between the on-the-run and off-the-run bonds. This coincided well with the overall widening of swap spreads since 1998. Relatively speaking, the 10-year sector saw a much larger increasing in spreads between the on-the-runs and the off-the-runs. This is also consistent with the fact that the term structure of swap spreads has been steepening (from 2-year to 10-year) ever since the LTCM crisis in 1998.

A number of papers have done work on swap spreads that are closely related to this paper. Nielsen and Ronn (1996) contain a model with a one-factor instantaneous spread process that values swaps as the appropriately discounted expected value of the instantaneous TED spreads (or financing spreads as defined in this paper). Collin-Dufresne and Solnik (1999) also argue that swaps shall be valued as default-free and use the concept to compare the LIBRO curve to the swap curve in a one-factor setting. Grinblatt (1999) introduces a concept of convenience yield for holding government bonds due to its liquidity advantage and models swap spreads as the present value of a flow of convenience yields. His approach is consistent with our liquidity premium argument. However, it ignores the value of short-term financing spreads as compensation for the extra cost that swap investors have to bear. Another interesting and related work is the recent paper by Liu, Longstaff and Mandell(2000) which also provides a multi-factor model to study the risk premium structure or market price of credit risk and estimates their model using on-the-run government bond yields and swap rates. Our approach differs from theirs in that we focus on explaining the term structure of swap spreads using the concept of risk premium and liquidity premium embedded in the off-the-run bonds and swaps, while their focus is on how to identify the short term liquidity premium from the default risk premium embedded in the on-the-run bond yields vs swap rates.

The majority of research work on swap spreads focus on proper modeling and measuring of default risk so that interest rate swaps can be valued with default risk being fairly accounted for. There are basically two classes of credit models that fall into this line of research. The first class of models takes a *structural* approach, modeling default events as the first-passage time some economic variables fall below or reach certain pre-specified triggering level, see Merton (1974), Black and Cox (1976), Cooper and Melllo (1991), Longstaff and Schwartz (1995), Leland (1994), and Leland and Toft (1995). Under this approach, defaults are endogenously triggered by the value of the underlying assets or firms, and are usually predictable. The second class of models takes a *reduce-form* approach, modeling default events as being triggered by an exogeneously specified jump process, see Das and Tufano (1996), Duffie and Huang (1996), and Duffie and Singleton (1999), Jarrow and Turnbull (1996), Jarrow, Lando, and Turnbull (1997), Madan and Unal (1994). Under the reduced-form approach, default events are typically unpredictable.

In addition to the theoretical work cited above, there is also a strand of empirical literature on the determinants of swap spreads. While these papers have shown some statistical relationships between swap spreads and level of interest rates or other economic variables, these relationships have not been found consistently over time, see Brown, Harlow, and Smith (1994), Chen and Selender (1994), Evans and Bales (1991), Minton (1993), Sun, Sundaresan and Wang (1993).

The rest of the paper is organized as follows. Section 2 provides some institutional materials on swap markets. Section 3 sets up the model and shows that the fair market swap spreads equal the present value of the short-term financing spreads properly amortized over the swap maturity. Section 4 provides models of term structures of swap rates using the 1-factor, 2-factor and 3-factor settings. Section 5 reports empirical results. We conclude the paper in Section 6.

2 Institutions and Recent Market Experiences

In this section we present institutional background on swap markets. Specifically, we explain the process by which short-term LIBOR rates are determined, and illustrate how short-term LIBOR rates may influence long-term swap rates or swap spreads. We then present historical evidence of swap spreads for three major currencies, USD, EUR and JPY. Finally, we present recent experiences of swap markets, including events such as the LTCM crisis in the Fall 1998, the Y2K liquidity crisis in the Summer 1999, and more recently, the US Treasury buy-backs in the Spring 2000.

2.1 Swap Contracts, LIBOR Fixing and Financing Spreads

Since the inception of swap contracts in 1981, markets for generic interest rate swaps, cross currency interest rate swaps, and related swap options (swaptions) have experienced tremendous growth in the past twenty years. Currently, swap contracts are popular for all major currencies, although USD, EUR and JPY represent the three most demanded currencies with swapping needs. The total notional amount of outstanding swaps is estimated around 49 trillion USD, a size that is much large than the size of global government bond markets combined. Indeed, for many of the overseas markets where either the size of government bonds is too small or the liquidity is not readily available for trading government bonds, swap contracts are more popular than government bonds. However, there is a major difference between owning a swap and owning a bond. When a bond position is sold, it leaves the trading book, whereas market participants tend to close out a swap position by entering a new offsetting swap contract. As a result, the notional amount of swaps outstanding may be significantly exaggerated.

When an investor enters a swap agreement as a fixed receiver in a generic fixed-for-floating swap, the investor is promised to receive from the counterparty a series of semi-annual fixed payments in exchange for paying the counterparty a series of semi-annual floating payments. While the fixed payments are determined at the outset of the swap agreement, the floating payments are to be determined at later dates, based on the six-month LIBOR rates prevailing at the beginning of each payment period. We have argued in the introduction that the spread between the six-month LIBOR rates and GC repo rates (or simply the short-term financing spread) represents an important factor in determining the term structure of swap spreads. Loosely speaking, the larger the short-term financing spreads, the larger the overall level of swap spreads. The term structure of swap spreads is closely related to market expectations of future short-term financing spreads.

The six-month LIBOR rate used for settling the floating leg of swap payments is based on a composite rate compiled by the British Banking Association (BBA) each day at 11:00am London time. The composite rate is calculated based on quotes provided by a basket of reference banks selected by BBA. A total of 16 banks are polled for their quotes of deposit rates from 1 week to 12 month. For each maturity, the highest four quotes and the lowest four quotes are dropped and the average of the rest of the eight banks is used as the official LIBOR fixing. For the three major currencies (USD, EUR and JPY), LIBOR rates are calculated based on the following set of reference banks:

- **USD**: BTM, Barclays, Citibank, CR Swiss, NetWest, BoA, Abbey, Norin BK Chase, Deutsche, Fuji, HSBC, Lloyds, West LB, RB Scot, RaboBank
- **EUR**: BTM, Barclays, Citibank, CR Swiss, NatWest, Chase, Lloyds, Halifax, BoA, Deutsche, MGT, HSBC, RBC, So Gen, UBS, West LB
- **JPY**: BTM, Sumitomo, Fuji, DKB, BOA, UBS, HSBC, NatWest, Chase, IBJ, Deutsche, Barclays, RaboBank, Norin, West LB, MGT, Sanwa

The deposit rates quoted by each reference bank incorporate the credit premium (over the default-free short-term interest rate) that investors may demand. By the way of its design, the six-month LIBOR rate, chosen as the benchmark index of the floating leg of a generic swap contract, serves as a market indicator of credit worthiness of the banking sector. Several points are worth emphasizing here. First, the credit quality among reference banks varies greatly, ranging from banks with AAA rating to banks with BBB rating. Second, there is a subset of brand-name banks serving as a part of the reference banks for all three major currencies. Third, in the past, banks with deteriorating credit quality in the LIBOR basket have been replaced by better banks. Thus, there is a tendency for LIBOR to be maintained at a constant and stable credit quality. Finally, since the credit quality for the basket banks is the average of the LIBOR rates offered by different banks, there is some diversification effect as well. In a perfect world in which the set of reference banks for all three major currencies are identical, short-term financing spreads (i.e., LIBOR over GC-repo) should be theoretically identical across different currencies. Consequently, swap spreads across different currencies ought to be theoretically identical, if the dynamics of short-term financing spreads is the only factor driving term structures of swap spreads. In such a world, variations in swap spreads may have to be explained by differences in investors' attitude towards risk or differences in liquidity premium and risk premium across difference countries.

Recent histories of short-term financing spreads (from 1998-2000) are reported in Figures 1(a), 1(b) and 1(c) for all three major currencies. The financing spreads for US are measured both as the difference between the 1-week dollar LIBOR rate and the Fed fund target rate and between the 1-week dollar LIBOR rate and the 1-week GC-repo rate. Note that Fed Fund target rate is usually used as the benchmark for pricing government bond repos. The financing spreads for Japan is calculated as the difference between the 1-week Yen LIBOR rate and the call rate, which is the Japanese equivalent of the Fed Fund rate and usually used as benchmark for pricing JGB repos. Finally, the financing spread for Germany is measured as the difference between the 1-week EURO LIBOR rate and the by-weekly repo target rate (which is currently set by the European Central Bank and previously set by the Bundesbank). All data are obtained from Bloomberg. Figures 1(a)-(c) show that short-term financing spreads have been well-behaved with a mean average of under 25 basis points. While there are a number of large spikes in these figures, the spikes appeared only at the term of the year, a well-understood phenomenon known as *the term effect*. Apart from those spikes, the short-term financing spread processes for all three currencies are not wildly volatile.

2.2 Historical Evidences

Historically, swap spreads have been volatile but reasonably well-behaved, exhibiting a strong degree of mean reversion. However, such historical relationship collapsed in 1998 in the aftermath of Russian default. Global swap spreads were blown out to a level that was never seen in recent years. Flight to quality and concern of systematic meltdown in the financial sector are forces behind the spread widening. While global swap spreads contracted in early 1999, they were blown out again in the second half of 1999 due to concerns over Y2K. In 2000, US swap spreads reached their historical highs due to government budget surpluses and US Treasury Department's decision to buy back 30 billion dollars worth of treasury bonds.

In US, swap rates are quoted simply as spreads over the on-the-run treasuries. As a result, swap spreads data can be obtained with ease. The following table provides a summary statistics on 10yr USD swap spreads in recent years.[1] These data are obtained from Bloomberg, and are reported in basis points.

	1996	1997	1998	1999	2000
Mean	37	40	63	84	101
Std	5	6	15	10	19
High	45	54	90	113	129
Low	31	32	47	66	72

Swap rates in Germany and Japan are quoted simply in terms of their actual rates. As such, swap spreads need to be calculated based on the difference between the swap rates and yields on maturity matched par government bonds. The yields on government bonds priced at par are usually obtained by fitting a discount curve to the prices of German Bunds or Japanese Government Bonds (JGBs). The par rates implied by the fitted discount curve are considered as the yields on government bonds priced at par. The following table provides a summary statistics of 10-year Euro/D-Mark swap spreads (over the German Bunds). These data are obtained from Bloomberg, and are reported in basis points.

[1] Swap spreads reported here are not adjusted for the on-the-run specialness. Had these spreads been adjusted for the specialness, we shall see a sizable reduction in these spreads.

	1996	1997	1998	1999	2000
Mean	–	24	36	40	42
Std	–	5	11	5	4
High	–	29	62	51	50
Low	–	15	20	31	36

The same statistical summary of the 10-year JPY swap spreads (over the Japan Government Bonds) is reported in the following table.

	1996	1997	1998	1999	2000
Mean	19	26	39	35	32
Std	5	5	6	10	3
High	27	35	55	51	36
Low	4	18	25	11	29

We note that while global swap spreads widened almost simultaneously in the Fall 1998, 10yr swap spreads in Japan collapsed to 11 basis points in early 1999 and resumed to its normal range of mid 30s by 1999. Swap spreads in Germany have been weakly correlated with US swap spreads, even though US swap spreads have maintained at their historical highs since 1998 while German swap spreads have resumed its normal range.

2.3 Global Swap Spreads Since 1998

We now take a closer look at term structures of swap spreads since the beginning of 1998. In the beginning of 1998, global swap spreads were at their normal level as shown in the following table:

	2yr	5yr	10yr	20/30yr
US	38	43	50	40
Germany	10	18	26	10
Japan	21	17	28	20

The term structure of swap spreads in US is typically humped, upward sloping from 0 to 10-year and downward sloping thereafter. The mean 10-year swap spreads level is around 40-50 basis points. Swap spreads in Germany and Japan are historically lower than their US counterpart. The term structure of swap spreads in Germany is also humped, while the term structure of swap spreads in Japan is decreasing (from 0-3yr)[2] and humped thereafter. Global swap spreads exploded in the Fall of 1998 in the aftermath of Russian default. Flight to quality and concern of systematic meltdown in the financial sector are forces behind the spreads widening. By Sep 1998, they reached a high level as shown below:

	2yr	5yr	10yr	20/30yr
US	73	78	97	94
Germany	17	30	54	27
Japan	23	33	55	55

[2] This phenomenon is due to the Japan Premium.

We note that term structure of swap spreads in Germany and Japan (between 0-10 year) became much steeper than it used to be.

While global swap spreads contracted in early 1999, they were blown out again in the second half of 1999 due to concerns over Y2K:

	2yr	5yr	10yr	20/30yr
US	62	86	102	108
Germany	16	17	46	23
Japan	27	33	60	32

Some of the real money investors refuse to lend out securities over the term of the century for fear of settlement risk. This triggered a serious short squeeze in the repo market. To alleviate liquidity concerns, central banks in various countries injected extra amount of liquidity into the system. In US, Fed enlarged the pool of securities eligible for repo transactions with the Fed. It also initiated liquidity options as an additional measure to secure Y2K funding. By historical standard, recent levels of swap spreads have been extraordinary wide. The last time swap spreads widened to the current level was in 1987 (stock market crash) and 1990 (S&L crisis). We note that this time the term structure of swap spreads in US became upward sloping.

Since the beginning of 2000, as US Treasury Department announced buying back 30 Billion dollars worth of US government bonds, swap spreads once again exploded to their highest level ever:

	2yr	5yr	10yr	20/30yr
US	74	94	129	140
Germany	17	24	37	35
Japan	20	22	30	20

The term structure of swap spreads in Germany as well Japan exhibits a humped shape, while the term structure of swap spreads in US is currently monotonically increasing, reflecting a strong demand for long-term bonds from real money investors. As a result, the risk premium demanded for holding long-term securities dropped significantly.

To summarize, we presented here recent experiences of swap spreads for three major currencies (USD, EUR and JPY). We illustrated the dynamic behavior of swap spreads in periods of normal economic environment and periods of financial crisis. We also showed a variety of term structures of swap spreads across different currencies under different economic environments (upward sloping (US), humped (US, Germany and Japan)). The analytical framework to be presented in the next section will help us analyze the various term structures of swap spreads observed in the marketplace.

3 Arbitrage-Free Valuation of Swap Spreads

In this section we formally setup our model suitable for constructing term structures of swap spreads. Taken as given is a probability space $(\Omega, P, \mathcal{F})$, where Ω denotes the states of nature, P the probability assessment for different states of nature, and $\mathcal{F}$ the information structure or filtration. Let r denote the stochastic process, defined on $(\Omega, P, \mathcal{F})$, for the instantaneous interest rate corresponding to the default-free government securities. In practice, this rate is best represented by the overnight interest rate charged on GC-repos (i.e., repurchase agreements for government bonds with general collateral). Let R be the stochastic process, also defined on $(\Omega, P, \mathcal{F})$, for the instantaneous interest rate corresponding to the LIBOR rates, which can be thought as the overnight deposit rate for banks with credit rating comparable to those in the reference banks of LIBOR fixing. Then,

$$\delta(t) = R(t) - r(t)$$

represents the instantaneous financing spread of LIBOR over GC repo. Given that banks are subject to the risk of default, it is expected that $\delta(t)$ is nonnegative.

Associated with the instantaneous interest rate processes r and R are the two discount curves that give the present value of one dollar to be paid at any future dates. Specifically, the discount curve corresponding to the government securities is given by

$$P_t(T) = \mathbf{E}_t^* \left[e^{-\int_t^{t+T} r(u)du} \right] \tag{1}$$

where $\mathbf{E}_t^*$ denotes an expectation under the equivalent martingale measure P^*, conditional on the information known at time t, and $P_t(T)$ denotes the price at time t of a discount bond that pays one unit at time $t+T$, see Harrison and Kreps (1979). Similarly, the discount curve associated with the LIBOR rates is given by[3]

$$Q_t(T) = \mathbf{E}_t^* \left[e^{-\int_t^{t+T} R(u)du} \right] = \mathbf{E}_t^* \left[e^{-\int_t^{t+T} (r(u)+\delta(u))du} \right] \tag{2}$$

The above formula is consistent with the reduce-form model of Duffie and Singleton (1999) for pricing defaultable securities. Indeed, if δ is interpreted as the product of hazard rate for default and fractional loss rate, then Q represents the price of a defaultable zero coupon bond. This interpretation is not necessary here for pricing swap rates, as all swaps considered in this paper are treated as default-free. However, we can use it to rationalize the current industry practice which treats the discount curve Q as the LIBOR rates quoted by the reference banks. Note that under our setting δ is observable from the financial market, which is extremely useful for model parameterization.

In the event that the spread process δ is independent of the default-free interest rate r,

$$Q_t(T) = P_t(T) \times \mathbf{E}_t^* \left[e^{-\int_t^{t+T} \delta(u)du} \right] = P_t(T) \times \Gamma_t(T)$$

where $\Gamma_t(T)$ is defined as

$$\Gamma_t(T) = \mathbf{E}_t^* \left[e^{-\int_t^{t+T} \delta(u)du} \right],$$

representing the credit adjustment implied by the LIBOR curve.

For purposes of understanding swap spreads, we take a look at various alternative forms of spreads that characterize more or less the same kind of credit differentials between LIBOR rates and government bond yields.

[3]At this point, we make a distinction between the discount curve corresponding to the LIBOR rates and the discount curve corresponding to the swap rates. This difference will be made clear below when we introduce the concept of par spreads and par swap spreads.

- (**Term Spreads**) Define the yields to maturity for the government and LIBOR curves as follows,

$$
\begin{aligned}
Y_T &= -\frac{1}{T}\ln P(T) \\
F_T &= -\frac{1}{T}\ln Q(T)
\end{aligned}
$$

where $P(T) = P_0(T)$ and $Q(T) = Q_0(T)$. The *term spread* for maturity T is defined as the spread between the yields on these two zeros, $F_T - Y_T$. In the event that the instantaneous financing spread process δ is independent of the instantaneous default-free interest rate process r, we have

$$
F_T - Y_T = -\frac{1}{T}\ln \mathbf{E}^*\left[e^{-\int_0^T \delta(u)du}\right] \tag{3}
$$

In this case, the term structure of term spreads is fully determined by the dynamic evolution of δ. For example, if δ is normally distributed, then

$$
F_T - Y_T = \frac{1}{T}\mathbf{E}^*\left[\int_0^T \delta(u)du\right] - \frac{1}{2T}\mathbf{Var}\left[\int_0^T \delta(u)du\right]
$$

In particular, if $\delta(u) = \overline{\delta}$, a constant, then

$$
F_T - Y_T = \overline{\delta}
$$

for every maturity T.

- (**Par Spreads**) An alternative way of looking at spreads between the government bond curve and the LIBOR curve is to compare the par rates implied by these two curves, defined here as *par spreads*. Specifically, we define par rates associated with the two zero curves as follows, for any maturity T,

$$
\begin{aligned}
Y_T &= \frac{2(1-P(T))}{\sum_{t=1}^{2T} P(t/2)} \\
F_T &= \frac{2(1-Q(T))}{\sum_{t=1}^{2T} Q(t/2)}
\end{aligned}
$$

The par spread between government bonds and LIBOR rates is defined as $F_T - Y_T$. While we tied the discount factors Q to LIBOR rates in our definition (for maturity up to 12 months), the industry practice is to construct Q based on market swap rates (for maturity over 12 months). Specifically, the discount factors Q (beyond 12 months) are constructed in such a way that the par rates implied by Q fit the market swap rates for all maturity. As such, par spreads defined here are called par swap spreads on the street when the discount factors Q are fitted to market swap rates, see Sundaresan (1991). We take a different approach here, as we will derive swap rates by assuming swaps are traded default-free.

- (**Par Swap Spreads**) We are now ready to define a par swap and its spread over the equivalent par bond. A (semi-annual) par swap is an exchange of a series of (semi-annual) floating payments for a series (semi-annual) fixed payments, where the floating legs are reset on a semi-annual basis. Specifically, the floating receiver receives, at the end of each payment period t ($t = 1, \cdots, 2T$), an amount equivalent to the six-month LIBOR interest, i.e.,

$$\frac{1}{Q_{t/2-\tau}(\tau)} - 1, \quad (\tau = \frac{1}{2})$$

whereas the fixed-side receiver receives a fixed amount of $\frac{F_T}{2}$. The notional of one dollar is exchanged at the maturity date T. Under the assumption of no-default, the fair market fixed rate for a par swap can be determined in such a way that the market value of all floating payments equals to the market value of all fixed payments:

$$\sum_{t=0}^{2T-1} \mathbf{E}^* \left[e^{-\int_0^{t/2+\tau} r(u)du} \times \left(\frac{1}{Q_{t/2}(\tau)} - 1 \right) \right] = \sum_{t=1}^{2T} \frac{F_T}{2} P(t\tau)$$

or equvalently,

$$\sum_{t=0}^{2T-1} \left(\mathbf{E}^* \left[e^{-\int_0^{t/2} r(u)du} \times \frac{e^{-\int_{t/2}^{t/2+\tau} r(u)du}}{\mathbf{E}_{t/2}[e^{-\int_{t/2}^{t/2+\tau} R(u)du}]} \right] - P(t/2+\tau) \right) = \frac{F_T}{2} \sum_{t=1}^{2T} P(t\tau) \qquad (4)$$

Let Y_T denote the yield of a government bond priced at par, then Y_T is Determined as

$$1 - P(T) = \frac{Y_T}{2} \sum_{t=1}^{2T} P(t\tau)$$

or equivalently,

$$\sum_{t=0}^{2T-1} \left(\mathbf{E}^* \left[e^{-\int_0^{t/2+\tau} r(u)du} \right] - P(t/2+\tau) \right) = \frac{Y_T}{2} \sum_{t=1}^{2T} P(t\tau) \qquad (5)$$

Subtracting (5) from (4), we obtain

$$\sum_{t=0}^{2T-1} \mathbf{E}^* \left[e^{-\int_0^{t/2} r(u)du} \times \left(\frac{\mathbf{E}_{t/2}[e^{-\int_{t/2}^{t/2+\tau} r(u)du}]}{\mathbf{E}_{t/2}[e^{-\int_{t/2}^{t/2+\tau} R(u)du}]} - 1 \right) \right] = \frac{F_T - Y_T}{2} \sum_{t=1}^{2T} P(t\tau)$$

The left-hand-side of the equation is roughly the present value of LIBOR-repo differentials or short-term financing spreads. Indeed, when δ is independent of r, we can simplify the above expression as

$$\sum_{t=0}^{2T-1} \mathbf{E}^* \left[e^{-\int_0^{t/2} r(u)du} \times \left(\frac{1}{\mathbf{E}_{t/2}[e^{-\int_{t/2}^{t/2+\tau} \delta(u)du}]} - 1 \right) \right] = \frac{F_T - Y_T}{2} \sum_{t=1}^{2T} P(t\tau) \qquad (6)$$

In particular, if $\delta(u) = \overline{\delta}$, a constant, then

$$\sum_{t=0}^{2T-1} \mathbf{E}^* \left[e^{-\int_0^{t/2} r(u)du} \times \left(e^{\delta\tau} - 1 \right) \right] = \frac{F_T - Y_T}{2} \sum_{t=1}^{2T} P(t\tau)$$

> In summary, par swap spreads can be interpreted as the present value of the stream of short-term financing spreads, properly amortized over the swap payment period. In particular, the positive spreads of swap rates over government bond yields can be attributed entirely to the positive spreads of LIBOR rates over GC-repo rates. Swap spreads, therefore, serve an indicator for the credit quality of banks involved in the LIBOR fixing, not the credit quality of the counterparty involved in the swap transaction.

We point out that although the above three forms of swap-related spreads are different by definition, the term structures of these spreads, i.e., spreads as a function of maturity, shall share more or less the same shape. In other words, if term spreads are upward sloping (downward sloping or humped), then par spreads as well as par swap spreads will also be upward sloping (downward sloping or humped).

We shall also point out that even though swaps are priced as default-free, there is strong correlation between corporate spreads and swap spreads. Loosely speaking, par spreads are essentially corporate spreads for the average banks involved in the LIBOR resetting. As such, the term structure of swap spreads is highly correlated with the term structure of corporate spreads. In practice, corporate spreads are defined for a combination of financial and non-financial firms while the credit quality of the reference banks in the LIBOR basket tends to be controlled by BBA which has the option to replace the bed names by good names. Consequently, corporate spreads are usually priced wider than swap spreads (see Collin-Dufresne and Solnik (1999)) and are highly correlated with swap spreads.

4 Term Structures of Swap Spreads: Models

In this section we construct term structures of swap spreads using the basic analytical framework outlined in the previous sections. Our eventual objective is to construct a 3-factor swap spread model that would allow us to examine the impact of short-term financing spreads, liquidity premium and risk premium on the term structure of swap spreads.[4] For ease of demonstrations, we start with a simple one-factor model, i.e. the one-factor Vasicek's model for the default-free instantaneous interest rate as well as for the instantaneous financing spread. We show how the default-free assumption of swaps generates the term structure of swap spreads. Next, we extend the one-factor model to a 2-factor framework so that we can examine the level and slope of swap spread curve and explore the notion of risk premium and liquidity premium. Finally, we introduce the 3-factor model that demonstrates how the dynamics of instantaneous financing spreads, risk premium and liquidity premium jointly determine the term structure of swap spreads.

4.1 Swap Spreads in a Single-Factor Setting

We first analyze term structures of swap spreads under the assumption of a one-factor Vasicek's model.[5] Specifically, we assume that the default-free instantaneous interest rate process r (i.e., the

[4]It is well-known that three factors are necessary in order to characterize movements of default-free yield curves, see Littleman and Scheinkman (1991). Similar conclusion can be confirmed from historical swap spreads.

[5]The use of Guassian models suffers from the usual blame for negative interest rates and negative swap spreads. Alternatively, we may choose a one-factor Cox, Ingersoll and Ross (1985)'s model as our basic interest rate process. Our results shall not be materially different.

overnight GC-repo rate) and the instantaneous financing spread process δ (i.e., the overnight LIBOR rate minus the overnight GC-repo rate) are determined by the following stochastic processes, see Vasicek (1977),

$$\begin{aligned} dr &= k_r(\overline{r} - r)dt + \sigma_r dw_r \\ d\delta &= k_\delta(\overline{\delta} - \delta)dt + \sigma_\delta dw_\delta \end{aligned}$$

where k_r and k_δ are (non-negative) constants for the mean-reversion of the two processes, σ_r and σ_δ (positive) constants for volatility, $\overline{r}$ and $\overline{\delta}$ (positive) constants for the long-run mean, w_r and w_s are two independent Brownian motions under the probability measure P.[6]

For the purpose of pricing under no-arbitrage, we seek an equivalent martingale measure P^* for r and δ so that under P^*,

$$w_r^* = w_r - \lambda_r dt, \quad w_\delta^* = w_\delta - \lambda_\delta dt$$

become two independent Brownian motions, where λ_r and λ_δ, respectively, are the market price of risk or the risk premium corresponding to the risk associated with each factor. Both risk premium parameters are usually positive. We assume for the purpose of this paper that λ_r and λ_δ are constants. Under the equivalent martingale measure P^*, we can write the stochastic dynamics of r and δ as

$$\begin{aligned} dr &= k_r(r^* - r)dt + \sigma_r dw_r^* \\ d\delta &= k_\delta(\delta^* - \delta)dt + \sigma_\delta dw_\delta^* \end{aligned}$$

where $k_r r^* = k_r \overline{r} + \lambda_r \sigma_r$ and $k_\delta \delta^* = k_\delta \overline{\delta} + \lambda_\delta \sigma_\delta$. Note that when $k_r > 0$, $r^* = \overline{r} + \lambda_r \sigma_r / k_r$. Thus, the long-run mean of r under P^* is adjusted upward by a constant term. Moreover, the larger the risk premium of the short rate factor, λ_r, the larger the long term mean r^*. For example, with $k_r = 0.5$, $\lambda_r = 0.2$ and $\sigma_r = 0.01$, the long-run mean of r is adjusted upward by about 40bp. Similar statement can be made for δ^*.

For ease of illustrations, we first consider the term structure of term spreads.

Proposition 1 *The term structure of term spreads under our 1-factor setting is given by*

$$F_T - Y_T = \psi(k_\delta T)\delta_0 + (1 - \psi(k_\delta T))\delta^* - \eta_T \tag{7}$$

where $\psi(x) \equiv \frac{1-e^{-x}}{x}$ *and* η_T *is a deterministic function of* T*, given by*

$$\eta_T = \frac{\sigma_\delta^2}{2Tk_\delta^2} \int_0^T (1 - e^{-k_\delta(T-u)})^2 du$$

A number of observations can be drawn from Proposition 1. The term structure of term spreads mean-reverts to their long run mean of δ^*. An increase in the initial short-term financing spreads increases the level of term spreads in the short-end, and thereby flattens the term structure of term spreads. An increase in the long-run mean of the short-term financing spread increases the level of

[6]Historically, correlation between changes in Treasury yields and changes in swap spreads has not been found to be either positive or negative consistently. As such, our assumption of zero correlation between w_r and w_δ is not unreasonable.

term spreads in the long-end, and thereby steepens the term structure of term spreads. When the current short-term financing spread is less (or greater) than its long run mean, the term structure of term spreads is *strictly* upward (or downward) sloping. The larger the volatility of short-term financing spreads, the smaller the overall level of term spreads. However, the volatility effect is almost negligible. For example, if $\sigma_\delta = 0.01$, $k_\delta = 0.45$ and $T = 10$, the volatility contribution to the 10yr term spread is a mere 1.5bp. Finally, the larger the risk premium λ_δ for δ, the larger the long-run mean δ^* and the steeper the term spread curve.

Figure 2 plots term structures of term spreads, par spreads and (default-free) par swap spreads for the following set of parameters:

$$k_r = 0.5,\ \sigma_r = 0.01,\ r(0) = 0.06,\ \overline{r} = 0.065,\ \lambda_r = 0.15$$
$$k_\delta = 0.5,\ \sigma_r = 0.0025,\ \delta(0) = 0.0025,\ \overline{\delta} = 0.0050,\ \lambda_\delta = 0.075$$

Par spreads are calculated based on the discount factors used for term spreads. In calculating par swap spreads, we make use of formula (6), which implies that,

$$F_T - Y_T = \frac{2}{\sum_{t=1}^{2T} P(t\tau)} \sum_{t=0}^{2T-1} P(t\tau)\Delta(t\tau)$$

where

$$\Delta(t) = \mathbf{E}^* \left[\frac{1}{\mathbf{E}_t[e^{-\int_t^{t+\tau} \delta(u)du}]} - 1 \right]$$

Under our assumption, $\delta(t)$ is normally distributed, allowing us to derive a close form expression for $\Delta(t)$:

$$\begin{aligned} \Delta(t) &= \mathbf{E}^* \left[e^{\tau\psi(k_\delta\tau)\delta(t) + \tau(1-\psi(k_\delta\tau))\delta^* - \gamma(\tau)} - 1 \right] \\ &= e^{\tau\psi(k_\delta\tau)\mathbf{E}^*[\delta(t)] + \frac{1}{2}\tau^2\psi(k_\delta\tau)^2 \mathbf{Var}^*[\delta(t)] + \tau(1-\psi(k_\delta\tau))\delta^* - \gamma(\tau)} - 1 \end{aligned}$$

where

$$\begin{aligned} \gamma(\tau) &= \frac{\sigma^2}{2k_\delta^2} \int_0^\tau (1 - e^{-k_\delta(\tau-u)})^2 du \\ \mathbf{E}^*[\delta(t)] &= e^{-k_\delta t}\delta_0 + (1 - e^{-k_\delta t})\delta^* \\ \mathbf{Var}^*[\delta(t)] &= \sigma_\delta^2 t \psi(2k_\delta t) \end{aligned}$$

We note that in Figure 2 the term structure of term spreads and the term structure of par spreads share much of the same shape. This is not surprising, as it is consistent with the observation that the spot yield curve usually has the same shape as the par yield curve. Figure 3 also indicates that the term structure of par spreads is almost exactly identical to the term structure of par swap spreads. The difference between the two, for the parameters chosen here, is less than 0.5 basis points. This is comforting, since it suggests that in the one-factor setting the current industry practice of fitting the par rates implied from the LIBOR zero curve to market swap rates is practically equivalent to fitting the default-free par swap rates to market swap rates.[7] Given the close relationship between

[7] While the difference between par spreads and par swap spreads may be larger in a multi-factor framework, the industry practice of fitting a few key points on the curve (such as fitting the 2yr, 10yr and 30yr swap rates) controls the difference between par and par swap spreads at all maturities.

term spreads and par spreads or par swap spreads, for the rest of the paper, we will focus primarily on the term structure of term spreads for our purposes of studying term structures of swap spreads.

It is useful to note that when the mean-reversion coefficient of the instantaneous financing spreads equals that of the default-free instantaneous interest rates (as in Figure 2), we can collapse the two state variables into a new state variable. This allows us to characterize the term structure of swap rates using the new state variable. Specifically, letting $R = r + \delta$, we have

$$dR = k_R(\overline{R} - R)dt + \sigma_R dw_R$$

where $k_R = k_r = k_\delta$, $\overline{R} = \overline{r} + \overline{\delta}$, $\sigma_R = \sqrt{\sigma_r^2 + \sigma_\delta^2}$, and $\sigma_R dw_R = \sigma_r dw_r + \sigma_\delta dw_\delta$. Under this setup, the instantaneous interest rate associated with swap rates can be equivalent modeled by a single-factor R, which has a starting value of $R_0 = r_0 + \delta_0$, a long-run mean of $\overline{R} = \overline{r} + \overline{\delta}$ under P and a long-run mean of $R^* = r^* + \delta^*$ under P^*, implying a risk premium of

$$\lambda_R = \frac{\lambda_r \sigma_r + \lambda_\delta \sigma_\delta}{\sqrt{\sigma_r^2 + \sigma_\delta^2}}$$

In Figure 2, given the parameters for processes r and δ, we have

$$k_R = 0.5,\ \sigma_R = 0.0103,\ R(0) = 0.0625,\ \overline{R} = 0.0700,\ \lambda_R = 0.1637$$

The usefulness of this approach will be demonstrated in the next two sub-sections when we introduce multi-factor models of swap spreads.

4.2 Swap Spreads in a 2-Factor Setting

One of the critical disadvantages of single-factor interest rate models is that these models can only generate term structures of interest rates and term structures of swap spreads that are either upward sloping or downward sloping. Yield curve shifts implied by single-factor models can only explain curve reshapings corresponding to either a steepening or a flattening move. It is empirically known that two or three independent factors are necessary in order to characterize the dynamic behavior of yield curve movements, e.g., Litterman and Scheinkman (1991). It is equally plausible that two or three factors are required to describe the dynamic behavior of swap spread movements.

In this section, we extend the single-factor swap spread model of the previous section into a 2-factor framework in which the instantaneous default-free interest rates as well as the instantaneous financing spreads are driven by two latent state variables. Specifically, we assume that the instantaneous short-rate is given by

$$r = x_1 + x_2$$

where the two stochastic latent factors are described by two mean-reverting processes:

$$\begin{aligned} dx_1 &= k_1(\overline{x}_1 - x_1)dt + \sigma_1 dw_1 \\ dx_2 &= k_2(\overline{x}_2 - x_2)dt + \sigma_2 dw_2 \end{aligned}$$

Here, k_1, k_2, $\overline{x}_1$, $\overline{x}_2$, σ_1 and σ_2 are (non-negative) constants, and w_1 and w_2 are two correlated Brownian motions with $\mathbf{Cov}(dw_1, dw_2) = \rho dt$ under P. With appropriately chosen parameters, we can relate factor x_1 to the overall interest rate level or long-term interest rate and factor x_2 to the slope factor or spread between the short- and long-term interest. The sum of x_1 and x_2 becomes the model-implied instantaneous short rate.[8] For the purpose of pricing, we again seek the equivalent martingale measure P^* so that under P^*,

$$w_1^* = w_1 - \lambda_1 t, \quad w_2^* = w_2 - \lambda_2 t$$

are two Brownian motions under P^* with $\mathbf{Cov}(dw_1^*, dw_2^*) = \rho dt$. The dynamics of x_1 and x_2 under P^* become

$$\begin{aligned} dx_1 &= k_1(x_1^* - x_1)dt + \sigma_1 dw_1^* \\ dx_2 &= k_1(x_2^* - x_2)dt + \sigma_2 dw_2^* \end{aligned}$$

where $k_1 x_1^* = k_1 \overline{x}_1 + \lambda_1 \sigma_1$ and $k_2 x_2^* = k_2 \overline{x}_2 + \lambda_2 \sigma_2$.

Similarly, we can extend our single-factor financing spread process into a two-factor setting such that the process for the instantaneous financing spreads is given by

$$\delta = \delta_1 + \delta_2$$

where δ_1 and δ_2 are two mean-reverting processes (independent of w_1 and w_2):

$$\begin{aligned} d\delta_1 &= k_{\delta_1}(\overline{\delta}_1 - \delta_1)dt + \sigma_{\delta_1} dw_{\delta_1} \\ d\delta_2 &= k_{\delta_2}(\overline{\delta}_2 - \delta_2)dt + \sigma_{\delta_2} dw_{\delta_2} \end{aligned}$$

with $\mathbf{Cov}(dw_{\delta_1}, dw_{\delta_2}) = \rho_\delta dt$, and k_{δ_1}, k_{δ_2}, $\overline{\delta}_1$, $\overline{\delta}_2$, σ_{δ_1} and σ_{δ_2} are constants. With properly chosen parameters, δ_1 can be related to the level of swap spreads, while δ_2 can be interpreted as the slope of swap spreads. Together, $\delta_1 + \delta_2$ defines the implied swap spread in the short end or the instantaneous financing spread. We assume that under the equivalent martingale measure P^*,

$$w_{\delta_1}^* = w_{\delta_1} - \lambda_{\delta_1} t, \quad w_{\delta_2}^* = w_{\delta_2} - \lambda_{\delta_2} t$$

are two Brownian motions with the same correlation structure and the dynamics of δ_1 and δ_2 under P^* become

$$\begin{aligned} d\delta_1 &= k_{\delta_1}(\delta_1^* - \delta_1)dt + \sigma_{\delta_1} dw_{\delta_1}^* \\ d\delta_2 &= k_{\delta_2}(\delta_2^* - \delta_2)dt + \sigma_{\delta_2} dw_{\delta_2}^* \end{aligned}$$

where $k_{\delta_1}\delta_1^* = k_{\delta_1}\overline{\delta}_1 + \lambda_{\delta_1}\sigma_{\delta_1}$ and $k_{\delta_2}\delta_2^* = k_{\delta_2}\overline{\delta}_2 + \lambda_{\delta_2}\sigma_{\delta_2}$.

We present our main result on the term structure of swap spreads in the following proposition.

Proposition 2 *The term structure of term spreads under our 2-factor setting is given by*

$$F_T - Y_T = \psi(k_{\delta_1}T)\delta_1(0) + \psi(k_{\delta_2}T)\delta_2(0) + (1 - \psi(k_{\delta_1}T))\delta_1^* + (1 - \psi(k_{\delta_2}T))\delta_2^* - \eta_T \qquad (8)$$

[8] In this 2-factor setup, the factor which has a slower speed of mean-reversion corresponds typically to the level factor; whereas the factor with a faster speed of mean-reversion is likely the slope factor.

where η_T is a deterministic function of T, given by

$$\begin{aligned}\eta_T &= \frac{\sigma_{\delta_1}^2}{2Tk_{\delta_1}^2}\int_0^T (1-e^{-k_{\delta_1}(T-u)})^2 du + \frac{\sigma_{\delta_2}^2}{2Tk_{\delta_2}^2}\int_0^T (1-e^{-k_{\delta_2}(T-u)})^2 du \\ &\quad + \frac{\rho_\delta \sigma_{\delta_1}\sigma_{\delta_2}}{Tk_{\delta_1}k_{\delta_2}}\int_0^T (1-e^{-k_{\delta_1}(T-u)})(1-e^{-k_{\delta_2}(T-u)})du\end{aligned}$$

We can draw a number of conclusions based on Proposition 2. To make our discussions more specific, we assume that $k_{\delta_1} << k_{\delta_2}$ and k_{δ_1} is close to zero so that δ_1 is highly correlated with the level of swap spreads while δ_2 is highly correlated with the slope of swap spreads. We claim that the term structure of term spreads mean-reverts to their long run mean of $\delta_1^* + \delta_2^*$. An increase in $\delta_2(0)$ (i.e., the slope factor) increases the term spreads in the short end, and thereby flatten the term structure of term spreads. Similarly, an increase in $\delta_1(0)$ has the effect of increasing the overall level of term spreads and slightly flatten the spread curve. An increase in δ_1^* increases the level of term spreads in the long-end, and thereby steepen the term structure of term spreads. Similarly, an increase in δ_2^* has the effect of increasing the overall level of term spreads and slightly steepen the spread curve. The term structure of term spreads can be *strictly* upward, downward sloping, or humped.

Figure 3 plots the term structure of term spreads for the following set of parameters:

$$\begin{aligned}&k_1 = 0.001,\ \sigma_1 = 0.010,\ x_1(0) = 0.06337,\ \overline{x}_1 = 0.06,\ \lambda_1 = 0.15 \\ &k_2 = 0.500,\ \sigma_2 = 0.015,\ x_2(0) = -0.0043,\ \overline{x}_2 = 0.0,\ \lambda_2 = 0.0,\ \rho = 0.0, \\ &k_{\delta_1} = 0.001,\ \sigma_{\delta_1} = 0.0050,\ \delta_1(0) = 0.0060,\ \overline{\delta}_1 = 0.0050,\ \lambda_{\delta_1} = 0.075 \\ &k_{\delta_2} = 0.500,\ \sigma_{\delta_2} = 0.0075,\ \delta_2(0) = -0.0045,\ \overline{\delta}_2 = 0.0,\ \lambda_{\delta_2} = 0.0,\ \rho_\delta = 0.0\end{aligned}$$

In contrast to single factor swap spread models, the 2-factor model can accommodate a variety of curve shapes including humps, i.e., upward sloping for the first part of the curve and downward sloping for the second part of the curve.

We point out that under the setting of Proposition 2, the term structure of default-free interest rates is given by

$$Y_T = \psi(k_1T)x_1(0) + \psi(k_2T)x_2(0) + (1-\psi(k_1T))x_1^* + (1-\psi(k_2T))x_2^* - \epsilon_T$$

where

$$\begin{aligned}\epsilon_T &= \frac{\sigma_1^2}{2Tk_1^2}\int_0^T (1-e^{-k_1(T-u)})^2 du + \frac{\sigma_2^2}{2Tk_2^2}\int_0^T (1-e^{-k_2(T-u)})^2 du \\ &\quad + \frac{\rho\sigma_1\sigma_2}{Tk_1k_2}\int_0^T (1-e^{-k_1(T-u)})(1-e^{-k_2(T-u)})du\end{aligned}$$

If we are given market yields of default-free bonds and swaps for certain maturity sectors, e.g., 2-year and 10-year, we can fit the above formula to the default-free bond yields at 2-year and 10-year, thereby obtaining the realization of $x_1(0)$ and $x_2(0)$. Once we find the $x_1(0)$ and $x_2(0)$, we can fit the formula in Proposition 2 to the swap rates at 2-year and 10-year, thereby obtaining the realization of $\delta_1(0)$ and $\delta_2(0)$. The following table illustrates a fit of bond and swap yields, based on market data on April 28, 2000 and the same set of model parameters as in Figure 3:

	2-yr	10-yr	$x_1(0)/\delta_1(0)$	$x_2(0)/\delta_2(0)$	$x_1(0)+x_2(0)$
Bonds	0.06676	0.06212	0.05254	0.02034	0.07288
Swaps	0.07299	0.07381	0.06493	0.01007	0.07500
Spreads (BP)	62.3	116.9	123.9	-103.3	20.6

The starting values of the two bond factors are 5.254% and 2.034 %, respectively, implying an instantaneous short-rate of 7.288%. The starting values of the two swap spread factors are 123.9 bp and -103.3 bp, respectively, implying an instantaneous financing spread of 20.6 bp. The instantaneous short-rate for the swap curve is 7.5%.

Similar to the single-factor case, we explore the special case when $k_1 = k_{\delta_1}$ and $k_2 = k_{\delta_2}$. Define $X_1 = x_1 + \delta_1$ and $X_2 = x_2 + \delta_2$. It is easily verified that

$$\begin{aligned} dX_1 &= k_1(\overline{X}_1 - X_1)dt + \sigma_{X_1} dw_{X_1} \\ dX_2 &= k_2(\overline{X}_2 - X_2)dt + \sigma_{X_2} dw_{X_2} \end{aligned}$$

where $\overline{X}_1 = \overline{x}_1 + \overline{\delta}_1$, $\overline{X}_2 = \overline{x}_2 + \overline{\delta}_2$, $\sigma_{X_1} = \sqrt{\sigma_1^2 + \sigma_{\delta_1}^2}$ and $\sigma_{X_2} = \sqrt{\sigma_2^2 + \sigma_{\delta_2}^2}$. Under the equivalent martingale measure P^*, we have

$$\begin{aligned} dX_1 &= k_1(X^* - x)dt + \sigma_{X_1} dw^*_{X_1} \\ dX_2 &= k_2(Y^* - Y)dt + \sigma_{X_2} dw^*_{X_2} \end{aligned}$$

where $X_1^* = x_1^* + \delta_1^*$ and $X_2^* = x_2^* + \delta_2^*$. In summary, similar to our 2-factor model for the government curve, we obtain a 2-factor model for the swap curve with exactly the same stochastic structure as for bonds. For the parameters given in Figure 3, we have

$$\begin{aligned} &k_1 = 0.001,\ \sigma_{X_1} = 0.0118,\ X_1(0) = 0.06940,\ \overline{X}_1 = 0.065,\ \lambda_X = 0.1677 \\ &k_2 = 0.500,\ \sigma_{X_2} = 0.01677,\ X_2(0) = -0.00884,\ \overline{X}_2 = 0.0,\ \lambda_Y = 0.0 \end{aligned}$$

The parameters in Figure 3 were chosen so that swaps and bonds can be fitted separately. We note when the term structure of swap spreads is upward sloping, a typical fit to bond yields and swap rates usually gives rise to $\delta_1(0) = X_1(0) - x_1(0) > 0$ while $\delta_2(0) = X_2(0) - x_2(0) < 0$. In other words, the implied level or long rate based on the swap curve is higher than that implied from the government curve, while the implied the slope factor based on the swap curve is steeper than that of the government curve. The processes δ_1 and δ_2 can be interpreted as follows.

Interpreting δ_1:

δ_1 represents the spread differential between long-term swap rates and long-term government bond yields. We argue that this spread is determined by the following two economic factors: 1) market expectations of future short-term financing spread and 2) risk premiums required for holding long-term bonds and long-dated swaps.[9]

- Market expectations of interest rates in the future affect long-term swap rates as well as long-term bond yields. Expectations of higher rates in the future generally result in higher

[9] In this paper, we only study the risk premium required for the level factors x_1 and X_1 while ignoring the risk premium associated with the slope factors x_2 and X_2.

long-term swap rates as well as higher long-term bond yields. However, the net effect on the level differential between swaps and bonds shall be negligible. However, expectations of future short-term financing spreads affect only the swap rates. Expectations of higher short-term financing spreads in the future can lead to higher long-term swap rates relative to bonds, thereby increasing the spread differential between the two level factors.

- The risk premium, also known as term premium, refers to the additional expected return fixed-income investors demand in order to compensate for the risk of holding longer term bonds (or swaps). Bond (swap) risk premiums make long-term interest rates higher than it would otherwise be. When fixed income investors demand more risk premium for holding longer term swaps than bonds of equal maturity, the long-term swap rates may be higher than it would other wise be. The risk premium differential between swaps and bonds is affected by the supply and the demand of government bonds vs the supply and the demand of swaps.

Interpreting δ_2:

δ_2 defines the slope differential between the swap curve and the bond curve. We argue that the slope differential characterizes the risk premium differential as well as the liquidity premium differential between swaps and government bonds. Note that the slope of the term structure of default-free interest rates is affected by two economic forces: 1) market expectation of future interest rates and 2) the term premium required for holding longer term bonds and the liquidity premium commended by government bonds (e.g., on-the-runs).

- Market expectations of interest rate movements in the near and distant future affect the slope of the yield curve in the short and intermediate sectors. But, such effect is applicable to both swaps and bonds. The net effect on the slope differential between the swap curve and the government curve may be trivial. In other words, the slope of swap spreads shouldn't be materially affected by the expectations of future interest rates.

- The risk premium and liquidity premium affect the slope of yield curve in a significant way.
  - Bond (swap) risk premiums make the yield curve steeper than it may otherwise be. When fixed income investors demand more risk premium for holding longer term swaps than for holding bonds of equal maturity, the term structure of swap rates becomes steeper than that of government bond yields, especially at the long-end.
  - The liquidity premium refers to the additional price investors are willing to pay for bonds with liquidity. In general, on-the-run bonds are far more liquid than swaps with equivalent maturity. If the 10 year on-the-runs commend more (or less) liquidity premium than the 2-year on-the-runs, then it may cause the bond yield curve to be flatter (or steeper) than it would otherwise be. The liquidity deferential in bonds and swaps may cause the swap curve to be steeper (or less steeper) than the bond curve, which implies a steeper (or flatter) term structure of swap spreads.
  - It is known that a large population of fixed-income money managers are mandated to invest a significant proportion of their assets in government bonds. In contrast, fixed-income money managers are typically forbidden from using swaps as an investment tool.

> The natural demand for swaps comes from corporations or dealers involving in long-term financing or hedging. Globally, the 10-year sector has traditionally been the most demanded sector among all maturity sectors, creating additional liquidity premium for the sector while causing the term structure of swap spreads more likely to be upward sloping.

In summary, the level factor δ_1 is affected by market expectations of financing spreads in the long run. Both δ_1 and δ_2 are influenced by the risk premium and the liquidity premium.

4.3 Swap Spreads in a 3-Factor Setting

In constructing term structures of swap spreads using the 2-factor model proposed in the previous section, we took the approach to fit bond yields and swap rates for maturities at 2-year and 10-year. Unfortunately, the instantaneous interest rates implied by fitting the 2-year and 10-year sectors may be ill-behaved and not consistent with the actual overnight rates observed in the market. Consequently, instantaneous finance spreads, upon which the term structure of swap spreads is constructed, may not be consistent with the actual financing spreads observed in the market. To resolve this problem, we extend our 2-factor model to include an additional factor which tracks the dynamic behavior of the instantaneous short rates.

Formally, we consider a 3-factor default-free term structure model under which the dynamics of the first two factors, economic latent variables x_1 and x_2, is given by

$$\begin{aligned} dx_1 &= k_1(\overline{x}_1 - x_1)dt + \sigma_1 dw_1 \\ dx_2 &= k_2(\overline{x}_2 - x_2)dt + \sigma_2 dw_2 \end{aligned}$$

where all parameters are the same as those defined in the previous section, while w_1 and w_2 are two correlated Brownian motions. As in the previous section, under suitable choice of the mean reversion parameters k_1 and k_2, the first factor x_1 tracts movements of long-term interest rates, while the second factor x_2 tracts movements of yield curve slope. Together, $\Theta = x_1 + x_2$ defines the model implied short-rate, which can be interpreted as the "equilibrium" short rate necessary to accommodate the prevailing state of the economy. For example, if the current state of the economy is such that it is heading towards recession, then the model implied short-rate may necessitate a level that is lower than the current market short-rate.

Alternatively, we interpret Θ as a *theoretical target rate*. Specifically, we model the evolution of the instantaneous default-free interest rate as a policy rule controlled by the central bank,

$$dr = k_r(\Theta - r)dt + \sigma_r dw_r$$

where k_r and σ_r are constants, and w_r is a Brownian motion independent of w_1 and w_2. That is, the instantaneous short-rate, controlled by the monetary policy authority, mean-reverts to the equilibrium target rate, subject to a random noise due possibly to transitory imbalance in the money market.[10] If the current short-term interest rate is below the equilibrium target rate, then there is a tendency that the central bank may hike the short-term interest rate to gradually catch

[10]See Piazessi (2000) for a similar specification with the central bank controlling short-term interest rates under a jump-diffusion setting.

up with the equilibrium short-term target rate. Similarly, if the current short-rate is higher than the equilibrium target rate, then it is more likely that the central bank will guide the short-rate gradually down towards its equilibrium target.

For purposes of pricing securities under the no-arbitrage condition, we assume there exists an equivalent martingale measure P^* under which,

$$w_i^* = w_i - \lambda_i t, \quad i = 1, 2, r$$

are converted into standard Brownian motions. In other words, under P^*, the dynamics of x_1, x_2 and r take the following form:

$$\begin{aligned} dx_1 &= [k_1(\overline{x}_1 - x_1) + \lambda_1\sigma_1]dt + \sigma_1 dw_1^* \\ dx_2 &= [k_2(\overline{x}_2 - x_2) + \lambda_2\sigma_2]dt + \sigma_2 dw_2^* \\ dr &= [k_r(\Theta - r) + \lambda_r\sigma_r]dt + \sigma_r dw_r^* \end{aligned}$$

Moreover, the no-arbitrage price at time t of a zero coupon bond maturing at time T can be determined as

$$P_t(T) = \mathbf{E}_t^* \left[e^{-\int_t^T r(s)ds} \right]$$

We need the following lemma for constructing term structures of government bond yields, swap rates and swap spreads.

Lemma 1 *Under the 3-factor setting specified above, the yield to maturity of default-free zero coupon bonds has the following closed form formula:*

$$Y_T = A_T r(0) + B_T x_1(0) + C_T x_2(0) + D_T \tag{9}$$

where

$$\begin{aligned} A_T &= \psi(k_r T) \\ B_T &= \frac{k_r}{k_r - k_1}(\psi(k_1 T) - \psi(k_r T)) \\ C_T &= \frac{k_r}{k_r - k_2}(\psi(k_2 T) - \psi(k_r T)) \\ D_T &= \lambda_r \sigma_r T H(k_r T) + \frac{k_r(k_1\overline{x}_1 + \lambda_1\sigma_1)T}{k_r - k_1}(H(k_1 T) - H(k_r T)) \\ &\quad + \frac{k_r(k_2\overline{x}_2 + \lambda_2\sigma_2)T}{k_r - k_2}(H(k_2 T) - H(k_r T)) - \epsilon_T \\ \epsilon_T &= \frac{\sigma_1^2 k_r^2}{2T(k_r - k_1)^2} \int_0^T (T-u)^2(\psi(k_1(T-u)) - \psi(k_r(T-u)))^2 du \\ &\quad + \frac{\sigma_2^2 k_r^2}{2T(k_r - k_2)^2} \int_0^T (T-u)^2(\psi(k_2(T-u)) - \psi(k_r(T-u)))^2 du \\ &\quad + \frac{\rho\sigma_1\sigma_2 k_r^2}{T(k_r - k_1)(k_r - k_2)} \int_0^T (T-u)^2(\psi(k_1(T-u)) - \psi(k_r(T-u)))(\psi(k_2(T-u)) - \psi(k_r(T-u)))du \\ &\quad + \frac{\sigma_r^2}{2T} \int_0^T (T-u)^2 \psi(k_r(T-u))^2 du \end{aligned}$$

with $H(x) = \frac{1-\psi(x)}{x}$ *for* $x > 0$ *and* $H(0) = \frac{1}{2}$.

Lemma 1 provides a complete solution to the term structure of zero coupon bonds. Specifically, given initial values of x_1, x_2, and r, we can generate yields to maturity for zero coupon bonds of all maturities. Formula (9) suggests that yields to maturity are linearly related to the three interest rate factors. Under the assumption that $k_1 << k_2$ and k_1 is close to zero, we can make the following observations. First, an increase in $x_1(0)$ increases the overall level of interest rates (especially in the long-end) while having no material impact on the short-end of the curve. Second, an increase in $x_2(0)$ flattens the yield curve. Finally, an increase in $r(0)$ sharply increases the short-end of the curve while having no material impact on the long-end of the curve.

Our 3-factor model also suggests that yields to maturity are linearly related to the three risk premium parameters, i.e., λ_1, λ_2 and λ_r. Each risk premium plays the role of steepening the yield curve for each of its relevant maturity sector. In other words, each demands additional yield compensation for risk bearing. In fact, it is easily shown that that for any zero coupon bond, its excess return can be explained by

$$\mathbf{E}\left[\frac{dP}{P}\right] - r = \lambda_r(\sigma_r T A_T) + \lambda_1(\sigma_1 T B_T) + \lambda_2(\sigma_2 T C_T)$$

Based on the loadings of A_T, B_T and C_T, bonds with very long maturity are primarily priced by λ_1 while bonds with very short maturity are primarily priced by λ_r, and the intermediate sector is priced by λ_2.

Given our experience with 1- and 2-factor models, we specify the dynamics of short-term financing spreads δ using the same analytical structure. Specifically, under the probability measure P, we assume that δ is governed by the following dynamic system,

$$\begin{aligned}
d\delta_1 &= k_{\delta_1}(\bar{\delta}_1 - \delta_1)dt + \sigma_{\delta_1} dw_{\delta_1} \\
d\delta_2 &= k_{\delta_2}(\bar{\delta}_2 - \delta_2)dt + \sigma_{\delta_2} dw_{\delta_2} \\
d\delta &= k_{\delta}(\delta_1 + \delta_2 - \delta)dt + \sigma_{\delta} dw_{\delta}
\end{aligned}$$

With a suitable choice of parameters, δ_1 represents the overall level of swap spreads (level factor) while δ_2 represents the slope of swap spread curve (slope factor). The sum of δ_1 and δ_2 is the model implied short-term financing spread, i.e., the implied financing spread that is consistent with the term structure of swap spreads in the intermediate and long maturity sectors. Finally, δ itself corresponds to the instantaneous financing spread. The 3-factor structure imposed here allows us to capture the over-all shape of the swap spread curve, including the short-end as well as the long-end. Moreover, we can study the dynamic behavior of the swap spread curve by tracking the dynamic behavior of each of the three swap spread factors. For the dynamics of δ under the equivalent martingale measure P^*, we assume that

$$\begin{aligned}
d\delta_1 &= [k_{\delta_1}(\delta_1 - \delta_1) + \lambda_{\delta_1}\sigma_{\delta_1}]dt + \sigma_{\delta_1} dw^*_{\delta_1} \\
d\delta_2 &= [k_{\delta_2}(\delta_2 - \delta_2) + \lambda_{\delta_2}\sigma_{\delta_2}]dt + \sigma_{\delta_2} dw^*_{\delta_2} \\
d\delta &= [k_{\delta}(\delta_1 + \delta_2 - \delta) + \lambda_{\delta}\sigma_{\delta}]dt + \sigma_{\delta} dw^*_{\delta}
\end{aligned}$$

where

$$w^*_{\delta_1} = w_{\delta_1} - \lambda_{\delta_1} t, \quad w^*_{\delta_2} = w_{\delta_2} - \lambda_{\delta_2} t, \quad w^*_{\delta} = w_{\delta} - \lambda_{\delta} t$$

are independent Brownian motions under P^*, and λ_{δ_1}, λ_{δ_2} and λ_δ are constant parameters for risk premiums. The following proposition constructs the term structure of term spreads using the formula derived in Lemma 1.

Proposition 3 *The term structure of term spreads under our 3-factor setting is given by*

$$F_T - Y_T = a_T\delta(0) + b_T\delta_1(0) + c_T\delta_2(0) + d_T \tag{10}$$

where a_T, b_T, c_T, and d_T are the same as A_T, B_T, C_T and D_T in Lemma 1 with all parameters for x_1, x_2 and r replaced by those of δ_1, δ_2 and δ.

Equation (10) provides the analytical formula necessary for constructing term structures of swap spreads. However, if we make the convenient assumption that the mean-reversion parameters corresponding to the 3-term structure factors in the swap space are identical to those of the bond space, i.e., $k_1 = k_{\delta_1}$, $k_2 = k_{\delta_2}$ and $k_3 = k_\delta$, then we can determine swap rates in a simple 3-factor framework as well. Specifically, the 3-factor model for swaps takes the following form:

$$\begin{aligned} dX_1 &= k_1(\overline{X}_1 - X_1)dt + \sigma_{X_1}dw_{X_1} \\ dX_2 &= k_2(\overline{X}_2 - X_2)dt + \sigma_{X_2}dw_{X_2} \\ dR &= k_3(X_1 + X_2 - R)dt + \sigma_R dw_R \end{aligned}$$

where $\overline{X}_1$, $\overline{X}_2$, σ_{X_1}, σ_{X_2} and σ_R are related to those parameters defining the default-free interest rate process and the short-term financing spread process as follows,

$$\overline{X}_1 = \overline{x}_1 + \overline{\delta}_1, \quad \overline{X}_2 = \overline{x}_2 + \overline{\delta}_2$$

$$\sigma_{X_1} = \sqrt{\sigma_{x_1}^2 + \sigma_{\delta_1}^2}, \quad \sigma_{X_2} = \sqrt{\sigma_{x_2}^2 + \sigma_{\delta_2}^2}, \quad \sigma_R = \sqrt{\sigma_r^2 + \sigma_\delta^2}$$

and the risk premium parameters for X_1, X_2 and R are determined by

$$\lambda_{X_1} = \frac{\lambda_{x_1}\sigma_{x_1} + \lambda_{\delta_1}\sigma_{\delta_1}}{\sigma_{X_1}}, \quad \lambda_{X_2} = \frac{\lambda_{x_2}\sigma_{x_2} + \lambda_{\delta_2}\sigma_{\delta_2}}{\sigma_{X_2}}, \quad \lambda_R = \frac{\lambda_R\sigma_R + \lambda_{\delta_R}\sigma_{\delta_R}}{\sigma_R}.$$

5 Term Structure of Swap Spreads: Empirical Results

In this section we turn to the empirical implementation of the 3-factor model outlined in the previous section. Our first task is to parameterize our model. This is achieved by adopting some of the existing empirical work. Given the parameterization, we then fit our model to historical bond yields and swaps rates. This allows us to examine empirically the impact on swap spreads of the short-term financing spreads, liquidity premium, and risk premium.

Our parameterization takes two steps. In the first step, following the empirical work of Duffie and Singleton (1997), Babbs and Nowman (1999), and Mendev-Vives and Naik (2000), we set $k_1 = 0$ and $k_2 = 0.5$, i.e., x_1 follows a random walk while x_2 is mean-reverting with a half-life of 1.4 years. In all these three studies, the mean reversion of the first factor is near zero while the mean reversion of the second factor is around 0.5. For k_3, which is a parameter characterizing the central bank's responsiveness to deviation from the market required short-rate, we assign it a value of 1.5, implying a half-life of about 6 months.[11] Similarly, we set $k_{\delta_1} = 0$, $k_{\delta_2} = 0.5$ and $k_\delta = 1.5$. Given that $k_1 = 0$, we can assume without loss of generality that $\overline{x}_1 = \overline{x}_2 = 0$ and $\overline{\delta}_1 = \overline{\delta}_2 = 0$ since all constant terms can be absorbed into the initial values of x_1 and δ_1. For specification of the volatility parameters, we again borrow the empirical estimates from Duffie and Singleton (1997), setting $\sigma_1 = 0.009$, $\sigma_2 = 0.014$ and $\rho = 0$, which is consistent with their estimates of the instantaneous volatility (of the square-root processes evaluated at the means).[12] For σ_3, we assign a value of 0.0050 or 50 basis points, which is consistent with that of Mendev-Vives and Naik (2000). For the spread volatility parameters, we set $\sigma_{\delta_1} = 0.0025$, $\sigma_{\delta_2} = 0.0035$, $\rho_\delta = 0$ and $\sigma_\delta = 0.0025$. This implies that $\sigma_{X_1} = 0.009341$, $\sigma_{X_2} = 0.014431$, and $\sigma_R = 0.00559$. Finally, for the risk premium parameters, we assume that $\lambda_2 = \lambda_r = 0$ and $\lambda_{\delta_2} = \lambda_{\delta_3} = 0$. The preliminary study by Meandev-Vives and Naik (2000) indicates that both λ_2 and λ_r are not equal to zero, while Longstaff (2000), using a more carefully chosen proxy for the instantaneous short rates, demonstrates that the term premium at the short-end is statistically insignificant. We choose to set these parameters to zero as we are not confident about their true values. In setting $\lambda_2 = \lambda_r = 0$, we can focus exclusively on the risk premium associated with holding long-term securities.

To generate our model yield curves that fit historical rates, we take as given the market yields for maturities at the short-end (the instantaneous short-rate), the 2-year sector, the 10-year sector and the long-end (the 30-year sector). While we have previously fitted x_1 and x_2 to rates at the 2-year and 10-year sectors in our 2-factor framework, we now expand our fitting of r to the short-end as well. In addition, we take the 30-year market rate as a benchmark for assessing the market-required risk premium for holding long-term government bonds or swaps. To summarize, for each given market yield curve, we search for a risk premium parameter λ_1 (or λ_{X_1}) so that the model yield curve generated by Lemma 1 fits the market yield curve jointly at the short-end, 2-year, 10-year and 30-year sectors.[13]

In the second step of parameterization, we modify the volatility and correlation parameters by the sample volatilities and sample correlations corresponding to the historical factor realizations obtained in the first step. Ideally, if the volatility and correlation parameters chosen for the first step are true, then the sample volatilities and sample correlations implied from the historical factors shall be reasonably close. By enforcing this second step of iteration, we can have a better chance to find the true volatility and correlation parameters. For fitting bond yields, we re-set σ_1 to 0.0125 and σ_2 to 0.0150 while allowing a negative correlation of -0.30 between x_1 and x_2. For fitting swap rates, we re-set σ_{X_1} to 0.0135 and σ_{X_2} to 0.0168 while allowing a negative correlation of -0.23 between X_1 and X_2. Finally, we also re-set σ_r to 0.005 and σ_R to 0.0075, based on the samlpe volatility of r and R observed from the first step.

In our first step fitting, we also find that the time series of the risk premium factor is negatively correlated with the realized level factor for both bonds and swaps. Specifically, we regress the

[11] Meandev-Vives and Naik (2000) report a value of 0.82. We choose a higher numerical value based on our intuition that the US Federal Reserve, under the leadership of Chairman Greenspan, is aggressive and proactive when it comes to setting its monetary policy in line with macroeconomic fundamentals.

[12] Both Babbs and Norman (1999) and Mendev-Vives and Naik (2000) obtain much higher estimates for σ_1 and σ_2 due to the negative correlation between the two factors.

[13] By fitting λ_{x_1} or λ_{X_1} for each weekly observation, we have implicitly allowed the risk premium parameter to be time-varying, even though long-term bonds are priced with a constant risk premium parameter on a given day. This approach is revised in the second step.

realized risk premiums against the realized level factors and find that for bonds, we have

$$\lambda_{x_1}(t) = 0.2480 - 2.026x_1(t), \quad R^2 = 0.25$$

while for swaps, we have

$$\lambda_{X_1}(t) = 0.2204 - 1.284X_1(t), \quad R^2 = 0.147$$

with very significant t-statistics for both regressions, -10.18 and -7.312 respectively. As a result, we impose in our second step a specification for the risk premium parameter using the above linear regression result. In doing so, the level factor for the bond curve becomes a mean-reverting process under the equivalent martingale measure:

$$dx_1 = (0.2480 - 2.026x_1 + \epsilon)\sigma_1 dt + \sigma_1 dw_1^*$$

The term ϵ represents a small error. We will take ϵ as a free parameter and fit to the data. Hopefully, ϵ will be zero on average. The level factor for the swap curve becomes

$$dX_1 = (0.2204 - 1.284X_1 + \epsilon)\sigma_{X_1} dt + \sigma_{X_1} dw_{X_1}^*$$

under the equivalent martingale measure.

The Data:

The data set used for our empirical study is obtained from Lehman Brothers, Inc. This data set includes weekly observations of government bond yields and swap rates from January 1995 to December 2000, based on the Friday's closes. For government bonds, the Lehman's data includes constant maturity treasury (CMT) rates for maturities of 2-year, 10-year and 30-year, for both the on-the-run and off-the-run bonds. The off-the-run yields are calculated as the constant maturity treasury (CMT) rates obtained by smoothing the off-the-run bond yields. We note that the yield spreads between the off-the-runs and on-the-runs provide us an unusual time series of specialness for the on-the-run bonds. The Lehman's data set also includes mid-market constant maturity swap (CMS) rates for maturities of 2-year, 10-year and 30-year. Combining our data for bonds and swaps, we can derive a time series of swap spreads associated with the on-the-run as well as the off-the-run bonds.

As an independent check on the data, we compare the Lehman data with Bloomberg which provides historical data for both government yields and swap rates. The two sources of data are generally consistent, although Bloomberg lacks historical data for the off-the-run bond yields and the 30-year swap rates. For short-term interest rates, we obtain the repo rates for general collateral from Lehman while the 1-month LIBOR rates are downloaded from Bloomberg. We note that our data set covers several economic and business cycles in which the Federal Reserve have both eased and tightened monetary policy. Except for the Mexico crisis in 1995, the first half the sample represents rather calm and normal economic environment. However, the second half of the sample covers several extraordinary events including the crisis of LTCM in the Fall 1998, the liquidity crisis of Y2K in the Summer 1999, and the Treasury buyback in the Spring 2000.

Results:

Table 1 presents summary statistics for the three interest rate factors (i.e., the short-rate factor, the long-rate factor, and the slope factor) and the risk premium factor (associated with the long-rate factor), based on fitting the GC-repo rates and the off-the-run treasury yields at the 2-year, 10-year, and 30-year sectors and on fitting the 1-month LIBOR rates, and swap rates at the 2-year, 10-year, and 30-year sectors. Table 1 also reports summary statistics for the spreads between the swap factors and bond factors, i.e., swap factors minus bond factors. Table 2 examines the correlation structure and volatility for the three bond factors and the three swap factors, and reports the sample correlation coefficients (off diagonal) and sample annualized volatilities (diagonal) of the weekly changes in these factors. Figures 4(a)-(d) plot the time-series history of the three interest rate factors and the risk premium factor associated with the long-rate factor, based on fitting the bond yields and the swap rates, and the time-series of the spread between these two histories of factors.

Volatilities and Correlations:

Table 2 reveals that the first differences or changes of the three bond factors are at best weakly correlated. The highest correlation coefficient is -0.342 (between the long-rate factor and the slope factor), i.e., higher long rates are more likely to be associated with a steeper yield curve. This coefficient matches our model correlation parameter of -0.30. Our assumption of zero correlation between the short-rate and long-rate factors and between the short-rate and slope factors is reasonably close to their sample correlations. Our assumption of volatilities (0.0125 for x_1, 0.0150 for x_2, and 0.005 for r) also matches those of the sample volatilities (0.0132 for x_1, 0.0153 for x_2, and 0.0045 for r). For swaps, the sample correlation between the long-rate factor and the slope factor is -0.2586, which again matches our model assumption of -0.23. However, the volatility assumption of 0.135 for X_1 is slightly larger than its sample volatility of 0.0129, while the volatility assumption of 0.0165 for X_2 and 0.0075 for R is largely in line with the sample volatilities. We also examine the correlation between the risk premium factor and the level factor. For both bonds and swaps, we find that the correlation between the risk premium factor and the level factor is near zero. Regressing the risk premium factor against the level factor results in insignificant t-statistics.

We note that given our parameterized volatility and correlation coefficients for the bond and swap factors, we can back out the implied model assumption for the spread process δ. Indeed, it can be easily shown that the volatilities implied by our assumptions on the bond and swap factors imply that $\sigma_{\delta_1} = 0.0050$ and $\sigma_{\delta_2} = 0.0070$ and $\sigma_\delta = 0.0055$. Unfortunately, these numbers are slightly off from their sample volatilities (0.0061 for σ_{δ_1}, 0.0120 for σ_{δ_2} and 0.0079 for σ_δ). The model correlation between δ_1 and δ_2 implied by our assumption for x and X is around -0.2, which is significantly below its sample correlation of -0.849. Table 2 suggests that when swap spreads widen, the term structure of swap spreads tends to steepen and that the swap spread in the short-end is not very correlated with the swap spread in the long-end. Altogether, we conclude that our model may still be slightly mis-specified.

Short-Rate and Long-Rate Factors:

Table 1 and Figures 4(a),(b) show that the short-rate and long-rate factors corresponding to swap rates are usually higher than those factors corresponding to government bond yields, with a mean difference of 19.5 and 30.9 basis points, respectively. The mean short-rate difference represents an average of short-term financing spreads, while the mean long-rate difference represents an average of long-term term spreads (or swap spreads). While the short-rate differences have been well behaved, fluctuating narrowly around its mean of 20 basis points (except for periods near the end of calendar year, known as the *term effect*), the long-rate differences have been steadily rising from around 20-30 basis points in 1995-1997 to near 110 basis point by late 2000. Clearly, this is related to the fact that long-term swap spreads have been steadily rising and reached their historical wides in the Spring 2000. The long-rate differentials peaked in the Fall 1998, the Summer 1999 and the Spring 2000. The correlation between the long-rate spread and the short-rate spread is around 0.25, implying a weak positive relationship between long-term swap spreads and short-term financing spreads.

For a brief period in early 1995 and late 1999, the long-rates implied by the swap curve falls below those of the bond curve. This happened at the time when the term structure of swap spreads was flat or near inverted.

Model-Implied Short-Rates:

The sum of the mean long-rate difference and the mean slope difference is about 44 basis points. This sum can be interpreted as the model-implied short-term financing spread, which may serve as the near-term target towards which the current short-term financing spread will converge. The model-implied financing spreads have been steadily rising during the period covered in our sample, from around 20 basis points in early 1998 to over 100 basis points in early 2000. In other words, based on the market prices in 2000, the US bond market was expecting the short-term financing spread to become much higher in the future. This is consistent with the view that bonds will become more special in the future as the US government continues to reduce the supply of Treasury bonds.

Slope Factors:

Over the sample period considered here, the slope factor corresponding to swap rates is slightly larger than that of bond yields, with a mean difference of 13.3 basis points. In other words, the yield curve corresponding to the off-the-run bonds is slightly steeper than that of swaps so that on average, the term structure of swap spreads (up to the 10-year maturity) is near flat or slightly downward sloping. However, the slope differential between swaps and bonds reached its low of -92.88 basis points in late 1998 in the aftermath of the LTCM crisis. The slope of the swap curve was much steeper than that of the bond curve so that the term structure of swap spreads became extremely upward sloping. The slope differential between the swap curve and the bond curve reached its local low level of near 0 basis point in the summer of 1999 when Y2K was a big concern, and -45 basis points in the Spring of 2000 when Treasury announced its buyback. The swap spread curve was also very steep in December 1995 when the slope differential reached -60 basis points.

We point out that the term structure of swap spreads would become much steeper had we fitted the bond yields using the on-the-runs (instead of the off-the-runs). In particular, the mean slope differential between the swap curve and the bond curve would be negative, implying an upward sloping term structure of swap spreads on average. The slope would slide even further towards its low during the periods of LTCK crisis in 1998, Y2K liquidity crisis in 1999 and Treasury buyback in 2000.

Risk Premium and Swap Spreads:

On average, the risk premium required by long-dated swaps is about 2.38% higher than that of long-term bonds. Figure 4(d) demonstrates that during the 1998 financial crisis, the risk premiums implied from the long-dated swaps and long-term bonds exploded to a high around 19-20%. The risk premium difference between the two curves dropped to -3.26% during that period. In other words, relative to long-dated swaps, the market was demanding a much higher risk premium for holding long-term government bonds. We note that during that time, the term structure of swap spreads had a humped shaped (upward sloping from 0 to 10-year and downward sloping from 10 to 30-year). This happened because long-term bonds (such as those of 30-year's) became less attractive relative to intermediate-term bonds (such as those of 10-year's) and the risk premium required by the swap market is much less than that required by the bond market. However, the term structure of swap spreads became strictly increasing since late 1999, especially since early 2000 after US Treasury Department announced its first-ever Treasury Buy-back program in recent history. The risk premium implied from bonds dropped from its peak of 20% in 1998 to a low of 4-5% in the beginning of 2000. The risk premium demanded for holding long-term bonds dropped significantly due to the strong demand for long-term government bonds (especially the 20-year plus sector). In the mean time, the risk premium associated with long-dated swaps reached a level around 8-9%, and the risk premium differential between swaps and bonds peaked to 7-8%. The sign change in the risk premium differential is the main reason why the term structure of swap spreads has become strictly upward sloping (even beyond 10-year) since the beginning of 2000.

Liquidity Premium and Swap Spreads:

We now take a closer look at the relationship between the liquidity premium associated with government bonds and the term structure of swap spreads. First, we point out that in general the liquidity premium is hard to quantify. Specifically, we don't have a variable that is observable yet provides a direct measurement for liquidity premium. Second, liquidity premium can affect the pricing of all government bonds relative swaps, widening or narrowing the overall spread between swaps and government bonds. Liquidity premium can also affect a specific sector of government bonds so that it may steepen or flatten the term structure of swap spreads. Third, the yield spreads between the on-the-run and off-the-run bonds serve a good indicator of the excess demand for the on-the-run bonds over the supply, and can therefore be used as a proxy for assessing the liquidity premium commended by government bonds. In other words, when the on-the-run bonds are in special and traded rich, there is a good chance that this sector of bonds is priced with liquidity premium. As such, it can put pressure on the swap spreads of this sector.

Figure 5 plots the time-series of yield spreads between the on-the-run and off-the-run bonds. The plot shows a clear widening in spreads between the off-the-run and on-the-run bonds, which coincides with the overall widening of swap spreads since 1998. The average on/off-the-run spreads are 3 basis points for the 2-year sector, 14.8 for the 10-year sector and 9.8 basis points for the 30-year sector. If we split the sample into two parts: pre-December 1997 and post-January 1998. Then, we find that prior to 1998, the on/off-the run spreads were 1.5 basis points for 2-year, 7.6 basis points for 10-year and 4.8 basis point for 30-year. However, since 1998, the on/off-the run spreads have increased drastically, averaging 4.6 basis points for 2-year, 21.9 basis points for 10-year, and 14.7 basis point for 30-year. Clearly, the 10-year sector has gained the most, relatively. This is consistent with the fact that the term structure of swap spreads has been steepenning ever since the LTCM crisis. As the 10-year bonds trade rich and commend more liquidity premium, the 10-year sector exhibits the widest swap spread, making the term structure of swap spreads steeper from the 2-year to 10-year. Note the brief drop of specialness in late 1999 coincided with the flattenning of the swap spread curve in late 1999.

6 Summary

In this paper we present a new approach of analyzing term structures of swap spreads using the notions of short-term financing spreads, liquidity premium and risk premium. Our approach departs from the traditional approach of attributing swap spreads to counterparty risk of default. We present an empirical analysis of swap spreads using data from the US market. Using our 3-factor term structure model as lenses and filters, we are able to explain some of the extreme movements in US swap markets in recent years. In particular, recent market data from bonds and swaps suggest that the financial market is rationally anticipating rising short-term financing spreads, rising liquidity premiums for holding government bonds, and lowering required risk premiums for holding long-term government bonds, as the expected US government surpluses may reduce the net supply of the government bonds in the future. The empirical results reported in the paper shall provide useful insights for both academic researchers and industry practitioners.

Reference

Brown, Keith, W. Harlow, and Donald. Smith, 1994, An empirical analysis of interest rate swap spreads, *Journal of Fixed Income* March, 61-78.

Babbs, Simon and K. Nowman, 1999, Kalman filtering of generalized Vasicek term struture models, *Journal of Financial and Quantitative Analysis* 34, 115-131.

Black, Fisher and John Cox, 1976, Valuing corporate securities: Some effects of bond indenture provisions, *Journal of Finance* 31, 351-367.

Chen, Andrew and Arthur Selender, 1994, Determination of swap spreads: An empirical analysis, working paper, Southern Methodist University, 1994

Collin-Dufresne, Pierre and Bruno Solnik, 1999, On the term structure of default premia in the swap and LIBOR markets, working paper, Carnegie Mellon University.

Cooper, Ian, and Antonio Mello, 1991, The default risk of swaps, *Journal of Finance* 46, 597-620.

Cox, John, Jonathan Ingersoll and Stephen Ross, 1985, A theory of the term structure of interest rates," *Econometrica* 53, 385-407.

Duffie, Darrell, and Ming Huang, 1996, Swap rates and credit quality, *Journal of Finance* 51, 921-949.

Duffie, Darrell, and Kenneth Singleton, 1997, An econometric model of the term structure of interest rate swap yields, *Journal of Finance* 52, 1287-1321.

Duffie, Darrell, and Kenneth Singleton, 1999, Modeling term structure of defaultable bonds, *Review of Financial Studies* 12, 687-720.

Das, Sanjay, and Peter Tufano, 1996, Pricing credit-sensitive debt when interest rates, credit Ratings and credit spreads are stochastic, *Journal of Financial Engineering* 5, 161-198.

Evans, E. and G. Bales, 1991, What drives interest rate swap spreads?, in *Interest Rate Swaps*, Carl Beidleman, ed., 280-303.

Grinblatt, Mark, 1999, An analytical solution for interest rate swap spreads, forthcoming in *Reivew of International Finance*.

Harrison, Michael, and David Kreps, 1979, Martingale and multiperiod securities markets, *Journal of Economic Theory* 20, 381-408.

Jarrow, Robert, and Stuart Turnbull, 1995, Pricing derivatives on financial securities subject to default risk, *Journal of Finance* 50, 53-85.

Jarrow, Robert, David Lando, and Stuart Turnbull, 1997, A Markov model for the term structure of credit spreads, *Review of Financial Studies* 10, 481-523.

Leland, Hayne, 1994, Corporate debt value, bond covenants, and optimal capital structure, *Journal of Finance* 49, 1213-1252.

Leland, Hayne and Klaus Toft, 1995, Optimal capital structure, endogenous, bankruptcy, and the term structure of credit spreads, *Journal of Finance* 51, 987-1019.

Litterman, Robert and Jose Scheinkman, 1991, Common factors affecting bond returns, *Journal of Fixed Income* 1, 51-61.

Litzenberger, Robert, 1992, Swaps: plain and fanciful, *Journal of Finance* 47, 831-850.

Longstaff, Francis and Eduardo Schwartz, 1995, A simple approach to valuing risky fixed and floating rate model, *Journal of Finance* 50, 789-821.

Longstaff, Francis, 2000, The term structure of very short-term rates: New evidence for the expectations hypothesis, *Journal of Financial Economics* 58, 397-415.

Madan, D., and H. Unal, 1994, Pricing the risks of default, working paper, University of Maryland.

Mendez-Vives, D., and V. Naik, 2000, The economic factor model: Theory and uses, working paper, Fixed Income Research, Lehman Brothers, Inc.

Merton, Robert, 1974, On the pricing of corporate debt: The risk structure of interest rates, *Journal of Finance* 29, 449-470.

Minton, B., 1993, An empirical examination of U.S. dollar swap spreads, working paper, University of Chicago.

Nielsen, Soren and Ehud Ronn, 1997, The valuation of default risk in corporate bonds and interest rate swaps, *Advances in Futures and Options Research*, 9, 175-196.

Piazzesi, Monika., 2000, An econometric model of the yield curve with macroeconomic jump effects, working paper, Stanford University.

Sorensen, Eric and Thierry Bollier, 1994, Pricing swap default risk, *Financial Analyst Journal*, May-June, 23-33.

Sun, Tong-Sheng, Suresh Sundaresan, and Ching Wang, 1993, Interest rate swaps: An empirical investigation, *Journal of Financial Economics* 34, 77-99.

Sundaresan, Suresh, 1991, Valuation of swaps, in *Recent Developments in International Banking and Finance*, S. Khoury, ed., Amsterdam: North Holland.

Vasicek, Oldrich, 1977, An equilibrium characterization of the term structure, *Journal of Financial Economics* 5, 177-188.

Table 1: Historical Yield Curve Factors

1995.1-2000.12		Short-Rate	Long-Rate	Slope	Risk Premium
Swaps	Mean	5.7619	5.9139	0.2371	0.1585
	Stdev	0.4610	0.6094	0.6397	0.0173
	Maximum	6.9275	7.0468	2.7987	0.2017
	Minimum	4.9060	4.2339	-1.8219	0.1087
Bonds	Mean	5.5673	5.6052	0.1037	0.1346
	Stdev	0.4542	0.7131	0.5264	0.0278
	Maximum	6.5554	7.0675	1.9158	0.1951
	Minimum	4.7215	3.5690	-1.4504	0.0461
Spreads	Mean	0.1946	0.3087	0.1334	0.0238
	Stdev	0.1462	0.2897	0.3568	0.0191
	Maximum	1.3042	1.1441	1.5236	0.0816
	Minimum	-0.0600	-0.2627	-0.9288	-0.0326
1995.1-1997.12		Short-Rate	Long-Rate	Slope	Risk Premium
Swaps	Mean	5.7317	6.1111	0.0272	0.1656
	Stdev	0.2542	0.4258	0.6741	0.0144
	Maximum	6.2492	6.9523	2.7987	0.2017
	Minimum	5.3453	5.3080	-1.8219	0.1372
Bonds	Mean	5.5774	5.9538	-0.0521	0.1445
	Stdev	0.2479	0.5020	0.4554	0.0107
	Maximum	6.1040	7.0675	1.9158	0.1716
	Minimum	5.1507	4.9721	-1.4504	0.1238
Spreads	Mean	0.1543	0.1573	0.0792	0.0211
	Stdev	0.0644	0.1332	0.2898	0.0127
	Maximum	0.4259	0.4435	0.8829	0.0554
	Minimum	0.0156	-0.2060	-0.5984	-0.002
1998.1-2000.12		Short-Rate	Long-Rate	Slope	Risk Premium
Swaps	Mean	5.7919	5.7178	0.4457	0.1513
	Stdev	0.5991	0.6965	0.5283	0.0170
	Maximum	6.9275	7.0468	1.5546	0.1989
	Minimum	4.9060	4.2339	-1.5550	0.1087
Bonds	Mean	5.5573	5.2588	0.2585	0.1248
	Stdev	0.5928	0.7240	0.5476	0.0351
	Maximum	6.5554	6.6263	1.5779	0.1951
	Minimum	4.7215	3.5690	-1.2246	0.0461
Spreads	Mean	0.2346	0.4590	0.1872	0.0266
	Stdev	0.1881	0.3234	0.4065	0.0236
	Maximum	1.3042	1.1441	1.5236	0.0816
	Minimum	-0.0597	-0.2627	-0.9288	-0.0326

Summary statistics of yield curve factors based on fitting the weekly off-the-run government bond yields and the weekly swap rates. The sample period is 1995:1-2000:12.

Table 2: Volatility and Correlation Coefficients

		Short-Rate	Long-Rate	Slope
Swaps	Short-Rate	0.0074		
	Long-Rate	0.1278	0.0129	
	Slope	-0.3917	-0.2586	0.01692
Bonds	Short-Rate	0.0045		
	Long-Rate	0.0960	0.0132	
	Slope	-0.1616	-0.3422	0.0153
Spreads	Short-Rate	0.0079		
	Long-Rate	0.2599	0.0061	
	Slope	-0.6455	-0.8493	0.0120

Annualized volatility (diagonal) and correlation coefficients (off-diagonal) for weekly changes of yield curve factors based on fitting the weekly on-the-run government bond yields and the weekly swap rates. The sample period is 1995:1-2000:12.

Figure 1(a): Short-Term Financing Spreads (US)

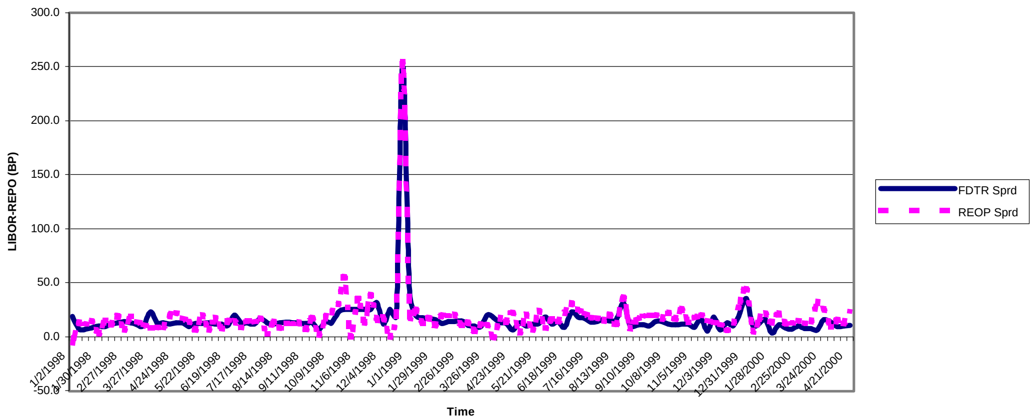


Figure 1(b): Short-Term Financing Spreads (EU)

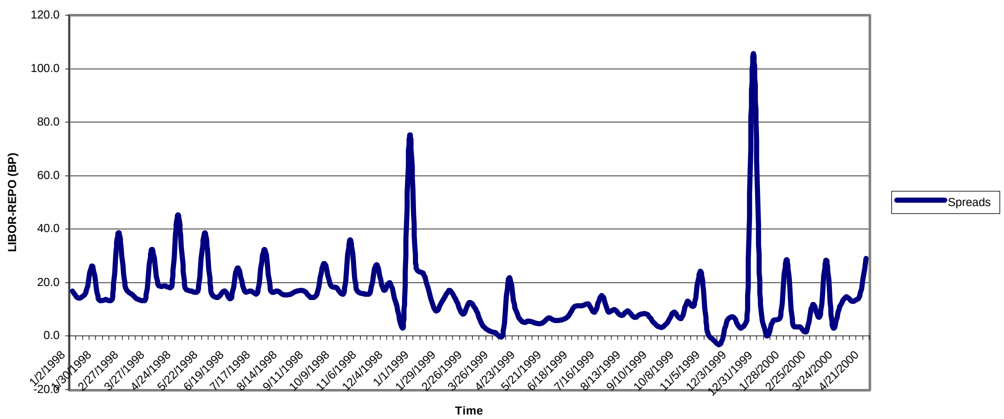


Figure 1(c): Short-Term Financing Spreads (JP)

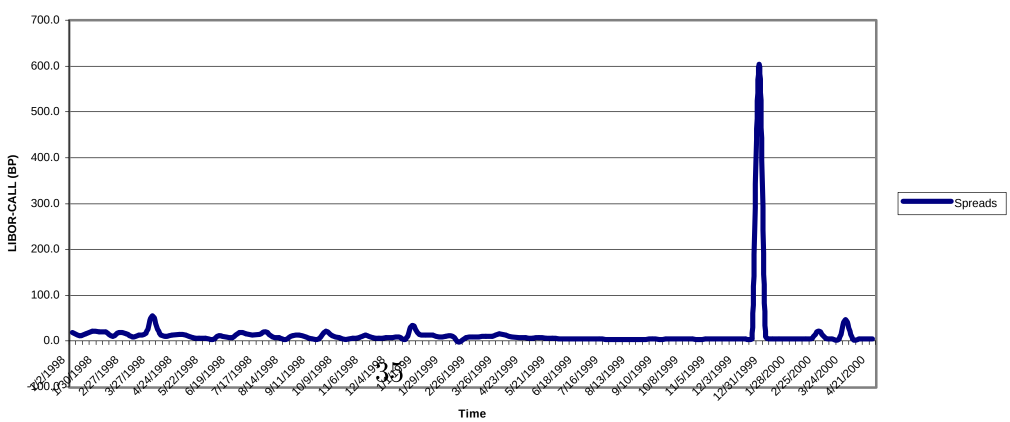


Figure 2: Term Structures of Swap Spreads (1 factor)

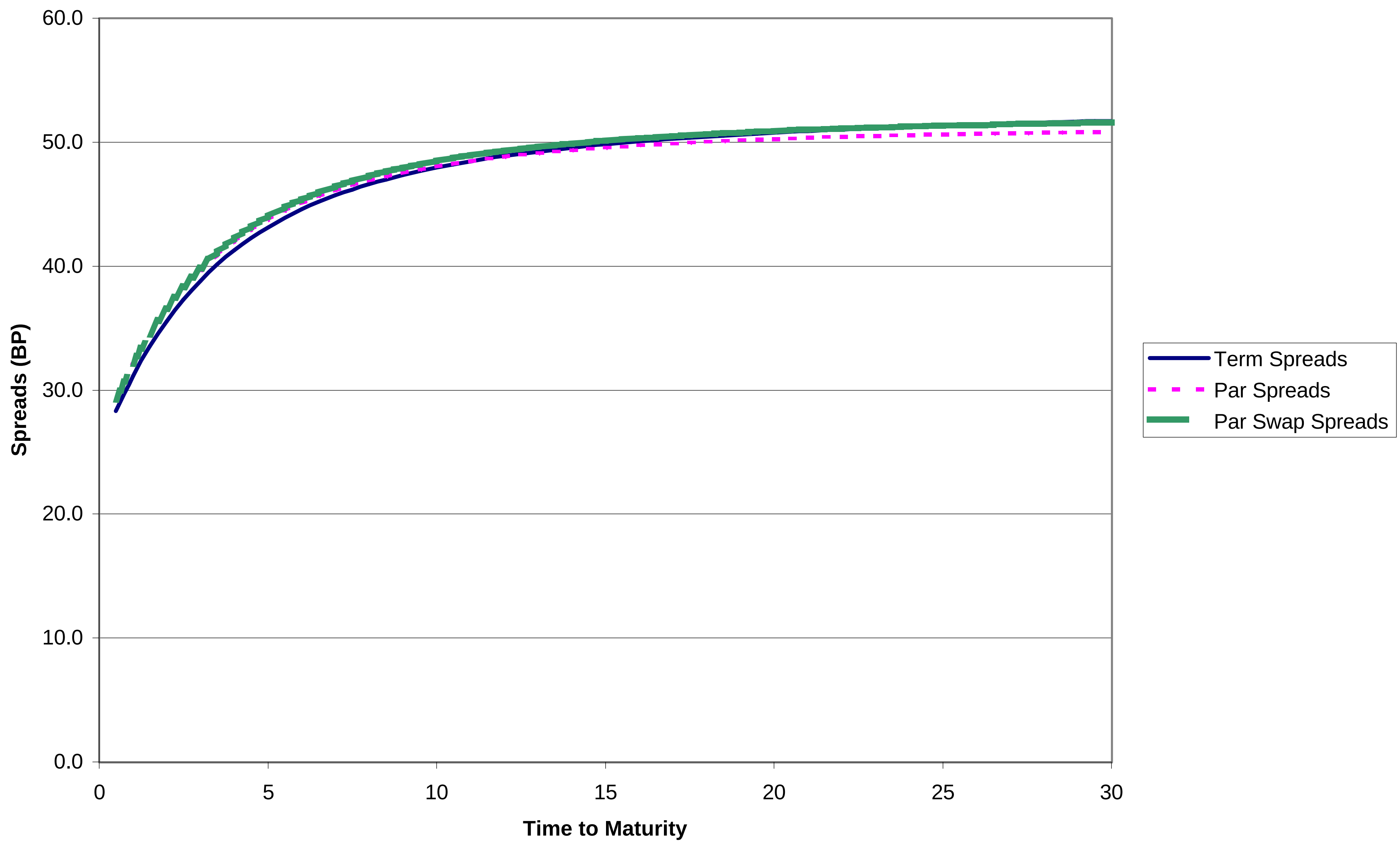
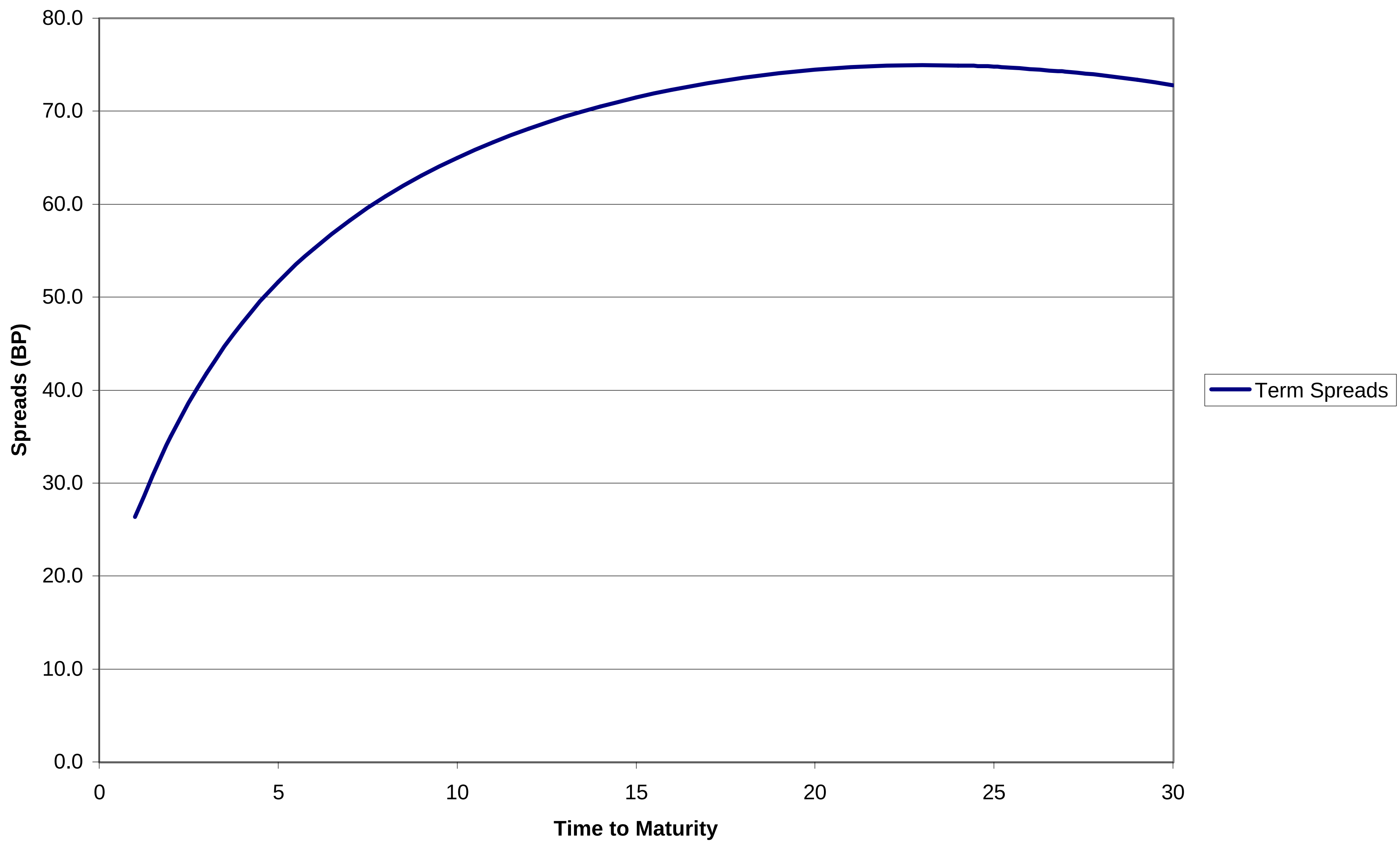


Figure 3: Term Structure of Term Spreads (2 factors)



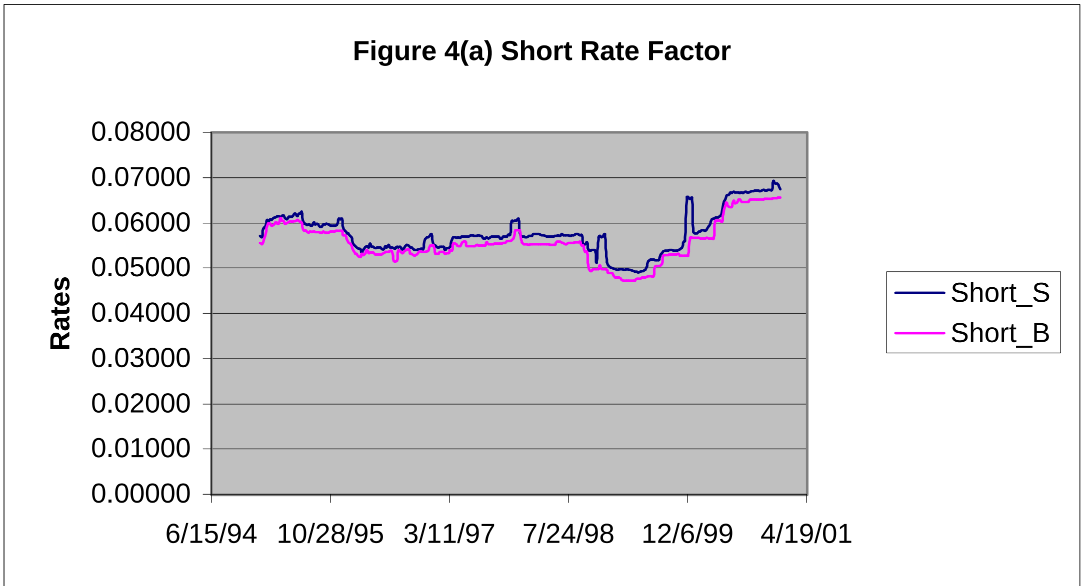
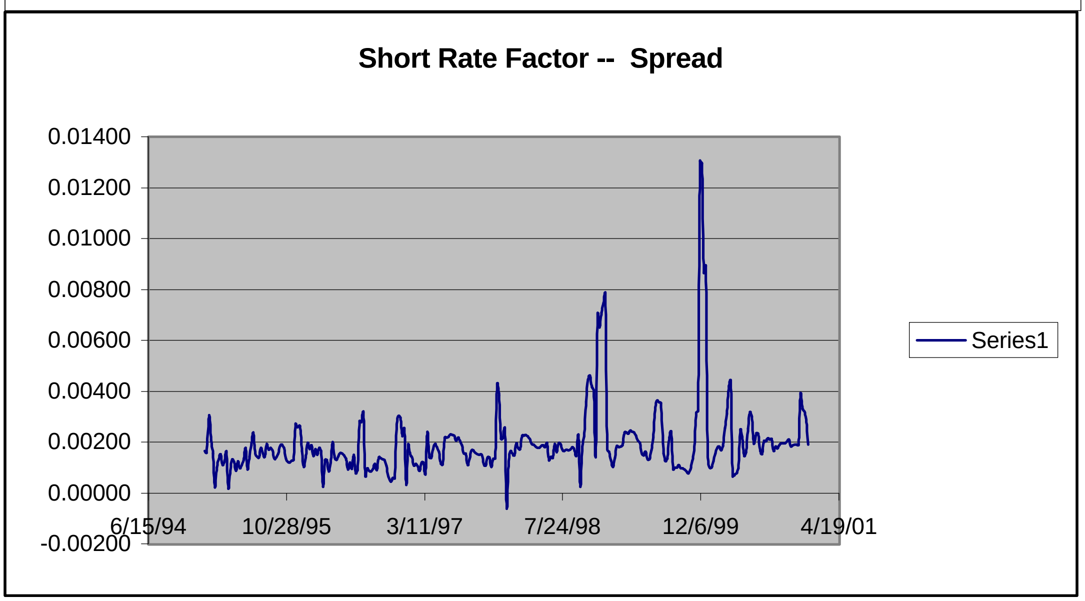


Figure 4(a) Short Rate Factor
Rates
0.08000
0.07000
0.06000
0.05000
0.04000
0.03000
0.02000
0.01000
0.00000
6/15/94
10/28/95
3/11/97
7/24/98
12/6/99
4/19/01
Short_S
Short_B


Short Rate Factor -- Spread
0.01400
0.01200
0.01000
0.00800
0.00600
0.00400
0.00200
0.00000
-0.00200
6/15/94
10/28/95
3/11/97
7/24/98
12/6/99
4/19/01
Series1

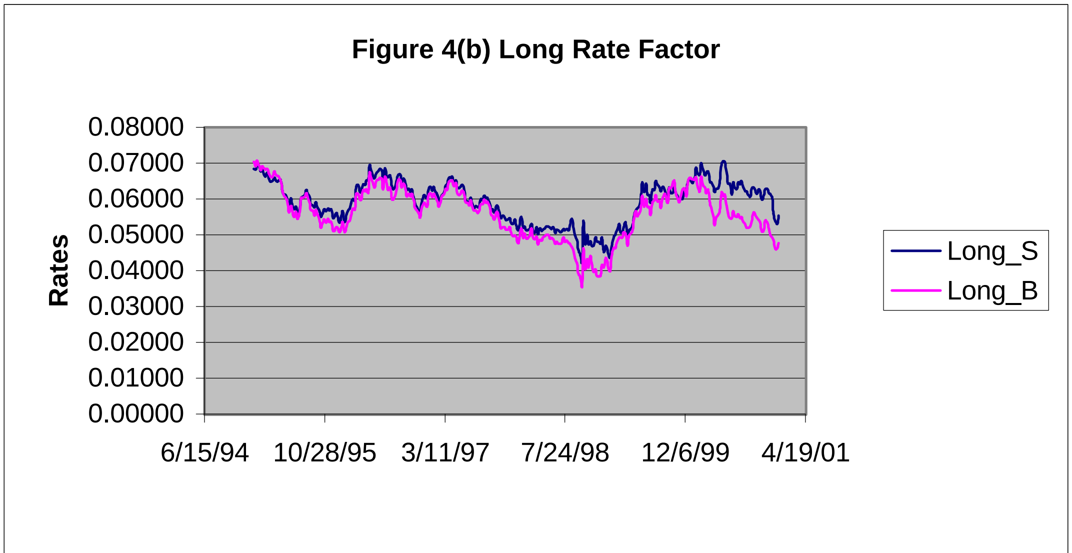
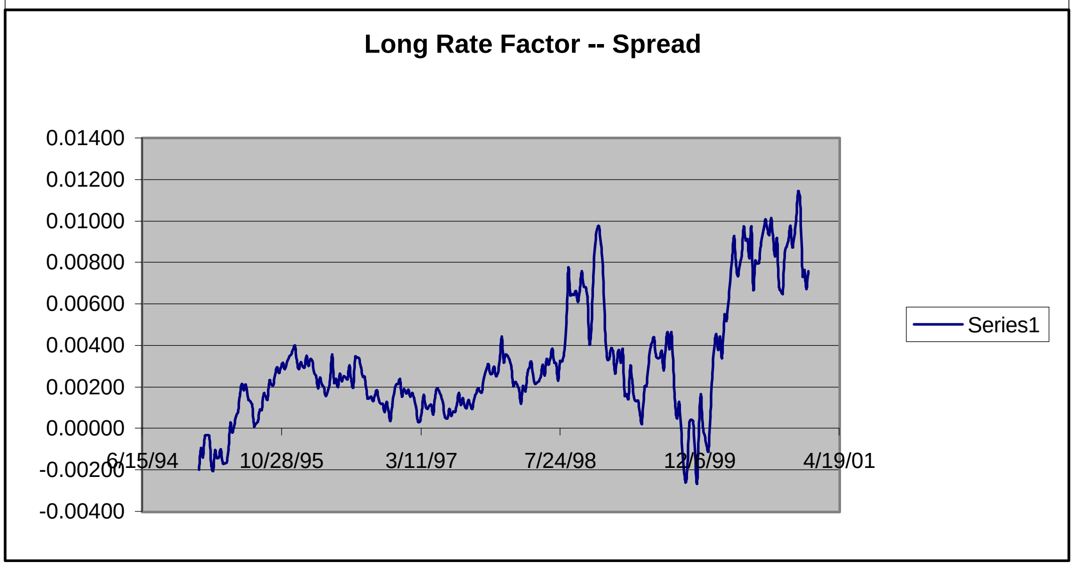


Figure 4(b) Long Rate Factor
Rates
0.08000
0.07000
0.06000
0.05000
0.04000
0.03000
0.02000
0.01000
0.00000
6/15/94
10/28/95
3/11/97
7/24/98
12/6/99
4/19/01
Long_S
Long_B


Long Rate Factor -- Spread
0.01400
0.01200
0.01000
0.00800
0.00600
0.00400
0.00200
0.00000
-0.00200
-0.00400
6/15/94
10/28/95
3/11/97
7/24/98
12/6/99
4/19/01
Series1

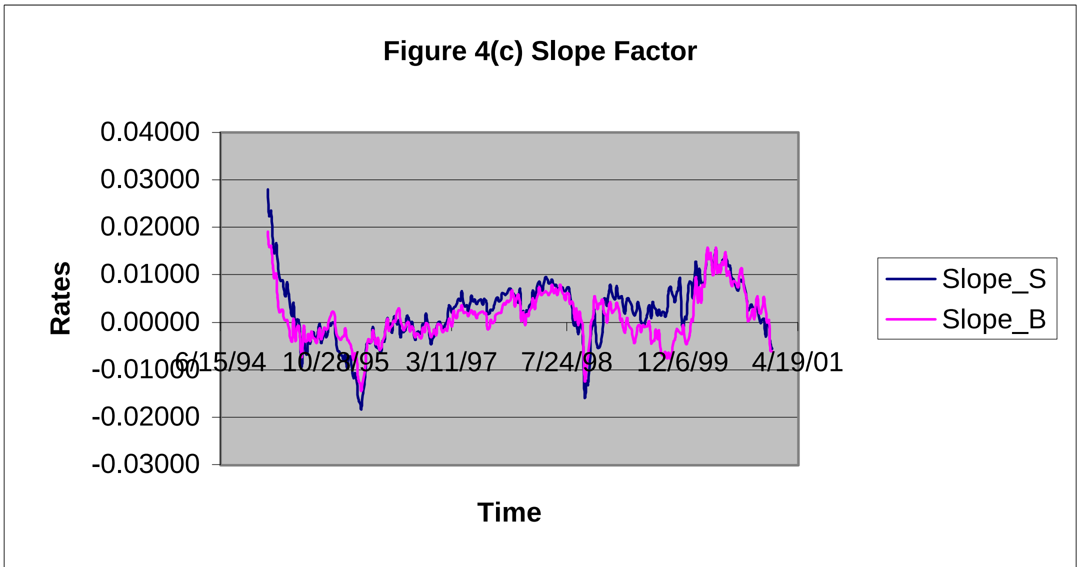
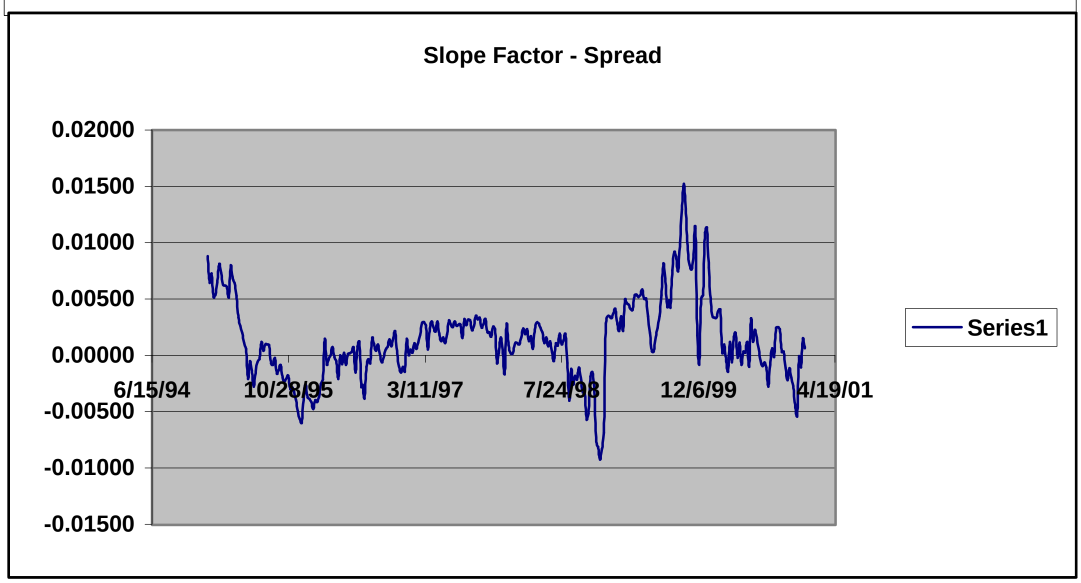


Figure 4(c) Slope Factor
0.04000
0.03000
0.02000
0.01000
0.00000
-0.01000
-0.02000
-0.03000
Rates
6/15/94
10/28/95
3/11/97
7/24/98
12/6/99
4/19/01
Time
Slope_S
Slope_B


Slope Factor - Spread
0.02000
0.01500
0.01000
0.00500
0.00000
-0.00500
-0.01000
-0.01500
6/15/94
10/28/95
3/11/97
7/24/98
12/6/99
4/19/01
Series1

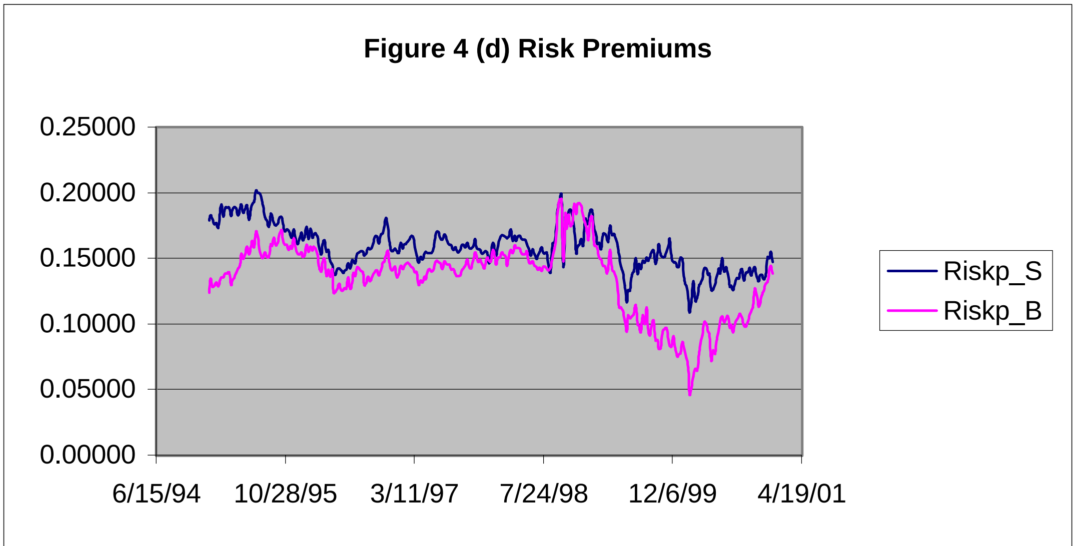


Figure 4 (d) Risk Premiums
0.25000
0.20000
0.15000
0.10000
0.05000
0.00000
6/15/94
10/28/95
3/11/97
7/24/98
12/6/99
4/19/01
Riskp_S
Riskp_B

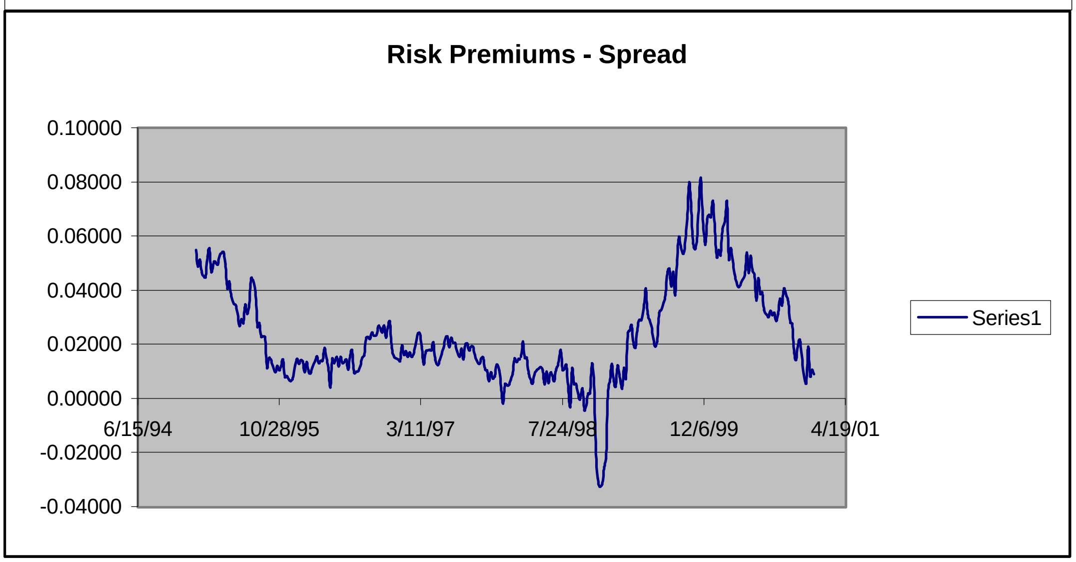

Risk Premiums - Spread
0.10000
0.08000
0.06000
0.04000
0.02000
0.00000
-0.02000
-0.04000
6/15/94
10/28/95
3/11/97
7/24/98
12/6/99
4/19/01
Series1

Figure 5. On/Off-the-Run Spreads