School of Management
Yale University
MGT 890
Final Exam
Answers

1.

$$PV = \frac{C_1}{1+i_1} + \frac{C_2}{(1+i_2)^2} + \frac{C_3}{(1+i_3)^3}$$

2.
(a)

$$PV = \sum_{t=3}^{12} \frac{15}{1.12^t} = \frac{1}{1.12^2}\sum_{t=1}^{15} \frac{15}{1.12^t} = \frac{84.75}{1.12^2} = 67.56$$

(b)

$$67.56 \times 1.12^{1/2} = 71.50$$

3.
(a)

$$r_p = .6 \times .1 + .4 \times .15 = .12$$

The portfolio's expected value therefore equals,

$$100{,}000 \times 1.12 = 112{,}000.$$

(b)
First you need the covariance. To get this use the relationship between the correlation, covariance, and standard deviations:

$$.3 = \frac{cov(\tilde{r}_{SP500}, \tilde{r}_{EM})}{.25 \times .4}$$

so the covariance between the SP500 and the Emerging Markets portfolio equals .03. Therefore the variance of the portfolio equals:

$$var(.6\tilde{r}_{SP500} + .4\tilde{r}_{EM}) = .6^2 \times .25^2 + .4^2 \times .4^2 + 2 \times .6 \times .4 \times .03 = .0625$$

and thus the standard deviation equals $.0625^{1/2} = .25$.

4.
(a)

$$r = .04 + 1.5(.08-.04) = 2.5(.04) = .10$$

(b)

$$120 = \frac{9}{.1} + PVGO$$

so PVGO = 30.

(c)
To find the growth rate use the fact that under the perpetual growth model the price satisfies the following relationship

$$P = \frac{Div_1}{r-g}.$$

Now, plug in 120 for P, 6 for Div_1, and .1 for r and then solve for g to get g = .05.

(d)

$$g = \text{ROE}\times\text{plowback ratio}$$

$$.05 = \text{ROE}\times(1/3)$$

$$\text{ROE} = .15$$

(e)

$$P_1 = \frac{Div_2}{r-g} = \frac{6(.105)}{.1-.05} = \frac{6.3}{.05} = 126.$$

(f)
Nothing. The price should remain the same. Investing in publically traded securities is an investment with a zero PVGO.

5.
(a)

$$P_A = \frac{105}{1.08} = 97.22$$

$$P_B = \frac{6}{1.09} + \frac{106}{1.09^2} = 94.72$$

(b)

$$94.72 = \frac{6}{1.08} + \frac{106}{1.08(1+f_2)}$$

solving for f_2 produces $f_2 = .10$.

6.

(a)

$$1.08 = 1.02(1+r_{real})$$

$$r_{real} = .059$$

(b)

To answer this question you need to calculate what the annual payments in real dollars will be starting in year 18. In *nominal* dollars they will equal $30\times1.0225^{18} = 44.78$. Thus, in real dollars the payment due in year 18 will equal $44.78/(1.02^{18}) = 31.35$. Since the first payment has gone from 30 to 31.35, the percentage change in real dollars of an Ivy League education will have increased by 31.35/30 = .045. Note, that if the first payment increases by 4.5% then so will all of the other payments.

(c)

Today the present value of an Ivy League education costs:

$$PV = 30 + \frac{30\times1.0225}{1.08} + \frac{30\times1.0225^2}{1.08^2} + \frac{30\times1.0225^3}{1.08^3}.$$

This is a growing annuity with four cash flows starting in year **0**, using the growing annuity formula the PV equals

$$PV = \frac{30}{.08-.0225}\left[1-\left(\frac{1.0225}{1.08}\right)^4\right]1.08 = 110.75.$$

Note, the 1.08 at the end comes from the fact that the standard formula assumes that the first cash flow arrives in year 1, whereas here it arrives in year zero.

The answer to part (b) showed that in *real* terms the cost of an Ivy League education would grow by 4.5% in year 18. Thus, the PV in real dollars in year 18 will be 110.75×1.045 = 115.74. Discounting this to the present (these are real dollars so you need to use the real rate) yields:

$$115.74/(1.059^{18}) = 41.24.$$

Thus, in real dollars your saving must satisfy

$$41.24 = \sum_{t=1}^{18} \frac{SAVE}{1.059^t}$$

which implies that you must save 3.78 per year. Since you start with $100 in income, this means you must save 3.78% of your income per year to pay for your daughter's future tuition.

7.
If a stock plots above the security market line then its return is too high for its associated market risk. If a stock has too high of a return its price must rise as people bid to acquire it. As people bid to acquire it, they will drive up the price until the security's return falls to the point where it actually falls onto the security market line.