

Answer Key MGT 890B Final Exam  
Spring 2000  
Professor Matthew Spiegel

1.

$$\text{WACC} = .3(.08)(1-.4) + .7(.14) = 11.24.$$

2.

$$A = 40 - 10 - 15 = 15$$

$$R = A + F + V$$

$$\beta_R = \beta_A(A/R) + \beta_F(F/R) + \beta_V(V/R)$$

Assuming  $\beta_F = 0$ , and using the problem's claim that  $\beta_R = \beta_V$  one has

$$1.5 = \beta_A(15/40) + 1.5(15/40) \text{ which implies } \beta_A = 1.667.$$

3.

a. Value of the video games

$$\beta = 1.2 \text{ so } r_{\text{video games}} = .05 + 1.2(.15-.05) = .17.$$

$$\text{PV} = -3,076.92 + 3000/(\.17-.04) = 20,000$$

Mega Toys will be worth  $100,000 + 20,000 = 120,000$ .

b. Value of the equity

The firm is 50% equity financed, so the equity is worth  $.5(120,000) = 60,000$ .

c. Beta of the equity

First you need the beta of the assets –

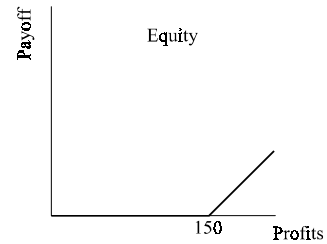
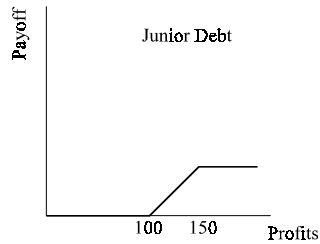
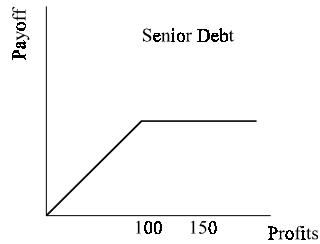
$\beta_{\text{total assets}} = \beta_{\text{original firm}} (\text{Value of the original firm/Value of the total assets}) + \beta_{\text{video games}} (\text{Value of the video games/Value of the total assets})$

$$\beta_{\text{total assets}} = .8(100/120) + 1.2(20/120) = 104/120.$$

Now use  $\beta_{\text{total assets}} = \beta_E(E/A)$  when the debt is risk free to get  $\beta_E = 2\beta_{\text{total assets}} = 208/120$ .

4.

a. Pictures



b. The junior debt is equal to a position that contains

- +1 Call option with a strike price of 100
- 1 Call option with a strike price of 150.

5.

$$r_m = .4(-.05) + .6(.2) = .1$$

$$\text{var}(r_m) = .4(-.05-.1)^2 + .6(.2-.1)^2 = .015.$$

$$\lambda = (.1-.05)/.015 = 3.33.$$

$$C_1 = .8[.4(6) + .6(10)] = 6.72 \text{ million.}$$

$$\text{Cov}(\tilde{C}_1, \tilde{r}_m) = .4(.8)(6-6.72)(-.05-.1) + .6(.8)(10-6.72)(.2-.1) + .4(.2)(0-6.72)(-.05-.1) + .6(.2)(0-6.72)(.2-.1) = .192.$$

$$\text{PV} = [6.72 - 3.33(.192)]/1.05 = 5.79.$$

$$\beta = 1.05(.192)/\{.015[6.72 - 3.33(.192)]\} = 2.21.$$

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a.  $.8 = .8\beta_E$  so  $\beta_E = 1$ .

b.  $.8 = .4\beta_E + .6(.2)$  so  $\beta_E = 1.7$

c.  $.8 = .3\beta_E + .4(.4)$  so  $\beta_E = 2.13$

d.

$$r_D = .05 + .3(.15-.05) = .08$$

$$\beta_E = .8 \left[ 1 + .5 \left( 1 - \frac{.4(.08)}{1.08} \right) \right] - .3(.5) \left( 1 - \frac{.4(.08)}{1.08} \right) = 1.14.$$

7.

$$APV = E_U + PV(\text{Tax Shields})$$

First calculate  $E_U$

The discount rate equals  $r_A = .05 + 1.2(.15-.05) = .17$ . The firm produces 5,000 in profits next period, of which the government keeps 40%. This leaves 3,000 for the firm. Since these profits are expected to grow at a rate of 2% per year the value of the unlevered equity equals

$$E_U = 3,000 / (.17 - .02) = 20,000.$$

Second calculate the value of the debt tax shield.

It is easier to answer this question by first writing down what the tax shields are year by year.

Year	Debt	Tax Shield
0	20,000	0
1	20,000	$20,000(.06)(.4) = 480$
2	$20,000(1.02^2)$	480
3	$20,000(1.02^2)$	$480(1.02^2)$
4	$20,000(1.02^4)$	$480(1.02^2)$

Note that the year  $t$  debt generates a tax shield in year  $t+1$  because the interest on a year  $t$  loan must be paid in year  $t+1$ . To calculate the present value of the tax shield it is convenient to calculate the present value of the tax shield in the odd years separately from the even years.

$$PV(\text{odd years}) = \frac{480}{1.06} + \frac{480(1.02^2)}{1.06(1.17^2)} + \frac{480(1.02^2)}{1.06(1.17^4)} + \dots$$

In year  $t-1$  you know the amount of debt the firm will have in year  $t$ . Therefore, in year  $t-1$  the only uncertainty is whether or not the firm will payoff on its debt. You should thus, bring the year  $t$  cash flows back to year  $t-1$  at the debt discount rate, which is 6% in this case. Prior to year  $t-1$  you do not know how much debt the firm will have. If it does well in subsequent years, it will have more debt (to keep the debt-equity ratio constant) and if it does poorly it will have less debt. Thus, you bring the money the rest of the way back at the discount rate for the firm's assets, which equals 17% in this case.

$$PV(\text{odd years}) = 1,887.00$$

Next calculate the present value of the tax shield in the even years.

$$PV(\text{even years}) = \frac{480}{1.06^2} + \frac{480(1.02^2)}{1.06^2(1.17^2)} + \frac{480(1.02^2)}{1.06^2(1.17^4)} + \dots$$

In an odd year  $t$  you know in year  $t-1$  what the firm will owe in interest. For an even year  $\tau$  you know in year  $\tau-2$  what the firm will owe in interest. Why? Because the firm only adjusts its debt every other year. Thus, whatever debt level it sets in year  $\tau-2$  it has the same amount of debt in year  $\tau-1$  and thus you know the year  $\tau$  interest payment. You should therefore discount the tax shield from year  $\tau$  to year  $\tau-1$  at the debt rate of 6%, and then again at the debt rate for year  $\tau-1$  to  $\tau-2$ . After that you should use the asset rate of 17% since in years prior to  $\tau-2$  you do not know exactly how much debt the firm will have outstanding in year  $\tau-1$  and thus you do not know what the tax shield will be in year  $\tau$ . Now, note that the  $PV(\text{odd years})$  equals the  $PV(\text{even year})/1.06$  and thus equals  $1887.00/1.06 = 1780.19$ .

$APV = 20,000 + 1887.00 + 1780.19 = 23,667.19$ . Since the firm has 20,000 in debt its equity has a value of 3,667.19.

8.

a. Filling the tree

		121
	110	
100		100
	90.91	
		82.65

b. Option value if the stock price rises to 110.

$$19 = C_u$$

110

$$0 = C_d$$

$$\Delta = (19-0)/[(1.1-.909)110] = .905$$

$$B = [1.1(0) - .909(19)]/[(1.1-.909)1.01] = -89.58$$

$$\text{option value} = .904(110) - 89.58 = 9.94.$$

c. Option value if the stock falls to 90.91.

$$0 = C_u$$

90.91

$$0 = C_d$$

$$\text{option value} = 0$$

d. Option value at the start

$$9.94 = C_u$$

100

$$0 = C_d$$

$$\text{Option value} = .521 (100) - 46.881 = 5.204$$