Chapter 10
Asset Pricing Implications of Equilibrium Business Cycle Models

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1. Introduction

Research problems on the boundary of finance and macroeconomics are rapidly emerging as central to the evolution of each field. In finance it is now well established that expected returns on securities do not depend on idiosyncratic risks, but rather on nondiversifiable components, as predicted by the standard Capital Asset Pricing Model (CAPM) of Sharpe (1964) and Lintner (1965). There is much less agreement on which aggregate factors are relevant to the determinants of expected returns or how these factors change over time. Rather, detailed analysis of asset pricing issues requires a general equilibrium model, as stressed by Cox, Ingersoll, and Ross (1985) and Breeden (1986). In macroeconomics, much recent research has been oriented toward development and simulation of small-scale dynamic equilibrium models, as in Kydland and Prescott (1982) and Long and Plosser (1983), but few papers have explored the asset pricing relationships of these general equilibrium macroeconomic models.1

A frequently debated issue in finance has been whether discount rates or expected returns are sufficiently constant to permit application of the CAPM of Sharpe (1964) and Lintner (1965) in research and decision-making. The early literature on stock price volatility, for instance, suggested that stock prices were too volatile relative to the present value of dividends, discounted at a constant rate (LeRoy and Porter 1981, Shiller 1981). Although most finance scholars will agree that discount rates vary over time, relatively little work has been done toward understanding the extent to which they vary, and the sources of this variation. Recently, a large number of authors have argued that there exists important variation in expected returns and return variances, and that this variation is linked to the stages of the business cycle. For instance, Schwert (1989) finds that equity returns are more volatile during recession periods. Fama and French (1988b, 1989) present evidence that the dividend-to-price ratio captures cyclical variation in returns and risk premiums on stocks.

The relationship between business cycles and asset prices poses a challenge for parsimonious equilibrium asset pricing models to explain. The starting point of this chapter is the standard neoclassical growth model, studied by Brock and Mirman (1972), modified to incorporate the endogenous choice of labor and leisure. This model has been relatively successful in replicating many features of business cycles, such as the comovement of output, consumption, and investment over time, as well as the relative variability of these aggregates (e.g., King, Plosser, and Rebelo 1988a, 1988b; Hansen 1985). By using arguments along the lines of Brock (1982) and Lucas (1978), these models of business cycles can easily be turned into asset pricing models. This illustrates the promise of developing the asset pricing implications of simple dynamic equilibrium models. Notably, the strategy can provide financial economists with a list of candidate state variables, or "factors," whose evolution determines the evolution of expected asset returns. Further, the strategy can provide macroeconomics with additional implications about cyclical variation in asset prices, which can be used to evaluate equilibrium macroeconomic models.

As is natural for an initial investigation, the model economy studied is very simple, with a single source of uncertainty (technology shocks). It abstracts completely from the influence of inflation, government spending and taxation, or incomplete markets, although it would be possible to study the implications of these modifications, by using the recent methodological advances of Baxter (1988) and Coleman (1988).

In long-run strategies of developing knowledge about asset pricing, however, the model studied here is a necessary first step.

The organization of the chapter is as follows. In Section 2, some basic facts about average returns on common stocks and bonds are reviewed by using quarterly data from the post-World War II United States. This section also discusses some empirical regularities in the cyclical behavior of returns and return volatility that have emerged from recent studies of Schwert (1989), Fama and French (1988a, 1988b, 1989), Poterba and Summers (1988), and Campbell and Shiller (1987). In Section 3, the economic model is described and the general nature of asset pricing relations is developed. In Section 4, a variety of simulation experiments are conducted within a parametric version of this model. Explorations with these simulations reveal how asset pricing implications of the model are influenced by variations in the parameters of technology and preferences. Related research, including that of Meltzer and Prescott (1985) on the equity premium puzzle, is reviewed in the light of these results. The chapter concludes with a brief summary of the main findings and suggestions for future research directions.

2. Stylized Facts about Returns

Before discussing the formal model, we present a set of "stylized facts" that have received much attention and debate in the finance literature. This set is by no means
exhaustive, but can serve as a benchmark to evaluate the asset pricing implications of equilibrium business cycle models. Part A of Table 10.1 gives the sample moments of U.S. postwar time series on real returns on stocks and bonds. The returns are measured at quarterly frequency and expressed in percentage per quarter. Data are taken from Ibbotson and Associates (1993). The average real return on one-month Treasury bills has been 0.23 percent per annum, slightly lower than the real return on long government bonds, which have provided 0.38 percent per annum. The return on long bonds has been much more volatile than Treasury bills, but investors have not been compensated for this extra variation during the postwar period. Stocks have grossed 2.23 percent in real terms, with a standard deviation of 7.74 percent. Part B of the table lists the correlations between these variables and provides the correlation between the various returns and risk premiums.

### Table 10.1
Sample Moments of U.S. Quarterly Time Series of Realized Real Returns

<table>
<thead>
<tr>
<th>A. Mean Returns, 1948I-1992IV</th>
<th>Mean</th>
<th>Rule</th>
</tr>
</thead>
<tbody>
<tr>
<td>Treasury bills</td>
<td>0.23</td>
<td>0.79</td>
</tr>
<tr>
<td>Long Government bonds</td>
<td>0.38</td>
<td>5.07</td>
</tr>
<tr>
<td>Corporate bonds</td>
<td>0.51</td>
<td>4.91</td>
</tr>
<tr>
<td>SP500</td>
<td>2.23</td>
<td>7.74</td>
</tr>
<tr>
<td>Government bond premium</td>
<td>0.14</td>
<td>4.76</td>
</tr>
<tr>
<td>Default premium</td>
<td>0.29</td>
<td>1.54</td>
</tr>
<tr>
<td>Equity premium</td>
<td>1.99</td>
<td>7.63</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>B. Correlations of Real Returns</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
<th>(7)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) Treasury bills</td>
<td>0.44</td>
<td>0.44</td>
<td>0.17</td>
<td>0.30</td>
<td>-0.64</td>
<td>0.06</td>
<td></td>
</tr>
<tr>
<td>(2) Long Government bonds</td>
<td>0.95</td>
<td>0.34</td>
<td>0.99</td>
<td>0.99</td>
<td>-0.25</td>
<td>0.29</td>
<td></td>
</tr>
<tr>
<td>(3) Corporate bonds</td>
<td>0.39</td>
<td>0.94</td>
<td>0.66</td>
<td>0.54</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(4) SP500</td>
<td>0.33</td>
<td>0.13</td>
<td>0.99</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(5) Government bond premium</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>-0.26</td>
<td>0.30</td>
</tr>
<tr>
<td>(6) Default premium</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.14</td>
</tr>
</tbody>
</table>

**Notes:** All returns are measured at the quarterly frequency and expressed in percent per quarter. The equity premium is computed as the ratio of one plus the return on the SP500 divided by one plus the return on Treasury bills. Other risk premiums are computed in a similar fashion. The government bond premium and default premium compare the return on long government bonds and corporate bonds to the return on Treasury bills. The averages are computed as arithmetic means. For a complete description of the data, see Ibbotson Associates (1993).

Several papers have discussed the inability of consumption-based asset pricing models to explain these cross-sectional differences in returns (Hansen and Singleton 1982; Mehra and Prescott 1985). This chapter will primarily focus on evidence about the time series variation in relative rates of return and return variability, in particular on those papers that suggest that these movements are related to the stage of the business cycle. Schwert (1989) provides evidence that equity returns are more variable during recession periods. His estimates indicate a major increase in volatility during recessions: 60 percent over the period 1953-1986 and 227 percent over 1927-1986. Schwert documents a similar increase in volatility in short-term interest rates, yields on corporate bonds, and the growth rate of industrial production. He concludes that the cyclical variation in return volatility can be only partially accounted for by movements in leverage, dividend yields, and macroeconomic variables. Flood, Hodrick, and Kaplan (1986), Fama and French (1988a, 1988b), and Poterba and Summers (1988) discuss the presence of negative autocorrelation in equity returns, which is more pronounced the longer the return interval. Fama and French (1988a, 1988b) argue that mean reversion in stock returns signals important time variation in expected returns, and that this time variation is tracked by the dividend and the earnings-to-price ratio. One explanation is that shocks to the economy have little or no effect on expected dividends or expected returns in the distant future, and the cumulative effect of the shocks has offsetting effects on expected returns and the current price. Since these negative autocorrelations are present in all size and industry portfolios, they argue that the explanation for this phenomenon can likely be found in a common factor, notably business cycles. In a subsequent paper, Fama and French (1989) find that excess returns on bonds and stocks can be forecast by the term spread, the default spread, and dividend yields, and that the predictive power increases with the return interval. Because the forecasting variables are related to business cycle conditions (typically high during recessions and low at business cycle peaks), Fama and French interpret this as evidence of capturing cyclical variation in expected returns. An important question is whether this variation of expected returns reflects rational pricing in an efficient market.

A final set of regularities links the information in asset prices to expectations about future real activity. Work by Stock and Watson (1988), and Estrella and Hardouvelis (1991) suggests that the slope of the term structure is an important leading indicator, and helps to predict future real activity.

These findings pose a challenge for parsimonious equilibrium asset pricing models to explain. The next section presents a simple equilibrium model commonly used in the literature on real business cycles, which can be turned into an asset pricing model with arguments along the lines of Lucas (1978) and Brock (1982). The evolution of the investment opportunity set is important for asset payoffs and valuation of these payoffs, but fundamentally these are governed by trade-offs agents make in production and consumption. Rather than posit a stochastic process for consumption, as in Mehra and Prescott (1985) or Kandel and Stambaugh (1991),
the model economy endogenizes the decisions to produce, invest, and consume. First, this allows pricing of portfolios of contingent claims apart from those that pay off the aggregate endowment, which leads from a finance perspective to a more natural definition of certain complex securities, such as equity shares. Second, this makes it possible to evaluate whether the stylized facts mentioned above can be accounted for by models that restrict the relations between the behavior of macroeconomic variables and returns in mutually consistent ways, and in which asset prices reflect rational optimizing behavior by economic agents.

3. Model Economies

The model we study is a version of the standard neoclassical growth model—which includes variable labor supply. The economy consists of many identical, infinitely lived households that derive utility from the consumption of goods and leisure. All production in the economy takes place in firms that own the stock of physical capital in the economy. Firms receive their income from supplying labor services to firms, and from dividends on the shares they hold in the firms. The market structure of the economy is as follows. At time 0 there exists a complete set of markets in which households and firms can trade contracts for future delivery of units of the single consumption good and labor services contingent on the state of the economy. The state of the economy at time 0 can be thought of as a "history" of the economy between dates 0 and t. Let s denote the state of the economy at time t, and let \( \pi(s_t) \) denote the date 0 probability of the history \( s_t \). The date 0 price of a unit of the consumption good in history \( t \) is \( p(s_t) \), and the date 0 price of one unit of labor services is \( p(s_t)w(s_t) \), where \( w(s_t) \) denotes the real wage rate.

Firms

Firms combine capital with labor services rented from households to produce a single good, according to a constant-returns-to-scale production function, \( y = zF[k, \lambda, h] \), where \( k \) represents the capital stock, \( h \) represents the amount of labor services, and \( \lambda \) is an index of technological progress, which, for now, is assumed to grow at a constant rate. \( F(\cdot) \) is assumed to be a concave, increasing, and twice continuously differentiable function of \( k \) and \( h \). The output of the economy is uncertain because of a random shock to total factor productivity, \( z \), which is assumed to follow a first-order Markov process. The information structure of the economy is such that the inputs into the production process have to be decided before the realization of this random shock is known. This dating is slightly different from the usual convention in the business cycle literature, which assumes that labor inputs can be chosen after the uncertainty about factor productivity has been resolved. Output in history \( s_t \) can therefore be written as

\[
y(s_t) = z(s_t)F[k(s_{t-1}), \lambda^{-1}h(s_{t-1})].
\]

The capital stock is assumed to depreciate at a constant rate, \( \delta \). Because firms own the capital, they decide on the amount of investment, \( x \), that is necessary to maintain the capital stock for production in future periods. The evolution of the capital stock is given by:

\[
k(s_t) = (1 - \delta)k(s_{t-1}) + x(s_t).
\]

The dividend of the firms, \( d \), is the residual of the value of the output produced after the factor payments to labor have been made and investment has been financed. Dividends in history \( s_t \) are given by

\[
d(s_t) = x(s_t) - w(s_t)h(s_{t-1}) - x(s_t).
\]

It will be assumed that firms maximize their net present value:

\[
V = \sum_{t=0}^{\infty} \sum_{s_t} \pi(s_t) p(s_t)[y(s_t) - w(s_t)h(s_{t-1}) - x(s_t)]
\]

s.t. (1) and (2).

The first-order condition for the efficient allocation of labor is

\[
z(s_{t+1})\lambda t D_t F[k(s_t), \lambda^{-1}h(s_t)] = w(s_{t+1}),
\]

where \( D_t \) denotes the derivative of a function with respect to its \( t \)th argument. Defining \( \pi(s_{t+1}; s_t) \) as the conditional probability of history \( s_{t+1} \) given \( s_t \), efficient allocation of capital requires that

\[
\sum_{s_{t+1}} \pi(s_{t+1}; s_t) p(s_{t+1})[z(s_{t+1})\lambda t D_t F[k(s_t), \lambda^{-1}h(s_t)]]
\]

\[
+ [1 - \delta] = p(s_t).
\]

These efficiency conditions state that the wage rate in history \( s_{t+1} \) equals the marginal product of labor, and that the price of one unit of capital equals the conditional expectation of its net marginal value product next period.

Households

The consumers in the economy are assumed to maximize their expected lifetime utility, defined over the consumption of goods, c, and leisure, l:

\[
\max U = \sum_{t=0}^{\infty} \beta^t \sum_{s_t} \pi(s_t) u(c(s_t), l(s_{t-1})).
\]

Instantaneous utility is such that both consumption and leisure are "goods." \( \beta \) is the discount factor applied to future utilities, reflecting the preference of agents, all else equal, to consume earlier rather than later. Household finance their consumption expenditures from the proceeds of renting labor services to firms and the dividends they receive from their ownership of shares. If we use \( P(s_t) \)
and \( Z(s_t) \) to denote the relative price and the number of shares in history \( s_t \), and normalize the endowment of time per period \( h + 1 = 1 \), the budget constraint of households can be written as

\[
\sum_{r=1}^{\infty} \sum_{s_t} \pi(s_t) p(s_t)[c(s_t) + P(s_t) Z(s_t)] \\
\leq \sum_{r=1}^{\infty} \sum_{s_t} \pi(s_t) p(s_t)[w(s_t)h(s_t-1) + Z(s_t-1)[P(s_t) + d(s_t)]] \tag{8}
\]

The consumer’s first-order conditions are as follows. Efficient consumption plans require

\[
\beta^t D_{1a}(c(s_t), 1 - h(s_t-1)) = \Lambda p(s_t); \tag{9}
\]

efficient allocation of work effort implies that

\[
\beta^{t+1} \sum_{s_{t+1}} \pi(s_{t+1}; s_t) D_{2a}(c(s_{t+1}), 1 - h(s_t)) \\
= \Lambda \sum_{s_{t+1}} \pi(s_{t+1}; s_t) p(s_{t+1}) w(s_{t+1}); \tag{10}
\]

and asset holdings are chosen such that

\[
p(s_t) P(s_t) = \sum_{s_{t+1}} \pi(s_{t+1}; s_t) p(s_{t+1})[P(s_{t+1}) + d(s_{t+1})], \tag{11}
\]

where \( \Lambda \) denotes the Lagrange multiplier on the household’s budget constraint. From Swan (1963) it is known that a constant returns to scale technology with labor-augmenting technological progress allows a feasible steady state (under certainty) in which all variables grow at the rate of technological progress, \( \lambda = 1 \). King et al. (1988a) demonstrate that additional restrictions are required on preferences in order for this feasible steady state to be compatible with the efficiency conditions of the economy. The reason is that effort cannot grow in the steady state, since time in a period is in fixed supply. We follow King, Plosser, and Rebelo (1988) by assuming that preferences belong to a class that is compatible with steady-state growth:

\[
u(c, 1 - h) = \begin{cases} 
\frac{1}{1 - \sigma} e^{1 - \sigma} v(1 - h), & \text{if } \sigma \neq 1, \\
\log(c) + \phi \log(1 - h), & \text{if } \sigma = 1
\end{cases}
\]

where \( \sigma \) is the inverse of the elasticity of intertemporal substitution, or the coefficient of relative risk aversion.

### Equilibrium

The equilibrium in the economy is a set of prices, \( p(s_t), P(s_t), w(s_t), \) and quantities, \( c(s_t), h(s_t), k(s_t), x(s_t), d(s_t), y(s_t), x(s_t), Z(s_t) \), that satisfies the efficiency conditions of firms, (5) and (6), and households, (9), (10) and (11); as well as the economywide resource constraints, (1), (2) and (3); and

\[
y(s_t) = c(s_t) + x(s_t) \tag{12}
\]

and

\[
Z(s_t) = 1, \tag{13}
\]

for all \( s_t, t = 1, 2, \ldots, \infty \), and given \( k_0, h_0, z_0, \) and \( Z_0 = 1 \).

### Asset Prices

Although the equilibrium prices and quantities in the economy are all determined in a single market-clearing operation at date 0, it is straightforward to derive the implied “shadow” asset prices at intermediate dates. Since the purpose of this chapter is to study the behavior of asset prices over time, we will derive the equilibrium prices that agents would be willing to pay for trading one unit of consumption between states \( s_t \) and \( s_{t+j} \). At date 0 this would be the relative price, \( \pi(s_{t+j}) p(s_{t+j})/\pi(s_t) p(s_t) \). However, not all histories \( s_{t+j} \) are feasible for a given history \( s_t \). For the purpose of studying bond prices, we will be interested in the relative price of one unit of consumption in state \( s_{t+j} \) given that the history \( s_t \) has been realized at date t. This price is \( \pi(s_{t+j}) \pi(s_{t+j}; s_t) p(s_{t+j})/\pi(s_t) p(s_t) = \pi(s_{t+j}; s_t) p(s_{t+j})/\pi(s_t) p(s_t) \). We are now ready to define the price of a j-period bond in state \( s_t \) as the relative price of a payoff of one unit of consumption at \( t+j \), by summing over all histories \( s_{t+j} \) that are feasible for a given history \( s_t \), up to time \( t+j \):

\[
B^t(s_t) = \sum_{s_{t+j}} \frac{\pi(s_{t+j}; s_t) p(s_{t+j})}{p(s_t)} = E_{s_t} p(s_{t+j}) \\
= \frac{E_{s_t} \beta^t D_{1a}(c(s_{t+j}), 1 - h(s_{t+j}-1))}{D_{1a}(c(s_t), 1 - h(s_t-1))}. \tag{14}
\]

By similar reasoning we can derive the relative price in state \( s_t \) of a dividend payment in state \( s_{t+j} \) conditional on the history \( s_t \), as \( \pi(s_{t+j}; s_t) p(s_{t+j}) d(s_{t+j})/p(s_t) \). The value of a claim in state \( s_t \) to all future dividends can be found analogously by summing over all feasible future histories:

\[
P(s_t) = \frac{\sum_{j=1}^{\infty} \sum_{s_{t+j}} \pi(s_{t+j}; s_t) p(s_{t+j}) d(s_{t+j})}{p(s_t)} \\
= \frac{E_{s_t} p(s_{t+j}) d(s_{t+j})}{p(s_t)}.
\]
Equation (15) is simply a restatement of the first-order condition (11) solved forward. It states that the price of a share is the present value of the expected future dividends. If we assume constant returns-to-scale in production, it is easy to show that the share price must also be equal to the capital stock. Substituting (2) and (5) into (3) and assuming constant returns to scale gives

\[ d(s_{t+1}) = y(s_{t+1}) - w(s_{t+1})k(s_{t+1}) - x(s_{t+1}) \]

\[ = k(s_{t+1})y(s_{t+1})D_1F[k(s_{t+1}), x' h(s_{t+1})] - k(s_{t+1}) - (1 - \delta)k(s_{t+1}) \] (16)

Rearranging terms yields

\[ \frac{d(s_{t+1}) + k(s_{t+1})}{k(s_{t+1})} = z(s_{t+1})D_1F[k(s_{t+1}), x' h(s_{t+1})] + (1 - \delta) \]. (17)

By substituting this into (6), we get (11) for \( P(s_{t+1}) = k(s_{t+1}) \).

**Time-Varying Expected Returns**

To understand why equilibrium business cycle models are natural candidates for the study of issues of time variation in expected returns and risk premiums, consider the net return, \( r_k \), from holding one unit of capital between states \( s_t \) and \( s_{t+1} \):

\[ r_k(s_{t+1}; s_t) = z(s_{t+1})D_1F[k(s_{t+1}), h(s_{t+1})] - \delta \]. (18)

The realized return can be decomposed into its expected and unexpected components:

\[ r_k(s_{t+1}; s_t) = E_n r_k(s_{t+1}; s_t) + \Delta \epsilon(s_{t+1}) \] (19)

where \( \Delta \epsilon(s_{t+1}) = D_1F[k(s_{t+1}), h(s_{t+1})] \), and \( E_n \) is the expected component of the factor realization, \( \epsilon \), multiplied by the sensitivity, \( h \), to this factor. This factor sensitivity, or "asset beta," is itself state dependent because it is a function of the time-varying marginal product of capital. The risk premium on productive capital can be derived by rewriting the Euler equation, (6), as

\[ E_n[1 + r_k(s_{t+1}; s_t)] \frac{p(s_{t+1})}{p(s_t)} = 1 \]. (20)

Using the definition of the covariance and substituting in (19), we get:

\[ 1 + E_n r_k(s_{t+1}; s_t) = \frac{p(s_{t+1})}{E_n p(s_{t+1})} - b(s_{t+1}) \frac{E_n \epsilon(s_{t+1})p(s_{t+1})}{E_n p(s_{t+1})} \] (21)

The conditional expected return on capital is therefore the sum of two components. The first is the inverse of the one-period bond price in state \( s_t \), as defined in equation (14) for \( j = 1 \). It is therefore equal to one plus the conditional one-period "risk-free" rate. The second term is the product of the factor sensitivity and the risk premium on the factor. The sign of the factor risk premium is determined by \( E_n \epsilon(s_{t+1})D_1F[k(s_{t+1}), h(s_{t+1})] \), or the conditional covariance between the technology shock and the marginal utility of consumption. Because an unexpectedly high realization of the technology shock will generally lead to an increase in consumption, this covariance will be negative, and the risk premium on the one-period bond price will be positive. It is important to note that all the components of the expected return, that is, the risk-free rate, the factor risk premium, and the "beta," are time varying. It is in this sense that the equilibrium business cycle models are natural candidates for the study of the links between variation in expected returns and fluctuations in the economic fundamentals.

**Firms and Leverage**

It has become common practice in the literature to compare the time series properties of equity shares in model economies to the behavior of stock market indices. Our current definition of equity shares has at least two aspects that do not mirror stock prices in reality. First, the share of capital and labor in the model are equally risky, as both represent constant fractions, \( \theta \) and \( 1 - \theta \), of output. As noted by Donaldson and Mehra (1984), in practice the share of capital is riskier than the share of labor, which is largely negotiated prior to the realization of output, which creates operating leverage. Because in the model economy, both inputs are chosen before the realization of the productivity shock, a natural way to capture this insurance aspect of labor services is to modify the contract structure and pay labor its expected utility-denominated marginal product instead of its actual marginal product. The fixed wage, \( \bar{w}(s_t) \), contracted for one period ahead solves

\[ E_n \bar{w}(s_{t+1})p(s_{t+1})D_1F[k(s_{t+1}), h(s_{t+1})] = \bar{w}(s_{t})E_n p(s_{t+1}) \] (23)

Incorporating operating leverage in this manner has the effect that capital income provides an insurance to labor income. This insurance is partial in the sense that it only pertains to the one-period-ahead realization of the productivity shock. Labor income will still vary with the level of the capital stock, but its share will covary negatively with the realization of \( z \).
A second unrealistic aspect of the equity definition is that it represents an unlevered claim to the stock of capital, whereas many firms rely in part on debt to finance productive investments. Because the economy is one in which the Modigliani and Miller propositions hold, there is, strictly speaking, no role for debt financing. By the same token, however, because capital structure does not alter the equilibrium allocations, we can simply assume a particular financial structure and use the complete markets framework to study two sets of contingent claims, corporate debt, and levered equity, that add up in value to the capital stock. Following this approach, we define a one-period risky bond in the firm that promises to pay a fraction, \( \psi \), of average firm value over all states, \( V \), or future firm value, whichever is lower. This definition emphasizes the option characteristics of corporate debt. If the value of the firm falls below a fraction of its average value, shareholders sell the firm to bondholders for the face value of the bond. The payoff to holders of the risky bond if the economy goes from state \( s_i \) to \( s_{i+1} \) is

\[
\min \left[ \psi V, P(s_{i+1}) + \theta(s_{i+1}; s_i) y(s_{i+1}) - x(s_{i+1}) \right].
\]

where \( \theta(s_{i+1}; s_i) \) is the share of capital after accounting for operating leverage. The price of this bond, \( B(s_i) \), is therefore given by

\[
F_{B(s_i)} \min \left[ \psi V, P(s_{i+1}) + \theta(s_{i+1}; s_i) y(s_{i+1}) - x(s_{i+1}) \right] / P(s_i).
\]

Note that the face value of the risky bond, \( \psi V \), is constant across all states. Consequently leverage, defined as the value of the risky bond relative to the value of the unlevered firm, will be high in states where firm value is low (recessions). The end of period cum dividend value of a levered equity share is given by the firm's cash flows net of the payments on the one-period risky bond:

\[
\max \left[ 0, P(s_{i+1}) + \theta(s_{i+1}; s_i) y(s_{i+1}) - x(s_{i+1}) - \psi V \right].
\]

The contingent ex-dividend value of the levered equity, \( P(s_i) - B(s_i) \), is simply the value of an unlevered share minus the value of the risky bond, \( P(s_i) - B(s_i) \). The dividend on the levered equity share, \( d(s_{i+1}) \), therefore equals the cum dividend value of the levered equity share minus its ex-dividend value:

\[
\max \left[ 0, P(s_{i+1}) + \theta(s_{i+1}; s_i) y(s_{i+1}) - x(s_{i+1}) - \psi V \right]
- \left[ P(s_{i+1}) - B(s_{i+1}) \right].
\]

It is perhaps important to point out that the component we call dividends in the model does not directly correspond to its empirical counterpart. It bears closer resemblance to a measure of net cash flows of the firm, after taking into account the effects of investment and debt financing. Approximating dividends by net cash flows is therefore likely to overstate their variability.

**Term Spread, Default Spread, and D/P-Ratio**

We conclude this section by defining the model equivalent of three financial variables that have been empirically shown to have predictive power for (excess) returns. The term spread is defined as the difference between the yield to maturity on a five-year (twenty-period) pure discount bond and that on a bond that matures in one quarter, which can be found from the bond pricing equation, (14), for \( j = 1, 20 \). The default spread compares the yield to maturity on the one-period risky bond to the yield on the pure discount bond of equal maturity. The dividend price ratio is defined as the dividend on the levered equity scaled by the price of levered equity.

**4. A Numerical Evaluation**

**Parameter Values**

This section contains a numerical simulation of the model. We choose parameters of technology and preferences such that the steady state under certainty coincides with the average behavior of per capita U.S. time series. The certainty steady state can be thought of as the limiting case of an economy with uncertainty, in which the shocks become arbitrarily small, and the stationary distribution degenerates to a single point. It will be assumed that the production function is Cobb-Douglas \( F [k, A] = k^\alpha A^{1-\alpha} \). First, \( 1 - \theta \) was chosen to be 0.65, on the basis of estimates of labor's share in GNP by Christiano (1988), and \( \lambda \) was set to 1.04, the average common (gross) growth rate of per capita output, consumption, and investment during the postwar period. Substituting the estimated investment-to-capital ratio of 0.0255 into (2), implies a depreciation rate of 2.2 percent per quarter. An estimate of the capital-to-output ratio of 10.57 can be used to compute the utility discount factor in the “transformed” economy \( \beta \) of 0.993. In Chapter 2, Hansen and Prescott show how to transform a growth economy into a stationary economy. We choose \( \phi \) such that the effort in the steady state equals 0.20, which is the average time devoted to market work in the postwar period. The remaining preference parameter, \( \sigma \), plays an important role in the determination of asset returns. The model will be simulated for \( \sigma = [1, 5] \), which is within the range suggested by Hall (1988a), but considerably lower than the value of 28 proposed by Kandel and Stambaugh (1991) for reasons that will become clear later. The degree of leverage, \( \psi \), is set to 0.40, as proposed in Kandel and Stambaugh (1991); this choice is close to estimates by Bodie, Kane, and McDonald (1983), who estimate the proportion of stocks in investors’ portfolios to be around 60 percent.
Technology shocks

On the basis of these parameter values, the productivity shock, \( z_t \), can be “inverted” from the data as \( \log(z_t) = \log(y_t) - \theta \log(k_{t-1}) - (1 - \theta) \log(k_{t-1}) \). The autocorrelation in these Solow residuals was estimated to be 0.96. Because it is not possible to reject the hypothesis of a unit root in technology, the model will be simulated under two assumptions about the persistence of \( \log(z_t) \). The deterministic trend (DT) model assumes that growth evolves according to a deterministic trend, \( \lambda t \). Deviations from this trend occur from stationary but correlated technology shocks with \( \rho = 0.96 \). In the difference stationary (DS) model the shock process has a unit root, which is equivalent to assuming that the growth rate of technology, \( \lambda \), is stochastic and independently distributed over time. In the DT model, the natural logarithm of \( z \) (or \( \lambda \) in the DS model) can take on three values, \( \{\mu - \epsilon, \mu, \mu + \epsilon \} \), and follows a first-order Markov process with transition probability matrix

\[
\Pi_x = \begin{bmatrix}
  p^2 & 2p(1-p) & (1-p)^2 \\
  p(1-q) & pq + (1-p)(1-q) & (1-p)q \\
  (1-q)^2 & 2q(1-q) & q^2
\end{bmatrix}.
\]

This process mimics a discrete first-order autoregressive model with autoregressive parameter \( p + q - 1 \), as discussed in more detail in the appendix to this chapter. I choose \( \epsilon \) to closely match the model’s implied variance of output to its U.S. postwar sample counterpart. This shock process is used for two reasons. First, its simplicity makes the interpretation of the parameters \( p \) and \( q \) transparent. The term \( p^2 \) is the probability that \( z_{t+1} = \mu - \epsilon \) given that \( z_t = \mu - \epsilon \), and \( (1-p)^2 \) is the conditional probability of a transition from the lowest state to the highest state. Similarly, \( q^2 \) is the probability that \( z_{t+1} = \mu + \epsilon \) given that \( z_t = \mu + \epsilon \) and \( (1-q)^2 \) is the conditional probability of a transition from the highest state to the lowest state. Hence, \( p \) governs the conditional probabilities of moving up after a low realization of the technology shock and \( q \) governs the conditional probabilities of moving down after a high realization. Second, it provides a simple description of business cycle conditions, because the choice of \( p \) and \( q \) affects the expected duration of contractions and expansions. In particular, choosing \( p \) to be smaller than \( q \) implies that bad shocks occur less frequently than good shocks, and that the conditional probabilities of leaving the lowest state are higher than the probabilities of leaving the highest state, which reduces the expected length of contractions relative to expansions. If \( p = q = 0.98 \), the stationary distribution of (28) is \( \{0.25 , 0.50 , 0.25 \} \), and hence the average time spent in the bad state and the good state are equal. The average time of reaching the highest state from the lowest state is about twenty periods, which is longer than the duration of the average postwar expansion. For \( p = 0.97, q = 0.99 \) the stationary distribution becomes \( \{0.0625, 0.3750, 0.5625\} \), making recessions shorter than expansions. Also, for a symmetric shock process \( \{\mu - \epsilon, \mu, \mu + \epsilon\} \), this would introduce conditional heteroskedasticity in the shocks. In the simulations of the DT model, \( p \) and \( q \) are both set to 0.98. In the DS model, \( z_t \) is assumed to have a unit root. This can be modeled by simulating a Markov process for \( \lambda \), as in (28), with \( p \) and \( q \) both equal to 0.50, where these probabilities now apply to the shifts of \( \lambda \).

Finally, the simulations assume that labor is inelastically supplied, with \( h \) set to 0.2, the average time devoted to market work during the postwar period. Utility reduces to \( u(c) = \frac{1}{1-e^{-\alpha}} \). Because this objective function is time separable, past consumption of goods and leisure does not affect current utility. Also, the time required to build new capital goods is one-period. These properties, combined with the fact that uncertainty is Markov, imply that the state of the economy can be described by the pair \( s' = (k, x') \).

Solution Methods

The model can be solved using a variety of numerical methods (see Chapter 2 for a discussion). We use a technique called value function iteration as described in Bertsekas (1976). This technique attempts to find a pointwise approximation to the solution for \( k' = k'(k, x') \), by restricting the capital stock to a discrete grid, \( \{k_1, k_2, \ldots, k_n\} \). In our numerical solutions we set \( n = 1,000 \). It is sometimes difficult to assess the accuracy of discrete approximation techniques. Taylor and Uhlig (1990) compare the relative performance of a variety of solution techniques, and report important discrepancies across methods. Also the relative accuracy may be different across policy functions. For instance, consumption is an order of magnitude smaller than capital, and is computed as the difference between output and investment. Because investment is approximately the time derivative of the capital stock, small approximation errors in the policy for capital become relatively less important for consumption and investment.

Table 10.2 gives an indication of the importance of approximation error in the model, by comparing on a state-by-state basis the accuracy of the Euler equation that plays a central role in the pricing of assets. Entries in the table give the left side of (20), which should be unity in the continuous economy. Although the average is close to unity, substantial deviations occur across states. The averages in Table 10.2 are not probability weighted. Because the largest deviations occur in the tails of the distribution of the capital stock and receive low weight in the stationary distribution, the table probably understates the importance of the approximation error. Nevertheless, these errors can contaminate some of the results, especially when we focus on small quantities, such as excess returns.

Quantity Dynamics

To understand the response of asset prices to shifts in the investment opportunity set, it is useful to briefly describe the dynamics of output, consumption, and investment. The model dynamics in this section were approximated by linearizing
Table 10.2
Approximation Errors in Euler Equation

<table>
<thead>
<tr>
<th></th>
<th>DT Model</th>
<th></th>
<th>DS Model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\sigma = 1$</td>
<td>$\sigma = 5$</td>
<td>$\sigma = 1$</td>
</tr>
<tr>
<td>Mean</td>
<td>1.0000</td>
<td>1.0001</td>
<td>1.0000</td>
</tr>
<tr>
<td>SD</td>
<td>0.0030</td>
<td>0.0115</td>
<td>0.0027</td>
</tr>
<tr>
<td>Minimum</td>
<td>0.9976</td>
<td>0.9550</td>
<td>0.9929</td>
</tr>
<tr>
<td>Maximum</td>
<td>1.0086</td>
<td>1.0433</td>
<td>1.0067</td>
</tr>
</tbody>
</table>

Note: The table lists for each of the four model variants summary statistics for the left side of the first-order condition (20) on a state-by-state basis. This ratio should be unity in the continuous economy.

The first-order conditions about the steady state and solving the associated system of linear difference equations (Kydland and Prescott 1982; King, Plosser, and Rebelo 1988a). This solution method was chosen for expositional purposes only, since it allows an easy interpretation of the expected adjustment of the economy to a shock by studying the impulse response functions. These impulse responses trace the effect on prices and quantities of a 1 percent increase in technology, assuming that the economy resides initially in the steady state. A detailed description of this method can be found in King, Plosser, and Rebelo (1987). Figure 10.1 presents the expected response to a temporary, but correlated shock to factor productivity in the DT model, starting from its steady-state level. In each panel, the lower dotted line represents the steady-state growth of the economy due to exogenous technological progress, $\lambda$, and the upper dotted line corresponds to a growth path 1 percent above the steady state. An increase in productivity increases consumption demand at all future dates through a wealth effect. The desire for smooth consumption profiles leads agents to save (invest) in the early periods when there is high output and the return on capital is high, and dissave in later periods by consuming part of the capital stock. Although the wealth effect of the shock increases consumption demand more at near than at more distant future dates, consumption initially keeps rising before it returns to its steady-state level. This intertemporal substitution is motivated by high interest rates immediately following the shock, because of the autocorrelation of the shocks. Over time the accumulation of capital and the gradual decline in productivity lower the marginal product, and interest rates fall below their long-run level as consumption growth rates slow down toward the long-run trend path (see Figure 10.2). Figure 10.3 shows that increasing risk aversion, or lowering the elasticity of intertemporal substitution, leads to smoother consumption profiles. This is one of the key properties that distinguishes production models from the endowment models studied by Lucas (1978), Mehra and Prescott (1985), and Kandel and Sianis (1991): optimal consumption choices are not invariant to changes in risk aversion, which feeds back to the behavior of asset returns and risk premiums. Increasing risk aversion tends to increase risk premiums, but this effect is mitigated by a smoothing of optimal consumption plans.

A permanent improvement in technology (in the DS model) allows agents in the long run to raise their consumption levels permanently. Figure 10.4 illustrates that after the productivity shock, the stock of physical capital is below its long run optimal level, stimulating investment demand. As capital is accumulated over time and its marginal product falls, agents consume a larger fraction of the extra output and reduce investment. Because consumption growth never falls below its long-run rate, the interest rate never falls below its long-run level either (see Figure 10.5).

The important conclusion from studying these quantity dynamics is that shifts in the investment opportunity set, temporary or permanent, give rise to temporary movements in (consumption) growth rates and discount rates. Although the figures show that the plans are smooth on an expected basis, they seldom are smooth ex post because new information arrives, that is, shocks occur, forcing agents to revise their plans. In the DT model, however, there exists no uncertainty about the long-run trend path, whereas in the DS model, agents have to continuously adjust...
to a shifting long-run equilibrium path. This difference between the long-run properties of the models is potentially important for the analysis of long-horizon returns, which will be discussed shortly.

Figures 2 and 4 already give some important clues about the cyclical behavior of returns. Positive productivity shocks increase expected growth rates of consumption and hence increase the return on short-term discount bonds. Also, consumption growth rates are larger in the short run than in the long-run. Consequently, short-term interest rates exceed the yield on long discount bonds. Although the yield on long-term bonds increases, the term structure slopes down following a positive productivity shock. The dividend-to-price ratio falls following a good shock in the DT model. Abstracting from the effects of operating leverage for the moment, the reason is that although output increases, the elastic investment (or “reinvested dividends”) response causes the dividend and the D/P ratio to decline. The behavior of dividends is sensitive to the persistence of the productivity shocks and the degree of risk aversion. In particular when the shocks become permanent, the investment response becomes smaller. The reason is that a temporary shock causes agents to increase their investment demand to save for future periods when productivity is expected to be lower. This intertemporal substitution effect becomes smaller as the persistence in the shocks increases. Figure 10.5 shows that for $\sigma = 5$ as shocks become more permanent, ‘reinvested dividends’ are sufficiently small that the dividend-to-price ratio increases in response to a good shock.

**Simulating Recessions and Expansions**

To link the predictions of the model more directly to the cyclical behavior of returns, we simulated the numerical solution to the model by drawing a time series of shocks to the level of technology in the DT model, and to the growth rate in the DS model, using the transition probabilities given in equation (28). Starting from a randomly drawn level of the capital stock, we used the policy function for capital to create a sample path for the state vector. The optimal consumption and investment plans, as well as the equilibrium asset returns are computed as functions of the simulated state vector. In order to obtain predictions about the cyclical behavior of returns, it is necessary to take a stand on what constitutes a recession or an expansion. A recession is defined as an episode that starts at the beginning of a period during which output growth is negative for two or more consecutive quarters. A recession turns into an expansion when economic growth is positive for at least two quarters. This definition of recessions allows one to
identify cycles only ex post, similar to the practice used by the NBER. Because the economy is growing, expansions will on average last longer than recessions.

**Average Returns**

Table 10.3 presents some summary statistics of simulating 100 time series of 1,000 observations in the DT model. Part A illustrates the point made by Hansen and Singleton (1983) and Mehra and Prescott (1985) that consumption-based models have great difficulty in explaining the cross-sectional difference in mean returns. The model in this chapter is no exception and has similar difficulties in generating sizable risk premiums, despite the change in the definition of the equity claim and the presence of operating and financial leverage. It is in some respects more difficult to explain risk premiums in a general equilibrium model than in an endowment economy. The reason is that the consumption choices are endogenously determined and not invariant to the level of risk aversion. Increasing risk aversion in a production economy leads to smaller changes in risk premiums because consumption itself becomes less variable, as can be seen by comparing Figures 10.1 and 10.3. Figure 10.3 suggests that increasing risk aversion even further, to around 30, as suggested by Kandel and Stambaugh (1991), would effectively remove any interesting consumption dynamics from the model. Although a sufficiently high level of risk aversion by itself can (perhaps) account for the mean return on equity, higher risk aversion will also raise the risk premium on long risk-free bonds, which has historically been small (or negative during the postwar period). It might be of some interest to see whether the proposed "solutions" to the equity premium puzzle (e.g., Rietz 1988; Constantinides 1990) can simultaneously rationalize a low excess return on long-term bonds and a high excess return on equity. Financial leverage effectively drives a wedge between the risk premium on equity and risk-free bonds. Unfortunately the current model does not assign an important role to leverage. This is best understood by returning to Figure 10.3 and examining the adjustment of the capital stock to a productivity shock. The adjustment of capital is extremely smooth, because technology allows agents to transform capital into consumption costlessly in each period. Consequently, the capital stock changes slowly between periods, which is reflected in the small variability of the return on unlevered equity shown in Table 10.3. Capital, or unlevered equity, is simply not a very risky claim in the model. Levering the firm to around 40 percent does not change this result. It takes unlikely high levels of leverage (> 0.9) for the bonds to even become risky and command a default premium.
Table 10.3
Return on Zero Coupon Bonds and Risky Equity: DT Economy

<table>
<thead>
<tr>
<th></th>
<th>( \sigma_t = 1 )</th>
<th>( \sigma_t = 5 )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>mean</td>
<td>SD</td>
</tr>
<tr>
<td>Bonds, 1 qtr.</td>
<td>1.112</td>
<td>0.254</td>
</tr>
<tr>
<td></td>
<td>(0.012)</td>
<td>(0.013)</td>
</tr>
<tr>
<td>Bonds, 5 yr.</td>
<td>1.112</td>
<td>0.379</td>
</tr>
<tr>
<td></td>
<td>(0.020)</td>
<td>(0.026)</td>
</tr>
<tr>
<td>Firm value</td>
<td>1.111</td>
<td>1.45</td>
</tr>
<tr>
<td></td>
<td>(0.010)</td>
<td>(0.011)</td>
</tr>
<tr>
<td>Levered equity</td>
<td>1.112</td>
<td>0.288</td>
</tr>
<tr>
<td></td>
<td>(0.009)</td>
<td>(0.018)</td>
</tr>
</tbody>
</table>

B. Cyclicality Variation in Returns

<table>
<thead>
<tr>
<th></th>
<th>( \sigma_t = 1 )</th>
<th>( \sigma_t = 5 )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>mean</td>
<td>SD</td>
</tr>
<tr>
<td>Bonds, 1 qtr.</td>
<td>1.092</td>
<td>0.246</td>
</tr>
<tr>
<td></td>
<td>(0.020)</td>
<td>(0.022)</td>
</tr>
<tr>
<td>Bonds, 5 yr.</td>
<td>1.023</td>
<td>1.104</td>
</tr>
<tr>
<td></td>
<td>(0.022)</td>
<td>(0.021)</td>
</tr>
<tr>
<td>Firm value</td>
<td>1.024</td>
<td>0.31</td>
</tr>
<tr>
<td></td>
<td>(0.023)</td>
<td>(0.031)</td>
</tr>
<tr>
<td>Levered equity</td>
<td>1.006</td>
<td>0.110</td>
</tr>
<tr>
<td></td>
<td>(0.022)</td>
<td>(0.027)</td>
</tr>
<tr>
<td>Leverage</td>
<td>0.004</td>
<td>0.006</td>
</tr>
<tr>
<td></td>
<td>(0.009)</td>
<td>(0.009)</td>
</tr>
<tr>
<td>D/P ratio (a)</td>
<td>0.008</td>
<td>0.006</td>
</tr>
<tr>
<td></td>
<td>(0.0062)</td>
<td>(0.0062)</td>
</tr>
<tr>
<td>D/P ratio (l)</td>
<td>0.009</td>
<td>0.005</td>
</tr>
<tr>
<td></td>
<td>(0.0009)</td>
<td>(0.0004)</td>
</tr>
</tbody>
</table>

Cyclical Variation in Returns

Short-term interest rates move procyclically in the model (see part B of Table 10.3). The return on bonds is higher during recessions. Recessions are typically periods with a sequence of bad shocks. Bad shocks cause downward revisions of the discount rate; good shocks tend to increase discount rates. A decline in short rates during a recession tends to drive up the prices of longer bonds, which explains their higher rate of return in recessions. This does not mean however that expected rates of return are also high in recessions. In fact, in a model with a single source of uncertainty expected returns are highly correlated. A less than perfect correlation of expected returns arises because the conditional covariance of the asset returns and the marginal rates of substitution changes over time because of nonlinearities in the model. As pointed out before, risk premiums are generally small at low levels of risk aversion. The small variation in expected excess returns implies that realized returns largely reflect discount rate revisions.
Recessions and expansions are defined only ex post, that is, after output growth has been negative or positive for at least two quarters. Part C of Table 10.3 shows the values yielded by the model for conditional returns, that is, average returns conditional on agents knowing whether a recession or expansion is underway.

Table 10.3 shows that the model predicts little variation in the cyclical volatility of returns, especially for $\sigma = 1$. Movements in leverage, stemming from variations in the relative value of bonds and equity, are small because relative rates of return show little variation. Consequently, leverage cannot explain the large changes in conditional volatility of equity returns reported in empirical work by Schwert (1989). Only for $\sigma = 5$ does volatility increase by about 15 percent during recessions in this model. It is not surprising that the price of risk, measured as the excess return on equity divided by the standard deviation of equity, is also virtually constant across states. The summary statistics shown in part B of Table 10.3 confirm the intuition of Figure 10.2 that the dividend-to-price ratio and the term spread move countercyclically in the DT model. Leverage slightly reinforces the cyclical movements in the dividend-to-price ratio. Table 10.4 shows that these conclusions carry over to the DS model, with a few exceptions. The dividend-to-price ratio becomes procyclical, as illustrated in Figure 10.5, and recessions show a sharper increase in the return on bonds.

Thus far, this discussion has centered around the behavior of mean returns and return variances over the cycle. The remaining part of this section will focus on the properties of low frequency movements in returns, in particular, on the ability of the term spread, $D/P$ ratio, and lagged returns to forecast future (excess) returns on stocks and bonds.

### Forecasting Returns

Fama and French (1988a, 1988b), Fama and Schwert (1987), and Campbell and Shiller (1988b) showed that the $R^2$ from regressing stock returns on their lagged values increases with the interval over which the returns are measured. The regression slopes are reliably negative, which is interpreted as evidence that stock price movements have temporary components. Time-varying expected returns can explain 25 to 40 percent of three- to five-year return variances. More recent studies have raised some caveats about these findings: Kim, Nelson, and Stark (1991) have reported that including the Great Depression in the sample period affected the results, and Richardson (1989) has questioned the testing methodology.

To evaluate the autocorrelation in returns over different horizons, 10 time series of returns of length 2,000 were simulated. To avoid bias in the regression slopes, overlapping observations were dropped, and consequently fewer observations were used to estimate the slope coefficients for long return intervals than for short return intervals. Figure 10.6 shows the slopes and $R^2$s obtained by regressing returns on their lagged values in the DT model. The left panels show the average regression
coefficients, and a band of twice the average standard error across the 10 replications. The top panels correspond to $\sigma = 1$, and the lower panels correspond $\sigma = 5$. The figures show that simulated returns are reliably positively correlated up to thirty quarters. Although returns are mean reverting, and stock prices have temporary components, the returns are positively autocorrelated. Increasing the parameter of the risk aversion only reinforces the positive correlation, because the increased desire for smooth consumption plans leads to slower and more persistent deviations of capital about its steady state. Figure 10.2 shows that in the DT model, only after capital has been accumulated do equity returns fall below their long-run mean. Mean reversion is therefore slow and does not take on a negative correlation in the regressions up to a thirty-quarter lag. Figure 10.7 shows that considering leverage does not alter this basic conclusion. A second aspect of the figures is that the autocorrelation is counterfactually large, reflecting the fact that movements in asset prices are largely driven by movements in expected returns. A drawback of studying equity returns in a model with deterministic trend is that the model necessarily understates the importance of the random walk component of stock prices. Because the economy is ultimately expected to return to its long-run trend path, there is no uncertainty about the long-run per se. In the DS model in which technology is a random walk, uncertainty increases with the forecast horizon. Although the long-run implications of a productivity shock are different in the DT and DS models, the pattern of autocorrelations is very similar, as illustrated in Figure 10.8. The conclusion is therefore that the model does not support a U-shaped correlation pattern.

Fama and French (1988b) have discussed the forecast power of dividends for future. Their explanation centers around what they call the discount rate effect: the offsetting adjustment of current prices triggered by shocks to discount rates and expected returns. As just pointed out, this mean reversion is not immediate in the model. First, productivity surprises cause both the stock price and the expected return to increase. Consequently, unexpected returns and subsequent expected returns are not negatively correlated. The neoclassical model, in which uncertainty stems from supply shocks, does not capture the mechanism suggested by Fama and French (1988b). Recent work by King (1991) indicates that interest rate dynamics in a Keynesian model of business cycle fluctuations driven by monetary shocks might not be fundamentally different from the interest rate dynamics presented here. This suggests that explanations for mean reversion may have to go beyond simple explanations based on demand or supply shocks.
Figure 10.7 Autocorrelation in Long-Horizon Levered Equity Returns in DT Model
Top panels show \( \sigma = 1 \); bottom panels show \( \sigma = 5 \). Broken lines show a two standard error band.

Figures 10.9 and 10.10 show that the sign of the \( D/P \) ratio in predicting future returns depends on the level of risk aversion. As discussed previously and illustrated in figures 10.2 and 10.5, risk aversion and stock persistence affect the elasticity of reinvested dividends and therefore the cyclical behavior of the dividend-to-price ratio.

**Forecasting Excess Returns**

Fama and French (1989) provide evidence that excess returns on bonds and stocks can be forecasted by the \( D/P \) ratio, the term spread, and the default spread, the last being the difference between the yield on low-grade bonds versus the yield on high-grade bonds. Moreover, they argue that these forecasting variables have a business cycle interpretation. Fama and French suggest that (1) the term spread proxies for a risk factor that varies in a similar fashion for all bonds and stocks, and (2) the \( D/P \)-ratio tracks variation in expected returns that is larger for stocks than bonds. Empirically, the term spread is high when business cycle conditions are poor and low or negative at business cycle peaks. Similarly, the dividend-to-price ratio is high around troughs and low around peaks.

Figures 10.11 through 13 present the ability of these variables to forecast future excess returns in the DS model. Exploring this issue is difficult because average risk premiums are generally small in consumption-based models. To reduce the potential bias caused by common approximation error in the excess return and the yield spread, excess returns starting at \( t \) were forecast by the yield spread at \( t - 1 \). The figures show that the forecast regressions perform poorly for both bonds and stocks, and do not support the empirical findings of Fama and French. Even over short horizons, the term spread does not predict future excess returns, which is at variance with the empirical evidence. Simulations with the DT model are not reported, but do not change this conclusion.

**Financial Leading Indicators**

Thus far the chapter has focused on the predictive power of the \( D/P \)-ratio and the term spread for future (excess) returns. In a recent paper, Stock and Watson (1988) compare a model of coincident economic indicators to the Index of Coincident Economic Indicators compiled by the U.S. Department of Commerce. This index includes measures of industrial production, personal income, manufacturing and trade sales, and employment. Stock and Watson find that financial prices and yields appear to have greater power to predict growth in the index than do measures of real output, real inputs or prices of foreign or domestic goods. Estrella and Hardouvelis (1991) document the ability of the term structure to forecast output several years into the future. Within my economy, a natural experiment to conduct is thus to regress output growth over various horizons on the yield on long bonds...
and the term spread. Figures 10.14 and 10.15 show that the long yield, but not the yield spread, forecasts future real activity. Also, the yield explains a larger fraction of future output growth if the forecast horizon increases. This is not surprising because the yield itself measures expected consumption growth over the remaining life of the bond (five years).

5. Conclusions

This chapter presented a simple intertemporal model of production and consumption. Despite the model's relative success reported in previous literature in describing certain key features of business cycles, its asset pricing implications were often at variance with the stylized facts. Nevertheless, the model provided important clues about elements that equilibrium models must incorporate for explaining the behavior of returns over the business cycle.

First, as was already known from earlier work by Mehra and Prescott (1985), the model was unable to replicate the cross-sectional differences in mean returns of bonds and stocks. In a way, it is more difficult to explain substantial risk premiums in a production economy, because consumption choices are endogenously determined and become smoother as risk aversion increases. Second, production economies allow for a more natural definition of complex securities, such as equity claims. This chapter studied the effects of operating and financial leverage on risk premiums, but the capital stock (or its value) varies too little to match either the mean or the variance of the return on equity. This, of course, does not disprove the importance of leverage for explaining risk premiums or cyclical variation in volatility. Movements in relative rates of return were simply too small for the influence of leverage to be significant. For instance, a number of recent studies have explored the avenue of changing preferences (e.g., Constantinides 1990) to explain relative rates of return by increasing the volatility of marginal rates of substitution. The effect on bond premiums, which have historically been quite small, have not yet been explored. Leverage might still be important because it effectively drives a wedge between the risk premium on bonds and stocks. Both findings on consumption smoothing and the low variability of equity returns motivate the study of technologies that allow less substitution in production, for instance by incorporating adjustment costs to drive a wedge between the replacement and the market value of equity capital.

Third, the problem of explaining the unconditional mean returns carries over to some extent into explaining the cyclical behavior of returns and volatility. Because expected returns move in a close lockstep, the role of leverage is small, and cannot
account for sharp volatility increases during recessions, as documented by Schwert (1989).

Fourth, the predictions of the model for low-frequency movements in returns were contrary to some empirical evidence that long-horizon returns are negatively autocorrelated. In both models with deterministic trends and those with stochastic trends, shocks induce temporary components in equity prices and mean reversion in returns, but the pattern of autocorrelations was uniformly positive for the variations of the model that were suggested by the data. However, recent work by King (1991) shows that Keynesian models of business cycles are likely to have return dynamics similar to those of the neoclassical model driven by technology shocks. This suggests that explanations for mean reversion in returns have to be found beyond simple models with demand and supply shocks.

Fifth, in the neoclassical model with a single source of uncertainty, expected returns closely follow variations in factor productivity, especially when (variation in the risk premium is low. Also, the fraction of return variation explained by variation in expected returns is very high; "noise" is virtually absent from the model despite the presence of shocks and leverage effects in operations and financing. Aside from technologies that allow less intertemporal substitution, other model elements, which emphasize higher frequency movements in returns, will have to be incorporated to successfully describe the behavior of financial markets over the business cycle. Richer models with multiple sources of uncertainty or imperfect information might be required to explain the correlations between returns over short and long horizons.

On the basis of its relative success in describing the behavior of output, investment, and consumption over the business cycle, the neoclassical model was a natural first candidate for the study of the behavior of asset returns over the business cycle. Although the neoclassical model with technology shocks often provided an incomplete description of the behavior of returns at business cycle frequencies, its limitations point to important model elements that have to be included to increase our understanding of business cycles. This chapter highlights the role variations in productivity can play in this explanation.

Appendix

This appendix describes the method of constructing the state transition matrices for the shock process [{\theta}_t^i}_{t=0}^\infty. It is often of interest to compare the discrete approximation to the solution of a dynamic program to the solution obtained from linearizing...
the first-order conditions of the program. Examples of the latter approach can be found in Kydland and Prescott (1982) and King, Plosser, and Rebelo (1988a). Since the former method requires discretization of the state space, one would like to find a discrete approximation to a continuous distribution. This appendix discusses a family of Markov processes that can be used to approximate an AR(1) process. Under certain conditions, the limiting distribution of this Markov process is a symmetric binomial, which for fine discretizations can approximate a normal distribution arbitrarily closely.

The single parameter does not influence the stationary distribution, but serves to set the first-order serial correlation of the process. The two-state member of this family is

\[
\Pi_2 = \begin{bmatrix} p & (1 - p) \\ (1 - q) & q \end{bmatrix},
\]

with stationary distribution \([\frac{1 - q}{(1 - p) + (1 - q)}, \frac{(1 - p)}{(1 - p) + (1 - q)}]\) and first-order serial correlation of \(p + q - 1\). The three-state member of this family is

\[
\Pi_3 = \begin{bmatrix} p^2 & 2p(1 - p) & (1 - p)^2 \\ p(1 - q) & pq + (1 - p)(1 - q) & q(1 - p) \\ (1 - q)^2 & 2q(1 - q) & q^2 \end{bmatrix},
\]

which also has a first-order serial correlation of \(p + q - 1\).

The \(\Pi_h\) case can be derived recursively from \(\Pi_{h-1}\) by applying the following two rules

**Rule I** Add the \((h \times h)\) matrices:

\[
p \left[ \begin{array}{cc} \Pi_{h-1} & 0 \\ 0' & 0 \end{array} \right] + (1 - p) \left[ \begin{array}{cc} 0 & \Pi_{h-1} \\ 0' & 0 \end{array} \right] + (1 - q) \left[ \begin{array}{cc} 0 & 0 \\ \Pi_{h-1} & 0 \end{array} \right] + q \left[ \begin{array}{cc} 0 & 0 \\ 0 & \Pi_{h-1} \end{array} \right],
\]

(29)
where $\mathbf{\theta}$ is a $(h-1) \times 1$ row vector.

**Rule 2.** Divide all but the top and bottom rows by two to restore the requirement that the conditional probabilities sum to one.

Recursive application of these rules yields the transition matrix for any number of states, while maintaining the property that the first-order autocorrelation is $p+q-1$. A special case is $p=q=1$, when the transition matrix is symmetric. It can be shown that in this special case the stationary distribution of the Markov process becomes independent of the single parameter $\pi$, which governs the first-order autocorrelation of the process: $2\pi - 1$. Assume that the shocks, $z$, can take on $h$ values, $z_1, z_2, \ldots, z_h$, which are symmetrically and evenly spaced over the interval $[-\epsilon, \epsilon]$. The stationary probability of being in state $z_j$ is given by

$$Pr(z = z_j) = \frac{(h-1)!}{2^{h-1}} \binom{h-1}{j-1},$$

where the variance of $z$ is

$$\sigma^2 = \frac{\epsilon^2}{(h-1)}.$$

The fourth central moment of $z$ is:

$$\mu_4 = \frac{(3h-5)\epsilon^4}{(h-1)^3},$$

so that the kurtosis of the distribution is given by

$$\frac{\mu_4}{\sigma^4} = 3 - \frac{2}{(h-1)}.$$

As $h \to \infty$, the kurtosis becomes 3. Therefore, for large $h$ and choosing $\epsilon = \sqrt{(h-1)}$, it is possible to obtain an arbitrarily close approximation to a standard normal distribution for the shocks, with autocorrelation $2\pi - 1$.

**Notes**

1. I thank Robert King, Charles Plosser, John Long, Jay Shanken, Marianne Baxter, John Campbell, Steven Cecchet, Tom Cooley, Mario Crucini, and Sergio Rebelo for helpful comments. Any errors are mine of course. Earlier versions of this chapter have circulated under the title *Asset Returns and Business Cycles: A General Equilibrium Approach*. This appendix draws heavily on joint work with Sergio Rebelo.


4. The arguments in this section closely follow Brock (1982).

5. The data used for the calibration were kindly provided by Larry Christiano and described in Christiano (1988).

6. Experimentation with specifications in which $p < q$ showed that conditional heteroskedasticity of the shocks will generally be translated into heteroskedastic asset returns. It is an empirical issue as to whether the shocks are indeed conditionally heteroskedastic. I applied Hamilton's (1989) regime-switching regression algorithm to the (detrended) Solow residuals computed from the data and found no evidence of conditional heteroskedasticity. Also, I thank Tim Bollerslev who spent some time estimating a GARCH model for the Solow residuals but found no evidence of time-varying volatility in the residuals. Choosing the distribution of shocks to be symmetric, helps to answer the question of whether the model contains propagation mechanisms to translate homoskedastic shock into heteroskedastic impulse responses.

6. During the postwar period, the average duration of a recession defined by the NBER was 4.6 quarters, and the average expansion was 14.3 quarters (Neflc 1984; Hamilton 1989). If the NBER dates recessions by a decline of output and output has an upward trend, recessions will be shorter than expansions even if the distribution of the shocks is symmetric. Therefore, recessions in the model will, on average, also be shorter than expansions.
7. This assumption was made for computational simplicity: simultaneously searching for the optimal choices for \( h \) and \( k \) increases the dimensionality of the problem considerably. Although labor supply is held fixed, the presence of labor still influences asset payoffs through operating leverage.

8. For a good survey of value function iteration and alternative numerical solution techniques, see Taylor and Uhlig (1990).

9. It should be pointed out, of course, that the claims studied in Mehra and Prescott (1985) and in this chapter do not exactly correspond to these empirical counterparts. For instance, a Treasury bill is not riskless in real terms because inflation is stochastic, and thus cannot directly be compared to a claim that pays off one unit of consumption with certainty. Incorporating money into general equilibrium models is a difficult problem, and no consensus has been reached about a satisfactory solution. An interesting problem is perhaps to turn the question around and ask what restrictions are required on the stochastic process for inflation to improve the model’s match of the data. Also, valuing the risky bond in this model includes valuing a put option on the “index,” which is different from the portfolio of puts required to value the aggregate value of all corporate bonds.

10. The number of replications was chosen to illustrate the basic workings of the model, rather than to try to replicate the sample moments reported by Fama and French (1988a, 1988b).

11. In empirical work the yield spread is more often used in forecasting regressions because it has the properties of an excess return and is therefore less sensitive to variations in expected inflation.